

# Warped Alternatives to Froggatt–Nielsen Models

with A. Iyer, [arXiv: 1304.3558 \[hep-ph\]](https://arxiv.org/abs/1304.3558)

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# FN models and RS

## Heavy Fermions

$$\mathcal{W} \supset Y_t Q_3 u_3 H_u + Y_1^u Q_2 F_1 H_u + Y_2^u \bar{F}_1 u_1 S + M_1 F_1 \bar{F}_1 + \dots$$

## Extra Dimension

$$S_{kin} = \int d^4x \int dy \sqrt{-g} (\bar{L}(i \not{D} - m_L)L + \bar{E}(i \not{D} - m_E)E + \dots)$$

$$S_{Yuk} = \int d^4x \int dy \sqrt{-g} (Y_U \bar{Q} U \tilde{H} + Y_D \bar{Q} D H + Y_E \bar{L} E H) \delta(y - \pi R)$$

## Integrating Out

$$W \supset Y_{ij}^u \left( \frac{S}{M_{Pl}} \right)^{c_{Q_i} + c_{u_j} + c_{H_u}} Q_i H_u U_j$$

$$(\mathcal{M}_F)_{ij} = \frac{v}{\sqrt{2}} (Y'_F)_{ij} e^{(1-c_i-c'_j)kR\pi} \xi(c_i) \xi(c'_j)$$

$$m_F = c_F k \quad \xi(c_i) = \sqrt{\frac{(0.5 - c_i)}{e^{(1-2c_i)\pi kR} - 1}}$$

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# FN models and RS

## U(1) Charges

fitting both  $O(1)$  as well  
U(1) charges

## Bulk Masses

fitting both  $O(1)$  as well  
bulk masses

## Additional Conditions

Anomalies should be  
cancelled, which leads to  
very strong constraints

Green Schwarz conditions

Unification of the gauge  
couplings leads to strong  
constraints on bulk masses

E. Dudas et.al, JHEP 1012  
(2010) 015

If one doesn't consider unification,  
reasonably relaxed framework

# FN models and RS

## Scale

Typically at Planck scale

$$\langle S \rangle \sim \lambda_c M_{Pl}$$

SUSY models have D-terms

Single flavon fields strongly constrained in SUSY

Ross, Lalak etc..

## Warp Factor

$$kR\pi \sim \mathcal{O}(11)$$

first KK scale around TeV

strong constraints from Hadronic and leptonic flavour

Iyer & Vempati, PRD 2012

LHLH : very large bulk masses,  $10^6 M_{Planck}$

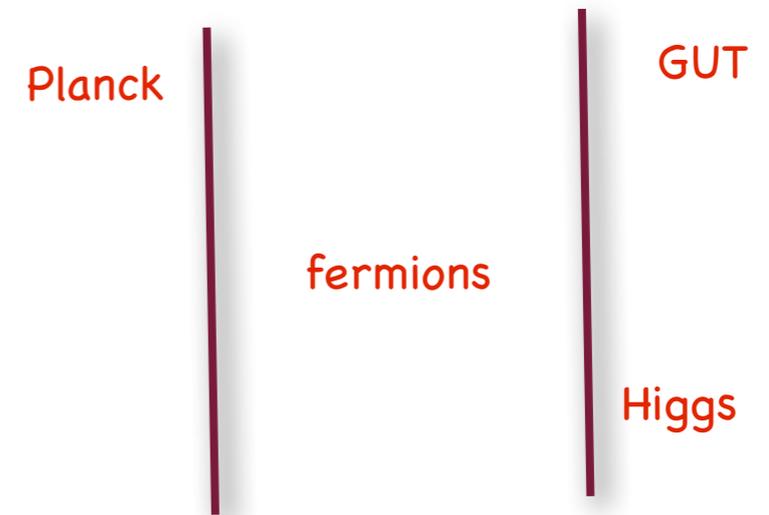
Dirac : strong flavour constraints

Majorana : strong flavour constraints

MFV ideas

Perez, Randall, M. C. Chen etc..

$$kR\pi \sim \mathcal{O}(1)$$



## Randall Sundrum at GUT scale

K. Choi et.al, Eur. Phys C 35 (2004) 267

Nomura et. al JHEP 0807 (2008) 055 ,

E. Dudas et.al, JHEP 1012 (2010) 015

F. Brummer et.al, JHEP 1112 (2011) 061

- RS as a theory of flavour at GUT scale
- Fit the fermion masses in SUSY and non-SUSY models
- Two cases for Neutrino Masses : Dirac and Planck scale Lepton number violation
- An interesting SUSY scenario where fermion hierarchy determines sfermion mass structure.

