THEORETICAL VIEW ON QUARKONIUM PRODUCTION

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Workshop on Heavy Flavor Physics
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OUTLINE

- Discussing prompt charmonium production (i.e. not from B-decay)
- Not an exhaustive review of latest theoretical calculation rather an attempt to give insight into the ingredients of the calculations
- Something is missing in the theory though there has been recent progress
- Distinguish model from theory

THE NON RELATIVISTIC NATURE OF QUARKONIUM

Quarkonium: Bound state of a heavy quark anti-quark pair

$$m_Q \gg \Lambda_{\rm QCD} \longrightarrow \quad v \ll 1$$

Charmonium	M	Bottomonium	M
$\eta_c \to c\bar{c}(n=1,{}^1S_0)$	2.98	$\eta_b \to b\bar{b}(n=1,{}^1S_0)$	9.39
$J/\psi \to c\bar{c}(n=1,{}^3S_1)$	3.096	$\Upsilon(1S) \to b\bar{b}(n=1,{}^3S_1)$	9.46
$\chi_{cJ} \to c\bar{c}(n=1,{}^3P_J)$	~ 3.5	$\chi_{bJ} \to b\bar{b}(n=1,{}^3P_J)$	~ 10

$v \sim 0.5$ (Charmonium)

$v \sim 0.3$ (Bottomonium)

NRQCD

(Caswell & Lepage, and Luke, Manohar & Rothstein)

Appropriate EFT for Describing Quarkonium Dynamics:
Non-Relativistic QCD

- Remove the heavy quark mass from QCD
- Power counting in relative velocity $v \ll 1$

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_Q} \right)\psi + \chi^\dagger \left(i\partial_0 - \frac{\nabla^2}{2m_Q} \right)\chi \; + \ldots$$

- Separates the scales: $m_Q, m_Q v, m_Q v^2$

QUARKONIUM DECAY

(Bodwin, Braaten & Lepage)

Inclusive & Semi-Inclusive Decay Rates Calculated via the O.P.E.

E.G. $J/\psi \to \gamma + X$

$$\frac{d\Gamma}{dE_\gamma}(J/\psi \to \gamma + X) = \sum_\beta \frac{d\Gamma}{dE_\gamma}(c\bar{c}[\beta] \to \gamma + X)\langle J/\psi|\mathcal{O}_\alpha|J/\psi\rangle$$

$$\mathcal{O}_1(^3S_1) = \psi^\dagger \boldsymbol{\sigma}\chi \cdot \chi^\dagger \boldsymbol{\sigma}\psi \qquad O(v^3)$$

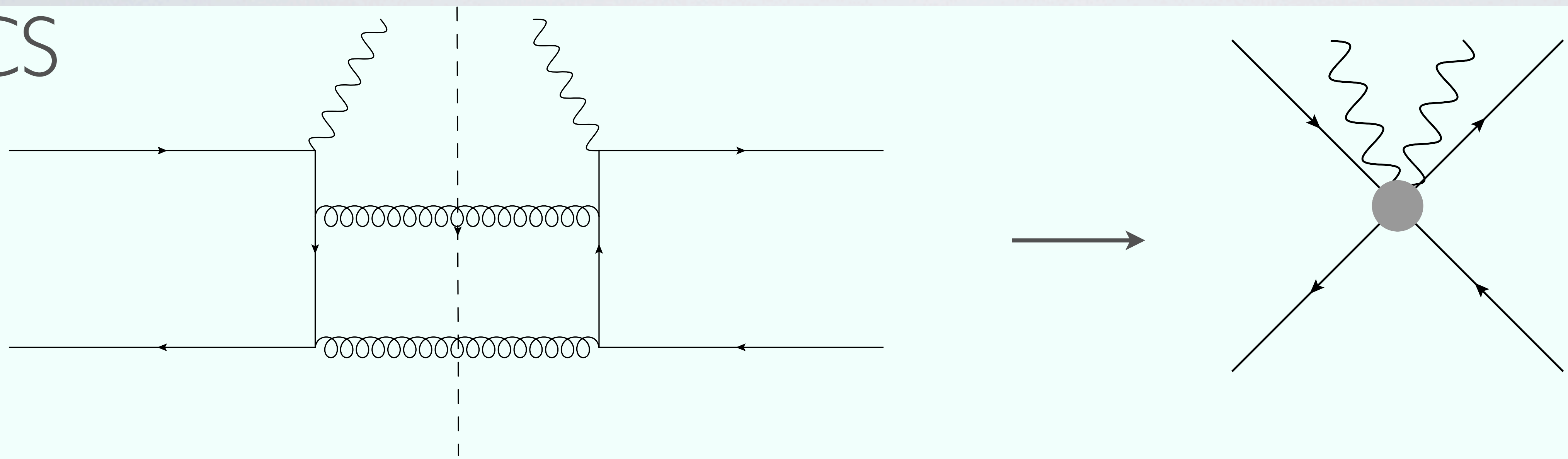
$$\mathcal{O}_8(^3S_1) = \psi^\dagger \boldsymbol{\sigma} T^A \chi \cdot \chi^\dagger T^A \boldsymbol{\sigma}\psi \qquad O(v^7)$$

