The infalling observer in AdS/CFT and the information paradox

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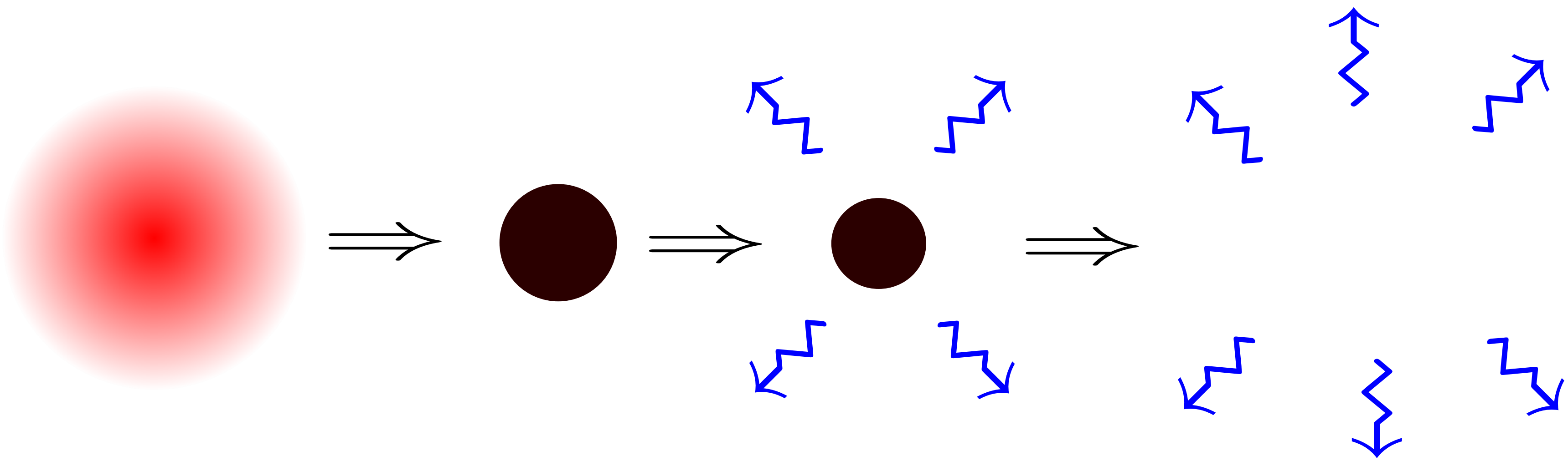
based on arXiv:1211.6767, K.P and Suvrat Raju
+ work in progress

Hawking Radiation vs Unitarity



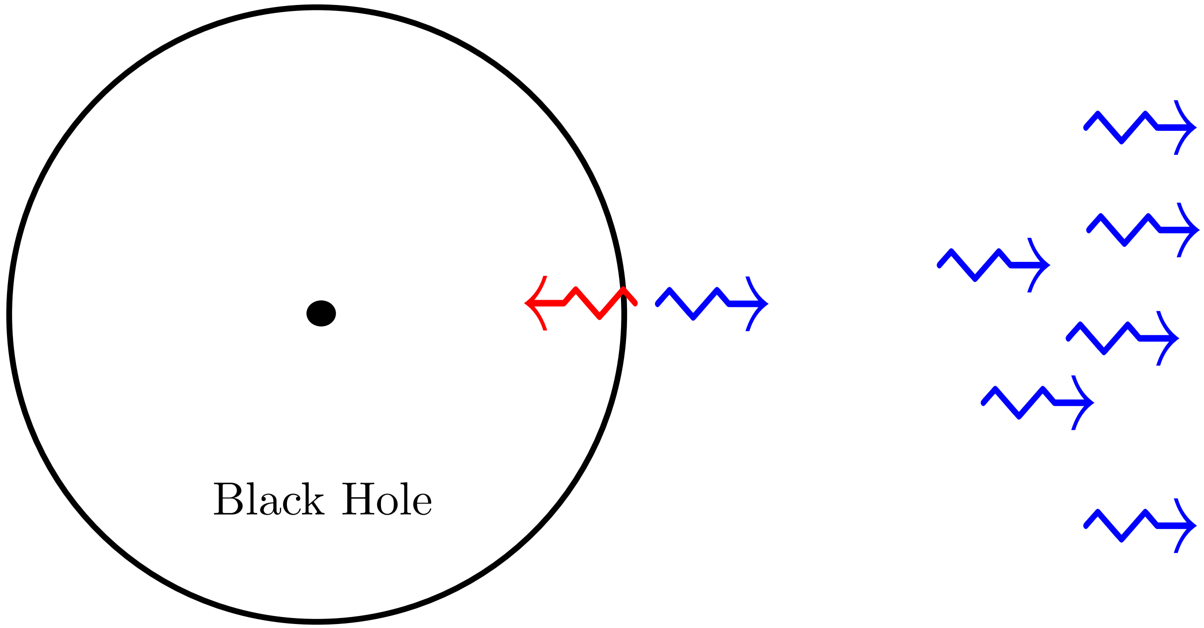
gas cloud
in pure state

Hawking radiation

$$|\Psi_0\rangle \quad \Longrightarrow \quad \cdots \quad \Longrightarrow \quad \rho_{\text{thermal}}$$

INCONSISTENT WITH UNITARY EVOLUTION

Information Paradox



- For unitarity: final state must carry information of initial state
- (In some sense) Hawking quanta are created near the horizon
- **If horizon is featureless and we have locality, how is information transfered to outgoing radiation?**

Information Paradox

We have tension between

- Unitarity
- Locality
- Equivalence Principle (smooth horizon)

CAN SMALL CORRECTIONS RESOLVE THE PARADOX?

