# SPIN DETERMINATION WITH INVISIBLE PARTICLES

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2013 PHENOMENOLOGY SYMPOSIUM

# OUTLINE

- Introduction
- Review of Wigner *d* functions, spin and the Feynman amplitude
- Kinematical method
- The angular distributions
- Summary and conclusion

# DARK MATTER AT A COLLIDER

- Substantial astronomical evidence points towards the existence of a nonluminous form of matter
- We anticipate dark matter production at a collider
- What are the properties of dark matter?
- We focus on the International Linear Collider (ILC)

# SPIN

- Given total initial angular momentum *j* and initial net spin *m* along the *z* axis and final net spin *m´* along the z´ axis then we define the Wigner *d* function as

$$d^j_{m,m'}(\theta) = \langle j, m', \theta \mid j, m \rangle = \langle j, m' \mid \exp\left(iJ_y\theta\right) \mid j, m \rangle$$

where the reaction takes place in the *x*-z plane, and $\theta$ is the angle between the z and z´ axes in the CM frame

- So for a particular spin combination *m*, *m´*

$$\mathcal{M}_{m,m'} = C_{m,m'} d^j_{m,m'}\left(\theta\right)$$

- The cross section is then

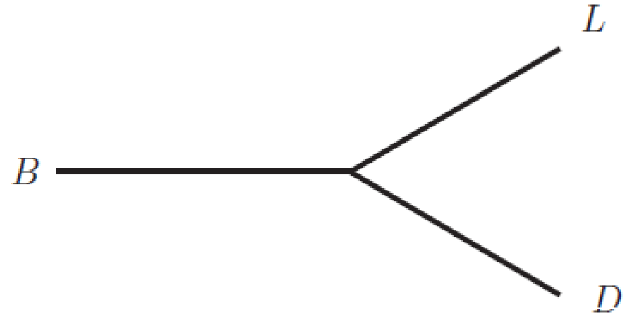
$$\sigma \propto \sum_{m,m'} \left|\mathcal{M}_{m,m'}\right|^2 = \sum_{m,m'} \left|C_{m,m'}\right|^2 \left(d^j_{m,m'}\left(\theta\right)\right)^2$$

# SPIN DIRECTLY FROM THE VERTEX

- In principle, we want to proceed as directly as possible to the dark matter vertex
- We need to calculate the angle of *L* with respect to *B* in the *B*'s CM frame:

$$\cos\theta_{LB} = \frac{\vec{p}_B \cdot \vec{p}_{L_{BCMS}}}{|\vec{p}_B||\vec{p}_{L_{BCMS}}|}$$

- From this, we can reconstruct which Wigner *d* functions are responsible and thus which spins are involved