# Fermion masses at GUT scale

Mass (MeV )	Mass (GeV)	Mass MeV	Mass squared Differences $eV^2$
$m_u = 0.48^{+0.20}_{-0.17}$	$m_c = 0.235^{+0.035}_{-0.034}$	$m_e = 0.4696^{+0.00000004}_{-0.00000004}$	$\Delta m_{12}^2 = 1.5^{+0.20}_{-0.21} \times 10^{-4}$
$m_d = 1.14^{+0.51}_{-0.48}$	$m_b = 1.0^{+0.04}_{-0.04}$	$m_\mu = 99.14^{+0.000008}_{-0.0000089}$	$\Delta m_{23}^2 = 4.6^{+0.13}_{-0.13} \times 10^{-3}$
$m_s = 22^{+7}_{-6}$	$m_t = 74.0^{+4.0}_{-3.7}$	$m_\tau = 1685.58^{+0.19}_{-0.19}$	-

mixing angles(CKM)	Mixing angles (PMNS)
$\theta_{12} = 0.226^{+0.00087}_{-0.00087}$	$\theta_{12} = 0.59^{+0.02}_{-0.015}$
$\theta_{23} = 0.0415^{+0.00019}_{-0.00019}$	$\theta_{23} = 0.79^{+0.12}_{-0.12}$
$\theta_{13} = 0.0035^{+0.001}_{-0.001}$	$\theta_{13} = 0.154^{+0.016}_{-0.016}$

# Results for SM

parameter	range	parameter	range	parameter	range
$c_{Q_1}$	[0,3.0]	$c_{D_1}$	[0.78,4]	$c_{U_1}$	[-0.97,3.98]
$c_{Q_2}$	[-1.95,2.36]	$c_{D_2}$	[0.39,3.02]	$c_{U_2}$	[-1.99,2.43]
$c_{Q_3}$	[-3,1]	$c_{D_3}$	[0.39,2.21]	$c_{U_3}$	[-4,1.0]

Dirac  
Case

parameter	range	parameter	range	parameter	range
$c_{L_1}$	[-1,2.9]	$c_{E_1}$	[0.39,3.62]	$c_{N_1}$	[5.29,8.97]
$c_{L_2}$	[-0.99,2.7]	$c_{E_2}$	[-1.0,2.63]	$c_{N_2}$	[5.31,8.99]
$c_{L_3}$	[-0.99,1.98]	$c_{E_3}$	[-0.99,1.93]	$c_{N_3}$	[5.12,8.97]

LHLH  
Case

parameter	range	parameter	range
$c_{L_1}$	[-1.5,-1.15]	$c_{E_1}$	[2.8,4.0]
$c_{L_2}$	[-1.5,-0.97]	$c_{E_2}$	[1.8,2.4]
$c_{L_3}$	[-1.5,-1.22]	$c_{E_3}$	[1.2,1.69]

# SUSY Set up

The 5D action is given by

$$S_5 = \int d^5x \left[ \int d^4\theta e^{-2ky} (\Phi^\dagger \Phi + \Phi^c \Phi^{c\dagger}) + \int d^2\theta e^{-3ky} \Phi^c \left( \partial_y + M_\Phi - \frac{3}{2}k \right) \Phi \right]$$

The 4D superpotential is given by with only zero modes

$$\begin{aligned} \mathcal{W}^{(4)} = & \int dy e^{-3ky} \left( e^{(\frac{3}{2}-c_{q_i})ky} e^{(\frac{3}{2}-c_{u_j})ky} Y_{ij}^u H_U Q_i U_j + e^{(\frac{3}{2}-c_{q_i})ky} e^{(\frac{3}{2}-c_{d_j})ky} Y_{ij}^d H_D Q_i D_j \right. \\ & \left. + e^{(\frac{3}{2}-c_{L_i})ky} e^{(\frac{3}{2}-c_{E_j})ky} Y_{ij}^E H_D L_i E_j + \dots \right) \delta(y - \pi R) \end{aligned}$$

$$\xi(c_i) = \sqrt{\frac{(0.5 - c_i)}{e^{(1-2c_i)\pi k R} - 1}}$$

$c$  are the bulk mass parameters

# Results for MSSM

$\tan\beta = 10$

parameter	range	parameter	range	parameter	range
$c_{Q_1}$	[-0.16,3.12]	$c_{D_1}$	[-0.5,4]	$c_{U_1}$	[-1.6,4.0]
$c_{Q_2}$	[-1.32,2.34]	$c_{D_2}$	[-1.9,2.5]	$c_{U_2}$	[-2,2.4]
$c_{Q_3}$	[-3,1]	$c_{D_3}$	[-2,1.7]	$c_{U_3}$	[-4,1.0]

Dirac  
Neutrinos

parameter	range	parameter	range	parameter	range
$c_{L_1}$	[-1,2.6]	$c_{E_1}$	[-0.86,3.46]	$c_{N_1}$	[5.68,8.9]
$c_{L_2}$	[-0.99,2.21]	$c_{E_2}$	[-1,2.24]	$c_{N_2}$	[5.67,8.99]
$c_{L_3}$	[-1,1.54]	$c_{E_3}$	[-1,1.49]	$c_{N_3}$	[5.64,8.99]

LHLH case

parameter	range	parameter	range
$c_{L_1}$	[-1.5,-0.22]	$c_{E_1}$	[2.6,3.7]
$c_{L_2}$	[-1.5,0.08]	$c_{E_2}$	[2.0,2.57]
$c_{L_3}$	[-1.5,0.04]	$c_{E_3}$	[1.1,1.8]

