Black hole horizons and quantum information
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High-energy gravitational scattering and black-hole quantum hair

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Outline

- The string-black hole correspondence: stringholes
- Stringhole production in HE gravitational scattering
- Probing stringhole's quantum hair
- Conclusions

The String-BH correspondence

Entropy of free string states

of physical string states @ vanishing string coupling

$$d(N) = N^{-p} e^{2\pi\sqrt{\frac{CN}{6}}} = M^{-2p} e^{2\pi\sqrt{\frac{\alpha' C}{6\hbar}M}}$$

Up to numerical factors this gives, at large M,

$$S_{st} \sim \frac{\alpha' M}{l_s} = \frac{M}{M_s} \ ; \ l_s = \sqrt{2\alpha'\hbar} \ ; \ M_s = \sqrt{\frac{\hbar}{2\alpha'}}$$

Possible interpretation of S_{st}: number of "string bits" contained in the total length of the string, $L = \alpha' M$.

Also? max. # of massive strings into which the highly excited one can decay.

Semiclassical BH entropy

Bekenstein-Hawking formula for arbitrary D

$$S_{BH} = \frac{\mathcal{A}}{4 l_D^{D-2}} \;\; ; \;\; \mathcal{A} \sim R_S^{D-2} \sim (G_D M)^{\frac{D-2}{D-3}} \Rightarrow S_{BH} = \frac{M R_S}{\hbar}$$

$$l_D^{D-2} = G_D \hbar$$

can be compared with previous

$$S_{st} \sim \frac{\alpha' M}{l_s} = \frac{M}{M_s} \;;\; l_s = \sqrt{2\alpha'\hbar} \;;\; M_s = \sqrt{\frac{\hbar}{2\alpha'}}$$

The two entropies look very different but can we trust both results everywhere in parameter space?

Let's assume for the moment that we can.

The correspondence curve

S_{BH} grows faster than S_{st} but latter starts higher at small M. Hence, the two entropies must meet at some finite value of M:

$$\frac{S_{BH}}{S_{st}} = \frac{M R_S/\hbar}{M/M_s} = \frac{M_s R_S}{\hbar} = \frac{R_S}{l_s}$$

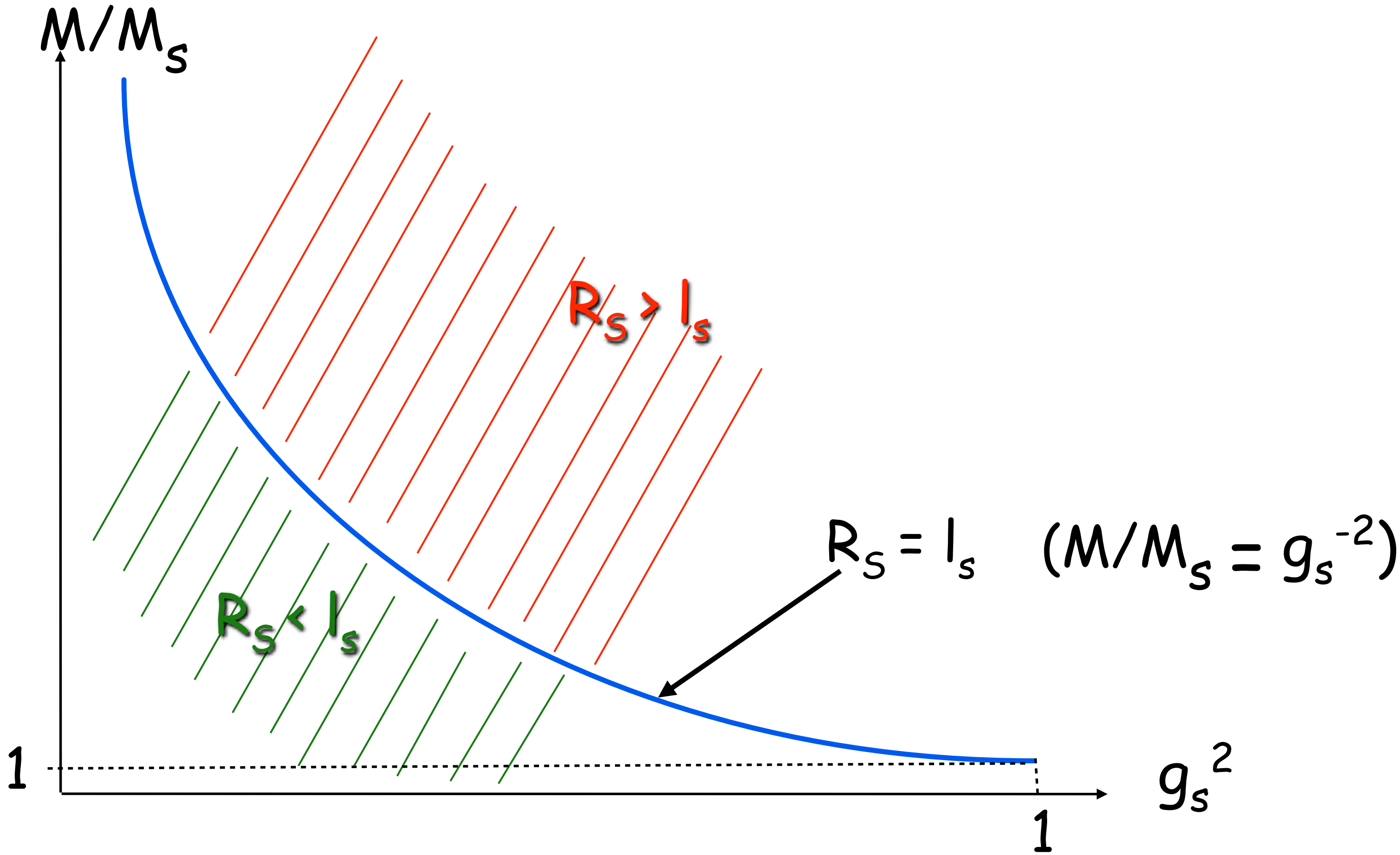
S_{BH} wins over S_{st} for $R > l_s$, the opposite is true for $R < l_s$. They coincide at $R = l_s$ (where $T_{BH} \sim T_{Hag}$) and take the value:

$$S_{BH} = S_{st} = \frac{l_s^{D-2}}{l_D^{D-2}} = g_s^{-2} \gg 1 \Rightarrow M = M_* \equiv g_s^{-2} M_s$$

NB: at very small string coupling $M_* \gg M_P \gg M_s$

$S_{BH} = S_{st}$ defines a hyperbola in the (g_s, M) plane called the correspondence curve.

The correspondence curve



Below the correspondence curve (CC)

Life is easy sincecorrections to the zero-coupling entropy can be argued to be parametrically small.

The Schwarzschild radius of the string is smaller than the string length scale.

The latter is believed to be the minimal size of any string.

Hence such strings are simply NOT BHs.

Old claim (GV '86): in QST there are no BHs whose $R_S < l_s$, i.e. whose Hawking temperature is higher than M_s