Not a model: theoretical errors understood and systematically improvable

QUARKONIUM DECAY

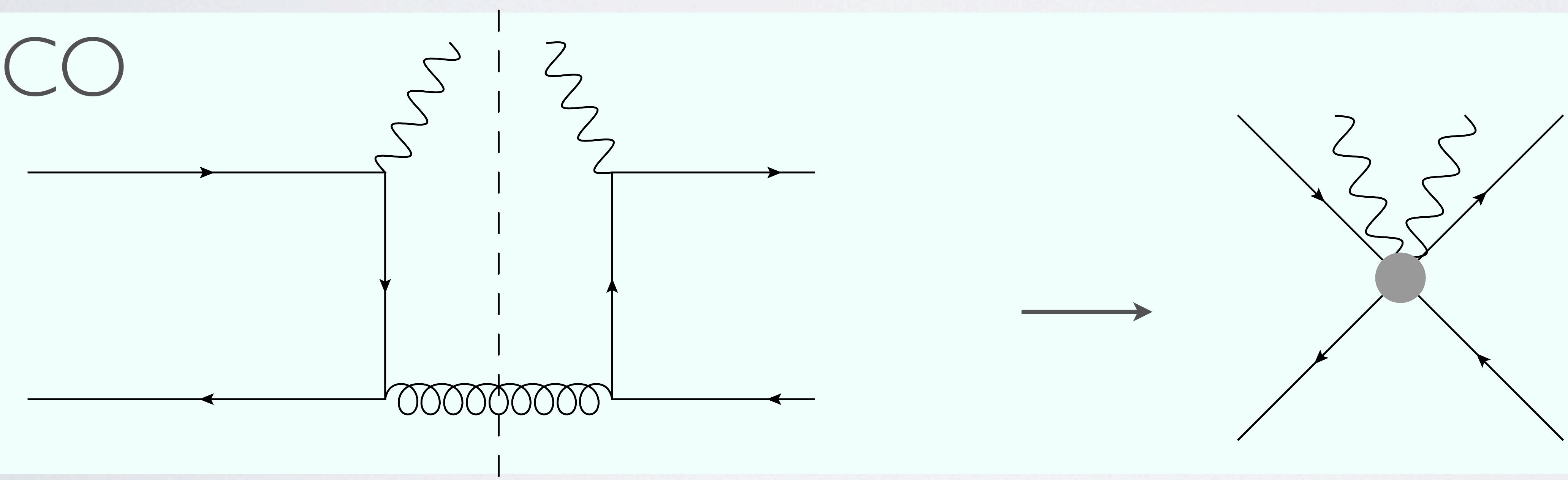
E.G. $J/\psi \to \gamma + X$

CS



$\alpha_s^2(2m_c)v^3$

CO

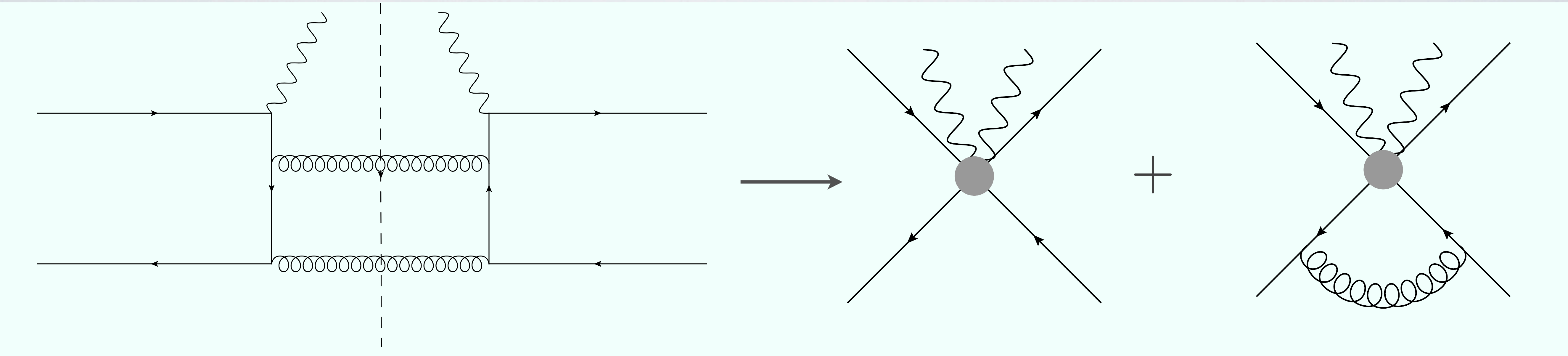


$\alpha_s(2m_c)v^7$

$$\mathrm{CO}/\mathrm{CS} = v^4/\alpha_s(2m_c) \approx 0.25$$

QUARKONIUM DECAY

$\chi_{cJ} \to \gamma + X$ at LO



$$\frac{d\Gamma}{dE_\gamma}(\chi_{cJ} \to \gamma + X) = \sum_\beta \frac{d\Gamma}{dE_\gamma}(c\bar{c}[\beta] \to \gamma + X)\langle \chi_{cJ}|\mathcal{O}_\alpha|\chi_{cJ}\rangle$$

$$\mathcal{O}_1(^3P_0) = \frac{1}{3}\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\chi\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\psi \qquad O(v^5)$$

$$\mathcal{O}_8(^3S_1) = \psi^\dagger\boldsymbol{\sigma}T^A\chi\cdot\chi^\dagger T^A\boldsymbol{\sigma}\psi \qquad O(v^3)$$

QUARKONIUM PRODUCTION

(Bodwin, Braaten & Lepage)

NRQCD Factorization for Production

- Inclusive Production Cross Section

$$\sigma(a+b\to H+X)=\sum_{\beta}\hat{\sigma}(a+b\to Q\bar{Q}(\beta)+X)\langle 0|\mathcal{O}_{\beta}^{H}|0\rangle$$

Universal(?)

$$\mathcal{O}_n^H=\chi^\dagger\mathcal{K}_n\psi\left(\sum_X\sum_{m_J}|H+X\rangle\langle H+X|\right)\psi^\dagger\mathcal{K}'_n\chi$$
$$=\chi^\dagger\mathcal{K}_n\psi\left(a_H^\dagger a_H\right)\psi^\dagger\mathcal{K}'_n\chi,$$

E.G. $\mathcal{O}_1^H(^3S_1)=\chi^\dagger\sigma^i\psi\left(a_H^\dagger a_H\right)\psi^\dagger\sigma^i\chi,$

$$\mathcal{O}_8^H(^1S_0)=\chi^\dagger T^a\psi\left(a_H^\dagger a_H\right)\psi^\dagger T^a\chi,$$