# SUSY Breaking

Higgs and X

scalar masses

$$(m_{\tilde{f}}^2)_{ij} = m_{3/2}^2 \hat{\beta}_{ij} e^{(1-c_i-c_j)kR\pi} \xi(c_i)\xi(c_j)$$

fermions

trilinear terms

$$A_{ij}^{u,d} = m_{3/2} A'_{ij} e^{(1-c_i-c'_j)kR\pi} \xi(c_i)\xi(c'_j)$$

PLANCK

GUT

gaugino masses

$$m_{1/2} = f m_{3/2}$$

SUSY breaking spurion and Higgs placed on GUT brane

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# Structure of Soft terms

Nomura et.al

$c$  are the bulk masses ,  
epsilon is small parameter

flavourful SUSY

$$\tilde{M}_{Q,U}^2 = m_{3/2}^2 (0.5 - c_{Q_3,U_3}) \begin{pmatrix} \epsilon^\alpha & \epsilon^\gamma & \epsilon^{\frac{\alpha}{2}} \\ \epsilon^\gamma & \epsilon^\beta & \epsilon^{\frac{\beta}{2}} \\ \epsilon^{\frac{\alpha}{2}} & \epsilon^{\frac{\beta}{2}} & 1 \end{pmatrix}$$

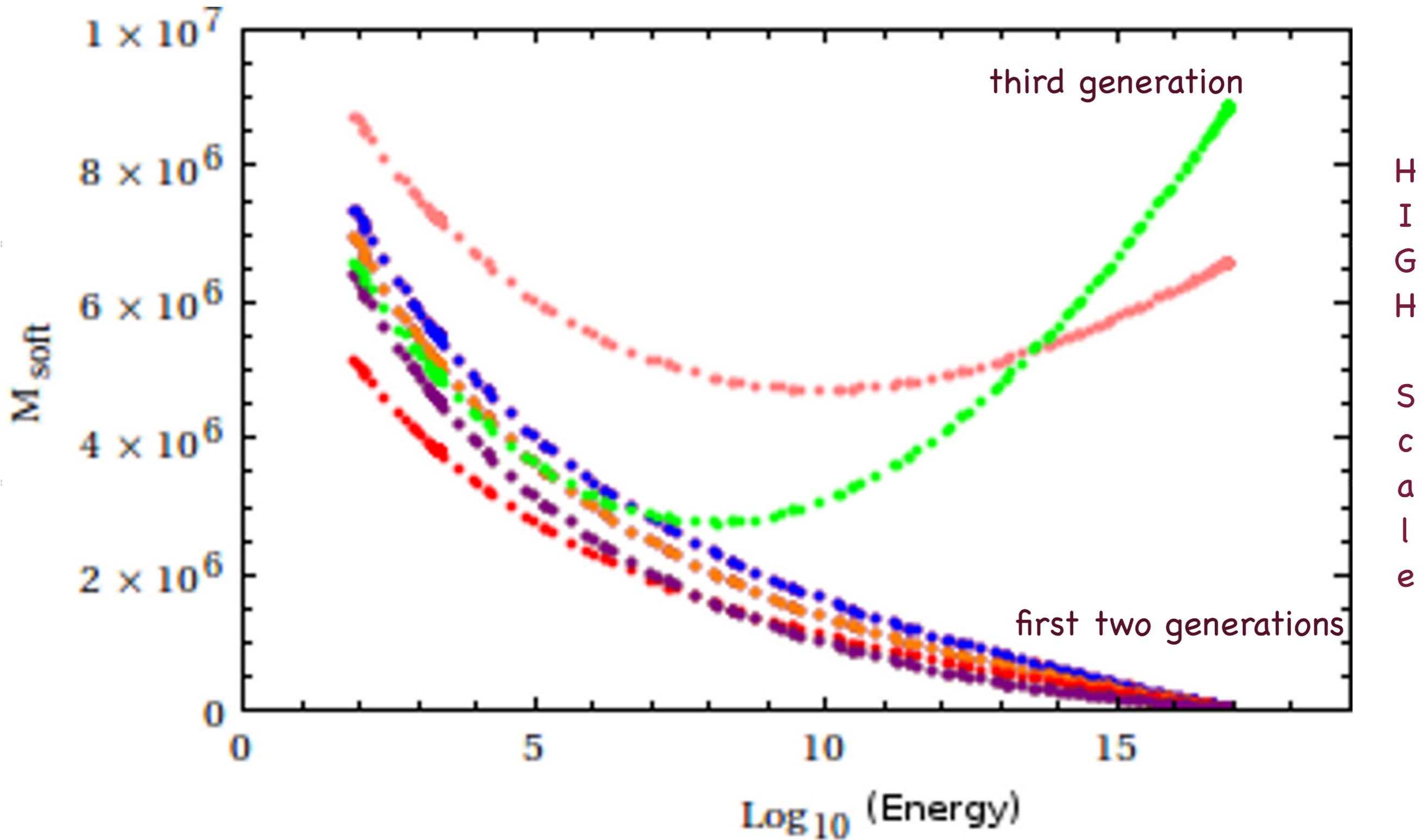
$$\alpha = 2c_1 - 1, \beta = 2c_2 - 1, \gamma = c_2 + c_1 - 1.$$

$$\tilde{M}_D^2 = m_{3/2}^2 e^{(1-2c_3)kr\pi} \begin{pmatrix} \epsilon^{2\alpha'} & \epsilon^{\gamma'} & \epsilon^{\alpha'} \\ \epsilon^{\gamma'} & \epsilon^{2\beta'} & \epsilon^{\beta'} \\ \epsilon^{\alpha'} & \epsilon^{\beta'} & 1 \end{pmatrix}$$