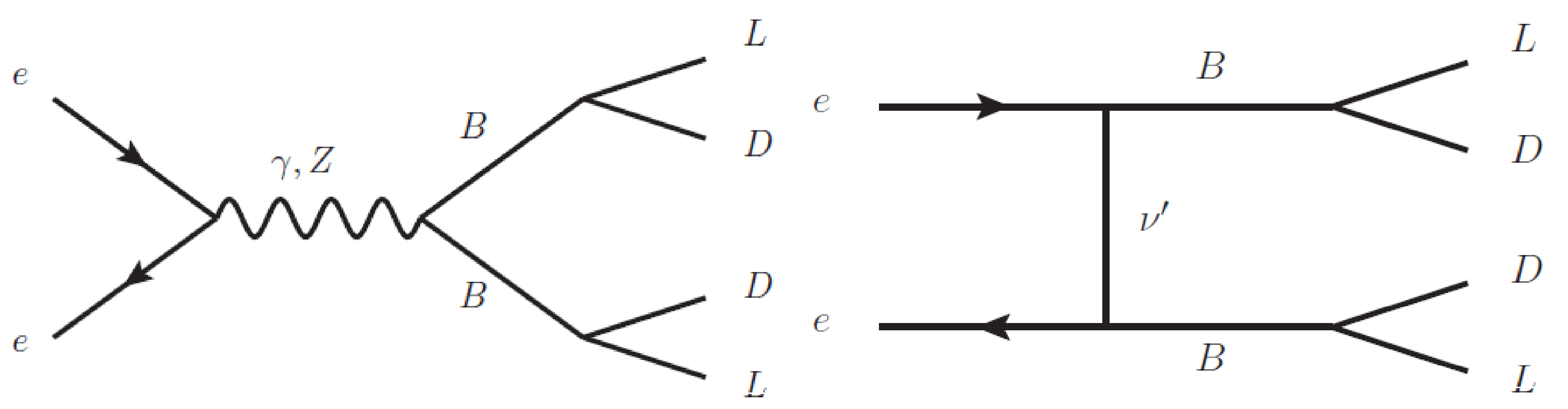


# WHY THE ILC

- The ILC is nice because
  - The collision CM frame is the LAB frame
  - We know $\sqrt{s}$
  - Have the ability to polarize the incoming beams

# SOME CONSIDERATIONS

- We assume the particles *B*, *L* and *D* are all on their mass shells
- The energy and momentum of *L* are completely known
- The masses of the *B* and *D* particles are known
  - Antler Diagram Topology
    - T. Han, I. –W. Kim and J. Song, Phys. Lett. B **693**, 575 (2010) [arXiv:0906.5009 [hep-ph]].
    - For example, in the MSSM, $e^+e^- \to \tilde{\mu}^+\tilde{\mu}^- \to \mu^+\mu^-\chi_1^0\chi_1^0$

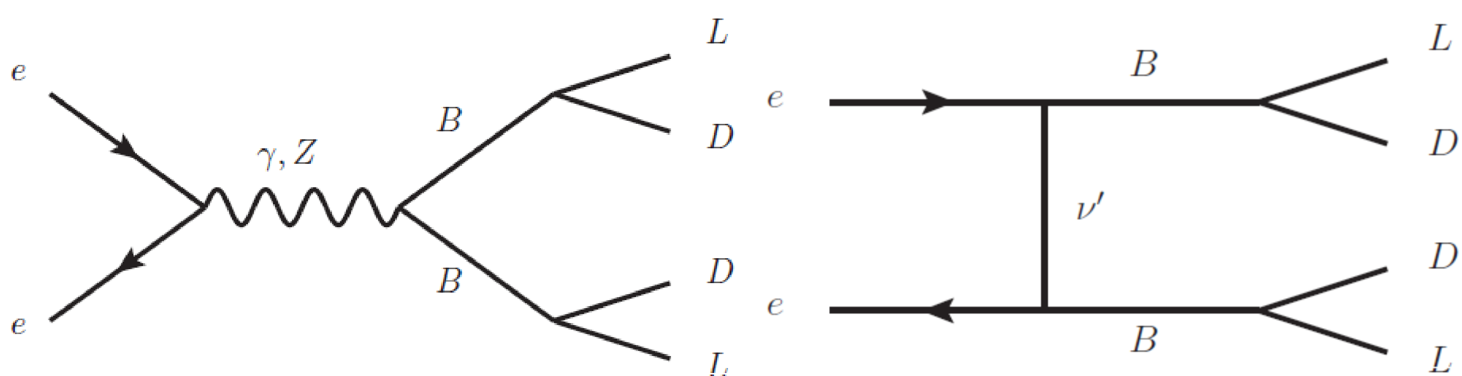


# KINEMATICS

- The angle of *L* with respect to *B*:

$$\cos\theta_{LB} = \frac{\vec{p}_B \cdot \vec{p}_{L_{BCMS}}}{|\vec{p}_B||\vec{p}_{L_{BCMS}}|}$$

- We found that we can calculate the magnitude of $\cos\theta_{LB}$ and partially reconstruct the sign
- To be published soon