QUARKONIUM PRODUCTION

- J/ψ Production at large $p_\perp$ in hadronic collisions

$$\frac{d\sigma}{dp_\perp}(p\bar{p} \to J/\psi(p_\perp) + X) = \int dx_1 f_{i/p}(x_1) \int dx_2 f_{j/\bar{p}}(x_2) \times \sum_\beta \hat{\sigma}(i\,j \to c\bar{c}(\beta, p_\perp) + X)\langle 0|\mathcal{O}_\beta^{J/\psi}|0\rangle$$

Leading contributions are a balance between powers of $\alpha_s(p_\perp)$ and powers of v

QUARKONIUM PRODUCTION

- J/ψ Production at large $p_\perp$ in hadronic collisions

LO

$$\mathcal{O}_1^H(^3S_1) = \chi^\dagger \sigma^i \psi \left(a_H^\dagger a_H\right) \psi^\dagger \sigma^i \chi \qquad \alpha_s^3(p_\perp) v^3$$

$$\mathcal{O}_8^H(^3S_1) = \chi^\dagger \sigma^i T^a \psi \left(a_H^\dagger a_H\right) \psi^\dagger \sigma^i T^a \chi \qquad \alpha_s^2(p_\perp)\alpha_s(2m_c) v^7$$

$$\mathcal{O}_8^H(^1S_0) = \chi^\dagger T^a \psi \left(a_H^\dagger a_H\right) \psi^\dagger T^a \chi, \qquad \alpha_s^3(p_\perp) v^7$$

$$\mathcal{O}_8^H(^3P_0) = \frac{1}{3}\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})T^A\psi(a_H^\dagger a_H)\psi^\dagger T^A(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\chi \qquad \alpha_s^3(p_\perp) v^7$$

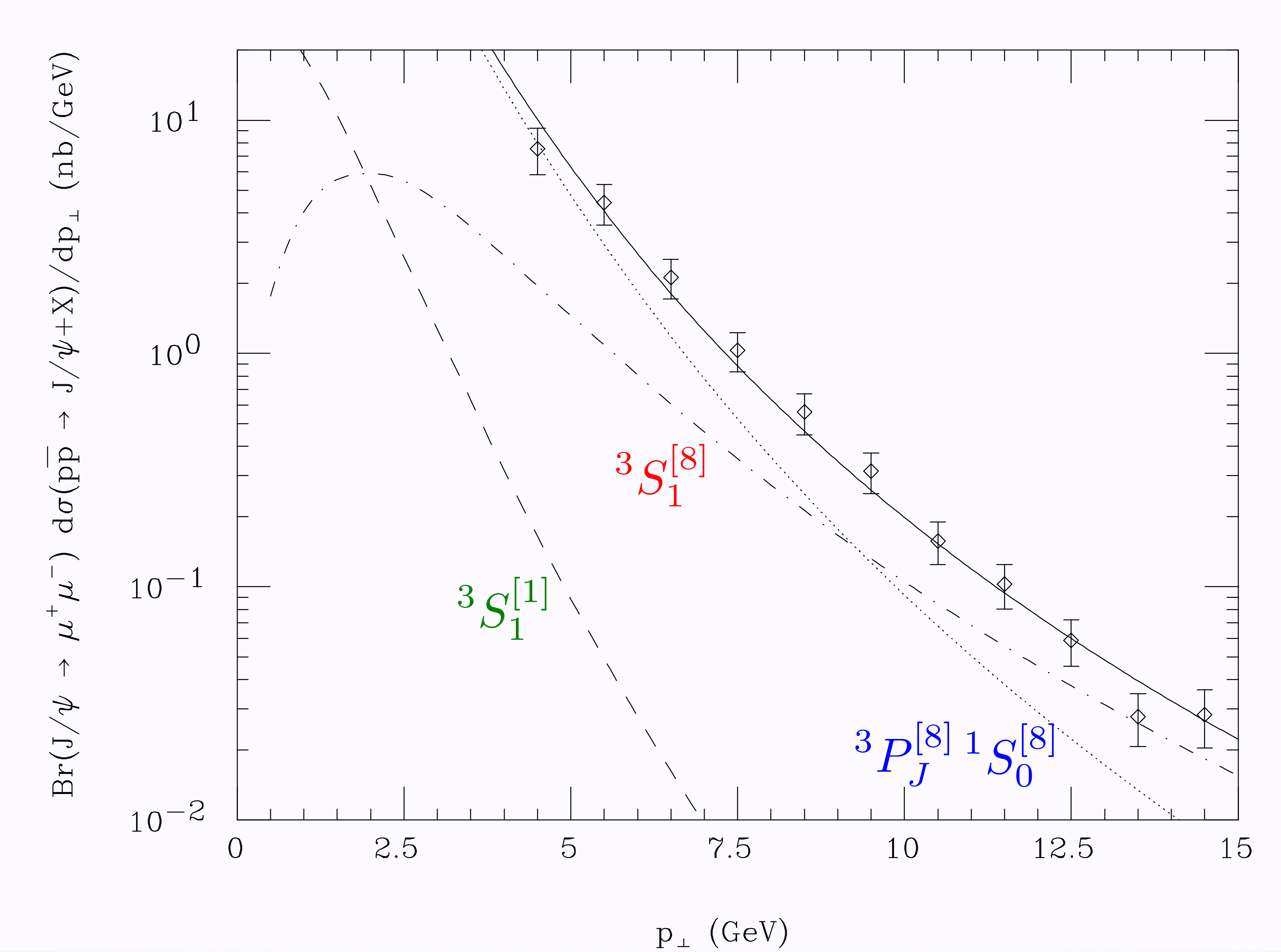
$$\mathcal{O}_8^H(^3P_1) = \frac{1}{2}\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma})^i T^A\psi(a_H^\dagger a_H)\chi^\dagger T^A(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma})^i\chi \qquad \alpha_s^3(p_\perp) v^7$$

$$\mathcal{O}_8^H(^3P_2) = \chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\sigma^{j)})T^A\psi(a_H^\dagger a_H)\chi^\dagger T^A(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\sigma^{j)})\chi \qquad \alpha_s^3(p_\perp) v^7$$

Different contributions give differential cross sections with different $p_\perp$ scaling

QUARKONIUM PRODUCTION

(Cho & Lebovich)



constrains a linear combination of $\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle \quad \langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle$