$$\alpha' = c_1 - c_3, \beta' = c_2 - c_3, \gamma' = c_2 + c_1 - 2c_3$$

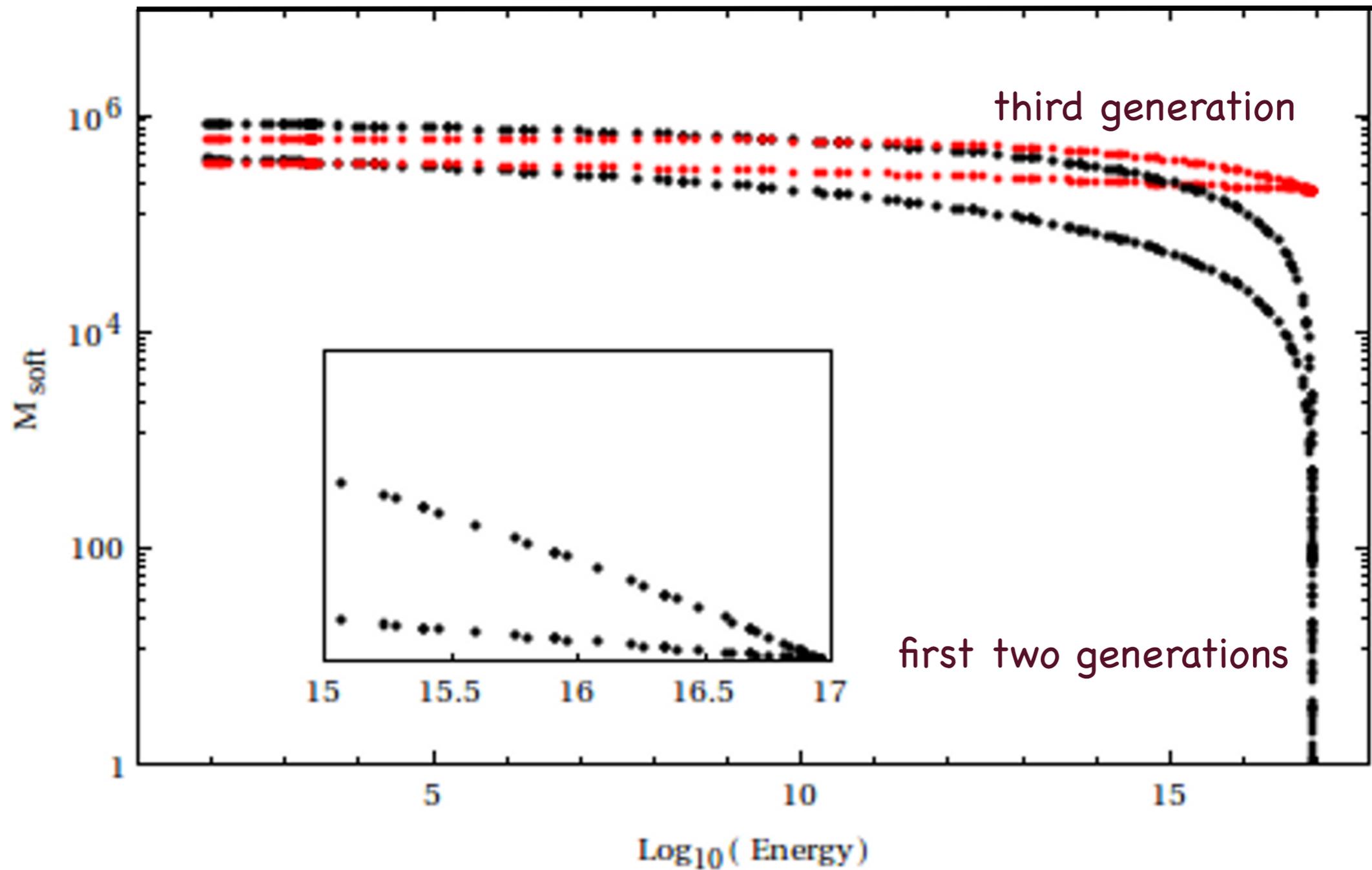
# weak scale spectrum

( squark mass squared )



# weak scale spectrum

(slepton mass squared)