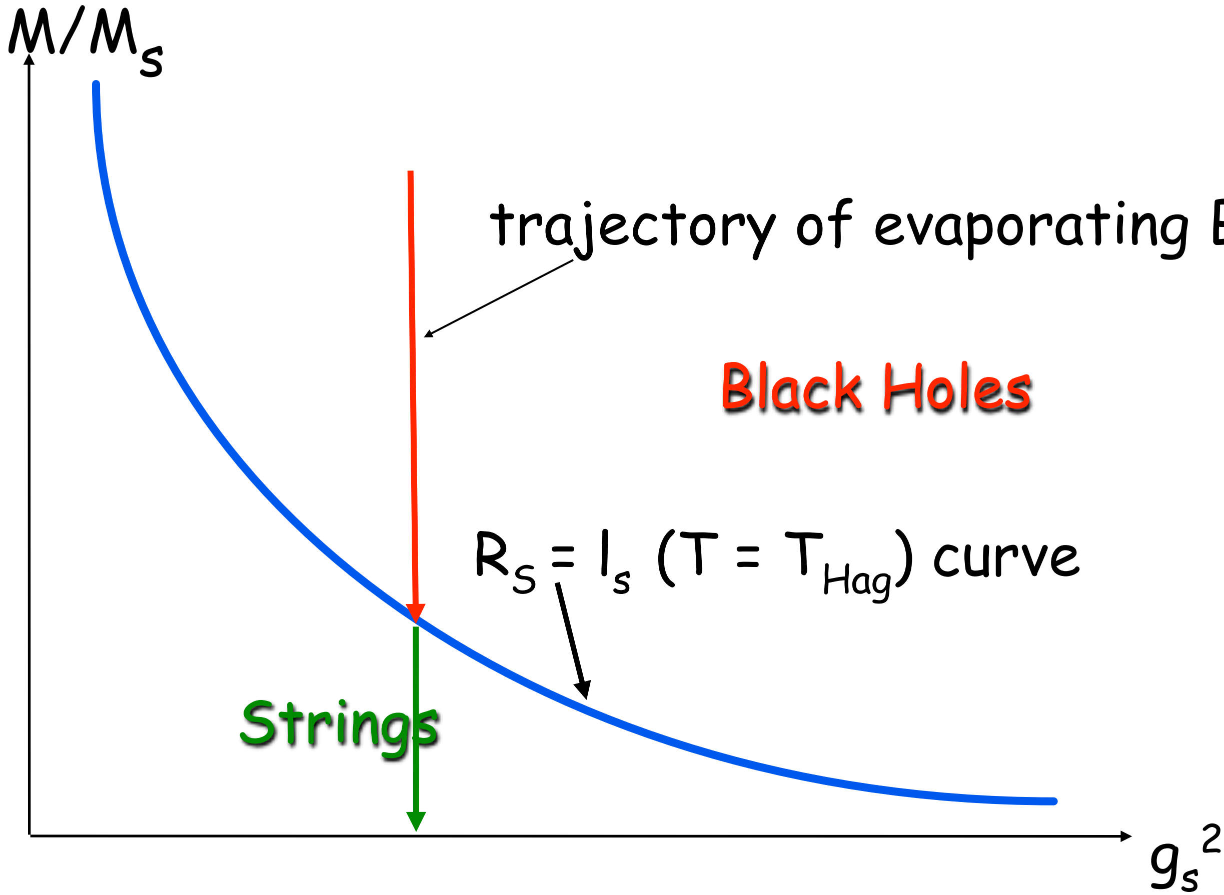
(NB: $T = M_s$ is believed ST's maximal temperature)

So far, everything looks consistent!

Also solves the problem of end-point of evaporation!

Evaporation of a BH at fixed g_s (Bowick et al. 1987)

Singularity at the end of evaporation avoided?



Approaching the correspondence curve: the random-walk puzzle

If we want to identify BH with FS above the CC, their properties should match as we approach the curve.

By definition the two entropies do match (up to O(1) factors) but there is still a "random-walk puzzle".

S_{st} can be understood in terms of a "random walk" but then a string on the CC being much longer (heavier) than l_s (M_s), will have a typical size much bigger than its Schwarzschild radius l_s.

But then it has nothing to do with a BH!

Size distribution of free strings

The resolution of the RW puzzle is quite simple.

One has to compute the distribution of sizes for a given M

(NB: M fixes length not size!).

This was done by T. Damour & GV (2000). The entropy of strings of given M and size R is given by

(c_1, c_2 are positive # O(1), calculation reliable for R > R_S):

$$S(M,R) \equiv \log d(M,R) = a_0 \frac{M}{M_s} f\left(\frac{R}{l_s}, \frac{\alpha' M}{l_s}\right);$$

$$a_0 = 2\pi\sqrt{\frac{D-2}{6}}\ ;\ f\left(\frac{R}{l_s}, \frac{\alpha' M}{l_s}\right) = \left(1 - \frac{c_1 l_s^2}{R^2}\right)\left(1 - \frac{c_2 R^2}{(\alpha' M)^2}\right)$$

Entropy is maximized for: $\frac{R}{l_s} \sim \sqrt{\frac{M}{M_s}} = \text{random walk value}$

But there is still an S of order M/M_s in strings of size $O(l_s)$!

We shall call such strings lying on the CC "stringholes"

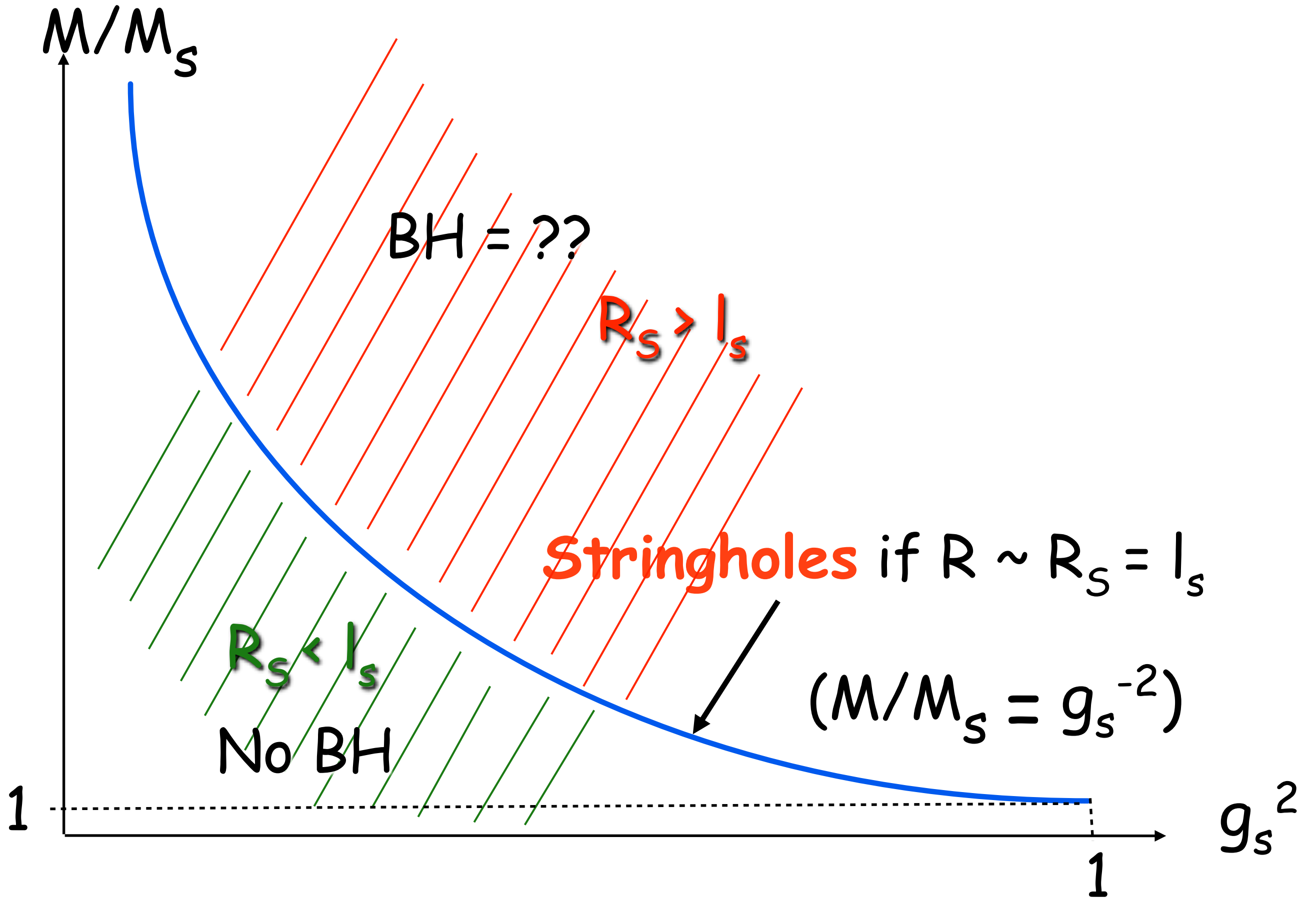
Stringholes can also be understood as string states in which only oscillators with $n > N^{1/2}$ are excited.
It is easy to compute the asymptotic behavior of such a restricted partition function and to find that it also gives an exponential degeneracy though with a smaller coefficient in the exponent.

$$P(z,K) = \prod_{k=K}^{\infty} \left(\frac{1}{1-z^k} \right)^{D-2} = \sum_N d(N,K) z^N$$

$$d\left(N, K \le \sqrt{N}\right) \sim e^{c_K \sqrt{N}} \quad ; \quad c_K = O(1)$$

$$\langle R^2 \rangle = l_s^2 \sum_{n > \sqrt{N}} \frac{1}{n} \langle a_n^\dagger a_n \rangle = O(l_s^2)$$

Stringholes



Above the correspondence curve

It is reassuring that the string-coupling corrections become O(1) just when we can reproduce BH properties up to factors O(1).