QUARKONIUM PRODUCTION

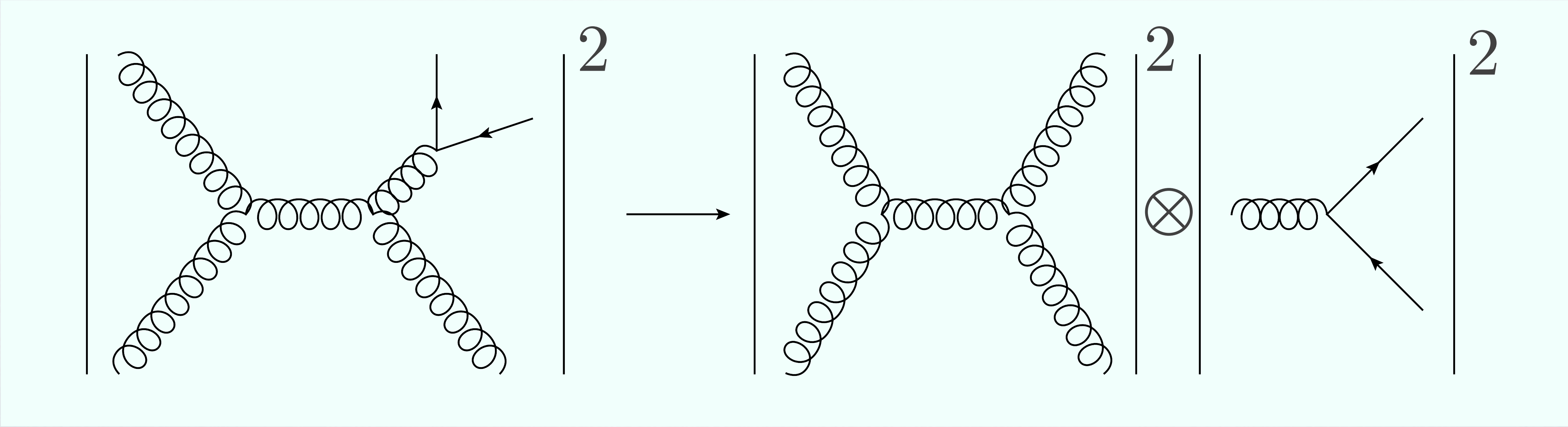
(Braaten & Fleming)

- J/ψ Production at large $p_\perp$ in hadronic collisions

$$\hat{\sigma}(a+b \to c\bar{c}(^3S_1^{[8]}) + X)\langle 0|\mathcal{O}^{J/\psi}(^3S_1^{[8]})|0\rangle \text{ is special}$$

Fragmentation: $$\frac{d\hat{\sigma}}{dp_\perp}(ij \to J/\psi + X)_{\text{octet}}$$

$$\overset{p_\perp \to \infty}{\longrightarrow} \int dz \frac{d\hat{\sigma}}{dp_\perp}(ij \to g(p_\perp/z) + X) D_{g\to J/\psi}(z)$$



QUARKONIUM PRODUCTION

Gluon Fragmentation

- Sum Logarthms: Run from $p_\perp$ to $2m_c$

$$\mu \frac{dD_{g\to\psi_Q}}{d\mu}(z,\mu) = \frac{\alpha_s(\mu)}{\pi}\int_z^1 \frac{dy}{y} P_{gg}(y) D_{g\to\psi_Q}(\frac{z}{y},\mu)$$

$$P_{gg}(y) = 6\Big[\frac{y}{(1-y)_+} + \frac{1-y}{y} + y(1-y) + \frac{33-2n_f}{36}\delta(1-y)\Big]$$

$$D_{g\to\psi'}(z,\mu) \;=\; \frac{\pi\alpha_s(2m_c)}{24m_c^3}\delta(1-z)\,\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle$$

These Logarithms are <u>not</u> summed in the other contributions!

COMMENT ON DIFFERENT APPROACHES

- color evaporation model (CEM) is taking $v \to 1$ and making additional assumptions (otherwise it would just be the equivalent of light meson production)
- color-singlet model is taking $v \to 0$
- The NRQCD approach just presented, while systematic in v has aspects of a model since the $p_\perp$ dependence is not treated systematically (and errors from corrections can not be **systematically** estimated)

STATE OF THE ART

NLO Analysis of J/ψ production

Chao, K-T et al, Phys.Rev.Lett. 108 (2012) 242004
Butenschoen & Kniehl, Nucl.Phys.Proc.Suppl. 222-224 (2012) 151-161

- 194 data points (over half from hadroproduction)
- 10 experiments (experiments at LHC, Tevatron, RHIC, HERA, LEP KEKB)
- 3 free parameters: $\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ $\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ $\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle$

  Hadroproduction only constrains a linear combination of $\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ $\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle$