Information Paradox

Will try to argue that the answer could be **YES**

"Small" amount of non-locality is sufficient to restore unitarity and at the same time preserve the smoothness of the horizon

Modification of black hole geometry?

- Proposals to modify interior of black hole (Fuzzballs, Firewalls, etc.)

-interior black hole geometry $\neq$ Schwarzschild solution

-infalling observer feels deviations from GR/burns when crossing the horizon

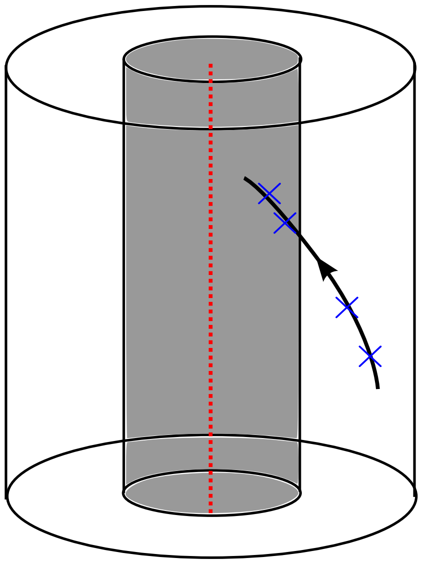
Free infall or not?

- Does an infalling observer notice something when crossing the horizon or not?

Main goals:

- Is the region behind the horizon encoded in the boundary CFT?
- Understand what happens to an observer falling into a black hole
- Address the information paradox

An infalling observer in AdS



- Consider a big black hole in AdS and an observer freely falling towards it
- The observer performs local experiments
- We will reconstruct these experiments from the boundary gauge theory
- We will argue that the results of these experiments are the same as those of semi-classical GR

Reconstructing local observables in empty AdS

In AdS/CFT we know that

"S-matrix elements" in AdS	$\Leftrightarrow$	Correlation functions in CFT
Local bulk correlators in AdS	$\Leftrightarrow$	?

Our first goal:

Construct local bulk observables from CFT

(based on earlier works: Banks, Douglas, Horowitz, Martinec, Bena, Balasubramanian, Giddings, Lawrence, Kraus, Trivedi, Susskind, Freivogel Hamilton, Kabat, Lifschytz, Lowe, Heemskerk, Marolf, Polchinski, Sully...)

Reconstructing local observables in empty AdS

- Large N CFTs contain in their spectrum **generalized free fields** i.e. (composite) local operators $\mathcal{O}(x)$ whose correlators factorize

$$\langle \mathcal{O}(x_1)...\mathcal{O}(x_{2n})\rangle = \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle \; ... \; \langle \mathcal{O}(x_{2n-1})\mathcal{O}(x_{2n})\rangle + \ldots$$

- Factorization ≈ "superposition principle". However, the operators $\mathcal{O}$ **do not satisfy any linear equation of motion in the CFT**.

- Hence, they are not **free fields**, but rather **generalized free fields**

- Excitations created by $\mathcal{O}$ behave like **ordinary free particles** in a higher dimensional AdS spacetime

Reconstructing local observables in empty AdS

- First we define the Fourier modes of $\mathcal{O}_{\omega,\vec{k}}$ by

$$\mathcal{O}(t,\vec{x}) = \int dt d\vec{x} \left(\mathcal{O}_{\omega,\vec{k}}\, e^{-i\omega t + i\vec{k}\vec{x}} + \text{h.c.}\right)$$

- Conformal invariance fixes the 2-point function to be

$$\langle \mathcal{O}(t,\vec{x})\mathcal{O}(0,\vec{0})\rangle = \left(\frac{-1}{t^2 - \vec{x}^2 - i\epsilon}\right)^{\Delta}$$

- From this we find

$$\mathcal{O}_{\omega,\vec{k}}|0\rangle = 0, \qquad \omega > 0$$

and

$$[\mathcal{O}_{\omega,\vec{k}}, \mathcal{O}^{\dagger}_{\omega',\vec{k}'}] = \mathcal{N}\theta(\omega^2 - \vec{k}^2)(\omega^2 - \vec{k}^2)^{\Delta - d/2}\delta(\omega-\omega')\delta(\vec{k}-\vec{k}')$$

Reconstructing local observables in empty AdS

- From this commutation relation we see that the modes $\mathcal{O}_{\omega,\vec{k}}$ create a **freely generated Fock space** of excitations.
- For an ordrinary free field we have dispersion relation $\omega^2 = \vec{k}^2 + m^2$.
- For the generalized free fields, excitations labeled by the **independent** parameters ω and $\vec{k}$.
- $\Rightarrow$ excitations behave like higher dimensional excitations
- Behave like ordinary free particles in AdS

Reconstructing local observables in empty AdS

- Consider AdS in Poincare patch

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

- and a scalar field satisfying $\Box\phi = m^2\phi$.