As we go farther and farther above the CC the discrepancy between free-string and BH entropy becomes larger and larger but, fortunately, also the corrections get out of hand.

In order to see whether we can have agreement there we would have to compute the effect of interactions when they become non-perturbative.

This is a hard & unsolved problem.

We shall try to get some hints below...

Transplanckian-energy strings collisions:

(ACV'87---'07+ many others)

A nice theoretical laboratory for studying deep questions about quantum string gravity.

We can hardly imagine a simpler pure initial state that could lead to BH formation and whose unitary evolution we would like to understand/follow.

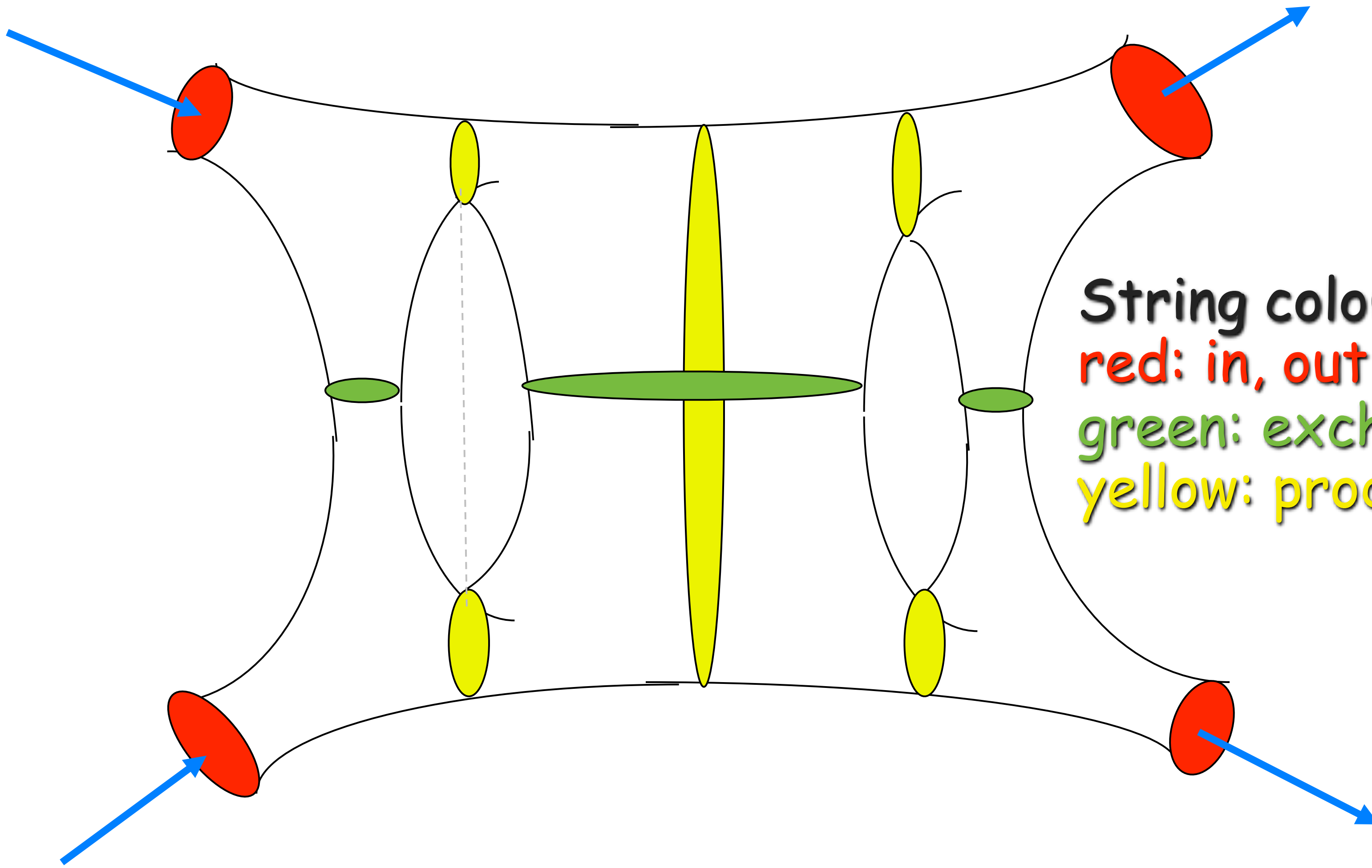
Calculations performed in flat spacetime & D =10.

An effective metric emerges at the end.

Recently extended (DDRV 2010 + ..) to HE string-brane collisions.

No time to review the subject.

TPE (closed)string-string collisions
(a two-loop contribution)



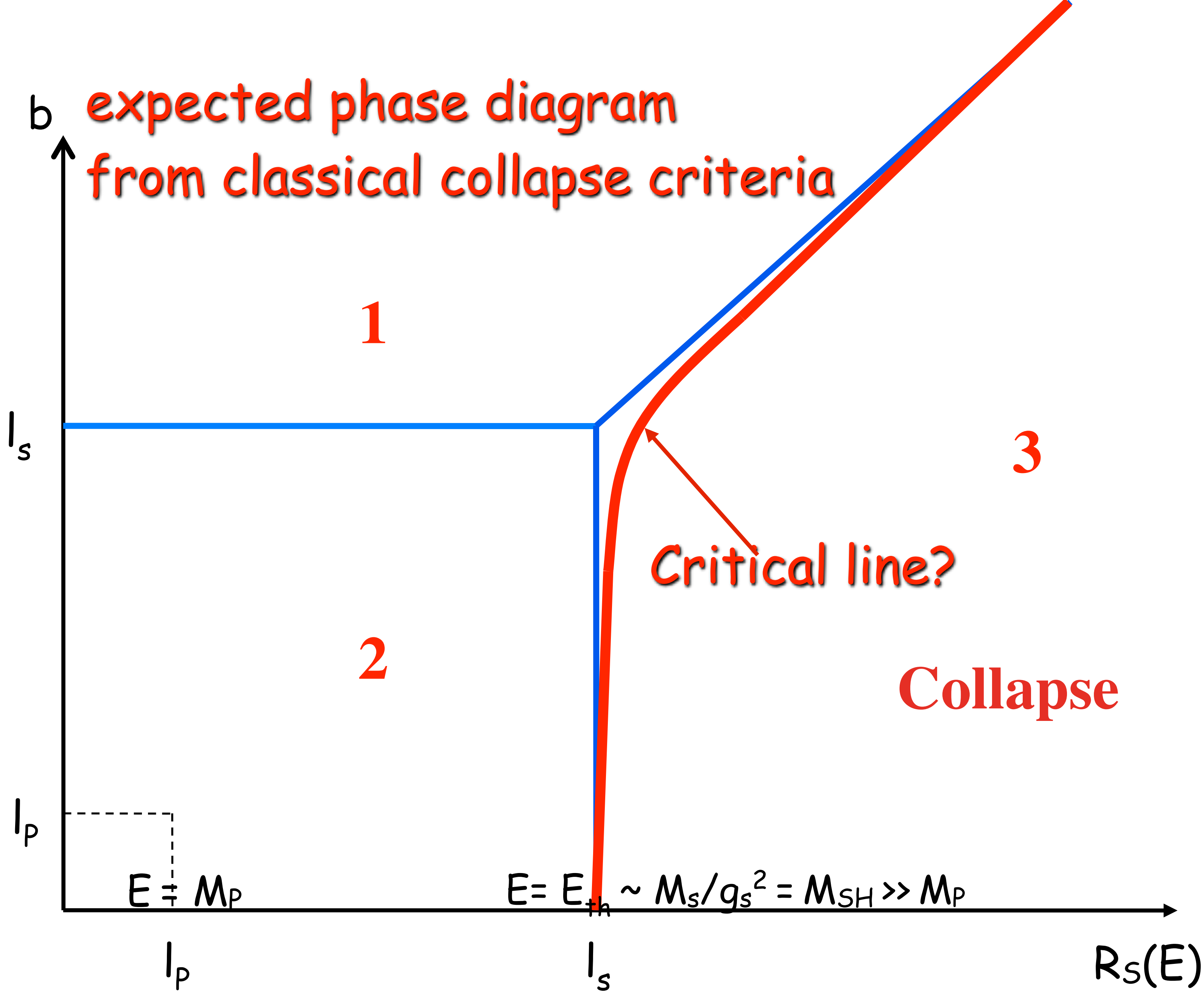
String colour code:
red: in, out
green: exchanged
yellow: produced

