

EFFECTIVE THEORY OF SELF INTERACTING **DARK MATTER**



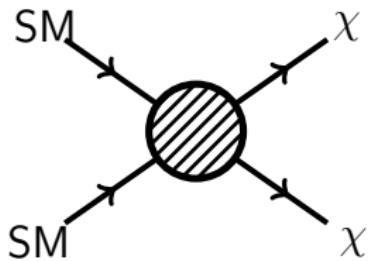
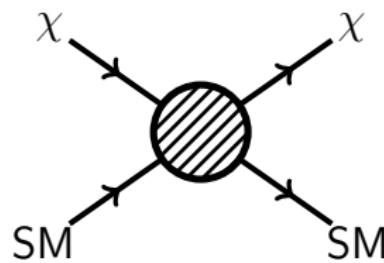
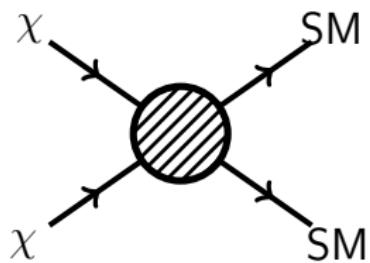
Mathieu Cliche, arXiv:1307.1129

In collaboration with Brando Bellazzini and Flip Tanedo

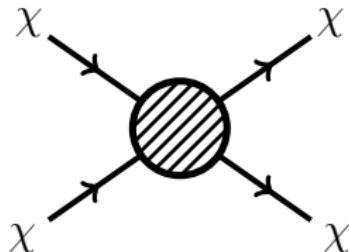
August 28, 2013

Introduction

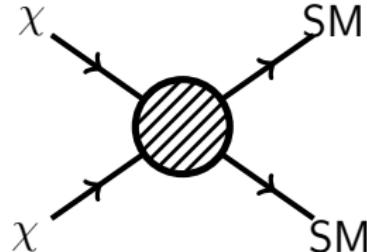
Dark matter interactions:



σ_{elast}



σ_{ID}



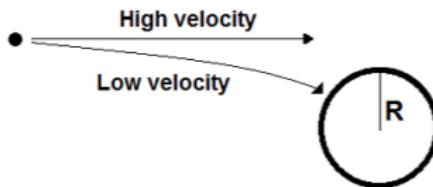
Indirect detection and elastic scattering can feel the dark matter self interaction at low velocity.

Possible consequences:

- Indirect detection: Positron excess and γ signals
- Small scale anomalies: Core vs cusp problem

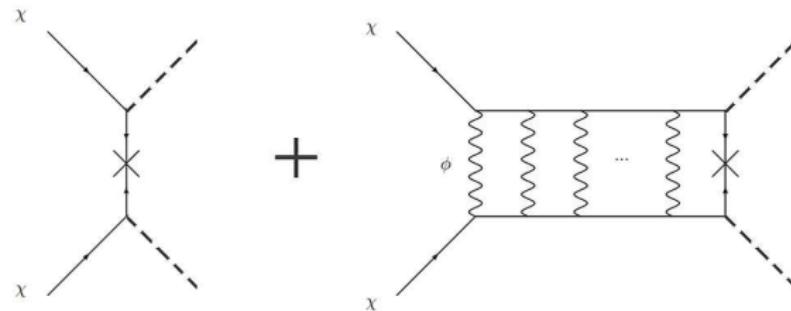
Sommerfeld enhancement: Intuition

Classical intuition:



$$\sigma = \pi R^2 \rightarrow \pi R^2 \left(1 + \frac{v_{esc}^2}{v^2} \right)$$

QFT intuition:

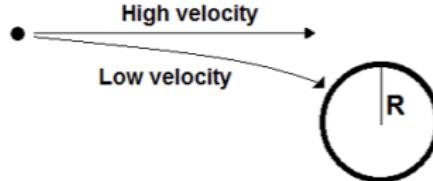


Sommerfeld enhancement: Derivation

- Annihilation interaction is **short range**: $H_{ann} = U_{ann}\delta^3(\vec{r})$.
- Solve the Schrodinger equation with the **long range** potential $V(r)$:

$$-\frac{1}{2M}\nabla^2\psi_k(r) + V(r)\psi_k(r) = \frac{k^2}{2M}\psi_k$$

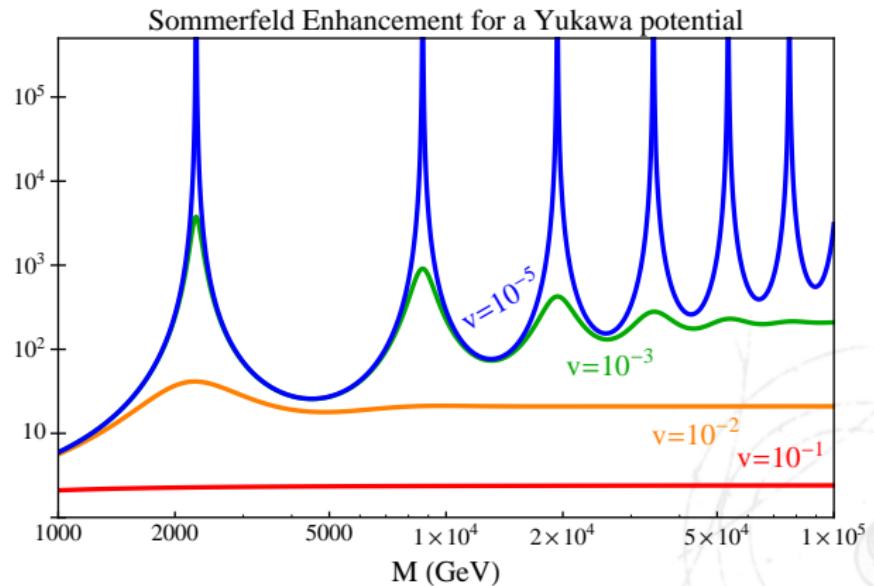
- Extract Sommerfeld enhancement S : $\sigma = |\psi_k(0)|^2\sigma_0$.



Sommerfeld enhancement: Previous results

Coulomb $\rightarrow \mathcal{L} = -e\bar{\psi}\gamma^\mu\psi A_\mu \rightarrow V(r) = -\alpha/r: S \approx 2\pi \left(\frac{\alpha}{v}\right)$.

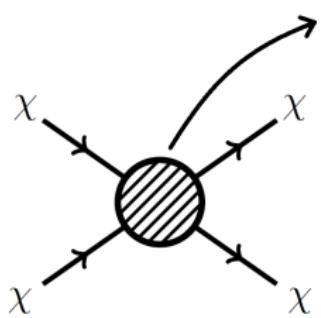
Yukawa $\rightarrow \mathcal{L} = -g\bar{\psi}\phi\psi \rightarrow V(r) = -\alpha e^{-m_\phi r}/r:$



Arkani-Hamed 0810.0713, Lattanzi 0812.0360

Effective self-interactions I

We want to be **agnostic** about the possible self interactions.



$$\int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{|\vec{q}|^2 + \mu^2}$$

$$\mathcal{M} = \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{|\vec{q}|^2 + \mu^2} \sum_i g_i \mathcal{O}_i \left(\frac{\vec{q}}{\Lambda}, \vec{s}_{1,2} \right)$$

Effective self-interactions II

List of P and T invariant QM operators

$$\mathcal{O}_1 = 1$$

$$\mathcal{O}_2 = \vec{s}_1 \cdot \vec{s}_2$$

$$\mathcal{O}_3 = -\frac{1}{\Lambda^2} (\vec{s}_1 \cdot \vec{q})(\vec{s}_2 \cdot \vec{q})$$

$$\mathcal{O}_4 = -\frac{i}{\Lambda} [(\vec{s}_1 + \vec{s}_2) \cdot (\vec{q} \times \vec{v})]$$

These terms give rise to an **effective potential**

$$V_{eff} = \frac{1}{4\pi r} \left\{ \left(\tilde{g}_1(r) - \frac{3}{4}\tilde{g}_2(r) \right) + \frac{1}{2}\tilde{g}_2(r)\vec{S}^2 + \frac{\tilde{g}_3(r)}{2\Lambda^2 r^2} \left[3(\vec{S} \cdot \hat{r})^2 - \vec{S}^2 \right] + \frac{2\tilde{g}_4(r)}{m_\chi \Lambda r^2} \vec{S} \cdot \vec{L} \right\}$$

Effective self interactions III

Example of exotic potentials:

- Tree level vector gives $V_0(r) \propto \frac{1}{r}$.
- Strong dynamics can generate $\rho(\mu^2)$.
- Weak coupling, but loop level $V_0(r) \sim \left\{ \frac{1}{r^3}, \frac{1}{r^5}, \frac{1}{r^7} \right\}$.