- We take m^2 to be related to the conformal dimension Δ of $\mathcal{O}$ by

$$\Delta = \frac{d}{2} + \sqrt{m^2 + d^2/4}$$

- For each value of $\omega, \vec{k}$ we find a solution of the Klein-Gordon equation of the form

$$f_{\omega,\vec{k}}(t,\vec{x},z) = e^{-i\omega t + i\vec{k}\vec{x}} z^{d/2} J_{\Delta - d/2}(\sqrt{\omega^2 - \vec{k}^2} z)$$

Reconstructing local observables in empty AdS

- We construct non-local CFT operators as

$$\phi_{\rm CFT}(t,\vec{x},z) = \int_{\omega>0} d\omega\, d\vec{k}\, \left(\mathcal{O}_{\omega,\vec{k}}\; f_{\omega,\vec{k}}(t,\vec{x},z) + {\rm h.c.}\right)$$

  Notice that while these are labeled by the coordinate z, they are really operators in the CFT. They are smeared, nonlocal operators.

- Using the previous results we can show that they satisfy

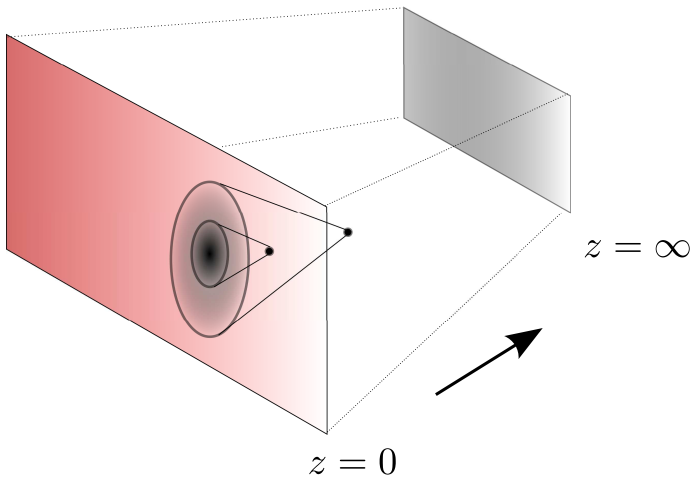
$$\Box_{\rm AdS}\phi_{\rm CFT} = m^2\,\phi_{\rm CFT}$$

  and

$$[\phi_{\rm CFT}(t,\vec{x},z)\,,\,\phi_{\rm CFT}(t',\vec{x}',z')] = 0$$

  for points $(t,\vec{x},z)$ and $(t',\vec{x}',z')$ spacelike **with respect to the AdS metric**.

Reconstructing local observables in empty AdS



- From the point of view of the CFT, coordinate z is an "auxiliary" parameter, which controls the smearing of the operators

- We can explicitly see how AdS space emerges from the lower dimensional CFT, as the combination of the coordinates $t, \vec{x}$ together with the extra parameter z

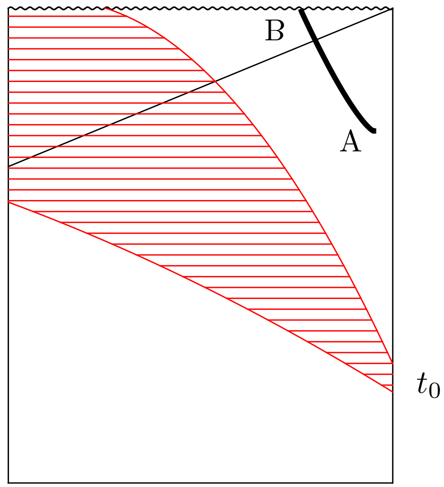
We can also interchange the order of the Fourier transforms to write

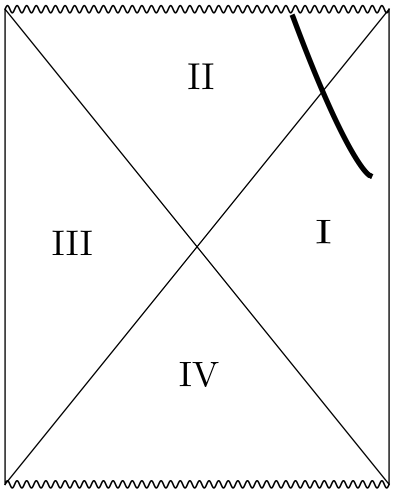
$$\phi_{\mathrm{CFT}}(t,\vec{x},z) = \int dt' d\vec{x}' \; K(t,\vec{x},z\;;\;t',\vec{x}')\mathcal{O}(t',\vec{x}')$$

where K is some kernel — sometimes called the *transfer function*.