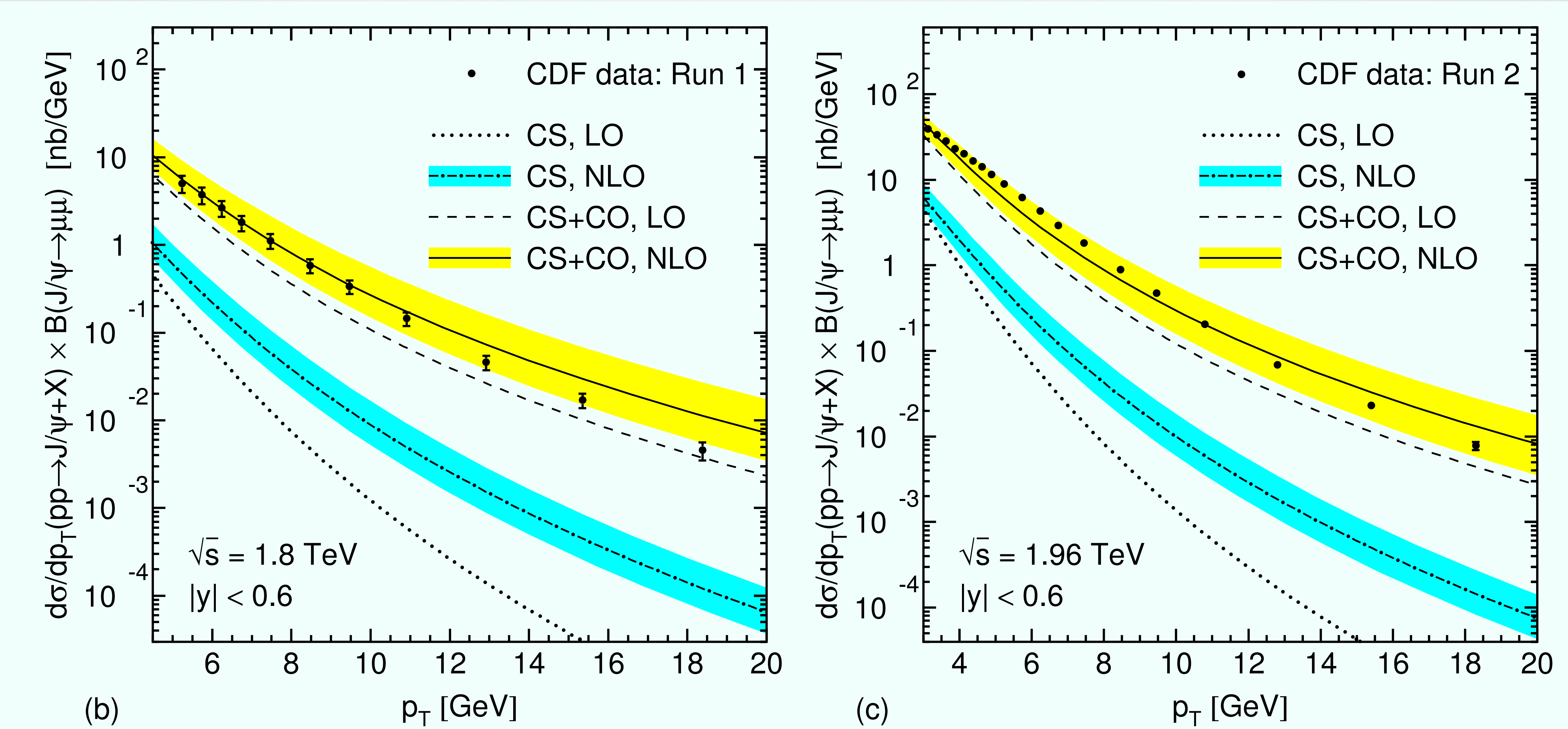
  Different linear combination measured at HERA (matrix elements universal)

- No resummation (even in fragmentation contribution)