Parameter-space for high-energy string-string collisions

3 relevant length scales (neglecting l_P @ g_s << 1)

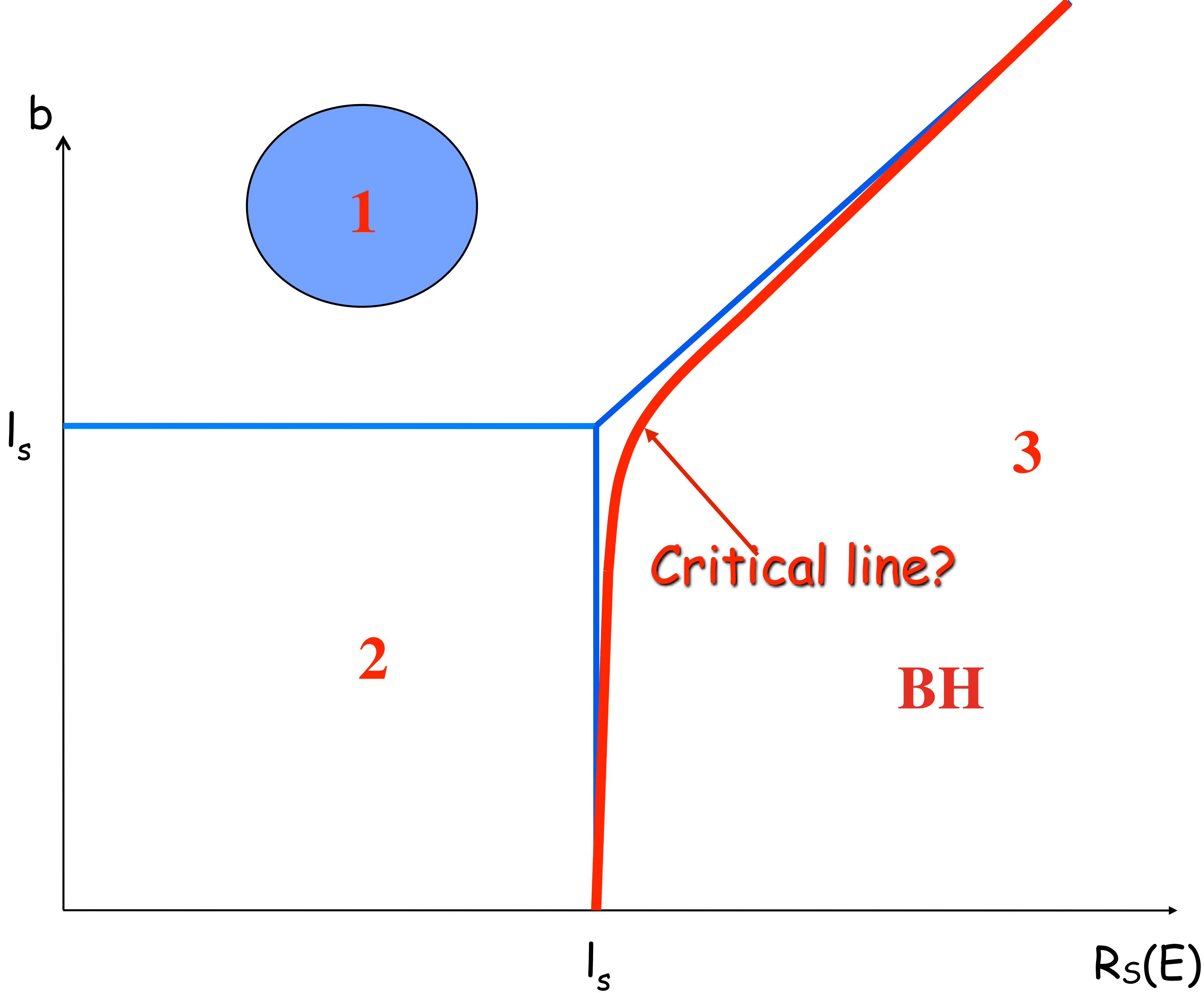
$$b \sim \frac{2J}{\sqrt{s}} \;\; ; \;\; R_S \sim \left(G\sqrt{s}\right)^{\frac{1}{D-3}} \;\; ; \;\; l_s \sim \sqrt{\alpha' \hbar} \;\; ; \;\; G\hbar = l_P^{D-2} \sim g_s^2 l_s^{D-2}$$

NB: Playing with s and g_s we can make R_S/l_s arbitrary



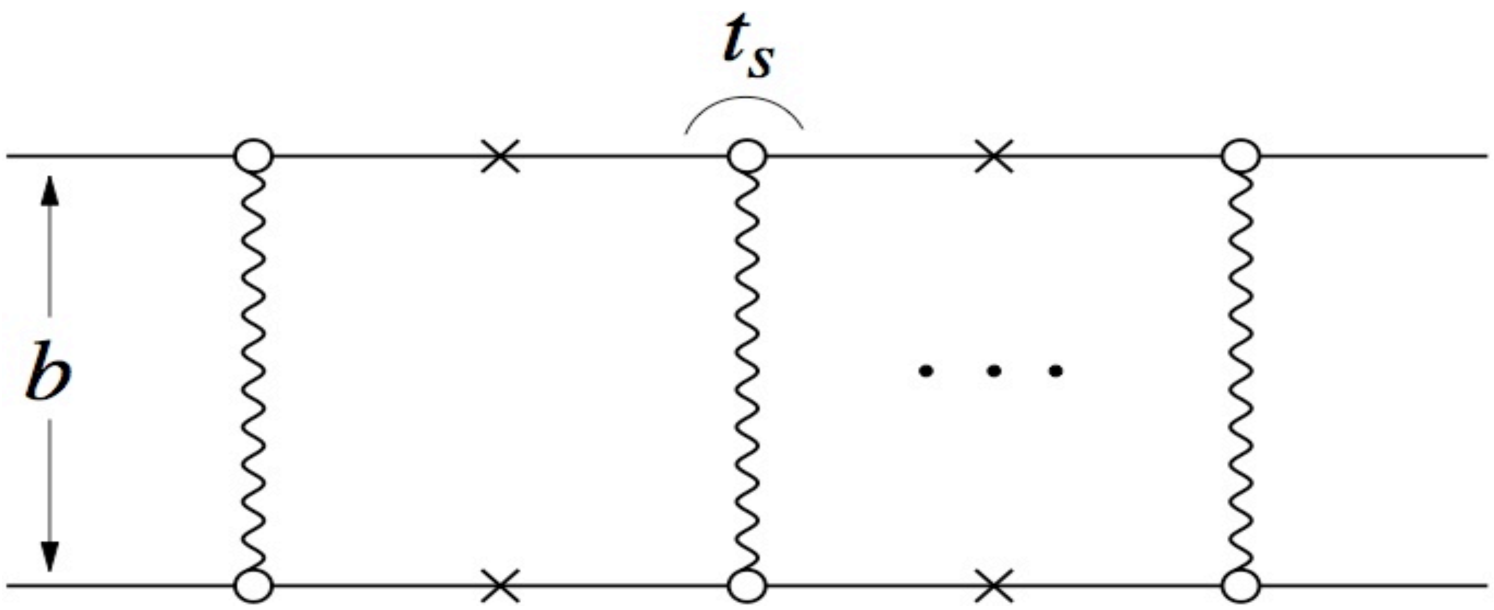
expected phase diagram
from classical collapse criteria
b
l_s
l_P
1
2
3
Critical line?
Collapse
E = M_P
E= E_{th} ~ M_s/g_s^2 = M_{SH} >> M_P
l_P
l_s
$R_S(E)$

The weak-gravity regime


b
1
l_s
Critical line?
2
3
BH
l_s
$R_S(E)$

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D} \left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \dots\right)$$

Leading eikonal diagrams (crossed ladders included)



PWU-bounds restored by resummation

Point-particle limit @ large b

$$S(E,b) \sim exp\left(i\frac{Gs}{\hbar}c_D b^{4-D}\right) \ ; \ S(E,q) = \int d^{D-2}b \ e^{-iqb} S(E,b) \ \ ; \ \ s = 4E^2 \ , \ q \sim \theta E$$

The integral is dominated by a saddle point at:

$$b_s^{D-3} \sim \frac{G\sqrt{s}}{\theta} \ ; \ \theta \sim \left(\frac{R_S}{b}\right)^{D-3} \ ; \ R_S^{D-3} \sim G\sqrt{s}$$

Generalization of Einstein's deflection formula to ultra-relativistic collisions and arbitrary D. It corresponds precisely to the relation between b and θ in the metric generated by a relativistic point-particle of energy E. This is an effective metric, NOT a class. one!

