The Higgs Mass, Superconnections, and the Emergence of the New Physics at the TeV scale

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SU(2/1)

Ne'eman (1979)

Internal supersymmetry with the graded gauged group
SU(2/1)⊃SU(2)xU(1)

$$SU(2/1) \swarrow \searrow$$

SU(2)xU(1) — Even Lie subgroup — 4 even generators (gauge)

Higgs fields — Odd Lie subgroup — 4 odd generators (ghosts)

Leptons –**3** representation $\begin{pmatrix} e_L \\ \nu_L \\ e_R \end{pmatrix} \longrightarrow$ (With right handed neutrino it is **4**)

Quarks–**4** $\begin{pmatrix} u_L \\ d_L \\ u_R \\ d_L \end{pmatrix}$

SU(2/1)

- Gives the right SM quantum numbers.

- A Geometric approach.

- Mixes boson with fermions—violates spin statistics—ghosts.

- <u>Cannot be ultraviolet complete.</u>

Can we still make use of it ? Can it give us more on beyond the SM?

Superconnection approach

Quillen (1985)

Ne'eman and Sternberg (1990)

- Don't impose SU(2/1) invariance.
- Consider it as a geometric pattern (accidental or maybe as remnant of high energy.
- The superconnection approach is a better way to go.
- Reduces to the SM with some constraints!
- Higgs terms come out naturally!

SU(2/1), superconnections and supercurvatures

We write the anti-commutative superconnection in the form

$$\mathcal{J} = \begin{pmatrix} M & \phi \\ \overline{\phi} & N \end{pmatrix}_{3x3}$$

M is 2x2 and N is 1x1 supermatrices-- SU(2/1) g-even (valued ever one-form).

ϕ and $\overline{\phi}$ are 1x2 and 2x1 supermatrices-- SU(2/1) g-odd (valued over zero-form).

$$\mathcal{J} = i\lambda_s^a J^a,$$

$$\mathcal{J} = i \begin{pmatrix} \mathcal{W} - \frac{1}{\sqrt{3}} B \cdot \mathbf{I} & \sqrt{2}\Phi \\ \sqrt{2}\Phi^\dagger & -\frac{2}{\sqrt{3}} B \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \qquad \mathcal{W} = W^i \tau^i$$

Supercurvature:

$$\mathcal{F} = \mathbf{d}\,\mathcal{J} + \mathcal{J} \cdot \mathcal{J}$$

by using Ne’eman-Sternberg rule for supermatrices.

SU(2/1) embedding

$$\mathcal{S} = \int \frac{-1}{4g^2} \mathrm{Tr}\left[\mathcal{F} \cdot \mathcal{F}^\star\right]$$

- $$\underline{\underline{\lambda = \frac{g^2}{2}}} \qquad M_H^2 = 8 M_W^2 \frac{\lambda}{g^2} \quad \Rightarrow \quad M_H = 2M_W$$

- $$\underline{\underline{\frac{g'}{g} = \frac{1}{\sqrt{3}}}}, \qquad \theta_W = 30^o, \qquad \sin^2\theta_W = 0.25.$$

Compare it to the exp. value
$\sin^2\theta_W = 0.2312$

Interpreting the $\sin^2\theta_W = 0.25$ is the value at Λ_s,

$$\underline{\underline{\Lambda_s \cong 4\ \text{TeV!}}}$$

UA, Minic, Takeuchi (2013)

SU(2/1) and the Higgs mass

$$\mu\frac{dh_t}{d\mu} = \frac{h_t}{(4\pi)^2}\left(\frac{9}{2}h_t^2 - \left(\frac{17}{12}g'^2 + \frac{9}{4}g^2 + 8g_s^2\right)\right),$$

$$\mu\frac{d\lambda}{d\mu} = \frac{1}{(4\pi)^2}\Big(\left(12h_t^2 - \left(3g'^2 + 9g^2\right)\right)\lambda - 6h_t^4 + 24\lambda^2 + \frac{3}{8}\left(g'^4 + 2g'^2g^2 + 3g^4\right)\Big),$$

B.C.s

$$\lambda = \frac{g^2}{2} \quad \text{at } \Lambda_s$$

$$h_t = \frac{\sqrt{2}M_t}{v} \quad \text{at } M_z$$

$$\lambda \cong 0.24$$

$$\underline{\underline{M_H \cong 170 \text{ GeV}}}$$

not good!

