Subleading Contributions to Thrust using EFT Techniques

Simon Michal Freedman, arXiv:1303:1558
PHENO 2013

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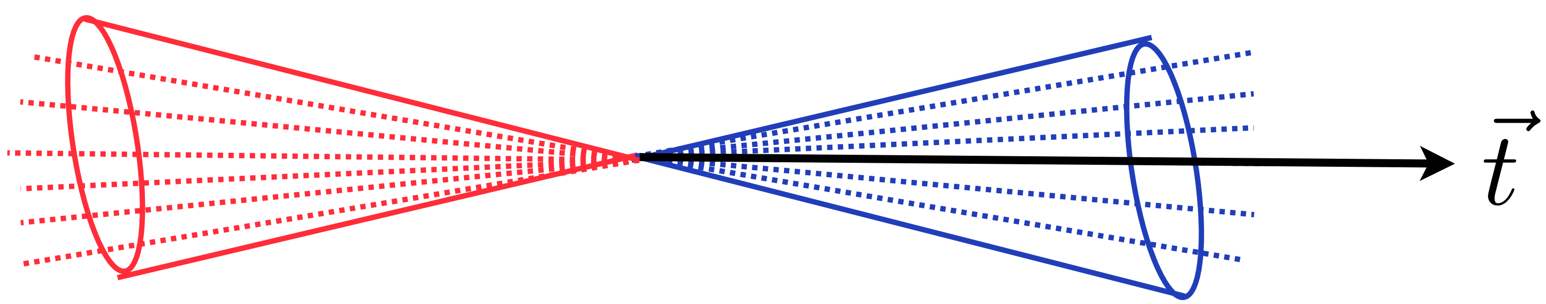
Outline

- Motivation for looking at subleading thrust
- Power corrections in Soft-Collinear Effective Theory (SCET)
- *Subleading Thrust using EFT techniques*
- Conclusion and Outlook

What is Thrust?

- Thrust is an example of an event shape
- Small thrust gives two "pencil-like" jets



[Farhi, 1977]

$$\boxed{\tau = \frac{1}{Q} \sum_{i \in X} \min(E_i \pm \vec{t} \cdot \vec{p}_i)}$$

Thrust axis: $\max_{\vec{t}} \sum_{i \in X} |\vec{t} \cdot \vec{p}_i|$

Initial Energy: Q

Final State: X

$$\boxed{R(\tau) = \frac{1}{\sigma_0} \int_0^{\tau} d\tau' \frac{d\sigma}{d\tau'} = R^{(0)}(\tau) + R^{(1)}(\tau) + O(\tau^2)}$$

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- Thrust is an easy example to look at subleading corrections
- Useful in measuring α_s at LEP *[Abbate, Fickinger, Hoang, Mateu, Stewart, 2011/12]*

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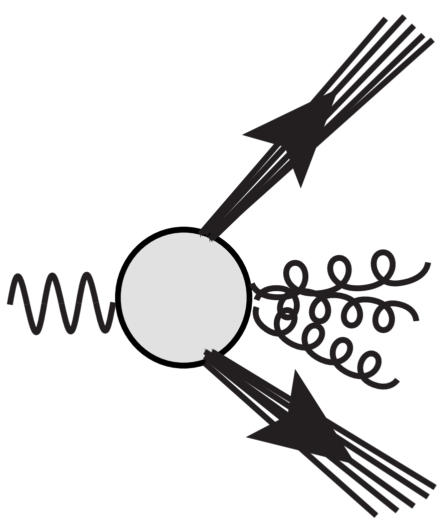
- Traditional methods (eg CSS) can systematically treat...
  - ✓ Large logarithms $R = C(\alpha_s) e^{\log(1/\tau) f_0(\alpha_s \log) + f_1(\alpha_s \log) + \ldots} + R^{(1)}(\tau) + \ldots$
  - ✓ Higher order in α_s
  - ~ Corrections in τ

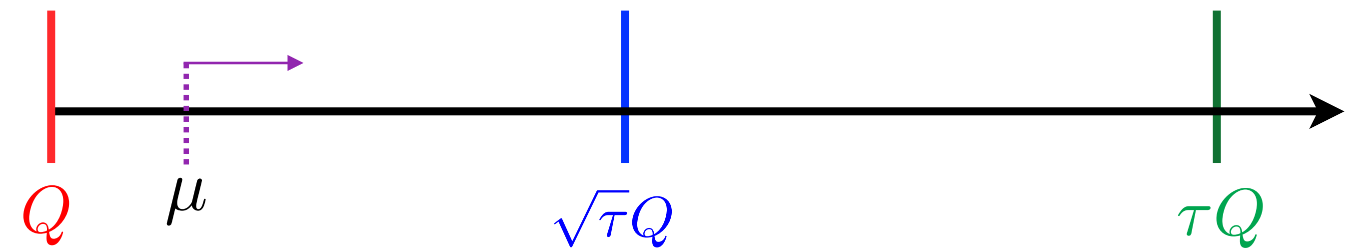
Effective Field Theory Approach

- Effective Field Theories treat physics at different scales separately (eg, perturbative, experimental extraction, lattice...)