Subtleties: 1/N expansion, gauge invariance....

Black Holes in AdS





BH formed by collapse $\approx$ Typical (QGP) pure state $|\Psi\rangle$

Eternal Black Hole in AdS $\approx$ Thermal ensemble in gauge theory

CFT Correlators at finite temperature

We use the notation

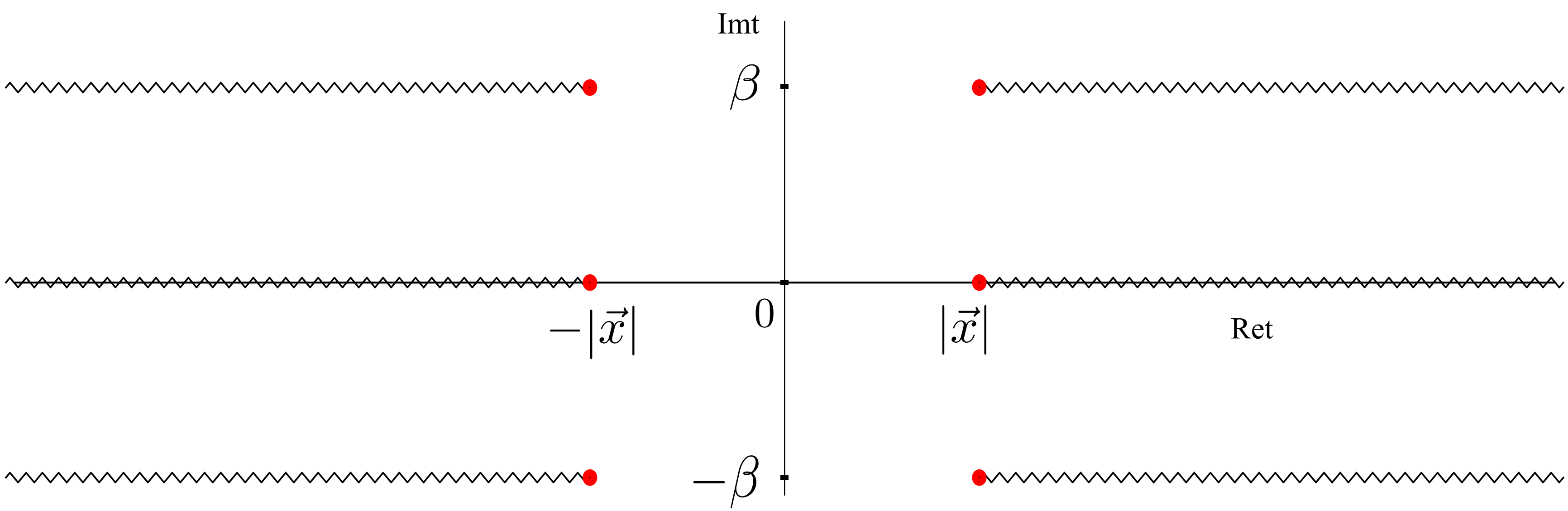
$$\langle A_1 ... A_n \rangle_\beta = \frac{1}{Z(\beta)} \mathrm{Tr} \left(e^{-\beta H} A_1 ... A_n \right)$$

- At large N thermal correlators factorize

$$\langle \mathcal{O}(x_1) ... \mathcal{O}(x_{2n}) \rangle_\beta = \langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_\beta \ldots \langle \mathcal{O}(x_{2n-1}) \mathcal{O}(x_{2n}) \rangle_\beta + \ldots$$

- Of course $\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_\beta \neq \langle 0 | \mathcal{O}(x_1) \mathcal{O}(x_2) | 0 \rangle$
- Factorization can fail if we scale the parameters of the correlator with N (for example: number of insertions, distances x_i, dimension of operators etc.)

CFT Correlators at finite temperature



- Consider the 2-point function $G_\beta(t, \vec{x}) = \langle \mathcal{O}(t, \vec{x}) \mathcal{O}(0, \vec{0}) \rangle_\beta$
- Satisfies the **KMS condition**

$$G_\beta(t - i\beta, \vec{x}) = G_\beta(-t, -\vec{x})$$

- In Fourier space

$$G_\beta(-\omega, \vec{k}) = e^{-\beta\omega} G_\beta(\omega, \vec{k})$$

CFT Correlators at finite temperature

- If we again define the Fourier modes $\mathcal{O}_{\omega,\vec{k}}$ by

$$\mathcal{O}(t,\vec{x}) = \int dt d^{d-1}x \left(\mathcal{O}_{\omega,\vec{k}}\, e^{-i\omega t + i\vec{k}\vec{x}} + \text{h.c.}\right)$$