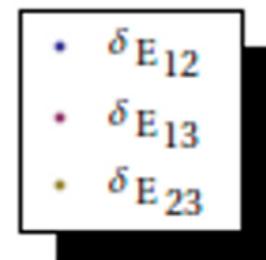
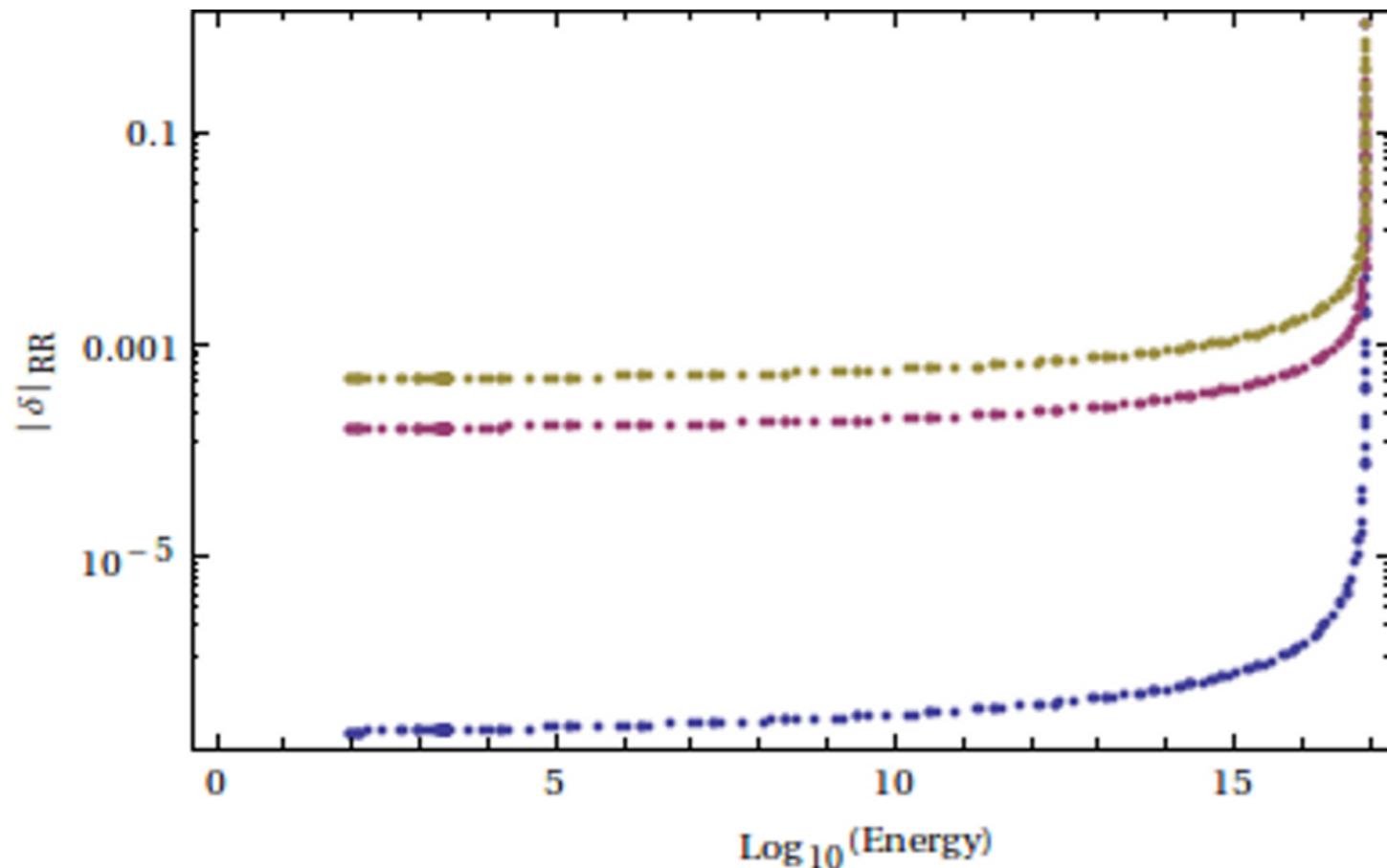
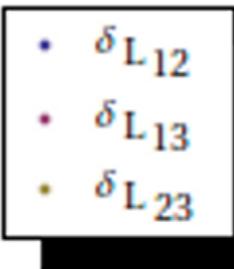
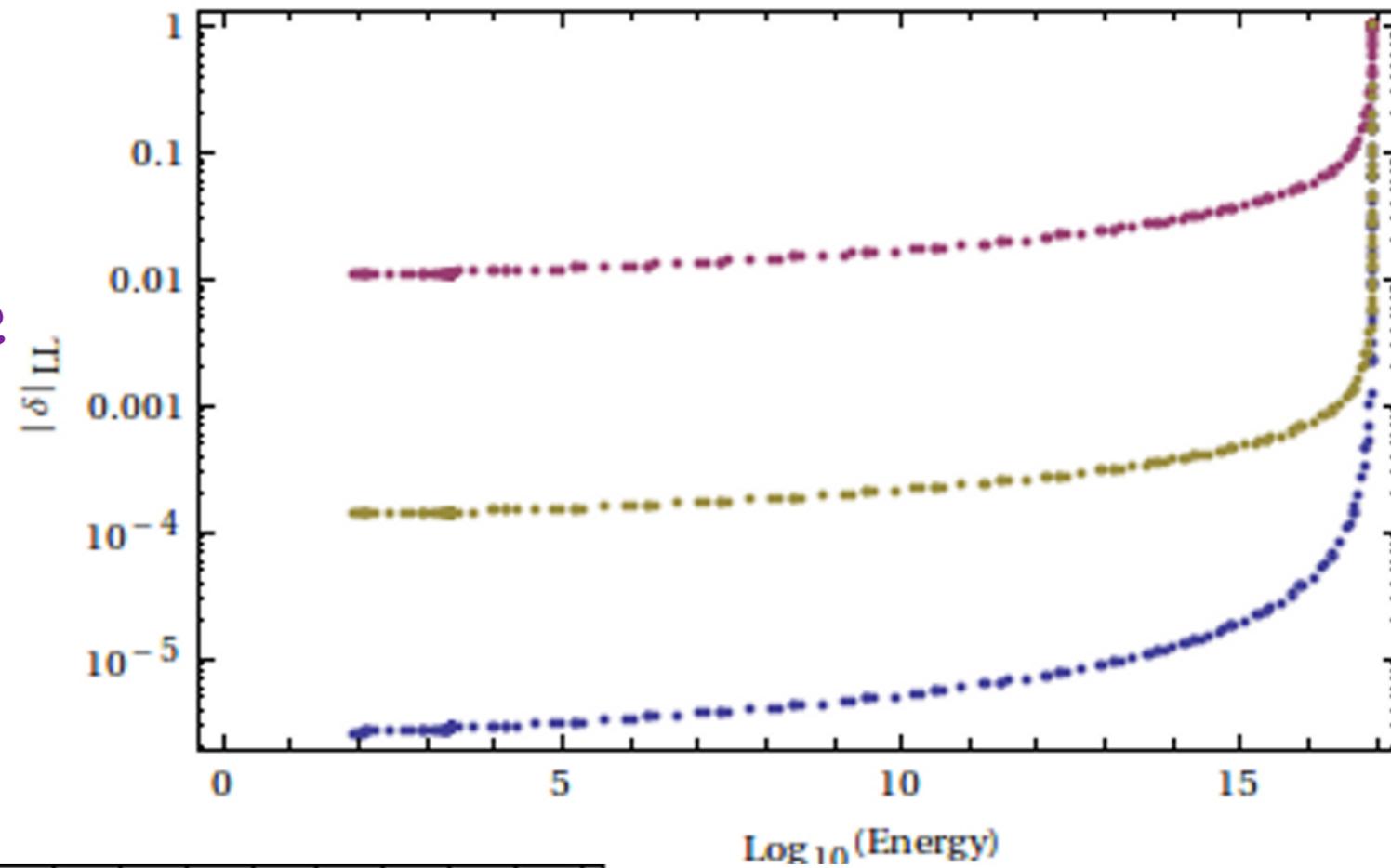