$\tau > \tau(X)$

$$R(\tau) = H(\mu) \times \langle 0|J(\mu)|0\rangle \otimes \langle 0|\bar{J}(\mu)|0\rangle \otimes \langle 0|S(\mu)|0\rangle$$



- Renormalization group allows logs to be summed
- Allows for systematic improvements to ...
  - ✓ Large logarithms
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- *Factorization of power corrections in dijet events in SCET?*

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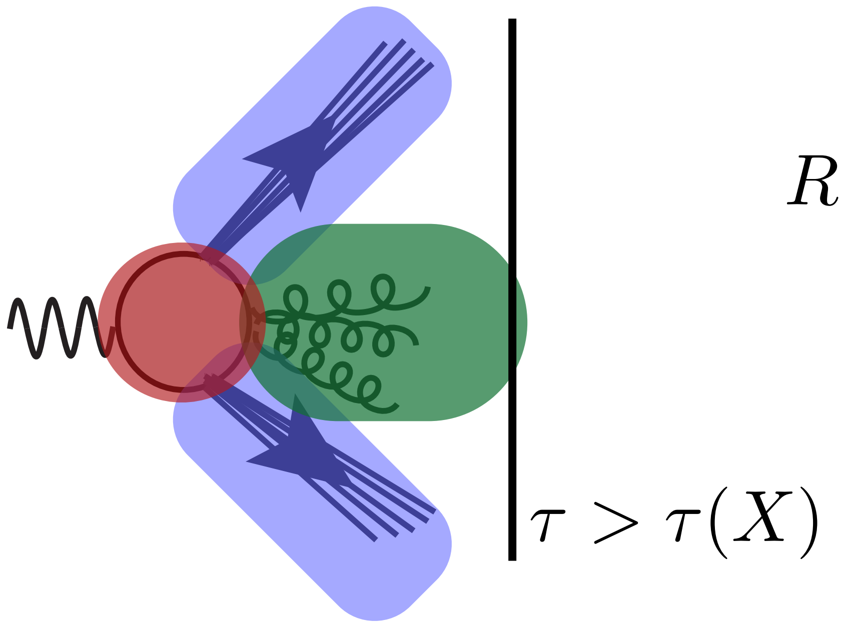

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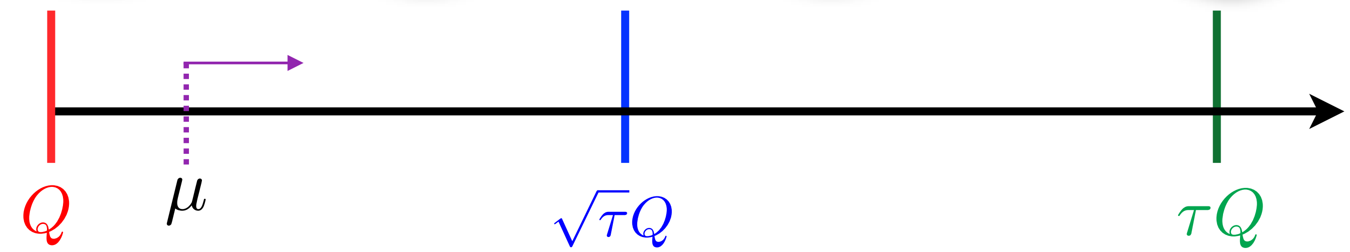

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SCET and Power Corrections

- Soft — Collinear Effective Theory

$p_s \sim Q(\lambda^2, \lambda^2, \lambda^2)$ $p_n \sim Q(\underset{\bar{n}^\mu}{\lambda^2}, \underset{n^\mu}{1}, \underset{\perp}{\lambda})$ $p_{\bar{n}} \sim Q(1, \lambda^2, \lambda)$ $\boxed{\lambda \sim \sqrt{\tau}}$

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$$\boxed{\mathcal{L}_{\mathrm{SCET}} = \mathcal{L}_{\mathrm{QCD}}^{\mathrm{soft}} + \mathcal{L}_{\mathrm{QCD}}^{n\text{-coll}} + \mathcal{L}_{\mathrm{QCD}}^{\bar{n}\text{-coll}}}$$

[SMF & Luke, 2012]

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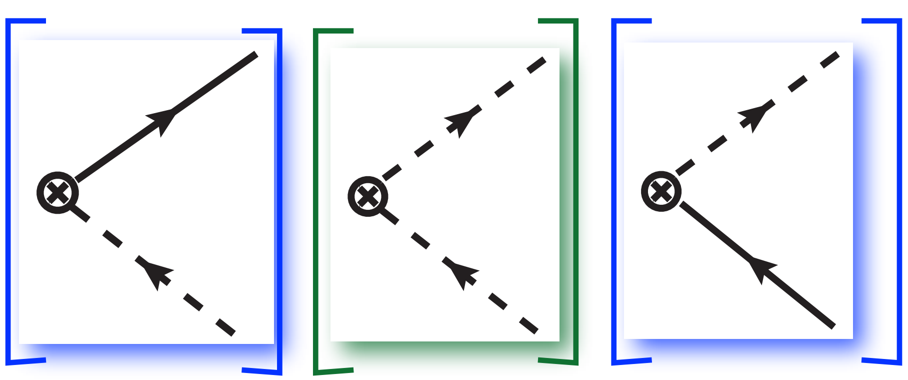
$$\boxed{J_{\rm QCD} \to C_2^{(0)} O_2^{(0)} + \frac{1}{Q} \sum_i C_2^{(i)} O_2^{(i)}}$$

$O_2(x) \sim$ 

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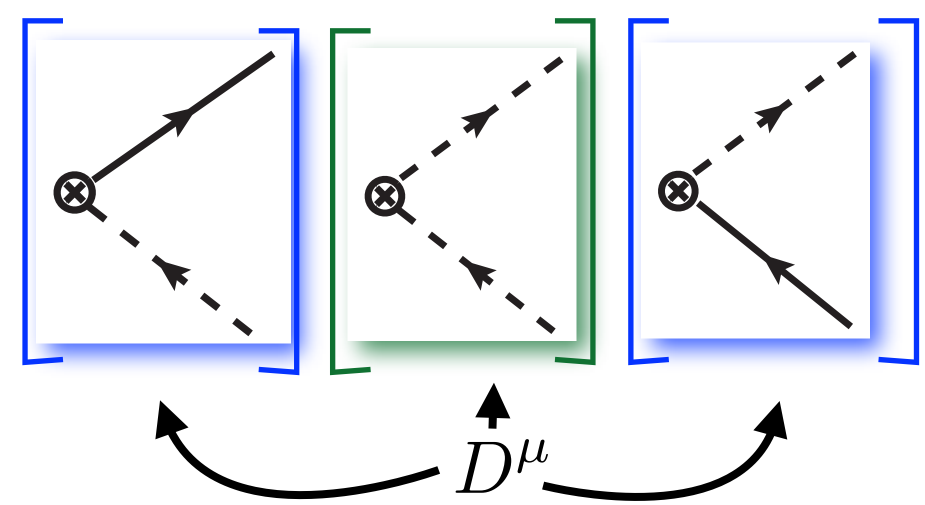