Example:

$$\mathcal{L} = \bar{f}\gamma^\mu(1 - \gamma^5)f\bar{\chi}\gamma_\mu(a - b\gamma^5)\chi$$



$$\rightarrow V(r) \propto \frac{1}{r^5} \left[a^2 - b^2 \left(\frac{3}{2} s_1 \cdot s_2 - \frac{5}{2} (\hat{r} \cdot s_1)(\hat{r} \cdot s_2) \right) \right]$$

Effective self interactions IV

Common tree level interactions:

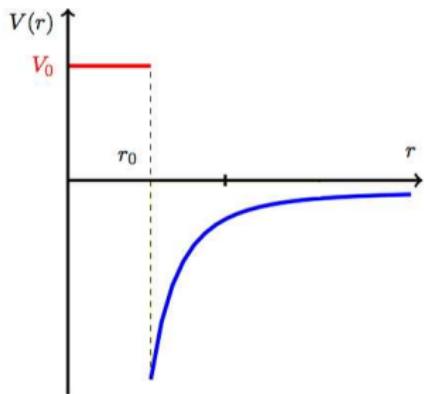
interaction	$\frac{1}{r}$	$\frac{1}{r} (\vec{s}_1 \cdot \vec{s}_2)$	$\frac{1}{r^3} [3(\vec{s}_1 \cdot \hat{r})(\vec{s}_2 \cdot \hat{r}) - \vec{s}_1 \cdot \vec{s}_2]$
$\lambda_s \bar{\chi} \chi \varphi$	$-\lambda_s^2$	0	0
$i\lambda_p \bar{\chi} \gamma^5 \chi \varphi$	0	$\frac{\lambda_p^2 m_\varphi^2}{3m_\chi^2}$	$\frac{\lambda_p^2}{m_\chi^2} h(m_\varphi, r)$
$\frac{1}{f} \bar{\chi} \gamma^\mu \gamma^5 \chi \partial_\mu \varphi$	0	$\frac{4m_\varphi^2}{3f^2}$	$\frac{4}{f^2} h(m_\varphi, r)$
$\lambda_v \bar{\chi} \gamma^\mu \chi A_\mu$	$\pm \lambda_v^2 \left(1 + \frac{m_A^2}{4m_\chi^2}\right)$	$\pm \frac{2\lambda_v^2 m_A^2}{3m_\chi^2}$	$\mp \frac{\lambda_v^2}{m_\chi^2} h(m_A, r)$
$\lambda_a \bar{\chi} \gamma^5 \gamma^\mu \chi A_\mu$	0	$-\frac{8\lambda_a^2}{3} \left(1 - \frac{m_A^2}{8m_\chi^2}\right)$	$\lambda_a^2 \left(\frac{1}{m_\chi^2} + \frac{4}{m_A^2}\right) h(m_A, r)$
$\frac{i}{2\Lambda} \bar{\chi} \sigma^{\mu\nu} \chi F_{\mu\nu}$	0	$\mp \frac{2m_A^2}{3\Lambda^2}$	$\pm \frac{1}{\Lambda^2} h(m_A, r)$

$$h(m_\phi, r) \equiv \left(1 + m_\phi r + \frac{m_\phi^2 r^2}{3}\right)$$

Singular potentials I

To deal with singular potentials we apply the following recipe:

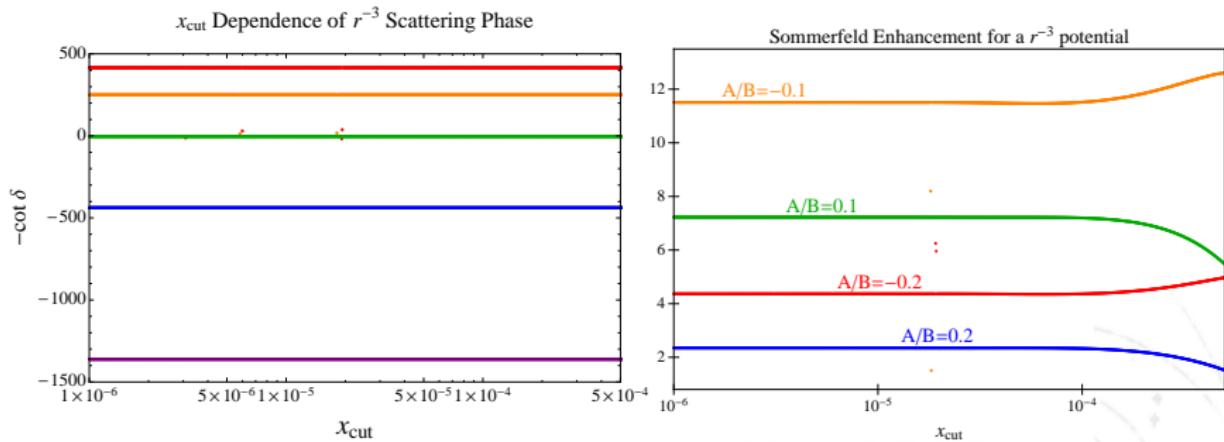
1. **Regularize** the potential.



2. Introduce **wavefunction renormalization**: $\psi_{\text{ren}} = Z_\psi \psi_{\text{bare}}$
3. **Renormalize** by matching with IR observables.

Singular potentials II

Cutoff independence with $V(r) = -\frac{\alpha}{f^2 r^3}$.