Perez, Ramond, Zhang, arxiv: 1209.6071

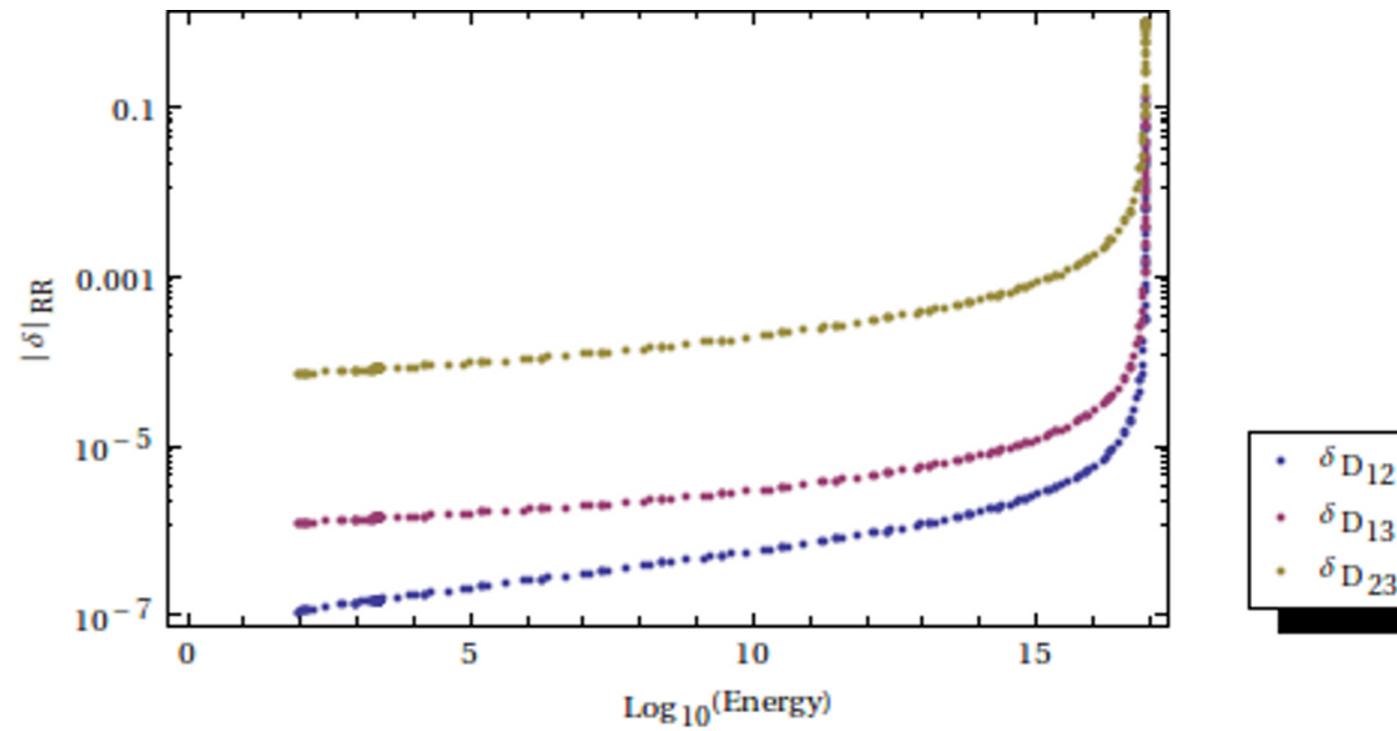
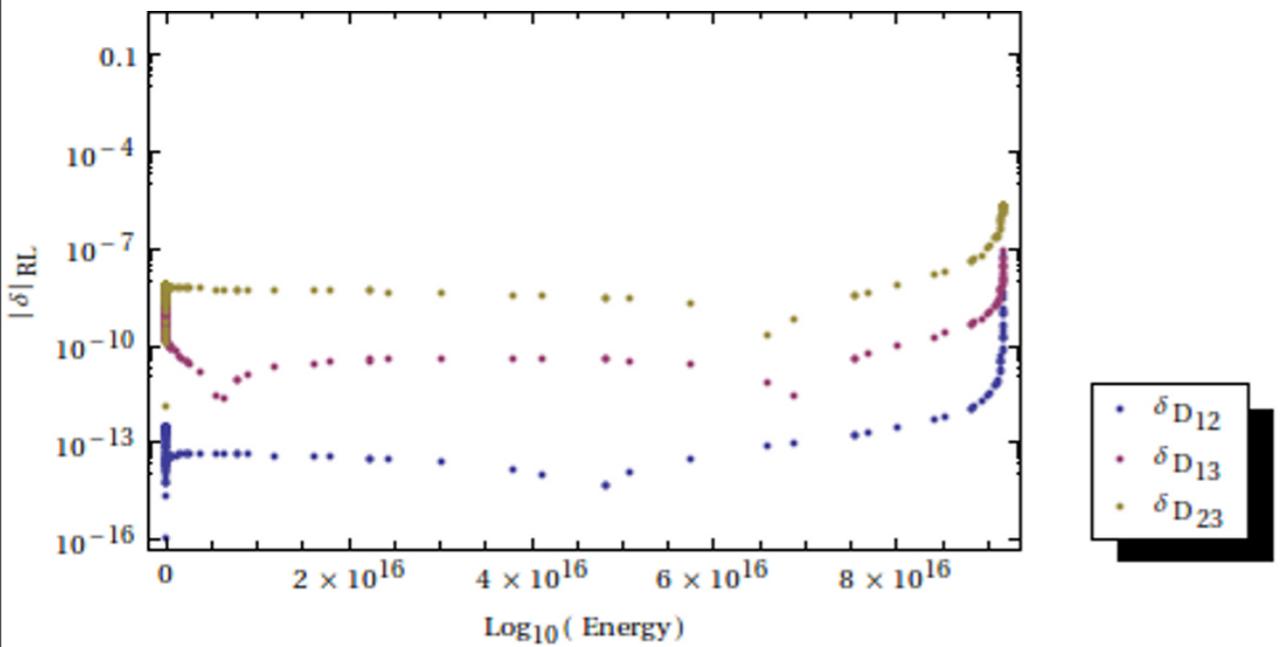