- At fixed θ, larger E probe larger b (i.e. the IR). How come?
- (Gs/h) b^{4-D} gives the average loop-number. The total q = θ E is shared among as many exchanged gravitons so that:

$$q_{ind} \sim \frac{\hbar q}{G \ s \ b^{4-D}} \sim \frac{\hbar \ \theta \ b^{D-4}}{R^{D-3}} \sim \frac{\hbar}{b_s}$$

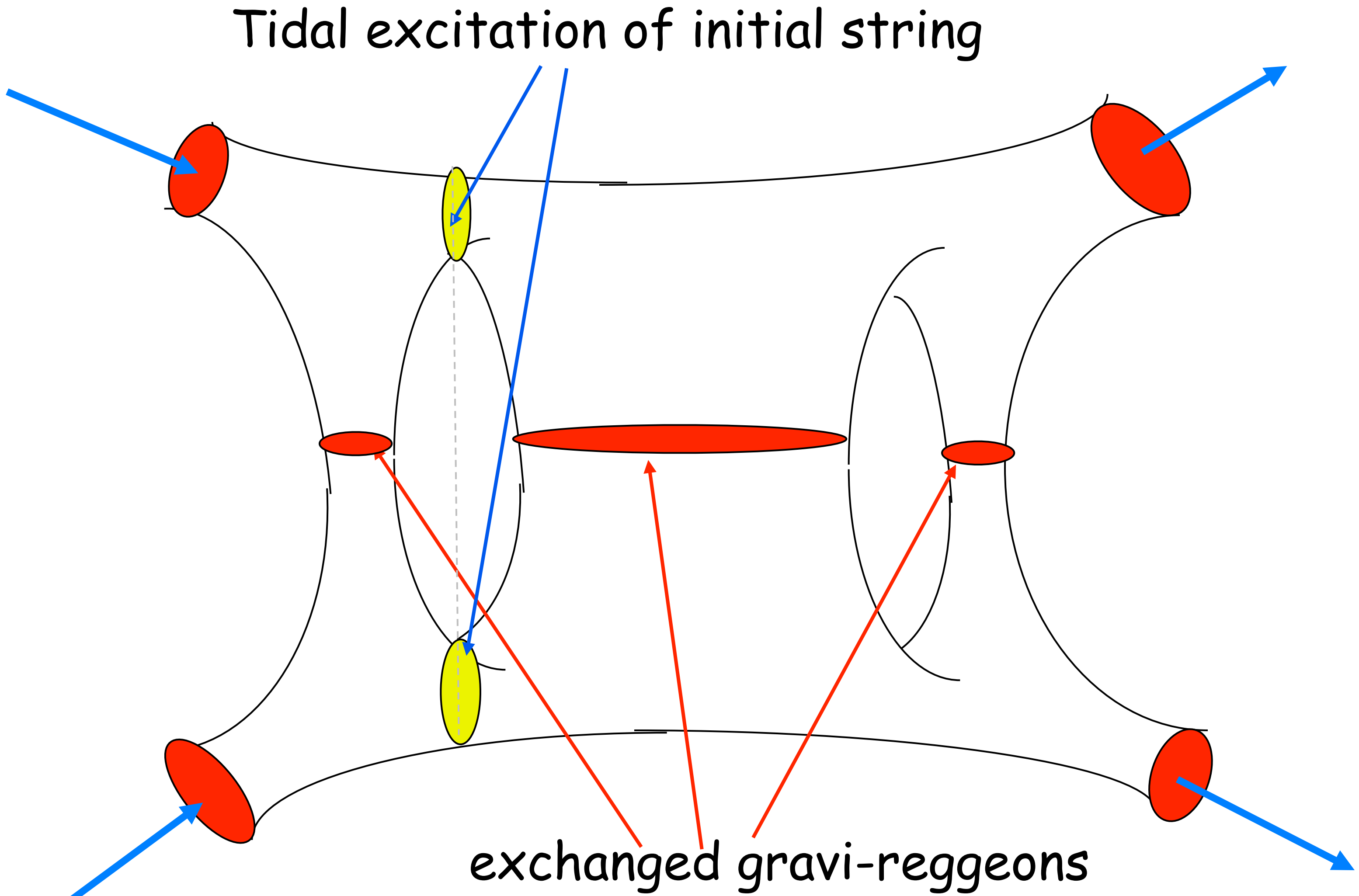
String-string scattering @ large b

(new effects because of imaginary part)

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D}\left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \ldots\right)$$

Graviton exchanges can excite one or both strings. Reason (Giddings '06): a string moving in a non-trivial metric feels tidal forces as a result of its finite size. A simple argument gives the critical impact parameter b_t below which the phenomenon kicks-in (as found by direct calculation by ACV). It is parametrically larger than l_s.

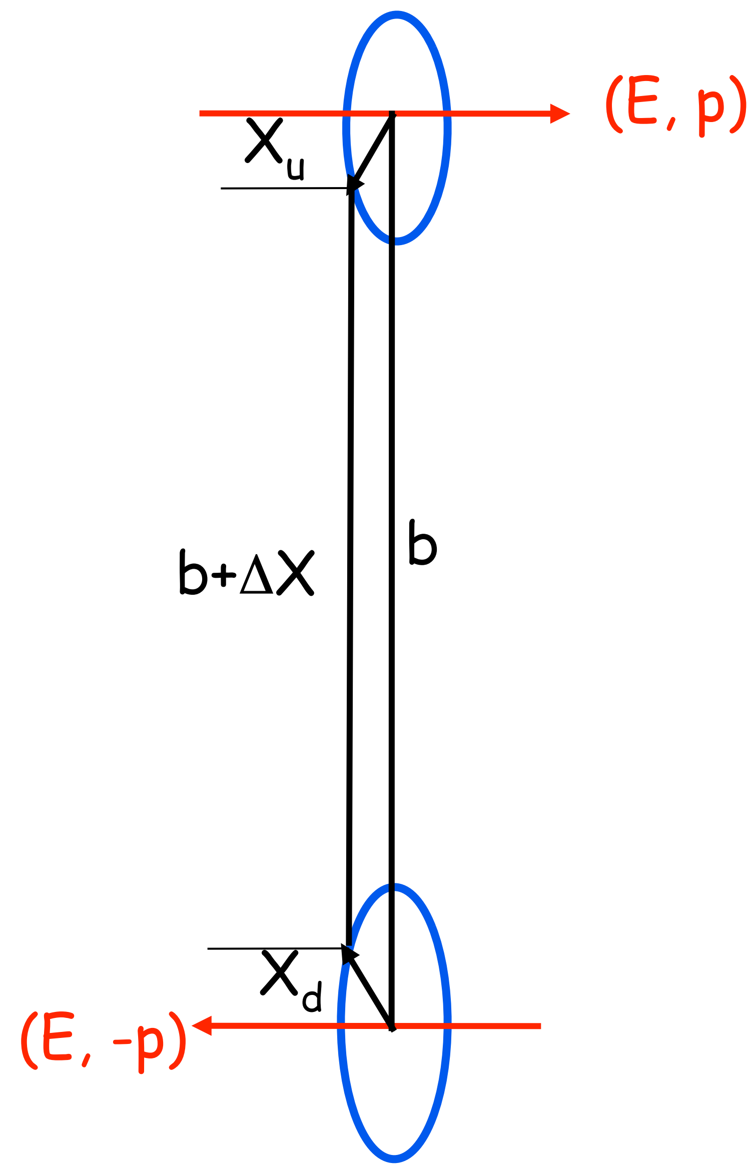
$$b_t \sim \left(\frac{Gsl_s^2}{\hbar}\right)^{\frac{1}{D-2}}$$


Tidal excitation of initial string
exchanged gravi-reggeons

These effects are neatly captured, at the leading eikonal level, by replacing the impact parameter **b** by a shifted impact parameter, displayed by each string's position operator (stripped of its zero modes) evaluated at $\tau = 0$ (= collision time) and averaged over σ.