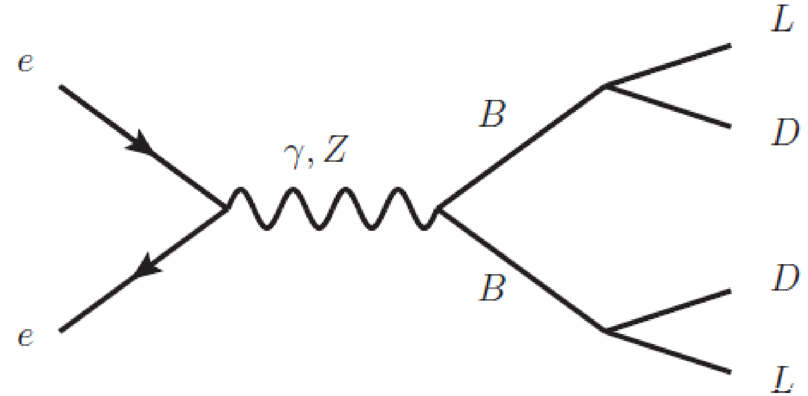


# NOTES ABOUT NOTATION

- All of the processes we will consider here proceed via

$$e^+e^- \to \gamma/Z \to B^+B^- \to \mu^+\mu^- DD$$

- Something like $sB \to \mu f D$ is simply