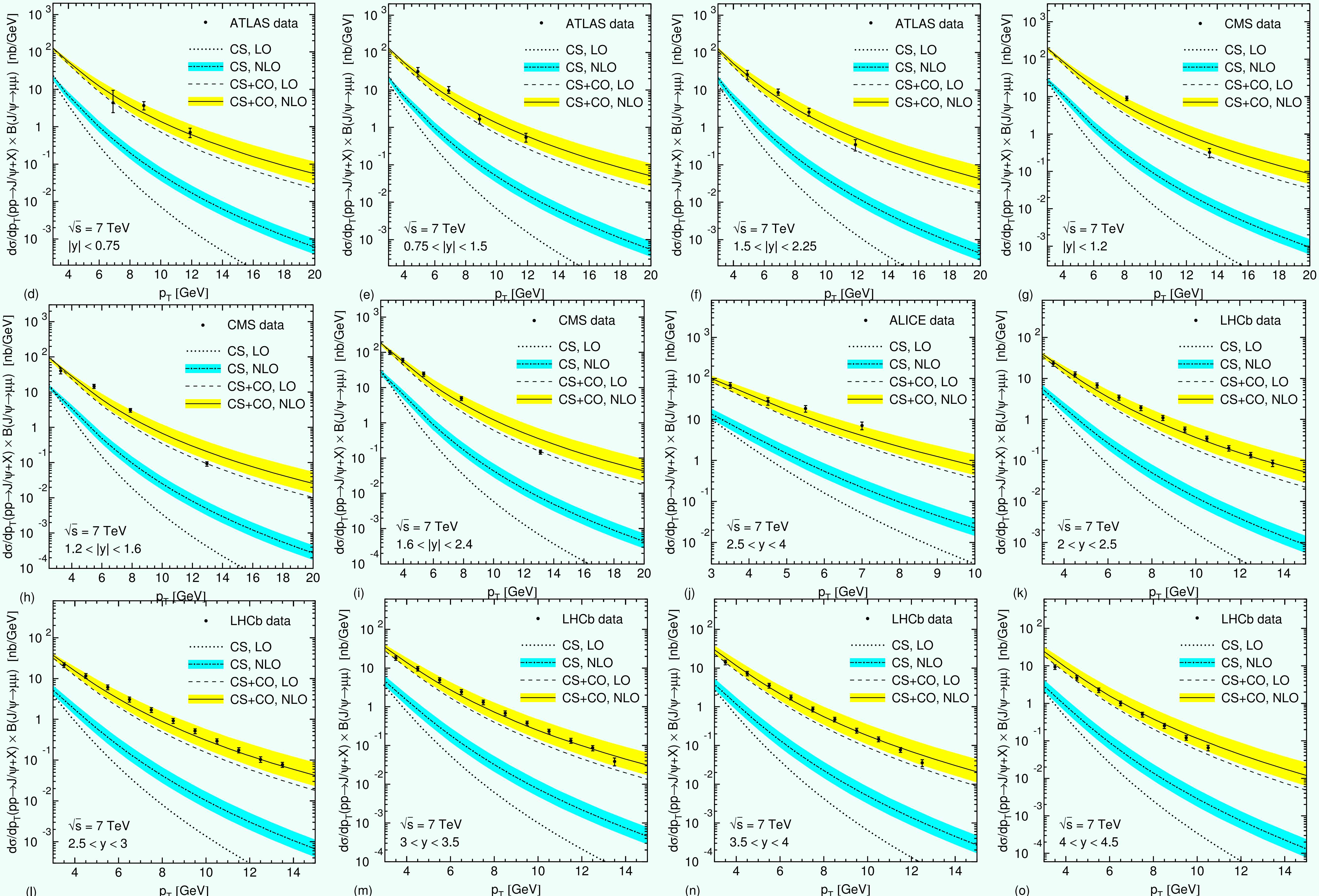
NLO J/ψ PRODUCTION

NLO Analysis

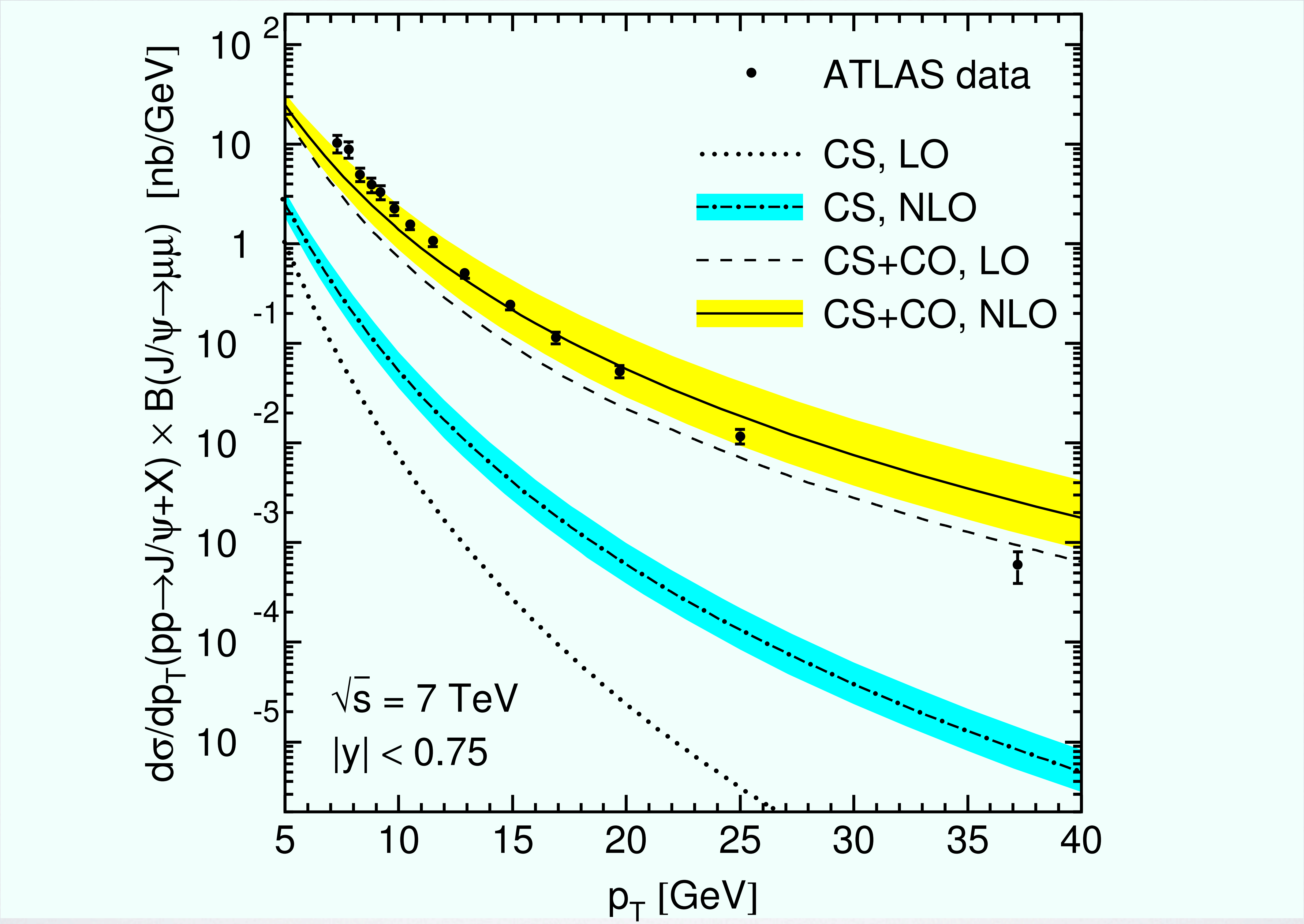
$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2}\ \mathrm{GeV}^3$
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3}\ \mathrm{GeV}^3$
$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2}\ \mathrm{GeV}^5$