UA, Minic, Takeuchi (2013)

SU(2/2) embedding

SU(2/2)⊃SU(2)xSU(2)xU(1)

$$u_R, d_R \rightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R \; ; \quad \nu_R, e_R \rightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\Phi = \begin{bmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{bmatrix}$$

Pati-Salam

$$SU(4) \times SU(2)_L \times SU(2)_R$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$\downarrow \quad g_L = g_R \text{ in this case}$$
$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

- **Correct quantum-numbers.**
- **Extra degrees of freedom which could explain the observed Higgs mass (as opposed to 170 GeV in SU(2/1) construction.**
- **Possible Planck scale connection.**
  Pati-Salam group appears in a large number of string vacua (Dienes 2006).

Superconnection and supercurvature

$$\mathcal{J} = i \begin{pmatrix} W_L - \frac{1}{\sqrt{2}} W_{BL} \cdot \mathbf{I} & \sqrt{2}\Phi \\ \sqrt{2}\Phi^\dagger & W_R - \frac{1}{\sqrt{2}} W_{BL} \cdot \mathbf{I} \end{pmatrix}$$

$$W_L = W_L^i \tau^i$$
$$W_R = W_R^i \tau^i$$

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$$

$$\mathcal{F} = \mathbf{d}\,\mathcal{J} + \mathcal{J} \cdot \mathcal{J} \qquad \mathcal{S} = \int \frac{-1}{4g^2} \mathrm{Tr}\left[\mathcal{F} \cdot \mathcal{F}^\star\right]$$

Constraints

1) $\lambda = \dfrac{g^2}{2}$ **same as in SU(2/1)!** (assuming κ≫κ')

2) $\dfrac{g'}{g} = \dfrac{1}{\sqrt{3}}$ $\qquad \dfrac{g_{BL}}{g} = \dfrac{1}{\sqrt{2}}$ $\qquad \Lambda_R = \Lambda_s$

again, **same as in the case of SU(2/1) !**

Symmetry breaking in SU(2/2) and triplets

- Emergent SU(2/2) is broken explicitly by the Higgs mass term.

- There is no room in the structure for extra particles such as the Higgs triplets to break the $SU(2)_L x U(1)_{B-L}$ to $U(1)_Y$. We introduce them by hand.

- If the low energy predictions of SU(2/2) turn out to be true, this brings in the possibility that these triplets may be remnant of a larger geometrical structure.

- This may predict new scales below Planck energies.

The scale of emergence of SU(2/2)

$$SU(2/2) \sim \Lambda_G$$
$$\downarrow$$
$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$\downarrow \; \Lambda_R$$
$$SU(2)_L \times U(1)_Y$$

- If $\Lambda_R = \Lambda_s$ exactly

$$\frac{g'}{g} = \frac{1}{\sqrt{3}}$$

With the particle content of SM

$\Lambda_s \cong 4$ TeV!

$$\frac{1}{[g_i(\Lambda_s)]^2} = \frac{1}{[g_i(\Lambda_0)]^2} - 2b_i \ln \frac{\Lambda_s}{\Lambda_0}$$

- If $\Lambda_R \leq \Lambda_s$

then the lower bound for the Λ_s is 4 TeV!.
However, could be pushed up to Λ_s higher energies but remains as $\Lambda_R \cong \Lambda_s$

$$\frac{1}{g_L^2(\Lambda)} = \frac{1}{g_2^2(M_W)} - 2b_2 \ln \frac{\Lambda_R}{M_W} - \underline{2b_L} \ln \frac{\Lambda}{\Lambda_R}$$
$$\frac{1}{g_{BL}^2(\Lambda)} = \frac{1}{g_{BL}^2(\Lambda_R)} - \underline{2b_{BL}} \ln \frac{\Lambda}{\Lambda_R}.$$

Indications are not clear.