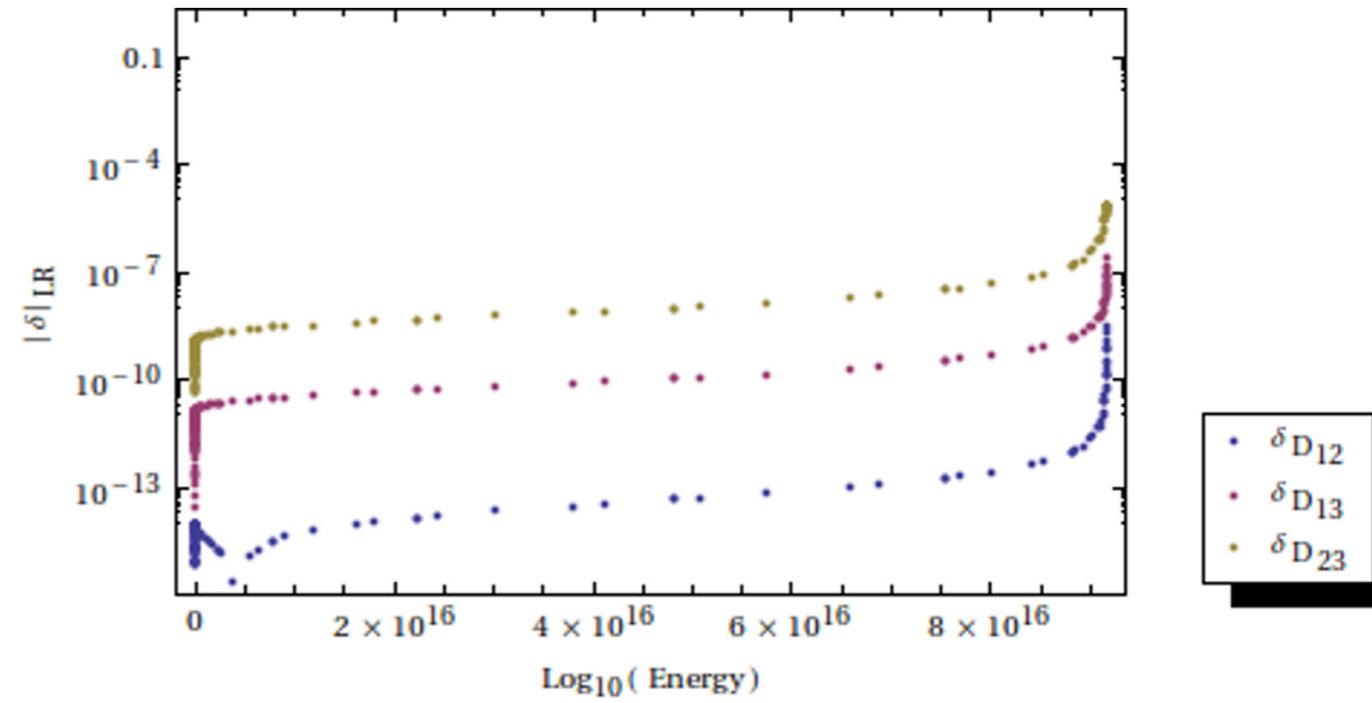
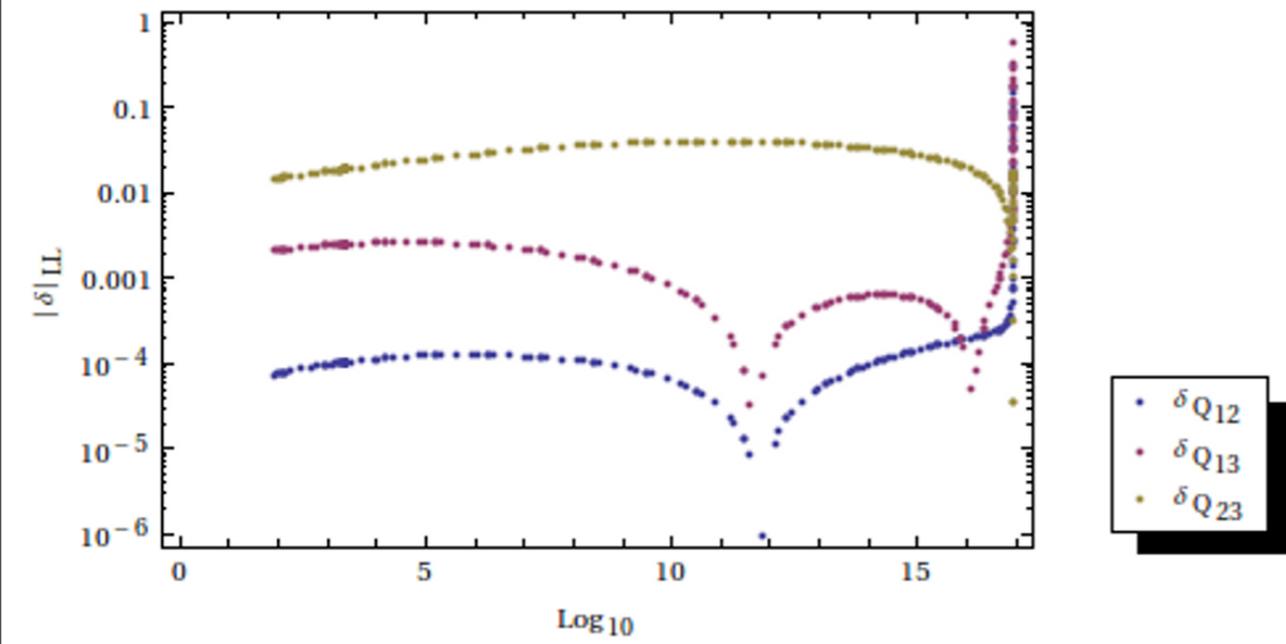
focussing feature similar to other flavour models