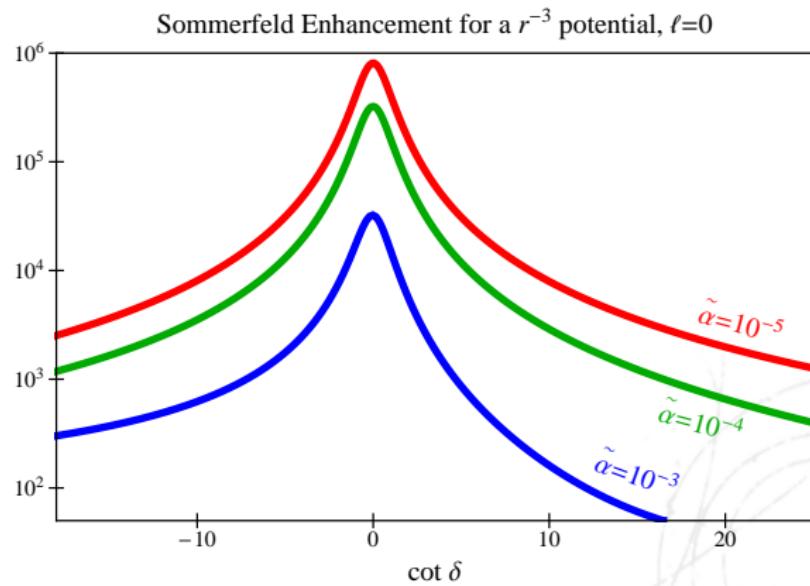


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Numerical results I

We can use this recipe to plot Sommerfeld enhancement for

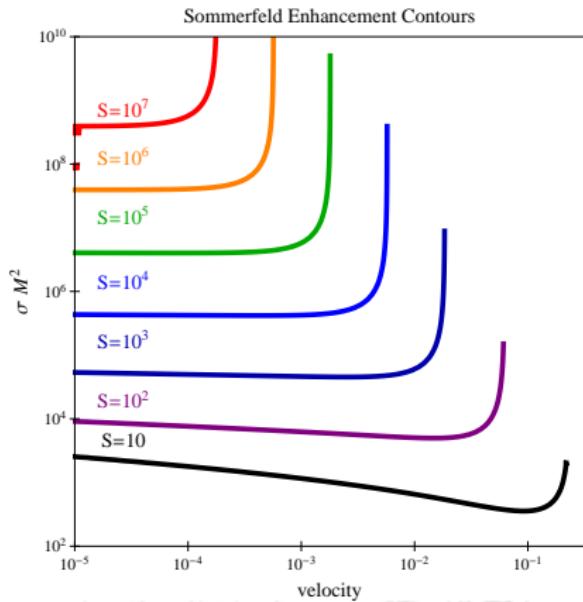
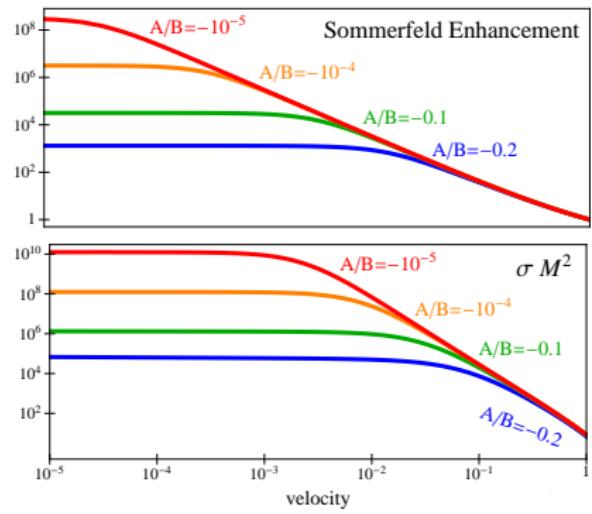
$$V(r) = \frac{-\alpha}{f^2 r^3}$$
 as a function of $\cot(\delta)$:



Recall $\sigma \approx \frac{4\pi}{k^2 \cot^2(\delta)}$

Numerical results II

The velocity dependence is important for phenomenology:

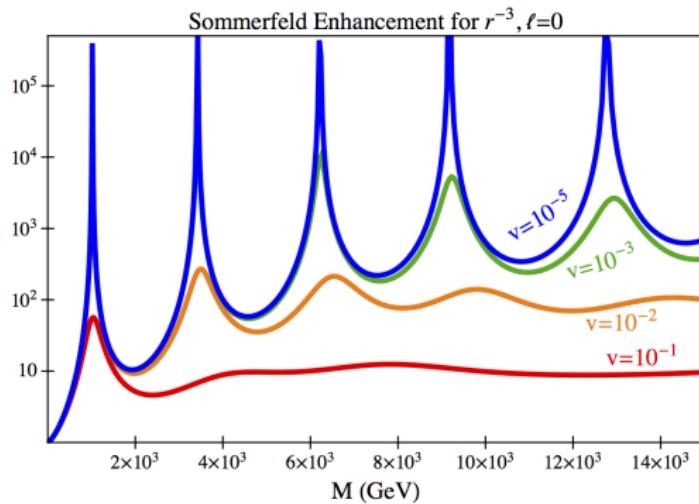


NDA approximations

Problem: $\cot(\delta)$ depends on the UV.

Solution: A NDA type of analysis.

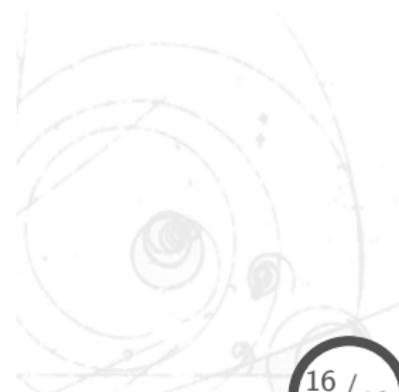
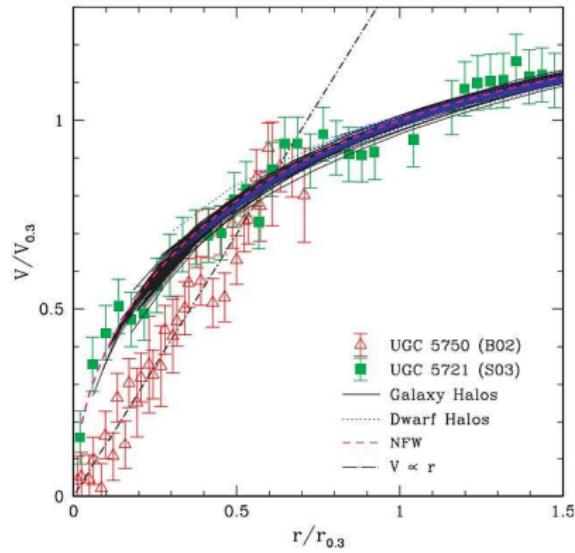
1. Physical cutoff: Relativistic $V(r_0) = M$
2. Fix V_0 using $V_0 = V(r_0)$.