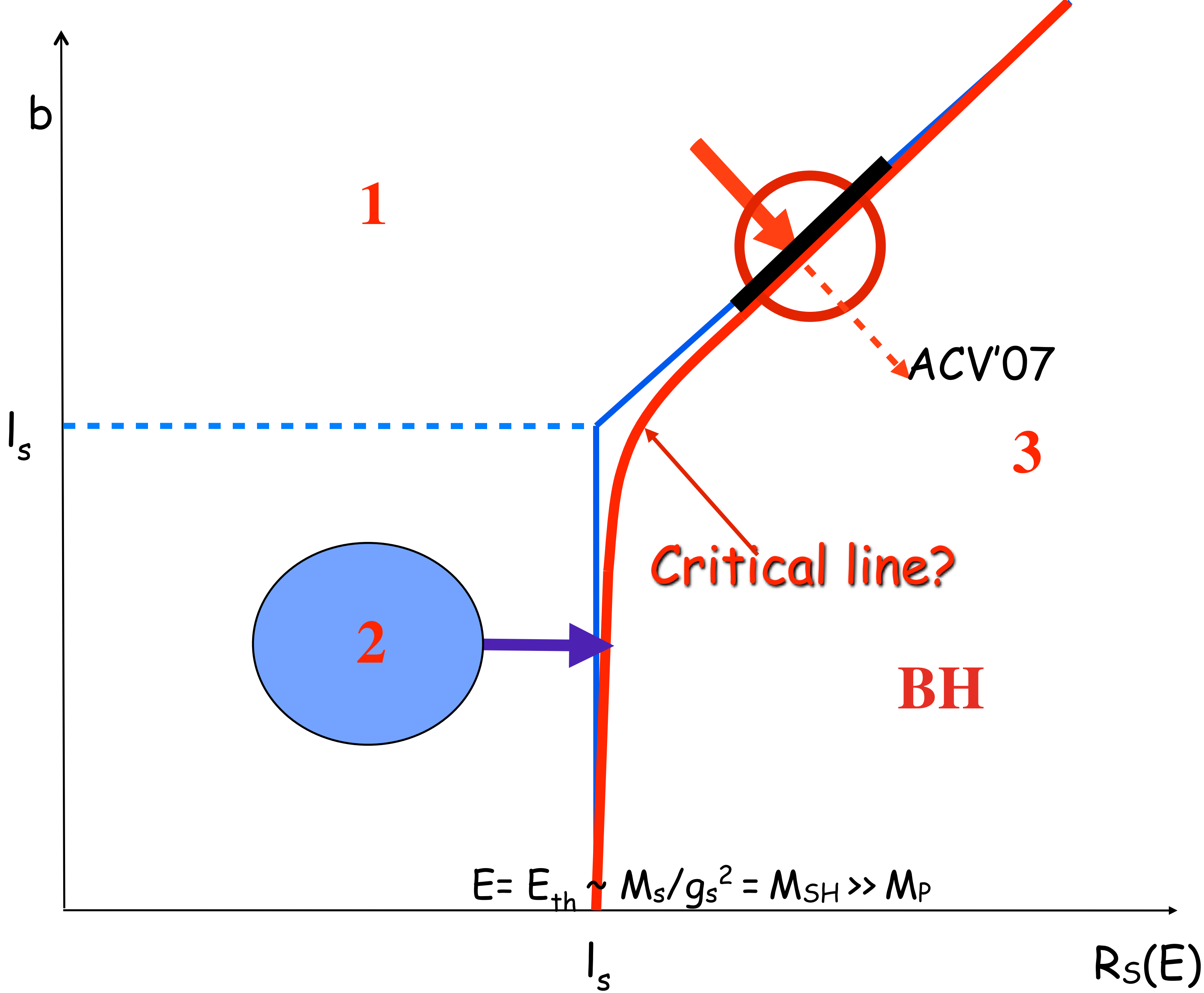
This leads to a unitary operator eikonal formula for the S-matrix

More details later...



The string-gravity regime: approaching stringhole production

(GV: 0410.166 and references therein)



b
1
ACV'07
l_s
3
Critical line?
2
BH
$E = E_{th} \sim M_s/g_s^2 = M_{SH} >> M_P$
l_s
$R_S(E)$

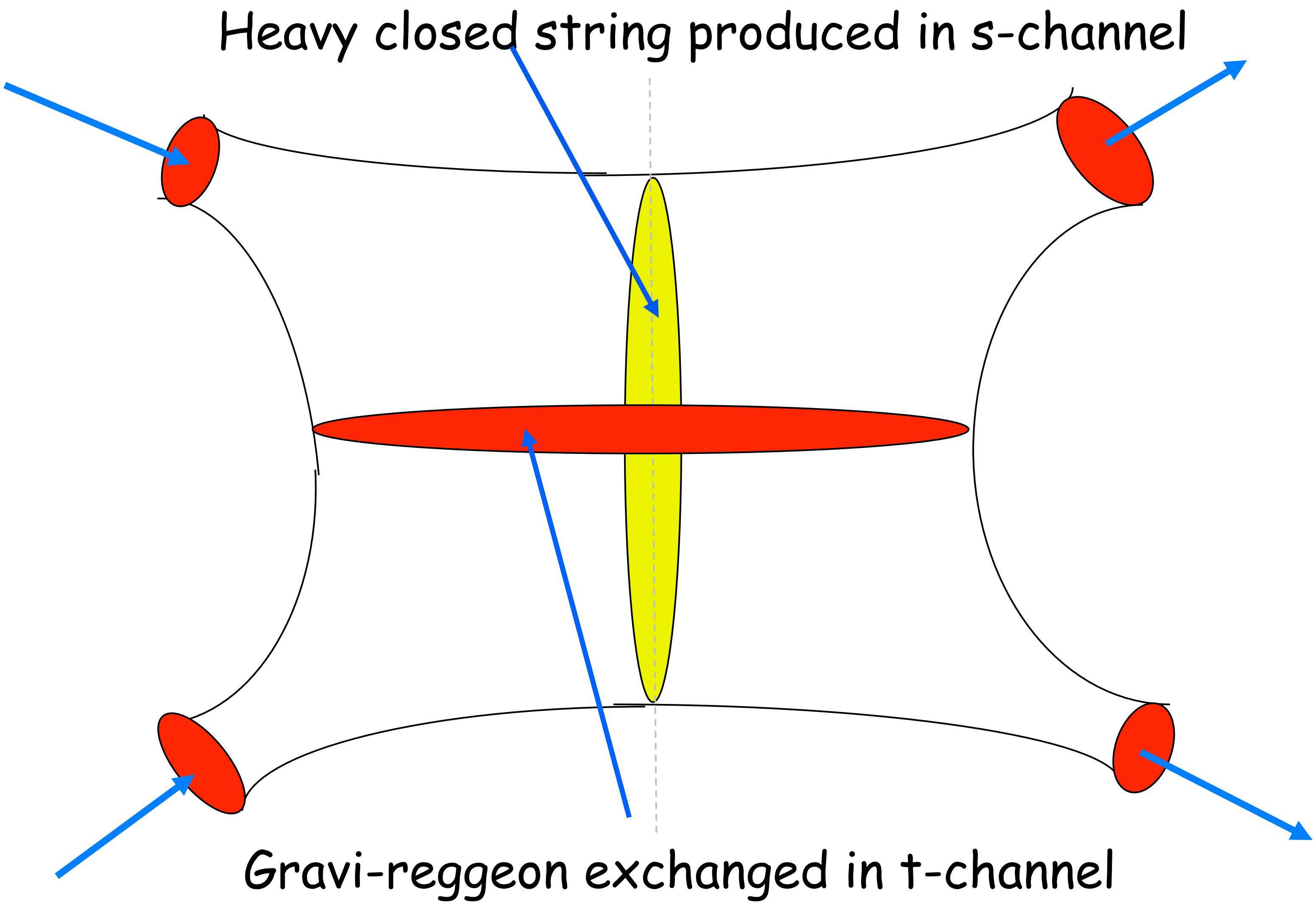
String-string scattering @ b, $R_S < l_s$