NLO J/ψ PRODUCTION



NLO J/ψ PRODUCTION

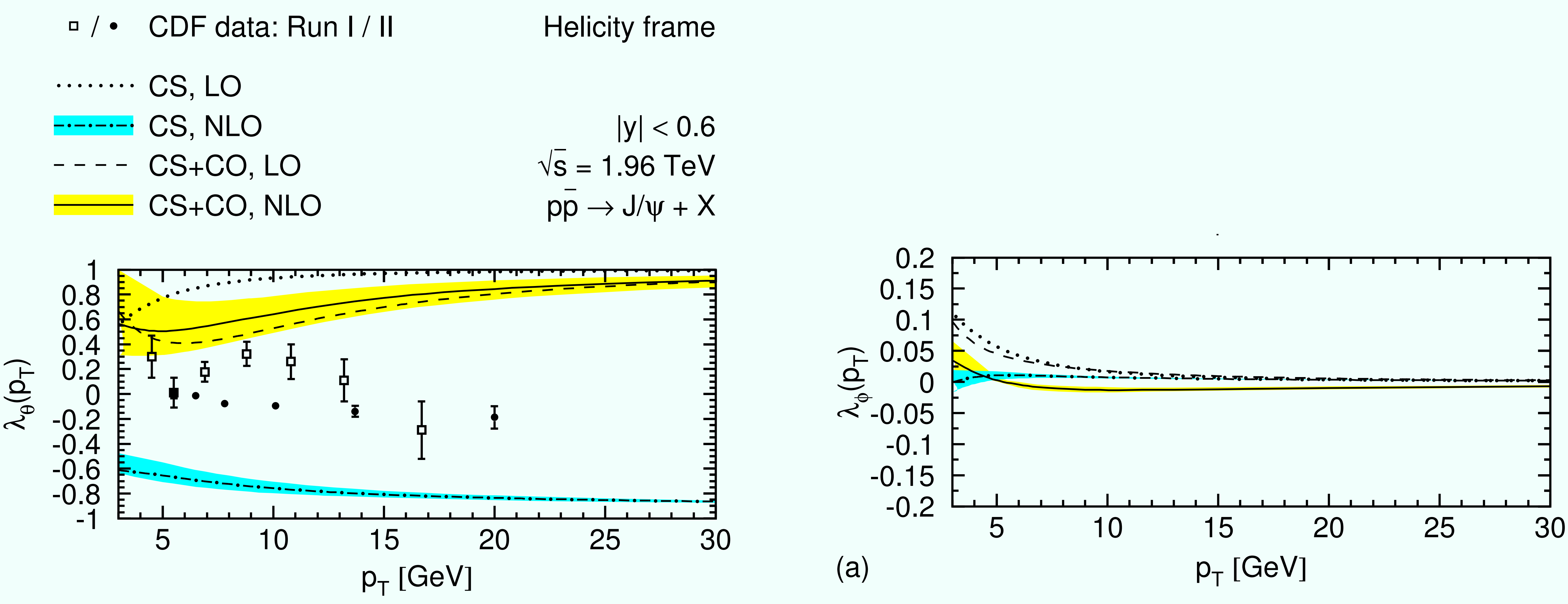


Atlas data not included in fit: large logs?!?!

NLO J/ψ PRODUCTION

Polarization reflect in distribution of J/ψ decay leptons

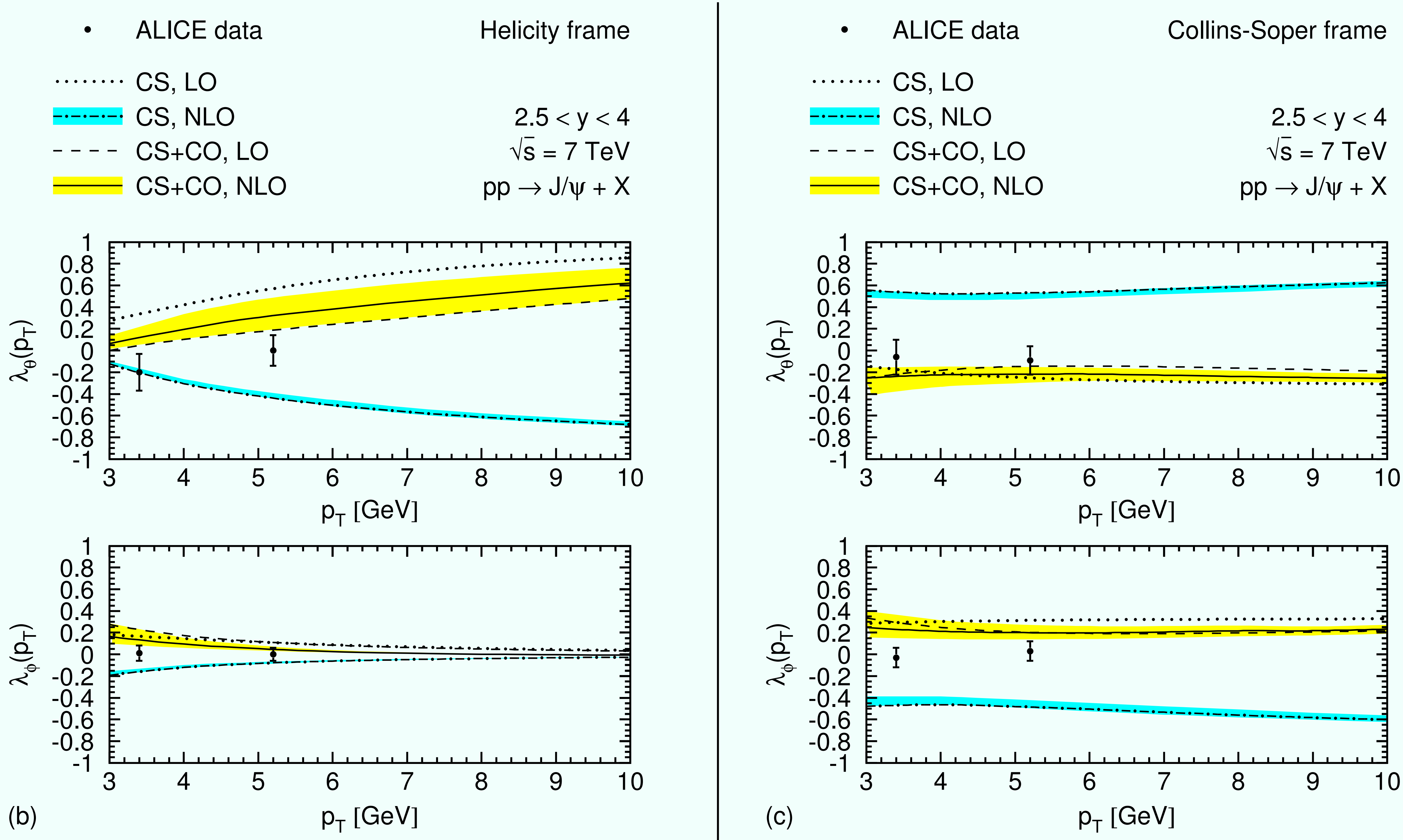
$$W(\theta,\phi) \quad \propto \quad 1+\lambda_\theta \cos^2\theta + \lambda_\phi \sin^2\theta \cos(2\phi) + \lambda_{\theta\phi} \sin(2\theta)\cos\phi$$



using fitted parameters (i.e. is a prediction, if MEs universal)

NLO J/ψ PRODUCTION

Polarization of J/ψ



OPEN QUESTIONS

Questions :

1. Can the non-systematic part of NRQCD factorization be fixed?
2. How about summing logarithms of $\frac{(2m_c)}{p_\perp}$ in all contributions?
3. Will the prediction for polarization change?