Small Scale Anomalies I

Core vs Cusp Problem: Disagreement for the dark matter distribution in the center of dwarf galaxies.

- **N body simulations:** CDM gives a cusp like behavior $\rho(r) \sim 1/r$.
- **Observations:** Core like behavior $\rho(r) \sim r^0$.



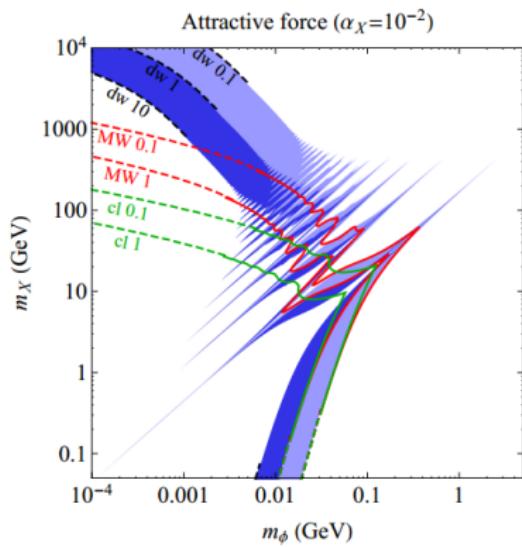
Small Scale Anomalies II

Solution: Dark matter self interaction.

- DM is collisional in dwarfs ($v \sim 10^{-5}$) with $\sigma_T/M_{DM} \in [0.1, 10] \text{ cm}^2/\text{g}$.

For $V(r) = -\alpha_X \frac{e^{-m_\phi r}}{r}$:

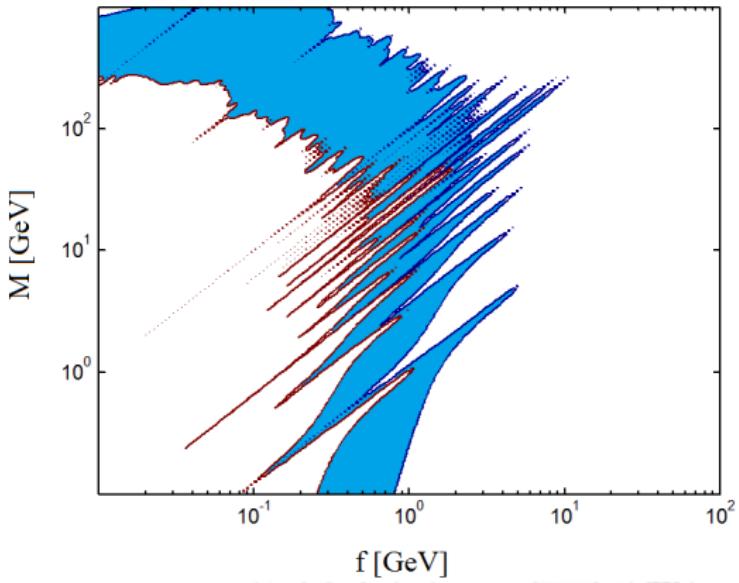
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Small Scale Anomalies III

Preliminary results:

For $V(r) = -\frac{\alpha}{f^2 r^3}$:



Conclusion

To sum up:

- We provided **the parametrization of self interacting dark matter** in the non-relativistic regime.
- We showed how to **renormalize singular QM potentials**.
- We began an analysis of the parameter space that **solves small scale anomalies**.

Future direction:

- Parameter space analysis for **indirect detection**.