Thrust at Leading Order

- The rate is given in QCD by

$$R(\tau) = \int d^d x e^{-iQ\cdot x} \langle 0 | J^{\mu\dagger}_{\rm QCD}(x) \hat{\mathcal{M}}_{\rm QCD}(\tau) J^{\mu}_{\rm QCD}(0) | 0 \rangle$$

- Where the measurement operator generates phase-space

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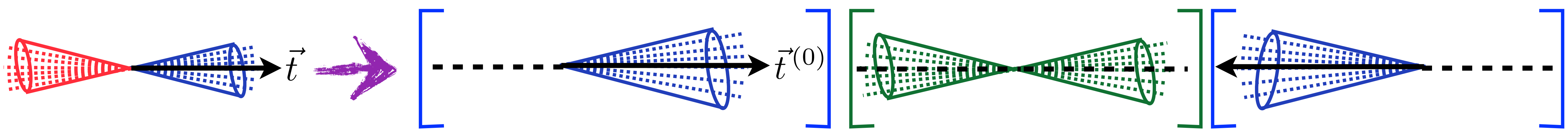
- Where the measurement operator generates phase-space

(1) Match onto SCET:

- Dijet operators (previous slide), including *momentum conservation*
- Measurement operator $\hat{\mathcal{M}}_{\rm QCD}(\tau) \to \hat{\mathcal{M}}^{(0)}_{\rm SCET}(\tau) + \hat{\mathcal{M}}^{(1)}_{\rm SCET}(\tau) + \ldots$
- Both dijet and measurement operators decouple



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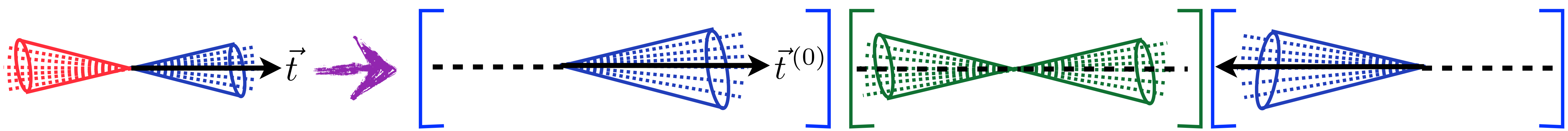


(2) Match onto jet and soft operators and renormalize

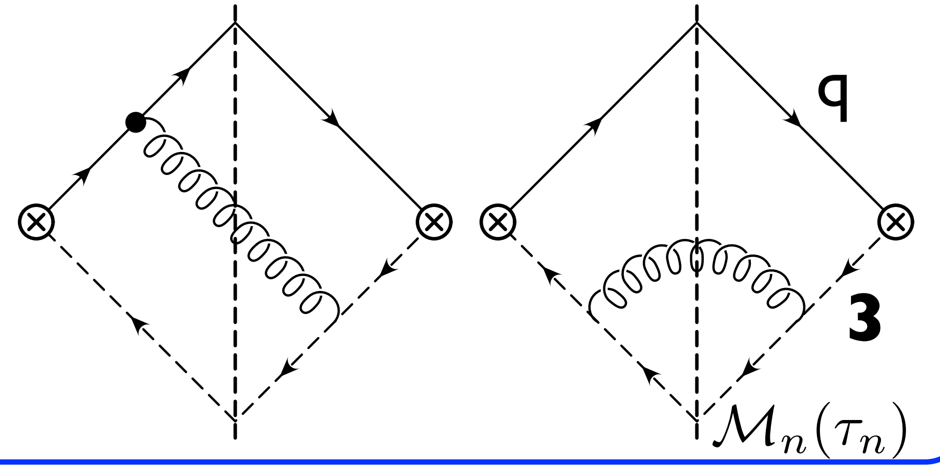
$$\boxed{\langle 0 | O_2^{(0)\dagger} \hat{\mathcal{M}}^{(0)}_{\rm SCET}(\tau) O_2^{(0)} | 0 \rangle \to \langle 0 | J^{(0)}(\tau_n) | 0 \rangle \otimes \langle 0 | \bar{J}^{(0)}(\tau_{\bar{n}}) | 0 \rangle \otimes \langle 0 | S^{(0)}(\tau_s) | 0 \rangle}$$

[Lee, Hornig, Ovanesyan, 2009]
[Fleming, Hoang, Mantry, Stewart, 2007]