- we find that they satisfy an oscillator algebra

$$[\mathcal{O}_{\omega,\vec{k}}\,,\,\mathcal{O}^{\dagger}_{\omega',\vec{k}'}] = \left(G_\beta(\omega,\vec{k}) - G_\beta(-\omega,\vec{k})\right)\delta(\omega-\omega')\delta(\vec{k}-\vec{k}')$$

- but now the (canonically normalized) oscillators are thermally populated

$$\langle \hat{\mathcal{O}}^{\dagger}_{\omega,\vec{k}}\,\hat{\mathcal{O}}_{\omega,\vec{k}}\rangle_\beta = \frac{1}{e^{\beta\omega}-1}$$

(this is the CFT analogue of the “thermal atmosphere” of the black hole)

Reconstructing the region outside the black hole

- Consider a black hole in AdS given by the metric

$$ds^2 = \frac{-h(z)dt^2 + dx^2 + h^{-1}(z)dz^2}{z^2} \quad , \quad h(z) = 1 - \frac{z^d}{z_0^d}$$

- Look for solutions of the Klein-Gordon equation of the form

$$f_{\omega,\vec{k}}(t,\vec{x},z) = e^{-i\omega t + i\vec{k}\vec{x}}\psi_{\omega,\vec{k}}(z)$$

- For every $(\omega, \vec{k})$ there is a unique solution, normalizable at the boundary $z = 0$.
- These are the usual "Schwarzschild modes" that we get when we quantize a scalar field near a black hole. We identify

$$f_{\omega\vec{k}}(t,\vec{x},z) \quad \Leftrightarrow \quad \mathcal{O}_{\omega,\vec{k}}$$

Reconstructing the region outside the black hole

- As before, we can write nonlocal CFT operators

$$\phi_{\text{CFT}}(t,\vec{x},z) = \int_{\omega>0} d\omega d\vec{k} \left(\mathcal{O}_{\omega,\vec{k}} f_{\omega,\vec{k}}(t,\vec{x},z) + \text{h.c.} \right)$$

- which behave like local fields around a black hole

$$(\Box - m^2)\phi_{\text{CFT}} = 0$$

$$[\phi_{\text{CFT}}(t,\vec{x},z)\,,\,\phi_{\text{CFT}}(t',\vec{x}',z')] = 0 \qquad , \quad \text{for spacelike points}$$

- and more generally

$$\langle \phi_{\text{CFT}}(P_1)...\phi_{\text{CFT}}(P_n)\rangle_\beta = \langle \phi_{gravity}(P_1)...\phi_{gravity}(P_n)\rangle_{\text{Hartle Hawking}}$$

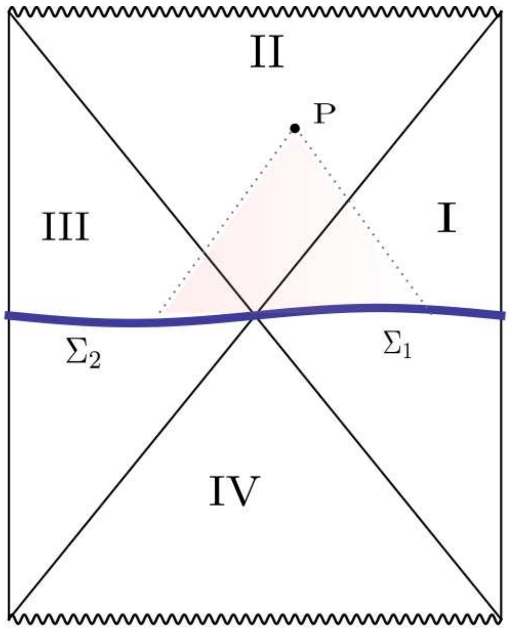
Reconstructing the region outside the black hole

- We have understood how to reconstruct the region outside the black hole from the point of view of the gauge theory

- We can write local observables in gravity as non-local operators in the gauge theory

Falling behind the horizon

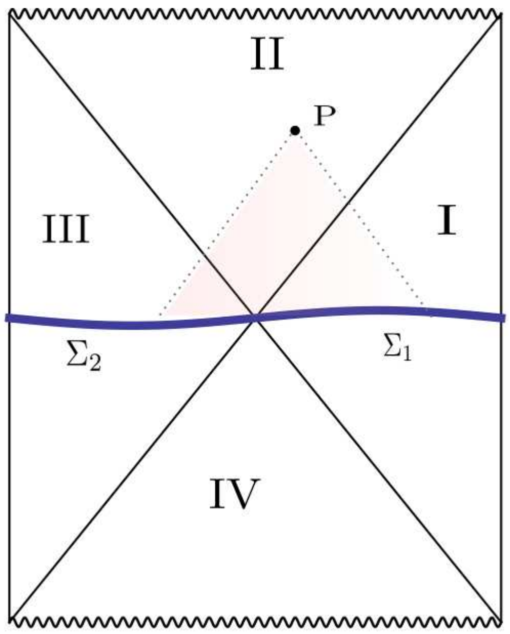
- Penrose diagram of (eternal) AdS black hole



- Cauchy slice for points in II is $\Sigma_1 \oplus \Sigma_2$
- To reconstruct local operator at P we need **both** modes on Σ_1 and Σ_2

Modes on Σ_1	$\Leftrightarrow$	$\mathcal{O}_{\omega,\vec{k}}$
Modes on Σ_2	$\Leftrightarrow$	$?$

Falling behind the horizon



- Maldacena: eternal black hole = 2 copies of CFT in entangled state
- In this formalism, modes on Σ_2 are the operators $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ in the second copy of the CFT
- Do we really need the two entangled copies?
- If we work **with a single CFT**, what is the meaning of the operators $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$?