SU(2/2) and the Higgs mass

- Low energy left-right symmetric model at 4 TeV.
- A simple case—a right handed neutrino and a singlet

Higgs Portal

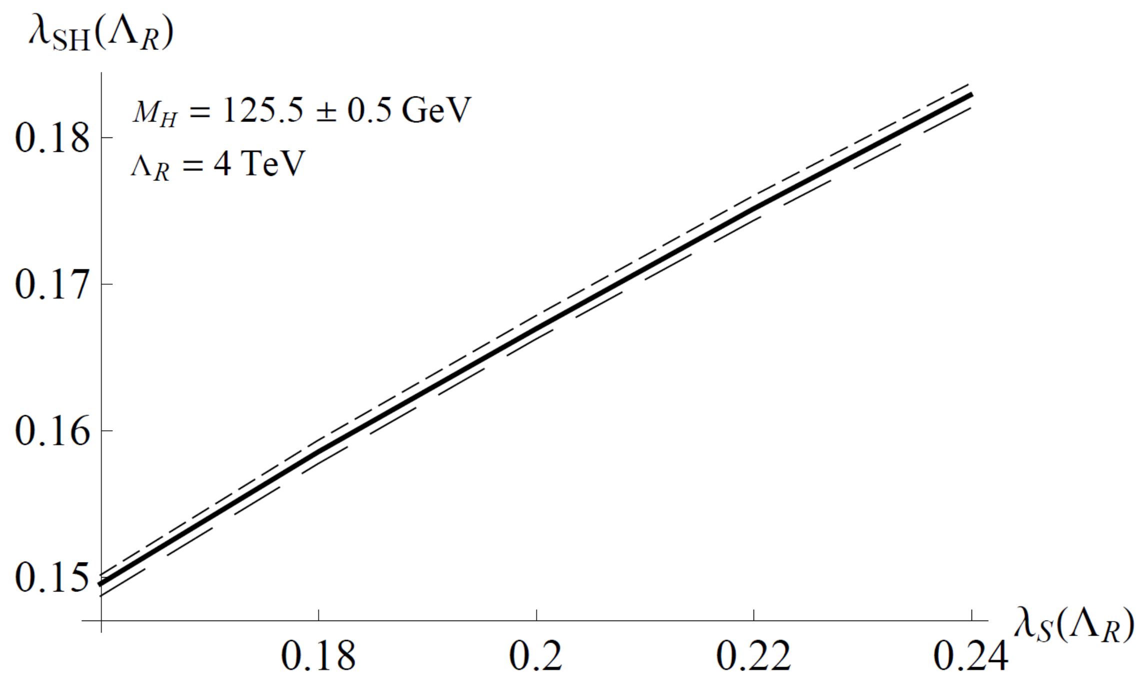
$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{m_0^2}{2}S^2 - \frac{\lambda_S}{4}S^4 - \lambda_{SH} S^2 H^\dagger H$$

$$h_t(M_Z) = 0.997$$

$$\lambda(M_R) = \frac{g_2^2(M_R)}{2}$$

$$\underbrace{h_\nu \sim 10^{-6}}_{\text{problem}}$$

$$m_H^2 \simeq 2\lambda v^2 \left(1 - \frac{\lambda_{HS}^2}{\lambda\,\lambda_S}\right)$$



Nondecoupling- UV/IR mixing

- In effective field theory approach, the low energy theory does not have much to offer for ‘new physics’.----decoupling.

- On the other hand in the geometric approach where we look for *emergent* geometric patterns at low energy, things could change-UV/IR mixing.

- Heavy Higgs does not decouple in LR symmetric models unlike gauge bosons.

- SU(2/2) construction offers a possible explanation.

LHC and the new physics from SU(2/2)

Heavy right handed gauge bosons at 4 TeV W_R and Z_R

$$M_{W_R} \simeq \sqrt{\frac{1}{2} g^2 \left(\kappa^2 + \kappa'^2 + 2\nu_R^2\right)} \cong 2.5\,\mathrm{TeV}$$

$$M_{W_Z} \simeq \sqrt{\frac{1}{2} g^2 \left(1 + \frac{g_{BL}^2}{g^2}\right) \left(\kappa^2 + \kappa'^2 + 4\nu_R^2\right)} \cong 4.3\,\mathrm{TeV}$$

- CMS reported that nothing seen up to **2.9 TeV**, 95% CL (April 2013)
- We will see what new data will tell when LHC starts running again.
- Recall that these predictions are of the simplest version of the model. Could be improved.

Other signals

- **Doubly Charged Higgs boson Δ^{++}**

$$\Delta_{L,R} = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

($e^- e^-$, $\mu^- \mu^-$, $\mu^- e^-$)

Lower bound set ∽ 450 GeV with 95% CL.