Example of Operators

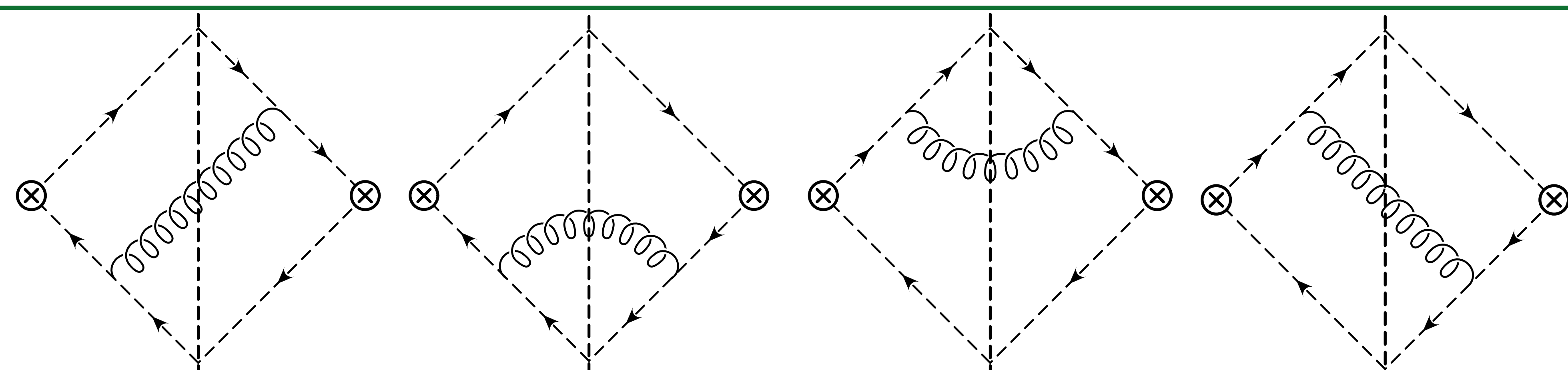
$$R^{(0)}(\tau) = H\langle 0|J^{(0)}(\tau_n)|0\rangle \otimes \langle 0|\bar{J}^{(0)}(\tau_{\bar{n}})|0\rangle \otimes \langle 0|S^{(0)}(\tau_s)|0\rangle$$

$\langle 0|J^{(0)}|0\rangle$



$\langle 0|\bar{J}^{(0)}|0\rangle$ ~ similar to above

$\langle 0|S^{(0)}|0\rangle$



Thrust at Subleading Order

- Same steps as leading order...

$$R(\tau) = \int d^d x e^{-iQ\cdot x} \langle 0 | J^{\mu\dagger}_{\rm QCD}(x) \hat{\mathcal{M}}_{\rm QCD}(\tau) J^{\mu}_{\rm QCD}(0)|0\rangle$$

$$\hat{\mathcal{M}}_{\rm QCD}(\tau) \to \hat{\mathcal{M}}^{(0)}_{\rm SCET}(\tau) + \hat{\mathcal{M}}^{(1)}_{\rm SCET}(\tau) + \ldots \qquad J_{\rm QCD} \to C_2^{(0)} O_2^{(0)} + \frac{1}{Q}\sum_i C_2^{(i)} O_2^{(i)}$$

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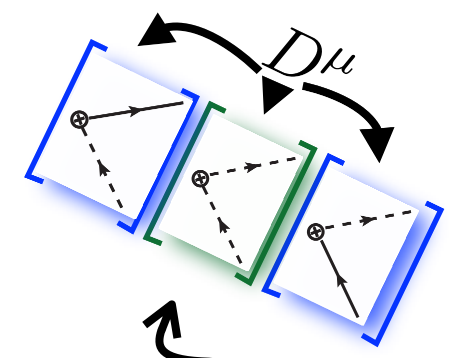

$$R(\tau) = \int d^d x e^{-iQ\cdot x} \langle 0 | J^{\mu\dagger}_{\mathrm{QCD}}(x) \hat{\mathcal{M}}_{\mathrm{QCD}}(\tau) J^{\mu}_{\mathrm{QCD}}(0) | 0 \rangle$$

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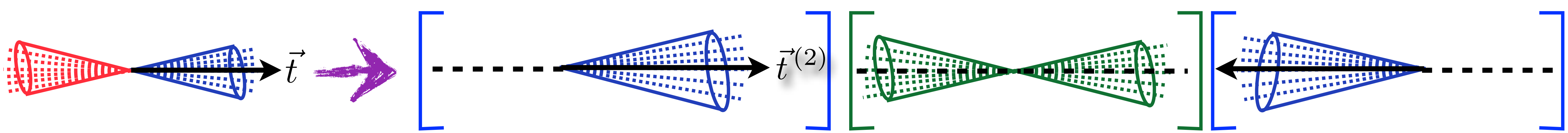

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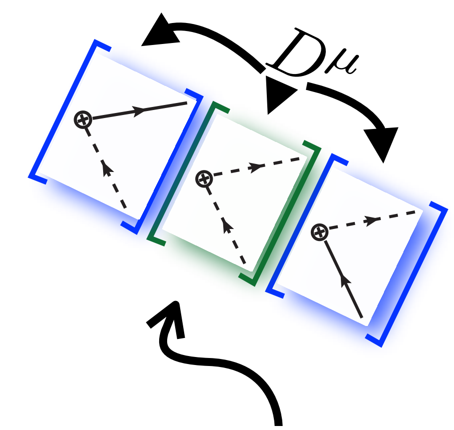

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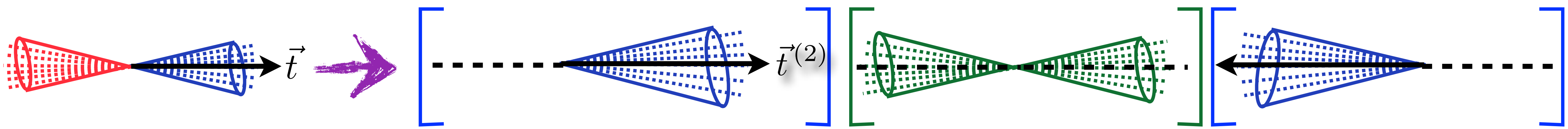

$$\langle 0 | O_2^{(i)\dagger}(x) \hat{\mathcal{M}}^{(j)}(\tau) O_2^{(k)}(0) | 0 \rangle \to \begin{cases} H_\alpha^{(i,j,k)} \times \langle 0 | J^{(l)}(\tau_n) | 0 \rangle \otimes \langle 0 | \bar{J}^{(m)}(\tau_{\bar{n}}) | 0 \rangle \otimes \langle 0 | S^{(n)}(\tau_s) | 0 \rangle \\ \tau H_0^{(i,j,k)} \times \langle 0 | J^{(0)}(\tau_n) | 0 \rangle \otimes \langle 0 | \bar{J}^{(0)}(\tau_{\bar{n}}) | 0 \rangle \otimes \langle 0 | S^{(0)}(\tau_s) | 0 \rangle \end{cases}$$

$$\boxed{R^{(2)}(\tau) \sim H(\mu) \times \langle 0 | J(\mu) | 0 \rangle \otimes \langle 0 | \bar{J}(\mu) | 0 \rangle \otimes \langle 0 | S(\mu) | 0 \rangle}$$

[SMF, 2013]

Example of Operators

- 13 jet operators, 16 soft operators, 38 matching coefficients!!!