Coarse-graining and doubling of operators

- Consider complicated (ergodic) system in pure state $|\Psi\rangle$
- Intuitive expectation $\Rightarrow$ system "thermalizes"
- For some observables $\{A_i\}$- called **coarse-grained observables**, their correlators on $|\Psi\rangle$ come close to thermal correlators

$$\langle\Psi|A_1..A_n|\Psi\rangle \approx \mathrm{Tr}\left(e^{-\beta H}A_1...A_n\right)$$

- This is not true for all observables, there are also **fine grained observables** which do not thermalize
- To simplify the language let us assume that the Hilbert space has the form

$$\mathcal{H} = \mathcal{H}_{\mathrm{coarse}} \otimes \mathcal{H}_{\mathrm{fine}}$$

(strictly speaking not true, but can be made more precise)
- $\mathcal{H}_{\mathrm{fine}}$ plays the role of a **heat bath** for $\mathcal{H}_{\mathrm{coarse}}$

Coarse graining and doubling of operators

- Every state $|\Psi\rangle$ can be written as

$$|\Psi\rangle = \sum_{ij} c_{ij} |\Psi_i^c\rangle \otimes |\Psi_j^f\rangle$$

where $|\Psi_i^c\rangle$, $|\Psi_j^f\rangle$ are orthonormal basis of $\mathcal{H}_{\mathrm{coarse}}$ and $\mathcal{H}_{\mathrm{fine}}$ respectively

- If $\mathcal{H}_{coarse}$ thermalizes, it means that the reduced density matrix

$$\rho_{\mathrm{coarse}} = Z_c^{-1} e^{-\beta H_{\mathrm{coarse}}}$$

- which means we can redefine our orthonormal basis such that

$$|\Psi\rangle = \sum_i \frac{e^{-\frac{\beta E_i^c}{2}}}{\sqrt{Z_c}} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$

Coarse graining and doubling of operators

- The state $|\Psi\rangle$ can be written as

$$|\Psi\rangle = \sum_i \frac{e^{-\frac{\beta E_i^c}{2}}}{\sqrt{Z_c}} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$

- Consider a **coarse-grained** operator acting on $\mathcal{H}_{\text{coarse}}$ as

$$A = \sum_{ij} a_{ij} |\hat{\Psi}_i^c\rangle \otimes \langle\hat{\Psi}_j^c|$$

- Then we **define** a new operator

$$\widetilde{A} = \sum_{ij} a_{ij}^* |\hat{\Psi}_i^f\rangle \otimes |\hat{\Psi}_j^f\rangle$$

acting on the fine-grained Hilbert space.

Coarse graining and doubling of operators

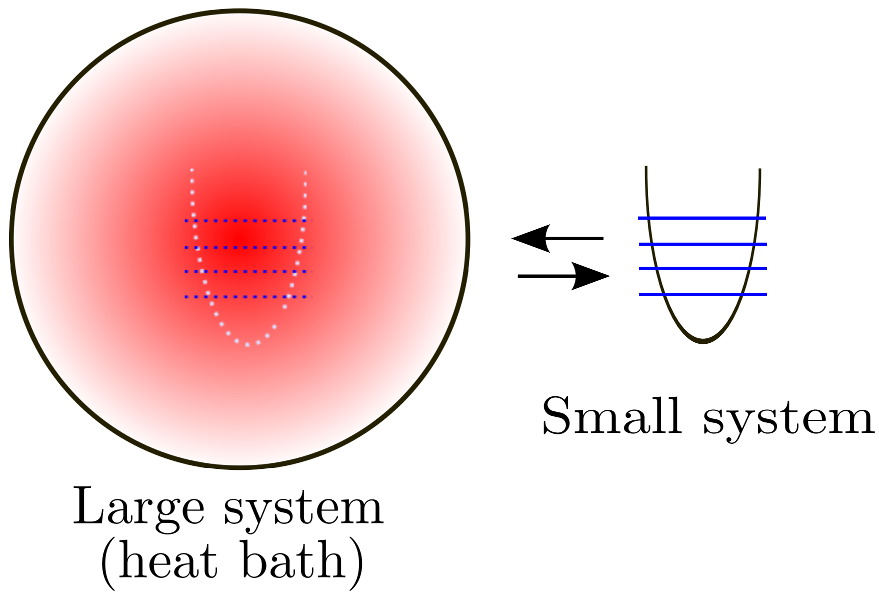
- We started with a set of coarse-grained operators A_i