this process in which we analyze the specific case where *B* is a spin-0 particle ($sB$) and *D* is a spin-1/2 particle ($fD$)



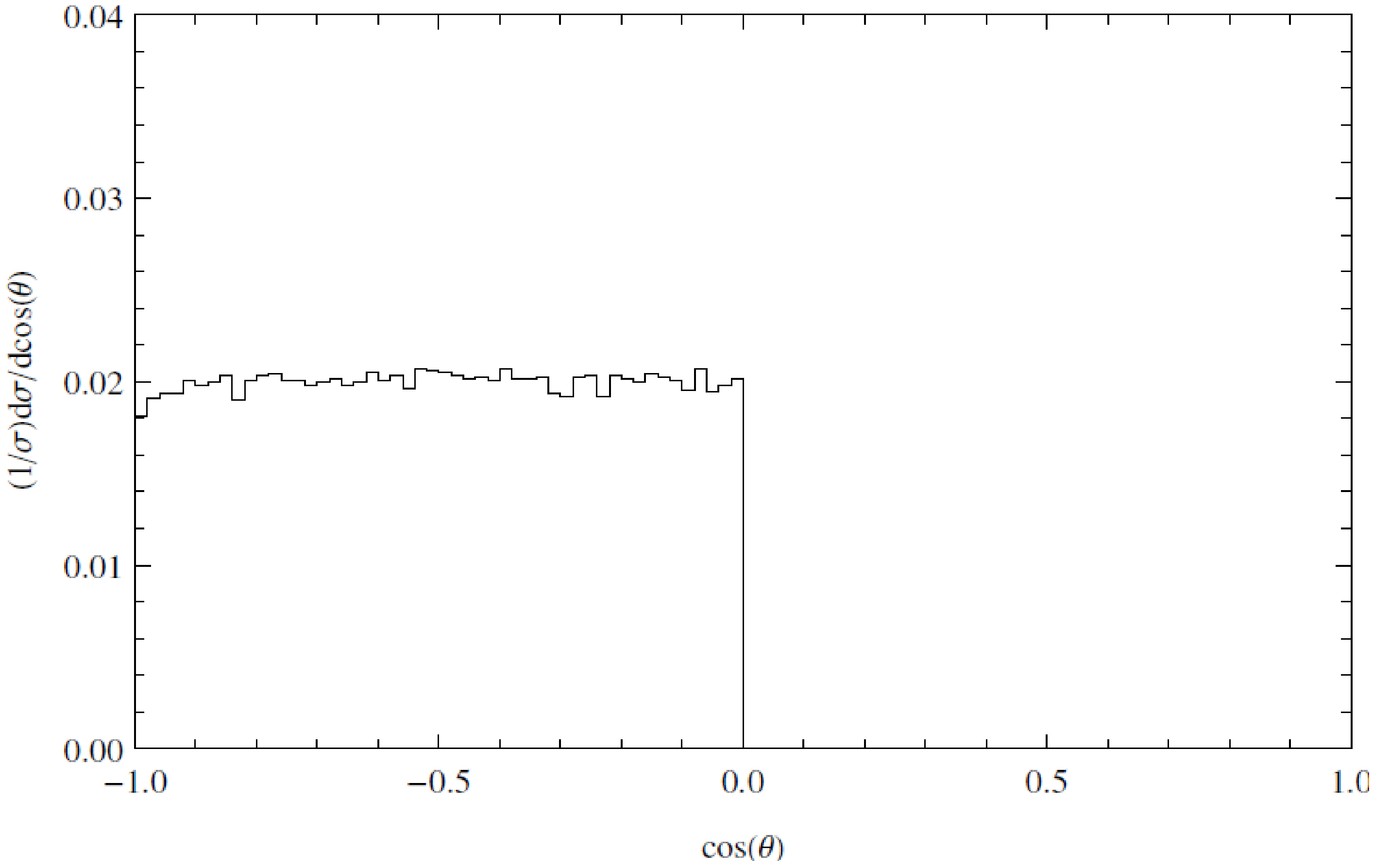
# ANGULAR DISTRIBUTIONS ($sB \rightarrow \mu f D$)