[Kramer, Lampe, 1989]

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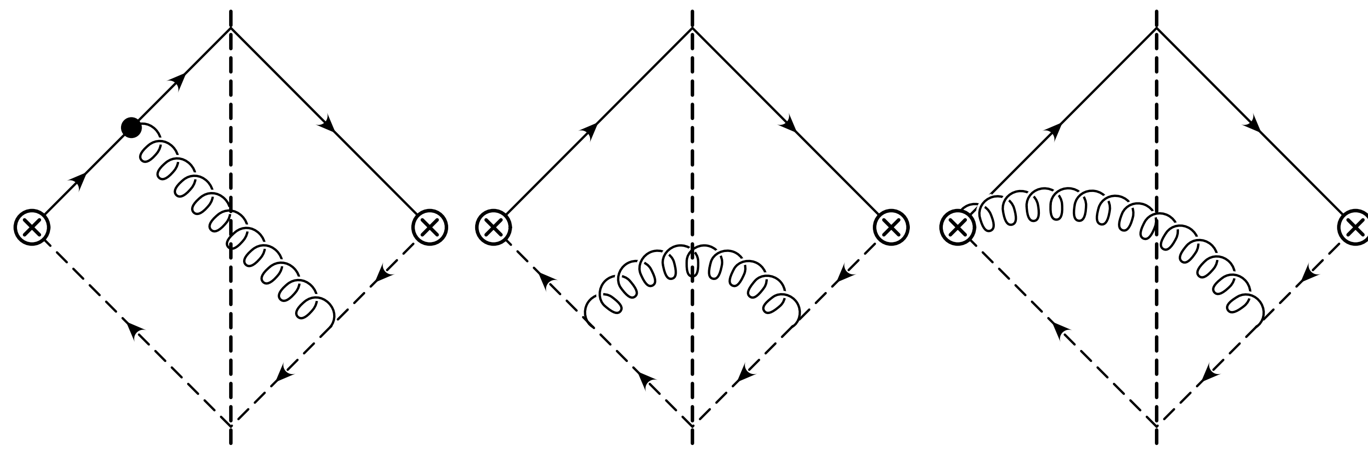
$\langle 0|J^{(2ab_n)}|0\rangle$  $\langle 0|\bar{J}^{(0)}|0\rangle \quad \langle 0|S^{(0)}|0\rangle$

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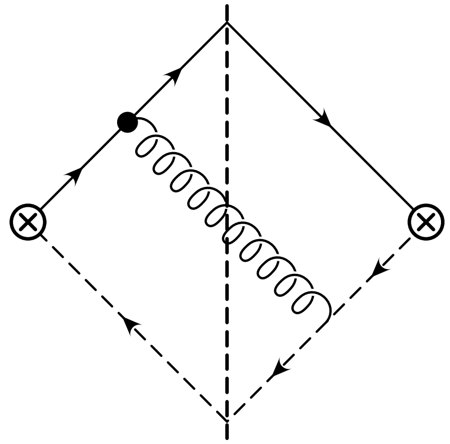

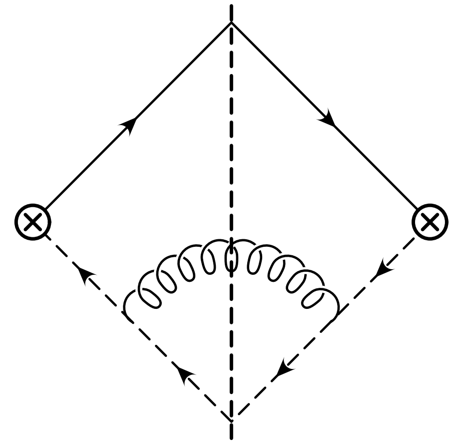

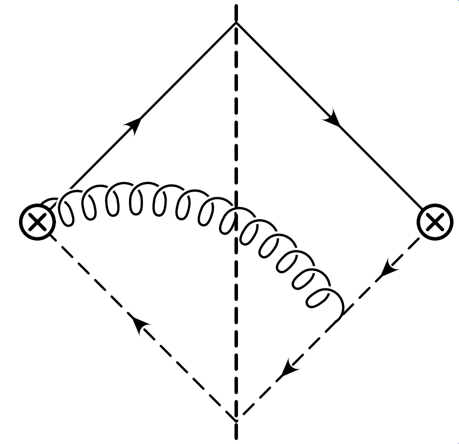

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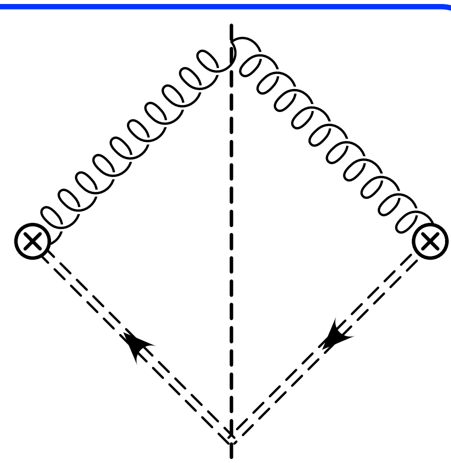
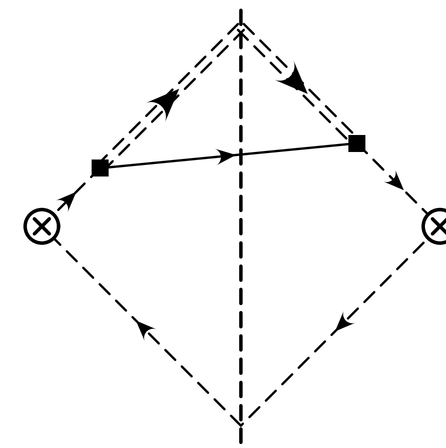
$\langle 0|J^{(0\mathbf{s})}|0\rangle$