- The operators $\widetilde{A}_i$ constructed as above, have the properties

1. The operator algebra $\widetilde{A}_i$ is isomorphic to that of A_i

2. Operators A_i commute with operators $\widetilde{A}_i$

Coarse graining and doubling of operators



SMALL SUBSYSTEM IS MIRRORED IN HEAT BATH

- For us the Quark-Gluon-Pasma is the heat bath
- The glueball operators $\mathcal{O}_i$ are the coarse-grained observables
- They are mirrored in the QGP, which leads to new operators $\widetilde{\mathcal{O}}_i$
- This mirroring involves the fine-degrees of freedom

Fine-grained observables

- The "tilde operators" are very special: they are **fine-grained** observables
- They are state-dependent operators, will not "click correctly" with different microstate $|\Psi'\rangle$ (which may be a good thing....)
- Among all possible fine-grained operators, the "tilde operators" are selected/protected via their entanglement with the coarse-grained ones
- They are "very sparse operators"

Coarse graining and doubling of operators

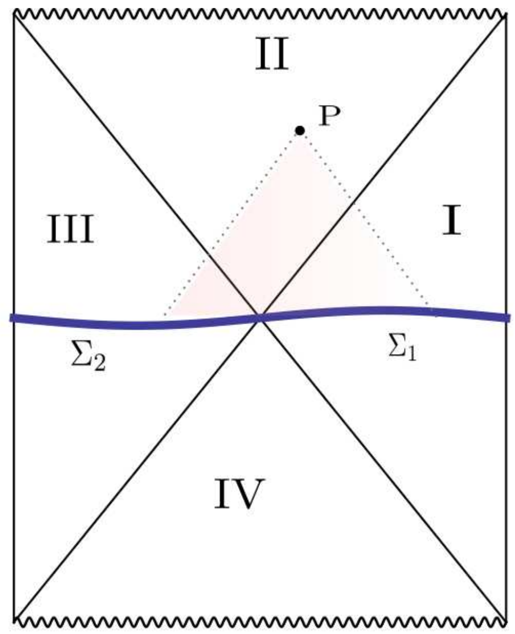
- At large N, correlation functions of the mirrored operators $\widetilde{\mathcal{O}}$ on a pure state, agree with those of analytically continued operators

$$\mathcal{O}(t + i\beta/2)$$

  and in particular correlators computed with the "thermofield doubling"

- However the $\widetilde{\mathcal{O}}$, **as operators acting on pure states**, were defined via the coarse/fine-grained decomposition

Falling behind the horizon



$$\text{Modes on } \Sigma_1 \quad \Leftrightarrow \quad \mathcal{O}_{\omega,\vec{k}}$$

$$\text{Modes on } \Sigma_2 \quad \Leftrightarrow \quad \widetilde{\mathcal{O}}_{\omega,\vec{k}}$$

where $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ are the Fourier transforms of the mirrored operators $\widetilde{\mathcal{O}}$

Local operators behind the horizon

Using both $\mathcal{O}_{\omega,\vec{k}}$ and $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ we can write local observables behind the horizon of the black hole.

$$\phi_{\rm CFT}(t,\vec{x},z) = \int_{\omega>0} d\omega d\vec{k} \left[\mathcal{O}_{\omega,\vec{k}}\; g^{(1)}_{\omega,\vec{k}}(t,\vec{x},z) + \widetilde{\mathcal{O}}_{\omega,\vec{k}}\; g^{(2)}_{\omega,\vec{k}}(t,\vec{x},z) + {\rm h.c.}\right]$$

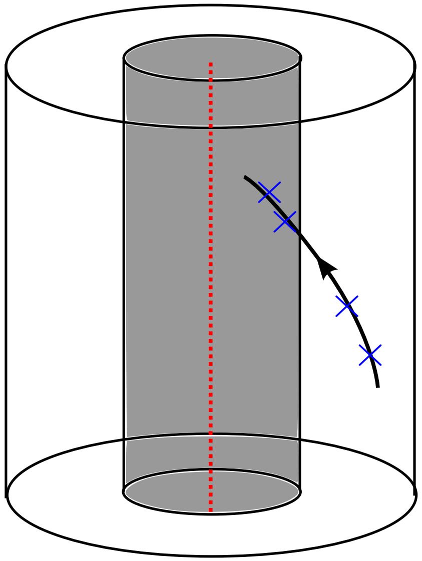
here $g^{(1),(2)}$ are solutions of the Klein-Gordon equation in region II

In the large N limit, correlators of $\phi_{\rm CFT}(t,\vec{x},z)$ on a typical pure state $|\Psi\rangle$ (corrseponding to a black hole microstate) agree with those computed in semiclassical gravity.