- For a scalar particle, there is no spin correlation
- The Wigner *d* function is

$$d^{0}_{0,0}\left(\theta\right) = 1$$

- We expect a flat line

# PLOT OF $sB \to \mu f D$



# ANGULAR DISTRIBUTIONS ($fB \to \mu s D$)

- The Wigner *d* functions associated with spin ½:

$$d^{1/2}_{m,m'} = \begin{pmatrix} \cos\theta/2 & \sin\theta/2 \\ -\sin\theta/2 & \cos\theta/2 \end{pmatrix}$$

- And, of course

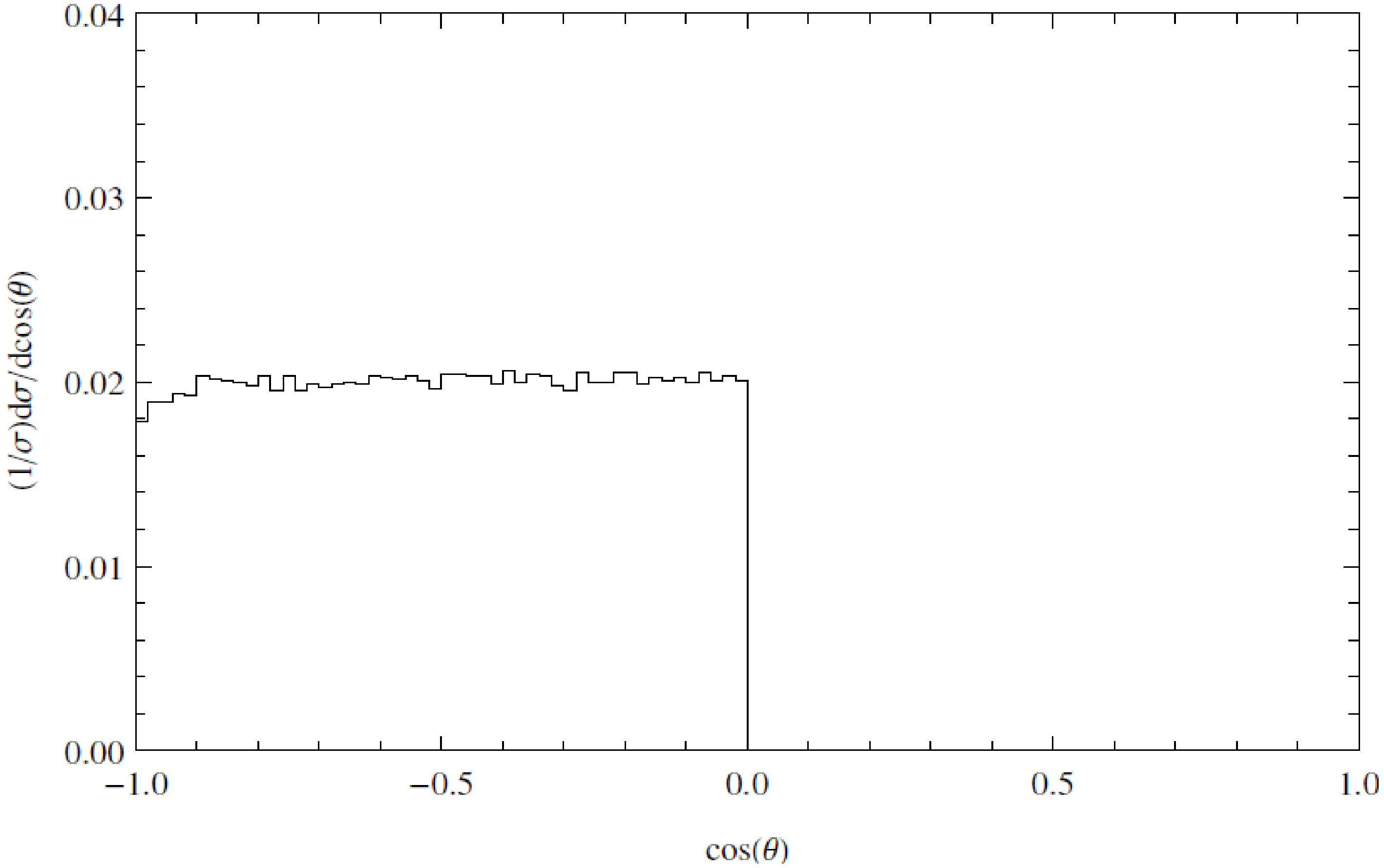
$$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2} \qquad\qquad \cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$$

- So as a linear combination we should have

$$\sum_{m,m'} |\mathcal{M}_{m,m'}|^2 = A\sin^2\theta/2 + B\cos^2\theta/2 = \frac{A}{2}(1-\cos\theta) + \frac{B}{2}(1+\cos\theta)$$