$\langle 0|\bar{J}^{(0)}|0\rangle$

$\langle 0|S^{(2d_{ns})}|0\rangle$



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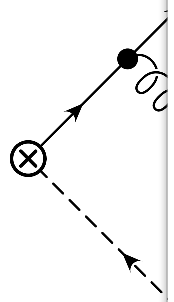
[Kramer, Lampe, 1989]

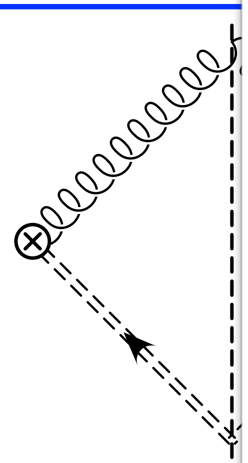
- But all

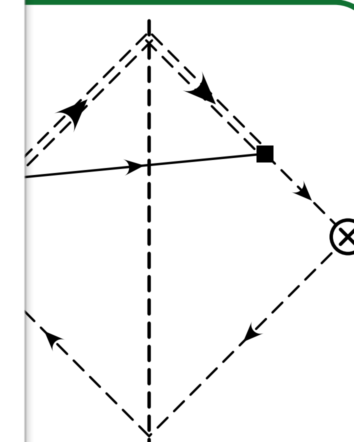
$\langle 0|J^{(2ab_n)}|0\rangle$



$\bar{J}^{(0)}|0\rangle$ $\langle 0|S^{(0)}|0\rangle$

$\langle 0|J^{(0_{\mathbf{8}})}|0\rangle$





(i,j,k)	(l,m,n)	$C_1^{(i,j,k)}$	$H_1^{(i,j,k)}$	$R(\tau)$
$(1a_n,1b_n,0)$	$(2ab_n,0,0)$	-1	$+1$	$\bar{\alpha}_s\tau/2$
$(1b_n,1a_n,0)$	$(2ab_n^\dagger,0,0)$	-1	$+1$	$\bar{\alpha}_s\tau/2$
$(2a_n,0,0)$	$(2a_n,0,0)$	1	$-1/2$	$\bar{\alpha}_s\tau(-1/\epsilon-L_J-9/4)$
$(0,2a_n,0)$	$(2a_n^\dagger,0,0)$	1	$-1/2$	$\bar{\alpha}_s\tau(-1/\epsilon-L_J-9/4)$
$(2A_n,0,0)$	$(2A_n,0,0)$	$-1/2$	$-1/4$	$-\bar{\alpha}_s\tau/4$
$(0,2A_n,0)$	$(2A_n^\dagger,0,0)$	$-1/2$	$-1/4$	$-\bar{\alpha}_s\tau/4$
$(2\delta_n,0,0)$	$(2\delta_n,0\delta_n,0)$	1	1	$\bar{\alpha}_s(4/\epsilon+4L_J+7)$
$(1e_n,1e_n,0)$	$(2e,0_{\mathbf{8}},0_{\mathbf{8}})$	$-1/2$	$-1/2$	$\bar{\alpha}_s\tau(-1/\epsilon-L_J-2)$
$(1f_n,1f_n,0)$	$(2f,0_{\mathbf{8}},0_{\mathbf{8}}$	$-1/2$	$-1/2$	$\bar{\alpha}_s\tau(-1/\epsilon-L_J-2)$
$(1a_{\bar{n}},1b_{\bar{n}},0)$	$(0,2ab_n,0)$	-1	$+1$	$\bar{\alpha}_s\tau/2$
$(1b_{\bar{n}},1a_n,0)$	$(0,2ab_{\bar{n}}^\dagger,0)$	-1	$+1$	$\bar{\alpha}_s\tau/2$
$(2a_{\bar{n}},0,0)$	$(0,2a_{\bar{n}},0)$	1	$-1/2$	$\bar{\alpha}_s\tau(-1/\epsilon-L_J-9/4)$
$(0,2a_{\bar{n}},0)$	$(0,2a_{\bar{n}}^\dagger,0)$	1	$-1/2$	$\bar{\alpha}_s\tau(-1/\epsilon-L_J-9/4)$
$(2A_{\bar{n}},0,0)$	$(0,2A_{\bar{n}},0)$	$-1/2$	$-1/4$	$-\bar{\alpha}_s\tau/4$
$(0,2A_{\bar{n}},0)$	$(0,2A_{\bar{n}}^\dagger,0)$	$-1/2$	$-1/4$	$-\bar{\alpha}_s\tau/4$
$(2\delta_n,0,0)$	$(2\delta_n,0\delta_n,0)$	1	1	$\bar{\alpha}_s(4/\epsilon+4L_J+7)$
$(1\delta_s,1c_{ns},0)$	$(0c_{ns}\delta_s,0,2c_{ns}\delta_s)$	$1/2$	1	$\bar{\alpha}_s\tau(-2/\epsilon-2L_S-2)$
$(2\delta c_{sn},0,0)$	$(0c_{ns}\delta_s,0,2\delta c_{ns})$	$1/2$	1	$\bar{\alpha}_s\tau(-2/\epsilon-2L_S-2)$
$(1d_{ns},1d_{ns},0)$	$0_{\mathbf{8}},0,2d_{ns})$	1	1	$\bar{\alpha}_s\tau(1/\epsilon+L_S+1)$
$(1d_{\bar{n}s},1d_{\bar{n}s},0)$	$(0_{\mathbf{8}},0,2d_{\bar{n}s})$	1	1	$\bar{\alpha}_s\tau(1/\epsilon+L_S+1)$
$(2A_{ns},0,0)$	$(0,0,2A_{ns})$	1	$1/2$	$\bar{\alpha}_s\tau(1/\epsilon+L_S+1)$
$(0,2A_{ns},0)$	$(0,0,2A_{ns}^\dagger)$	1	$1/2$	$\bar{\alpha}_s\tau(1/\epsilon+L_S+1)$
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$(2\delta_{s-},0,0)$	$(0,0\delta_n,2\delta_{s-})$	1	1	$\bar{\alpha}_s\tau(-4/\epsilon-4L_S-4)$
$(2\delta_{s\perp},0,0)$	$(-2\delta_s,0,4\delta_{s\perp})$	$1/2$	$1/2$	$4\bar{\alpha}_s\tau$
$(1\delta_s,0,1n)$	$(-2\delta_s M_n,0_{M1},4\delta_s M_n)$	1	1	$4\bar{\alpha}_s\tau$
$(0,0,2n_b)$	$(-2M_{2n_b},0_{M2},4M_{n_b})$	1	1	$-2\bar{\alpha}_s\tau$
$(0,0,2\bar{n}_b)$	$(0,-2M_{\bar{n}_b},4M_{n_b})$	1	1	$2\bar{\alpha}_s\tau$
$(0,0,2s_1)$	$(0,0_M,2M_{s1})$	1	1	$-4\bar{\alpha}_s\tau$
$(0,0,2s_{+t})$	$(0,0_M,2M_{+t})$	1	1	$-4\bar{\alpha}_s\tau$