- **Right handed neutrino** $W_R \rightarrow N_R e$

- **Neutrinoless double β decay** $(\beta\beta)_{0\nu}$

$M_{WR} \cong 3$ TeV and $M_N \cong 10$ TeV compatible with $M_\nu = 1$eV

Outlook:

- We discussed the superconnection approach to EW part of the SM including the supergroup SU(2/1) .

- We related the observed Higgs mass with the emergent new physics SU(2/2).

- Prediction of the theory is basically left-right symmetric model at TeV scale (around 4 TeV or above).

- This (geometrical) approach may offer a road towards understanding the possible implications of Plank scale physics via decoupling in the Higgs sector.

Back up slides

Das et.al. (2012)

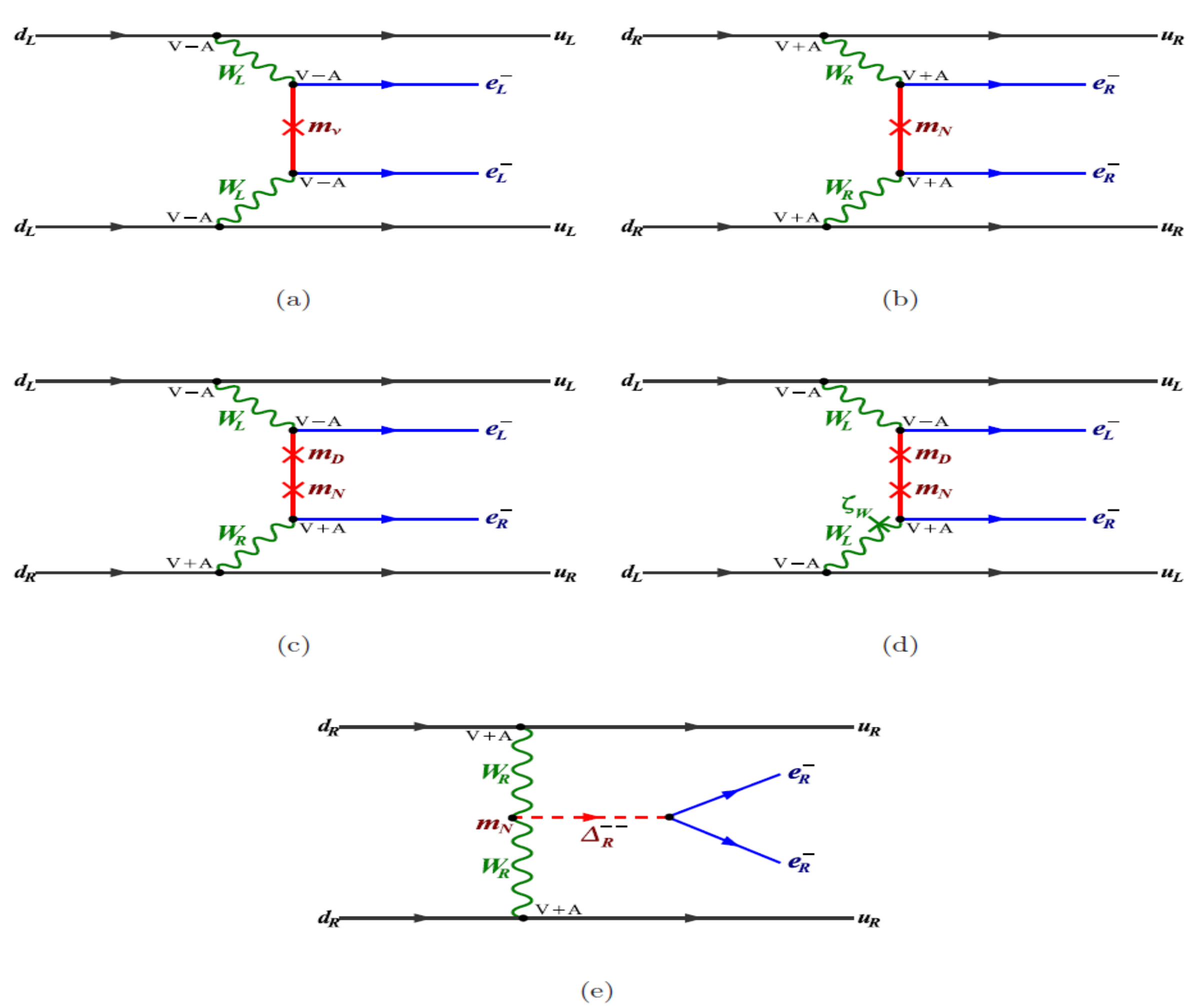


FIG. 2: Diagrams contributing to $0\nu\beta\beta$ decay in left-right symmetry: (a) Light neutrino exchange (standard mass mechanism), (b) Heavy neutrino exchange, (c) Neutrino and heavy W exchange with Dirac mass helicity flip (λ mechanism), (d) Neutrino and light W exchange with Dirac mass and W mixing suppression (η mechanism), (e) Doubly charged Higgs Triplet exchange.