# flavour constraints

leptonic sector  
is well within the  
bounds



# hadronic flavour constraints



# Example Point

Parameter	Mass(TeV)	Parameter	Mass(TeV)	Parameter	Mass(TeV)	Parameter	Mass(TeV)	Parameter	Mass(TeV)
$\tilde{t}_1$	0.702	$\tilde{b}_1$	2.06	$\tilde{\tau}_1$	0.480	$\tilde{\nu}_\tau$	0.570	$N_1$	0.465
$\tilde{t}_2$	2.31	$\tilde{b}_2$	2.32	$\tilde{\tau}_2$	0.802	$\tilde{\nu}_\mu$	0.624	$N_2$	0.928
$\tilde{c}_R$	2.25	$\tilde{s}_R$	2.36	$\tilde{\mu}_R$	0.608	$\tilde{\nu}_e$	0.625	$N_3$	4.26
$\tilde{c}_L$	2.45	$\tilde{s}_L$	2.45	$\tilde{\mu}_L$	0.902	-	-	$N_4$	4.26
$\tilde{u}_R$	2.25	$\tilde{d}_R$	2.36	$\tilde{e}_R$	0.610	-	-	$C_1$	0.894
$\tilde{u}_L$	2.45	$\tilde{d}_L$	2.45	$\tilde{e}_L$	0.903	-	-	$C_2$	4.32
$m_{A^0}$	4.18	$m_H^\pm$	4.18	$m_h$	0.1235	$m_H$	3.96	-	-

(ij)	$ \delta_{LL}^Q $	$ \delta_{LL}^L $	$ \delta_{LR}^D $	$ \delta_{LR}^U $	$ \delta_{RL}^D $	$ \delta_{RL}^U $	$ \delta_{RR}^D $	$ \delta_{RR}^E $	$ \delta_{RR}^U $
12	0.0003	$10^{-6}$	$10^{-10}$	$10^{-8}$	$10^{-8}$	$10^{-5}$	$10^{-7}$	$10^{-7}$	0.00005
13	0.01	0.007	$10^{-8}$	$10^{-8}$	$10^{-5}$	0.002	$10^{-6}$	$10^{-4}$	0.06
23	0.06	$10^{-4}$	$10^{-6}$	$10^{-5}$	$10^{-5}$	0.01	$10^{-4}$	0.0006	0.001

**Dirac case**  $m_{3/2} = 800$  GeV;  $M_{1/2} = 1200$  GeV

# Summary

Randall Sundrum is considered as a theory of flavour only, between Planck and GUT scale.

The framework is good for lepton mass fitting, and also for GUTs.

Supersymmetry can be a solution of hierarchy problem at low scales.

The nature of soft terms is like “Flavourful Supersymmetry” if SUSY broken on the GUT brane. Viable spectrum at weak scale.