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$(1e_n,1e_n,0)$	$\bar{\alpha}_s$	$\bar{\alpha}_s$	$\bar{\alpha}_s\tau$
$(1f_n,1f_n,0)$	$\bar{\alpha}_s$	$\bar{\alpha}_s$	$\bar{\alpha}_s\tau$
$(2\delta_{\bar{n}},0,0)$	$-4\bar{\alpha}_s$	$-4\bar{\alpha}_s$	$-4\bar{\alpha}_s\tau$
$(1d_{ns},1d_{ns},0)$	$-\bar{\alpha}_s$	$-\bar{\alpha}_s$	$-\bar{\alpha}_s\tau$
$(1d_{\bar{n}s},1d_{\bar{n}s},0)$	$-\bar{\alpha}_s$	$-\bar{\alpha}_s$	$-\bar{\alpha}_s\tau$
$(2\delta_{s-},0,0)$	$4\bar{\alpha}_s$	$4\bar{\alpha}_s$	$4\bar{\alpha}_s\tau$

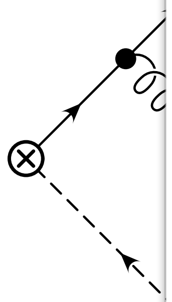
Example o

- 13 jet c ... oefficients!!!

[Kramer, Lampe, 1989]