Superconnection and supercurvature

$$\mathcal{J} = i \begin{pmatrix} W_L - \frac{1}{\sqrt{2}} W_{BL} \cdot \mathbf{I} & \sqrt{2}\Phi \\ \sqrt{2}\Phi^\dagger & W_R - \frac{1}{\sqrt{2}} W_{BL} \cdot \mathbf{I} \end{pmatrix}$$

$$W_L = W_L^i \tau^i$$
$$W_R = W_R^i \tau^i$$
$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$$

$$\mathcal{F} = \mathbf{d}\,\mathcal{J} + \mathcal{J} \cdot \mathcal{J}$$

$$\begin{aligned} \mathcal{L} &= \frac{1}{4g^2} \mathrm{Tr}\left[\mathcal{F} \cdot \mathcal{F}^\star\right] \\ &= -\frac{1}{4} F^i_{L\,\mu\nu} F_L^{i\,\mu\nu} - \frac{1}{4} F^i_{R\,\mu\nu} F_R^{i\,\mu\nu} - \frac{1}{4} F_{BL\,\mu\nu} F_{BL}^{\mu\nu} + \mathrm{Tr}\left[(D_\mu\Phi)^\dagger (D^\mu\Phi)\right] \\ &- \widetilde{\lambda}\left(\mathrm{Tr}\left[\left(\Phi^\dagger\Phi\right)^2\right] + \mathrm{Tr}\left[\left(\Phi\Phi^\dagger\right)^2\right]\right). \end{aligned}$$

$$\widetilde{\lambda} = g^2/4$$

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M is 2x2 and N is 1x1 supermatrices-- SU(2/1) g-even (valued ever one-form).

ϕ and $\overline{\phi}$ are 1x2 and 2x1 supermatrices-- SU(2/1) g-odd (valued over zero-form).

Supercurvature:

$$\mathcal{J} = i\lambda_s^a J^a,$$

$$\mathcal{F} = \mathbf{d}\,\mathcal{J} + \mathcal{J} \cdot \mathcal{J}$$

by using Ne'eman-Sternberg rule for supermatrices

$$\begin{pmatrix} A & C \\ D & B \end{pmatrix} \cdot \begin{pmatrix} A' & C' \\ D' & B' \end{pmatrix} = \begin{pmatrix} A \wedge A' + (-1)^{|D'|} C \wedge D' & A \wedge C' + (-1)^{|B'|} C \wedge B' \\ (-1)^{|A'|} D \wedge A' + B \wedge D' & (-1)^{|C'|} D \wedge C' + B \wedge B' \end{pmatrix}$$

Constraints: $\sin^2\theta_W$

$$\mathcal{L}_f = -\frac{1}{2}g\left[\overline{\psi}\gamma^\mu \boldsymbol{\lambda}^s \cdot \widetilde{\mathbf{W}}_\mu \psi\right] = \frac{1}{2}g\left[\bar{l}_L\gamma^\mu\left(\boldsymbol{\tau}\cdot\mathbf{W}_\mu\right)l_L - \frac{1}{\sqrt{3}}\left(\bar{l}_L\gamma^\mu l_L + 2\,\bar{l}_R\gamma^\mu l_R\right)B_\mu\right]$$

Identifying $\mathbf{W}_\mu$ as $\widetilde{W}^{1,2,3}$ and B_μ as $\widetilde{W}^8$ we obtain

$\psi = \begin{pmatrix} l_L \\ l_R \end{pmatrix}$ is a triplet

Comparing to the SM Lagrangian

$$\boxed{\frac{g'}{g} = \frac{1}{\sqrt{3}}}, \qquad \theta_W = 30^o\,, \qquad \sin^2\theta_W = 0.25$$

Compare it to the exp. value $\sin^2\theta_W = 0.2312$

Connes' non-commutative geometry and the Spectral standard model

Connes and Chamseddine

- Spectral Action—Spectrum of Dirac operator D
- 16 fermions of the SM—right handed neutrino and see-saw mechanism.
- Symmetries of the SM emerge.
- Boundary conditions from the action—RG running from GUT scale
- A singlet (in addition to RH neutrino) naturally appears.
- Without the singlet and right handed neutrino, the Higgs mass is <u>**170 GeV**</u>
- With the additional dof it is possible to reduce it down to125-126 GeV.