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D} \left(1 + \cancel{O((R/b)^{2(D-3)})} + O(l_s^2/b^2) + \cancel{O((l_P/b)^{D-2})} + \dots\right)$$

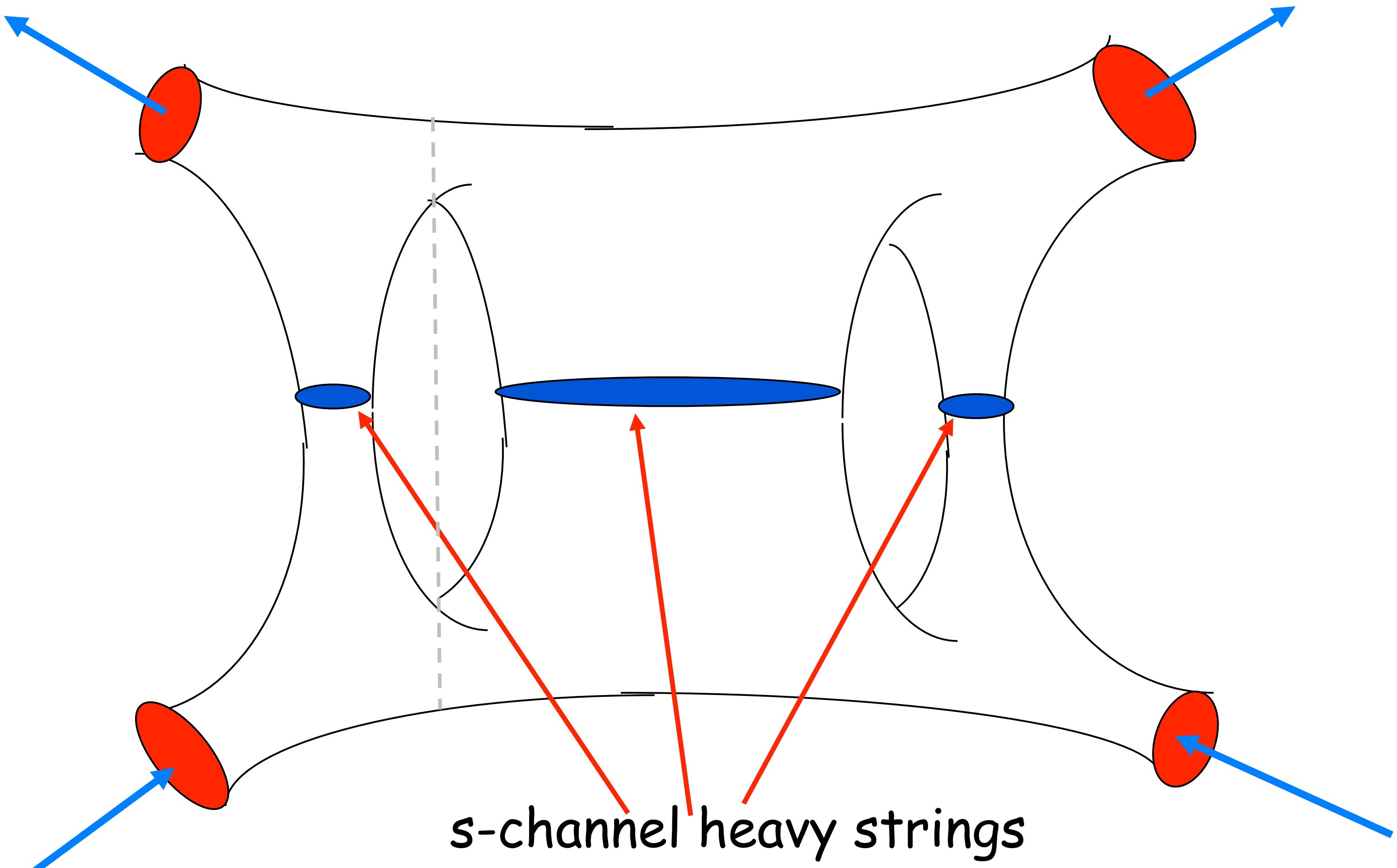
Because of (good old DHS) duality even single graviton exchange does not give a real scattering amplitude. The imaginary part is due to formation of closed-strings in the s-channel.

It is exponentially small at large impact parameter (hence irrelevant in region 1, important in region 2)

Im A is due to closed strings in s-channel (DHS duality, '67)



At higher loop order many strings produced in s-channel
Average number given by imaginary part of the phase shift



Turning the previous diagram by 90°

$$\mathrm{Im} A_{cl}(E,b) \sim \frac{G\ s\ l_s^{4-D}}{\hbar} \exp\left(-\frac{b^2}{l_s^2\ \log s}\right)$$

At impact parameters below the string scale one starts producing more and more strings. Their average number grows like $Gs \sim E^2$ (Cf. # of exchanged strings) so that, above $E = M_s/g$, the average energy of each final string starts decreasing as the incoming energy grows

$$\langle E_{final} \rangle \sim \frac{M_s^2}{g^2\sqrt{s}} \to M_s \text{ at } \sqrt{s} = E_{th} \ \text{with} \ \langle n \rangle \to g_s^{-2} \sim S_{SH}$$

Similar to what we expect in BH physics!

Fast growth of <n> & consequent softening: an interesting signature even below the actual threshold of BH production?

If extrapolated to $R_S > l_s$ this gives only massless string modes (Hawking radiation?). Can it be trusted?

A hint on the nature of BHs in String Theory?

If extrapolation to $R_S > l_s$ can be qualitatively trusted it would indicate that above the correspondence line it becomes entropically preferable to break up the heavy string/black hole into its massless decay products.

Can these form a gravitationally bound system (a geon?)

As argued by Dvali and Gomez the number of massless quanta ("gravitons") whose energies add up to the total mass M, and which can bind gravitationally in a region of size R_S, is of order $M R_S/h$, i.e. of order S_{BH}.

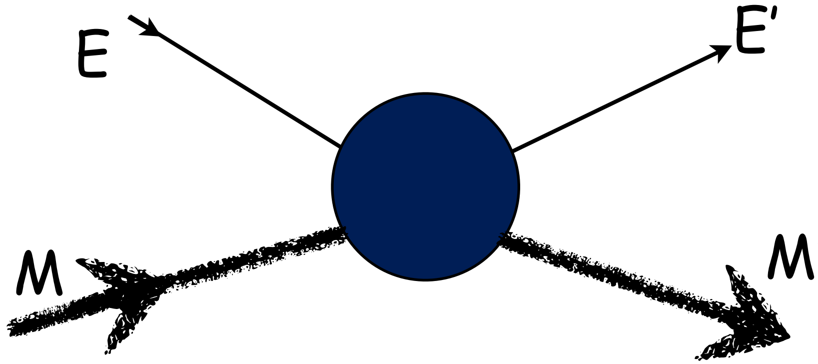
Our results appear to lend some credibility to their picture (not necessarily in its details).

Stringholes are hippies!

(GV: 1212.2606)

She asks me why *I'm just a hairy guy I'm* hairy noon and night *Hair* that's a fright *I'm* hairy high and low *Don't ask me why*, don't know ...