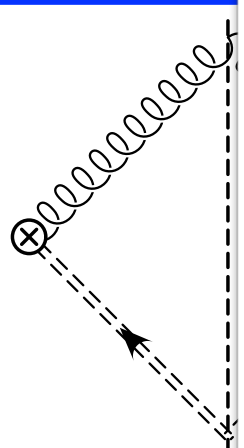
- But all

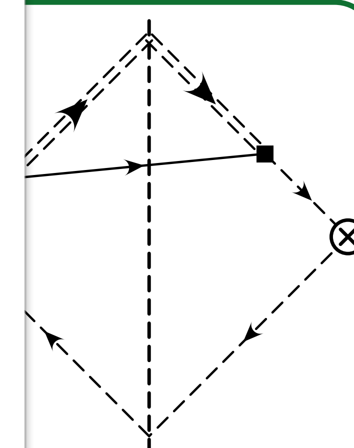
$\langle 0 | J^{(2ab_n)} | 0 \rangle$



$\bar{J}^{(0)} | 0 \rangle \quad \langle 0 | S^{(0)} | 0 \rangle$

$\langle 0 | J^{(0_\mathbf{8})} | 0 \rangle$





(i,j,k)	(l,m,n)	$C_1^{(i,j,k)}$	$H_1^{(i,j,k)}$	$R(\tau)$
$(1a_n, 1b_n, 0)$	$(2ab_n, 0, 0)$	-1	$+1$	$\bar{\alpha}_s \tau/2$
$(1b_n, 1a_n, 0)$	$(2ab_n^\dagger, 0, 0)$	-1	$+1$	$\bar{\alpha}_s \tau/2$
$(2a_n, 0, 0)$	$(2a_n, 0, 0)$	1	$-1/2$	$\bar{\alpha}_s \tau(-1/\epsilon - L_J - 9/4)$
$(0, 2a_n, 0)$	$(2a_n^\dagger, 0, 0)$	1	$-1/2$	$\bar{\alpha}_s \tau(-1/\epsilon - L_J - 9/4)$
$(2A_n, 0, 0)$	$(2A_n, 0, 0)$	$-1/2$	$-1/4$	$-\bar{\alpha}_s \tau/4$
$(0, 2A_n, 0)$	$(2A_n^\dagger, 0, 0)$	$-1/2$	$-1/4$	$-\bar{\alpha}_s \tau/4$
$(2\delta_n, 0, 0)$	$(2\delta_n, 0\delta_n, 0)$	1	1	$\bar{\alpha}_s(4/\epsilon + 4L_J + 7)$
$(1e_n, 1e_n, 0)$	$(2e, 0_\mathbf{8}, 0_\mathbf{8})$	$-1/2$	$-1/2$	$\bar{\alpha}_s \tau(-1/\epsilon - L_J - 2)$
$(1f_n, 1f_n, 0)$	$(2f, 0_\mathbf{8}, 0_\mathbf{8}$	$-1/2$	$-1/2$	$\bar{\alpha}_s \tau(-1/\epsilon - L_J - 2)$
$(1a_{\bar{n}}, 1b_{\bar{n}}, 0)$	$(0, 2ab_{\bar{n}}, 0)$	-1	$+1$	$\bar{\alpha}_s \tau/2$
$(1b_{\bar{n}}, 1a_{\bar{n}}, 0)$	$(0, 2ab_{\bar{n}}^\dagger, 0)$	-1	$+1$	$\bar{\alpha}_s \tau/2$
$(2a_{\bar{n}}, 0, 0)$	$(0, 2a_{\bar{n}}, 0)$	1	$-1/2$	$\bar{\alpha}_s \tau(-1/\epsilon - L_J - 9/4)$
$(0, 2a_{\bar{n}}, 0)$	$(0, 2a_{\bar{n}}^\dagger, 0)$	1	$-1/2$	$\bar{\alpha}_s \tau(-1/\epsilon - L_J - 9/4)$
$(2A_{\bar{n}}, 0, 0)$	$(0, 2A_{\bar{n}}, 0)$	$-1/2$	$-1/4$	$-\bar{\alpha}_s \tau/4$
$(0, 2A_{\bar{n}}, 0)$	$(0, 2A_{\bar{n}}^\dagger, 0)$	$-1/2$	$-1/4$	$-\bar{\alpha}_s \tau/4$
$(2\delta_{\bar{n}}, 0, 0)$	$(2\delta_n, 0\delta_n, 0)$	1	1	$\bar{\alpha}_s(4/\epsilon + 4L_J + 7)$
$(1\delta_s, 1c_{ns}, 0)$	$(0c_{ns}\delta_s, 0, 2c_{ns}\delta_s)$	$1/2$	1	$\bar{\alpha}_s \tau(-2/\epsilon - 2L_S - 2)$
$(2\delta c_{sn}, 0, 0)$	$(0c_{ns}\delta_s, 0, 2\delta c_{ns})$	$1/2$	1	$\bar{\alpha}_s \tau(-2/\epsilon - 2L_S - 2)$
$(1d_{ns}, 1d_{ns}, 0)$	$0_\mathbf{8}, 0, 2d_{ns})$	1	1	$\bar{\alpha}_s \tau(1/\epsilon + L_S + 1)$
$(1d_{\bar{n}s}, 1d_{\bar{n}s}, 0)$	$(0_\mathbf{8}, 0, 2d_{\bar{n}s})$	1	1	$\bar{\alpha}_s \tau(1/\epsilon + L_S + 1)$
$(2A_{ns}, 0, 0)$	$(0, 0, 2A_{ns})$	1	$1/2$	$\bar{\alpha}_s \tau(1/\epsilon + L_S + 1)$
$(0, 2A_{ns}, 0)$	$(0, 0, 2A_{ns}^\dagger)$	1	$1/2$	$\bar{\alpha}_s \tau(1/\epsilon + L_S + 1)$
$(2A_{\bar{n}s}, 0, 0)$	$(0, 0, 2A_{\bar{n}s})$	1	$1/2$	$\bar{\alpha}_s \tau(1/\epsilon + L_S + 1)$
$(0, 2A_{\bar{n}s}, 0)$	$(0, 0, 2A_{\bar{n}s}^\dagger)$	1	$1/2$	$\bar{\alpha}_s \tau(1/\epsilon + L_S + 1)$
$(2\delta_{s-}, 0, 0)$	$(0, 0\delta_n, 2\delta_{s-})$	1	1	$\bar{\alpha}_s \tau(-4/\epsilon - 4L_S - 4)$
$(2\delta_{s\perp}, 0, 0)$	$(-2\delta_s, 0, 4\delta_{s\perp})$	$1/2$	$1/2$	$4\bar{\alpha}_s \tau$
$(1\delta_s, 0, 1n)$	$(-2\delta_s M_n, 0_{M1}, 4\delta_s M_n)$	1	1	$4\bar{\alpha}_s \tau$
$(0, 0, 2n_b)$	$(-2M_{2n_b}, 0_{M2}, 4M_{n_b})$	1	1	$-2\bar{\alpha}_s \tau$
$(0, 0, 2\bar{n}_b)$	$(0, -2M_{\bar{n}_b}, 4M_{n_b})$	1	1	$2\bar{\alpha}_s \tau$
$(0, 0, 2s_1)$	$(0, 0_M, 2M_{s1})$	1	1	$-4\bar{\alpha}_s \tau$
$(0, 0, 2s_{+t})$	$(0, 0_M, 2M_{+t})$	1	1	$-4\bar{\alpha}_s \tau$

(i,j,k)	$C_1^{(i,j,k)}$	$H_1^{(i,j,k)}$	$R(\tau)$
$(1d_{\bar{n}s}, 1d_{\bar{n}s}, 0)$	$-\bar{\alpha}_s$	$-\bar{\alpha}_s$	$-\bar{\alpha}_s \tau$
$(2\delta_{s-}, 0, 0)$	$4\bar{\alpha}_s$	$4\bar{\alpha}_s$	$4\bar{\alpha}_s \tau$

$$R^{(1)}(\tau) = \frac{\alpha_s C_F}{2\pi} \tau (2 \ln \tau - 4)$$

Conclusion & Outlook

- Formally derived operators necessary for subleading corrections to thrust
- Factorization continues to hold at $O(\tau)$
  - Automatic decoupling of sectors in the SMF + Luke picture helped in derivation
- Expanding phase space important
  - Momentum conservation expansion was necessary
- Renormalizing operators to sum $\tau \log(1/\tau)$?
- Apply to other observables

  eg) angularities, anti-kT for R = 0.6

Thank You!