- We find that $A = B$

# PLOT OF $fB \to \mu s D$



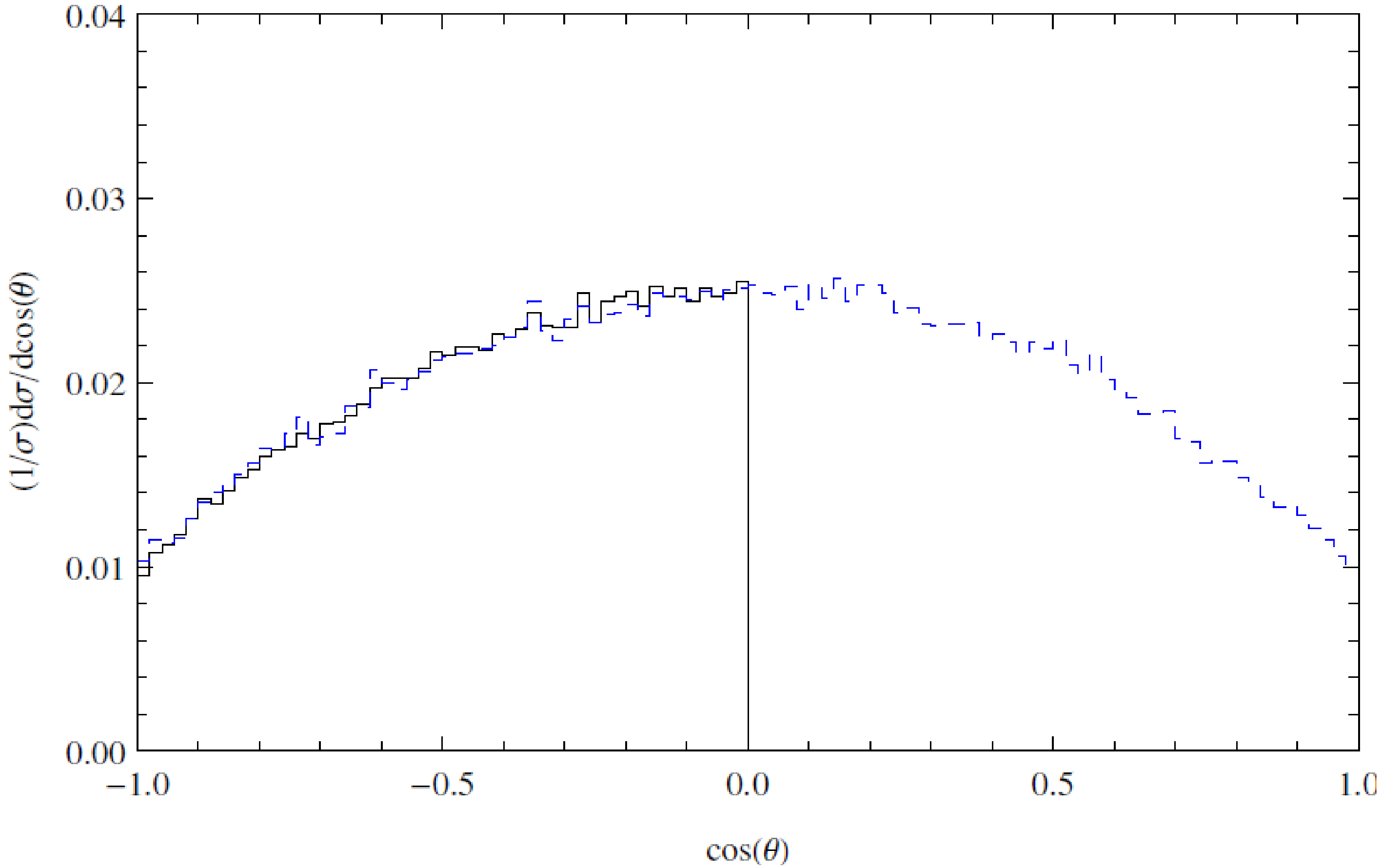
# ANGULAR DISTRIBUTIONS ($\nu B \to \mu f D$)

- The Wigner *d* functions corresponding to the spin configurations are

$$d^1_{m,m'}(\theta) = \begin{pmatrix} \frac{1}{2}(1+\cos\theta) & \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1-\cos\theta) \\ -\frac{1}{\sqrt{2}}\sin\theta & \cos\theta & \frac{1}{\sqrt{2}}\sin\theta \\ \frac{1}{2}(1-\cos\theta) & -\frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1+\cos\theta) \end{pmatrix}$$

- There is an unequal contribution from the longitudinal term
- There is a dominance of $\sin^2\theta$

# PLOT OF $vB \rightarrow \mu f D$



# SUMMARY AND CONCLUSION

- The ILC is an opportune place to make measurements on the fundamental properties of dark matter such as its mass and spin
- Our method of kinematics is model independent
- From it we can determine the magnitude of $\cos\theta_{LB}$ and partially reconstruct its sign from only visible momenta
- We can apply our method in other situations in which there is "missing" energy and momentum in the final state
  - Drell Yan process at the LHC
    - C. -W. Chiang, N. D. Christensen, G. -J. Ding and T. Han, Phys. Rev. D85, 015023 (2012)[arXiv:1107.5830 [hep-ph]].
- So far, our angular distributions produced from our kinematics seem to fit expectations

Thank You

# FURTHER ANALYSIS OF (VB→ $\mu$FD)

- Spin contributions