Scattering of a massless string on a heavy one



kinematical region:

$$M_s M \ll s - M^2 = -2p \cdot P = 2EM \ll M^2$$

Light string acting as a hair-detecting probe

Leading eikonal generalizing ACV and DDRV (R. Russo private comm.)

$$S(E,M,b) \sim \exp(i\frac{\mathcal{A}_{cl}}{\hbar}) = \exp\left(i\frac{4GEM}{\hbar}c_D b^{4-D}\right) \equiv e^{2i\delta(E,M,b)}$$

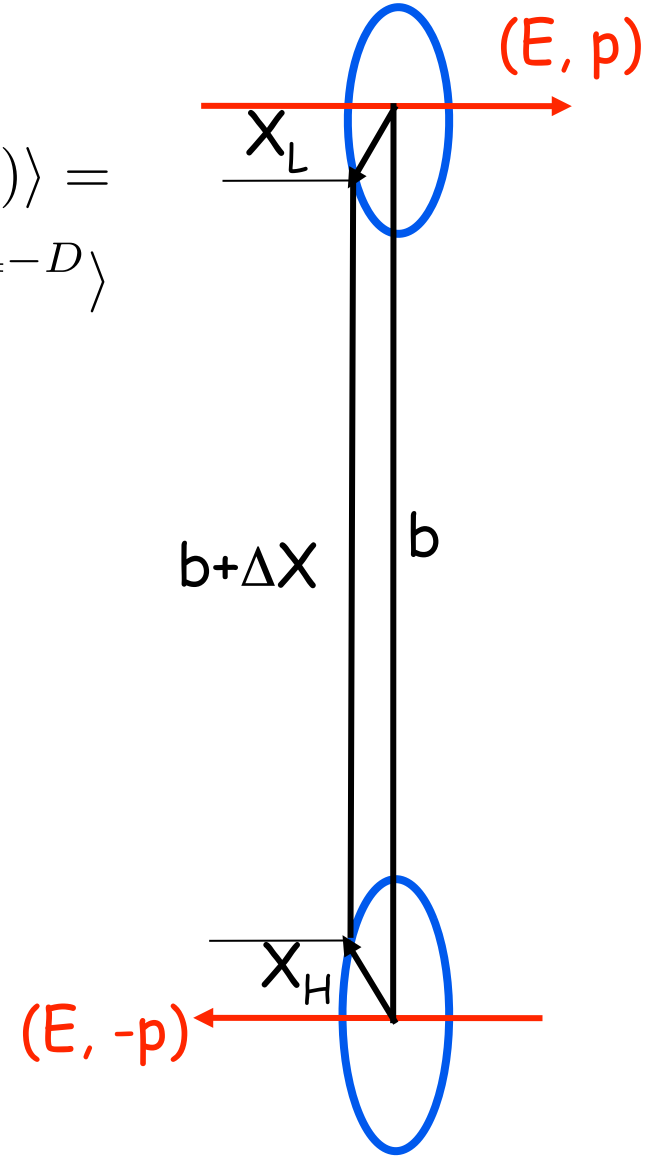
$$c_D = \Omega_{D-4}^{-1} \equiv \frac{\Gamma(\frac{D-4}{2})}{2\pi^{\frac{D-4}{2}}}$$

Check of deflection angle @ saddle point

$$\theta = \frac{8\pi GM}{\Omega_{D-2}b^{D-3}} \sim \left(\frac{R_S}{b}\right)^{D-3} \ll 1 \;\; ; \;\; (GM)^{\frac{1}{D-3}} \sim R_S \ll b$$

$$\delta(E,M,b) \to \hat{\delta}(E,M,b) = \langle \delta(b + \hat{X}_H - \hat{X}_L) \rangle = 2GEM\hbar^{-1} c_D \langle (b + \hat{X}_H - \hat{X}_L)^{4-D} \rangle$$

Adding tidal excitation a la ACV-DDRV



$$\langle (b + \hat{X}_H - \hat{X}_L)^{4-D} \rangle \equiv \int_0^{2\pi} \frac{d\sigma_L}{2\pi} \int_0^{2\pi} \frac{d\sigma_H}{2\pi} : \Big(b + \hat{X}_H(\sigma_H, 0) - \hat{X}_L(\sigma_L, 0) \Big)^{4-D} :$$

Expansion of phase shift operator in l_s/b:

$$2(\hat{\delta}-\delta)=\frac{2\pi GEM(D-2)}{\hbar\Omega_{D-2}b^{D-2}}\langle Q_H^{ij}+Q_L^{ij}\rangle\hat{b}_i\hat{b}_j$$

$$Q_H^{ij}=\hat{X}_H^i\hat{X}_H^j-\frac{\delta_{ij}}{D-2}\sum_{i=1}^{D-2}\hat{X}_H^i\hat{X}_H^i$$

also:

$$Q_H^{ij}\hat{b}_i\hat{b}_j=\hat{X}_H^i\hat{X}_H^j\left(\hat{b}_i\hat{b}_j-\frac{\delta_{ij}}{D-2}\right)\equiv\Pi_{ij}\hat{X}_H^i\hat{X}_H^j$$

b-projection of Lorentz-contracted quadrupole operator!

Higher multipoles appear at higher orders.

We can rewrite the S-matrix in the form

$$S(E, M, b) = \exp(2i\delta)\ \Sigma_L\ \Sigma_H\ ;\ \Sigma_{L,H} = \exp\left(i(D-2)\Delta\ \tilde{Q}^{ij}_{L,H}\ \hat{b}_i\hat{b}_j\right)$$

where $\Delta = \frac{2\pi GEMl_s^2}{\hbar\Omega_{D-2}b^{D-2}}$; $\tilde{Q} = l_s^{-2}Q$ $\qquad \Pi_{ij} \equiv \hat{b}_i\hat{b}_j - \frac{\delta_{ij}}{D-2}$

$$\tilde{Q}^{ij}\ \hat{b}_i\hat{b}_j = \Pi_{ij}\ \sum_{n=1}^{\infty}\frac{1}{n}\left(a_n^{\dagger i}a_n^j + \tilde{a}_n^{\dagger i}\tilde{a}_n^j + a_n^i\tilde{a}_n^j + a_n^{\dagger i}\tilde{a}_n^{\dagger j}\right)$$

Using standard techniques we can get a normal-ordered Σ (useful between coherent states) as:

$$\begin{aligned}
\Sigma_H &= \Sigma^{(univ)}\ \Sigma^{(hair)}\ ;\ \Sigma^{(univ)} = \Gamma(1+i\Delta)^{D-3}\ \Gamma(1-i(D-3)\Delta) \\
\Sigma^{(hair)} &= :\exp\left(\sum_{n=1}^{\infty}(a_n^{\dagger i}+\tilde{a}_n^i)(a_n^j+\tilde{a}_n^{\dagger j})\left[C_n(\Delta)(\delta_{ij}-\hat{b}_i\hat{b}_j)+\tilde{C}_n(\Delta)\hat{b}_i\hat{b}_j\right]\right): \\
C_n(\Delta) &= -\frac{i\Delta}{n+i\Delta}\ ;\ \tilde{C}_n(\Delta) = C_n(-(D-3)\Delta)\,.
\end{aligned}$$

We finally take the heavy string to be a "stringhole" the idea being to interpret the result now in terms of BH properties (unfortunately we are presently unable to make reliable calculation much above the SH mass scale). Then:

$$\Delta = \frac{GEMl_s^2}{\hbar\, b^{D-2}} \to \frac{El_s}{\hbar}\left(\frac{l_s}{b}\right)^{D-2}$$

with

$$1 \ll \frac{El_s}{\hbar} \ll g_s^{-2} \qquad \left(\frac{l_s}{b}\right)^{D-2} \sim \theta^{\frac{D-2}{D-3}} \ll 1$$

in our kinematical region $\quad \theta^{\frac{D-2}{D-3}} \ll \Delta \ll g_s^{-2}\theta^{\frac{D-2}{D-3}}$

and there is a lot of parameter space for Δ to be large

The resulting S-matrix has many universal (i.e. no-hair) factors but it also has terms that probe the quadrupole (and also other multipoles) of the SH. At leading order in Δ/n:

$$\Sigma^{(hair)} =: \exp\left(-i(D-2)\Delta\sum_{n=1}^{\infty}\frac{1}{n}(a_n^{\dagger i}+\tilde{a}_n^{i})(a_n^{j}+\tilde{a}_n^{\dagger j})\Pi_{ij}\right):$$

This is the quantum hair of the SH as "seen" by the probe string via our thought experiment.
It turns out to be relatively large, possibly only a power of g_s^2 smaller than the no-hair terms.
If we apply the S-BH correspondence idea, we would conclude that also BHs should have such a large amount of quantum hair in agreement with Dvali-Gomez's recent papers, but:
Q_1: Are SHs good representatives of BH?
Q_2: Can the situation suddenly change above the CC?
For the moment I have no answer...

Summarizing

- The string-black hole correspondence (and stringholes microstates) can be useful tools for testing quantum-string gravity ideas in a regime still under reasonable control.

- Definite conclusions on the information puzzle will have to wait for a better understanding of how the correspondence works particularly much above the correspondence curve.

Thank you!