Left-right symmetric model

Pati-Salam,
Mohapatra-Senjanovic

G=SU(2)$_L$ x SU(2)$_R$ x U(1)$_{BL}$

$$Q = I_L^3 + I_R^3 + I_{BL}$$

$$u_R,\, d_R \to \begin{pmatrix} u \\ d \end{pmatrix}_R \; ; \quad \nu_R,\, e_R \to \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$I_{BL} = \frac{(B-L)}{2}$$

$$(I_L, I_R, I_{BL}) = \underset{q_L}{(2,1,\tfrac{1}{6})}, \quad \underset{q_R}{(1,2,\tfrac{1}{6})}, \quad \underset{l_L}{(2,1,-\tfrac{1}{2})}, \quad \underset{l_R}{(1,2,-\tfrac{1}{2})}$$

heavy RH gauge bosons

$$\mathcal{L} = -\bar{l}_L \gamma_\mu \left[g_L \frac{\boldsymbol{\tau} \cdot \mathbf{W}_L^\mu}{2} - \frac{g_{BL}}{2} W_{BL}^\mu \right] l_L - \bar{l}_R \gamma_\mu \left[g_R \frac{\boldsymbol{\tau} \cdot \mathbf{W}_R^\mu}{2} - \frac{g_{BL}}{2} W_{BL}^\mu \right] l_R$$

quark terms…

Bidoublets and Triplets

(2,2*,0)

$$\mathcal{L}_{\Phi}^{kin} = \mathrm{Tr}\left[(D_\mu\Phi)^\dagger (D^\mu\Phi)\right] \qquad \Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \quad \widetilde{\Phi} = \begin{pmatrix} \phi_2^{0*} & -\phi_2^+ \\ -\phi_1^- & \phi_1^{0*} \end{pmatrix}$$

Acquires VEVs $\langle\Phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}, \quad \langle\widetilde{\Phi}\rangle = \begin{pmatrix} \kappa'^* & 0 \\ 0 & \kappa^* \end{pmatrix}$

We also need a Higgs multiplet with t_{2R}, $t_{BL} \neq 0$ to break the $SU(2)_R \times U(1)_{BL}$ to U(1)

$$\Delta_{L,R} = \frac{1}{\sqrt{2}}\tau\cdot\delta_{L,R} = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}_{L,R} \qquad \delta_{L,R} \equiv \begin{pmatrix} \delta^{++} \\ \delta^+ \\ \delta^0 \end{pmatrix}_{L,R}$$

$$\langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ \nu_{L,R} & 0 \end{pmatrix}_{L,R}$$

Fermions in SU(2/2):Yukawa coupling universality

Use the isomorphism btw the Clifford algebra and the exterior algebra.
Define the Dirac operator

$$\not{D} = \not{\partial} \cdot \mathbf{I} + \frac{g}{2} \not{\mathcal{J}} \qquad \not{\mathcal{J}} = i \begin{pmatrix} \boldsymbol{\tau} \cdot \not{\mathbf{W}}_L - \frac{1}{\sqrt{2}} \not{W}_{BL} \cdot \mathbf{I} & \sqrt{2}\Phi \\ \sqrt{2}\Phi^\dagger & \boldsymbol{\tau} \cdot \not{\mathbf{W}}_R - \frac{1}{\sqrt{2}} \not{W}_{BL} \cdot \mathbf{I} \end{pmatrix}$$

$$\mathcal{L}_l = \overline{\psi}_m i \not{D} \psi_m \longrightarrow \frac{g}{\underline{\sqrt{2}}} \left(\bar{l}_L \phi l_R + \bar{l}_R \phi^\dagger l_L \right)$$

$$Y = \frac{g}{\sqrt{2}}$$

a problem and must be resolved .