We have reconstructed both the exterior and the interior of the black hole from the dual gauge theory

Fate of the infalling observer

Using the operators $\phi_{\rm CFT}$ we can reconstruct the experiments of the infalling observer



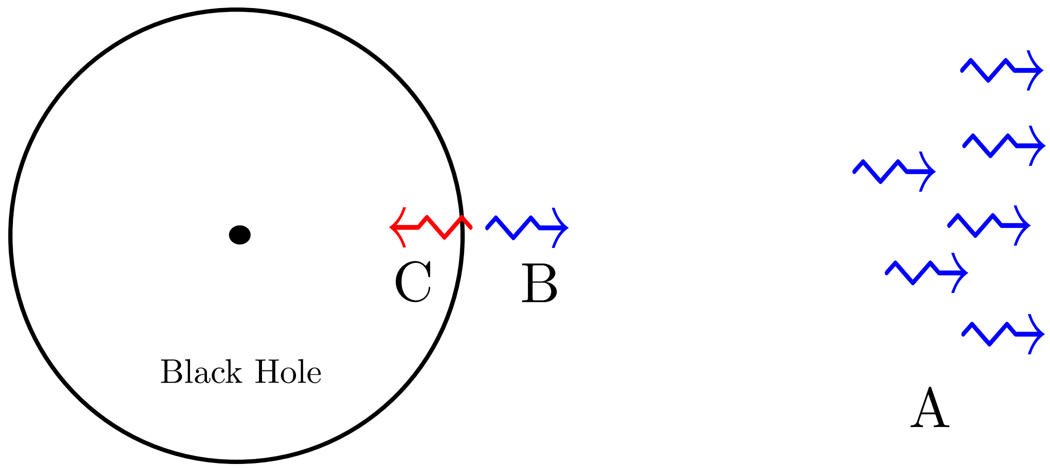
MAIN CONCLUSION: For a big AdS black hole, an infalling semi-classical observer does not notice anything special when crossing the horizon

Various subtleties

- Sensitivity to pure state $|\Psi\rangle$?
- Including $1/N$ corrections?
- Spread of transfer function as we approach the horizon?
- Sensitivity to late times - Poincare recurrences?

If large N expansion at finite temperature holds, we can show that they are under control.

Sharpened version of the information paradox (Mathur, AMPS)



Unitarity: after Page time we need $S_{AB} < S_A$

Strong subadditivity theorem: for 3 **independent** systems A,B,C we have

$$S_{AB} + S_{BC} \geq S_A + S_C$$

For the Hawking pair production we have $S_{BC} \approx 0$ and $S_C > 0$ which would imply

$$S_{AB} > S_A$$

Proposed resolution

- In our language the C's are fine-grained operators defined via their entanglement with coarse grained operators
- The early radiation A plays the role of the heat bath
- Hence C's are "highly scrambled" combinations of A's

  Systems A,B,C are not really independent

$$\Downarrow$$

Strong subadditivity theorem cannot be applied to A,B,C

Consistent with locality?

We would like to satisfy simultaneously:

- System C is not independent, but rather a highly scrambled version of (part of) A
- Locality for **simple measurements** is preserved. For P_1 inside horizon and P_2 outside

$$[\phi(P_1)\,,\,\phi(P_2)] \approx 0$$

  up to very small corrections.
- Of course, if $\mathcal{W}$ is a complicated operator acting outside the horizon which measures many of the As then we allow

$$[\phi(P_1)\,,\,\mathcal{W}] \neq 0$$

Are these statements consistent?

Coarse-grained vs fine-grained observables

- Fine-grained are “sparse” (very few nonzero eigenvalues)
- Coarse-grained are “not-sparse“

The fact that C's are fine-grained (state-dependent) makes it easier to simultaneously satisfy

$$[\phi(A), \phi(C)] \approx 0$$

while at the same time $C \subset A$.

(spin chain toy-model + scrambling)

On complementarity

How we understand complementarity

- There is a large Hilbert space describing **both** the interior and the exterior of the black hole
- We can construct operators acting on this Hilbert space and describing observables outside the black hole
- We can construct operators acting on the same Hilbert space and describing observables inside the black hole
- For few, light observables, they approximately commute.
- But not for too accurate measurements, or measurements involving too many insertions

Quantum cloning?

- Thought experiment: extract scrambled qubit from A and then jump in to meet ”interior copy“
- If possible to be performed, does this really lead to a paradox of Quantum Mechanics ?
- Is the interior qubit ”different“ from the scrambled qubit?
- Operator identifications vs ”copies of qubits“

Summary

- **Local bulk physics from CFT**: local observables both outside and inside the black hole
- **Infalling observer:** does not notice anything special
- **Interior of black hole:** coarse-grained observables effectively doubled in fine grained degrees of freedom. Black hole interior is a combination of both.
- **Information paradox:** small corrections can restore unitarity.
- **Strong subadditivity argument (Mathur, AMPS)**: A,B,C are not independent systems. C is a highly scrambled rewriting of A
- **Non-locality:** The amount of non-locality required is small...(?)

Future directions

- Dynamics, "stability" of fine-grained operators
- Thermalization (stages)
- Meaning of singularity
- $1/N$ corrections etc.
- Bounds on non-locality, toy model
- ...

THANK YOU