Answers :

1. Yes! Using improved NRQCD factorization.
2. Running of operators sums logs in improved formalism.
3. Maybe: reorganizes calculation at large $p_\perp$, has something to say about the universality of MEs...

IMPROVED FACTORIZATION APPROACH

(Kang, Qiu & Sterman and Fleming, Leibovich, Mehen & Rothstein)

Improved NRQCD factorization:

First organizes $\frac{d\sigma}{dp_\perp}$ in powers of $\frac{(2m_c)}{p_\perp}$

Then organizes $\frac{d\sigma}{dp_\perp}$ in powers of v

Systematic perturbative expansion in $\alpha_s(p_\perp)$ and $\alpha_s(2m_c)$

Then we have to consider to what order we work in for each parameter

IMPROVED FACTORIZATION APPROACH

Example: J/ψ production

LO in $\frac{(2m_c)}{p_\perp}$: single parton fragmentation

$$\frac{d\sigma}{dp_\perp}(h_1 h_2 \to J/\psi(p_\perp) + X) = \int dx_1 dx_2 dz \frac{d\hat{\sigma}(z, x_1, x_2)}{dp_\perp}$$

$$\times f_{i/h_1}(x_1) f_{j/h_2}(x_2) D_{k \to J/\psi}(z)$$

Leading contributions in v scales as ($v^2 \approx \alpha_s(2m_c) \approx 0.3$)

$$\mathrm{CS}\ {}^3S_1 \sim \alpha_s^2(p_\perp)\alpha_s^3(2m_c)v^3$$

$$\mathrm{C0}\ {}^1S_0,\ {}^3P_J \sim \alpha_s^2(p_\perp)\alpha_s^2(2m_c)v^5$$

$$\mathrm{C0}\ {}^3S_1, \sim \alpha_s^2(p_\perp)\alpha_s(2m_c)v^7$$

IMPROVED FACTORIZATION APPROACH

Example: J/ψ production

NLO in $\frac{(2m_c)}{p_\perp}$: double parton fragmentation

$$\frac{d\sigma}{dp_\perp}(h_1 h_2 \to J/\psi(p_\perp) + X) = \int dx_1 dx_2 du dv dz \frac{\hat{\sigma}(u, v, z, x_1, x_2)}{dp_\perp}$$

$$\times f_{i/h_1}(x_1) f_{j/h_2}(x_2) D_{J/\psi}^{(Q\bar{Q})}(u, v, z)$$

Leading contributions in v scales as

$$CS \sim \alpha_s^3(p_\perp) v^3 \frac{(2m_c)^2}{p_\perp^2}$$

$$CO \sim \alpha_s^3(p_\perp) v^7 \frac{(2m_c)^2}{p_\perp^2}$$ subleading, generated by RGE

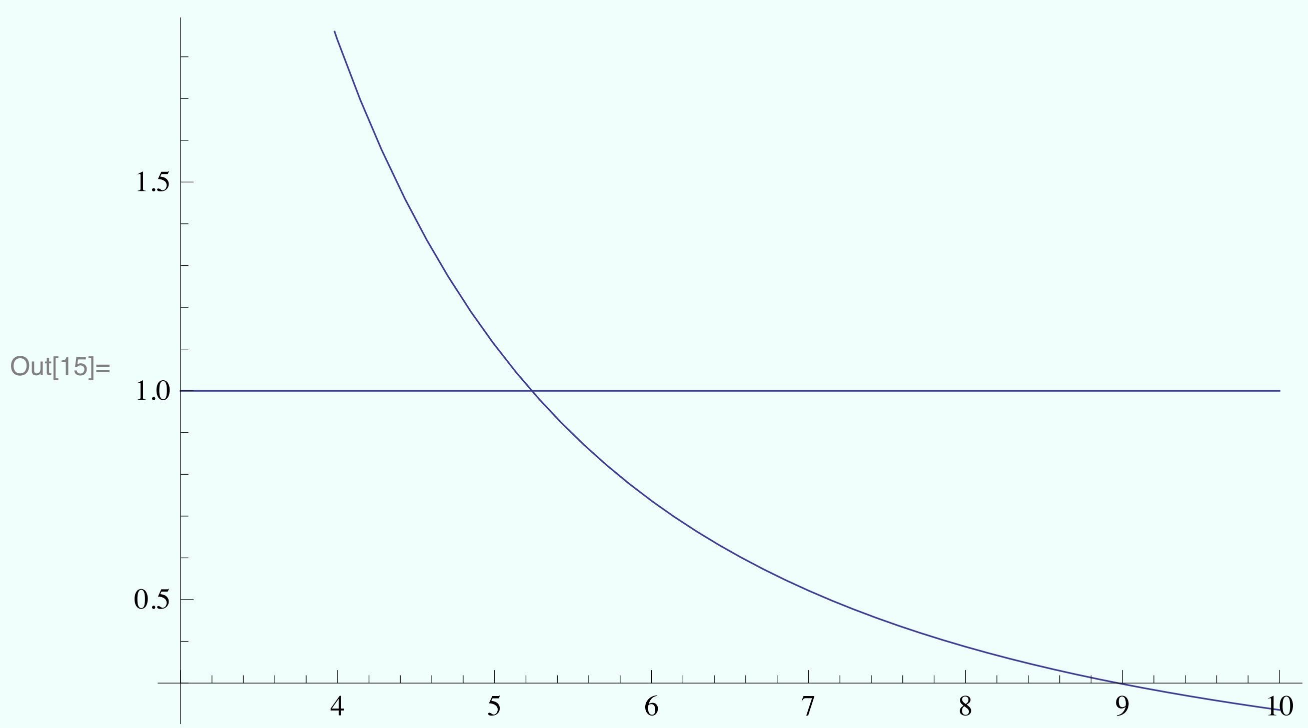
IMPROVED FACTORIZATION APPROACH

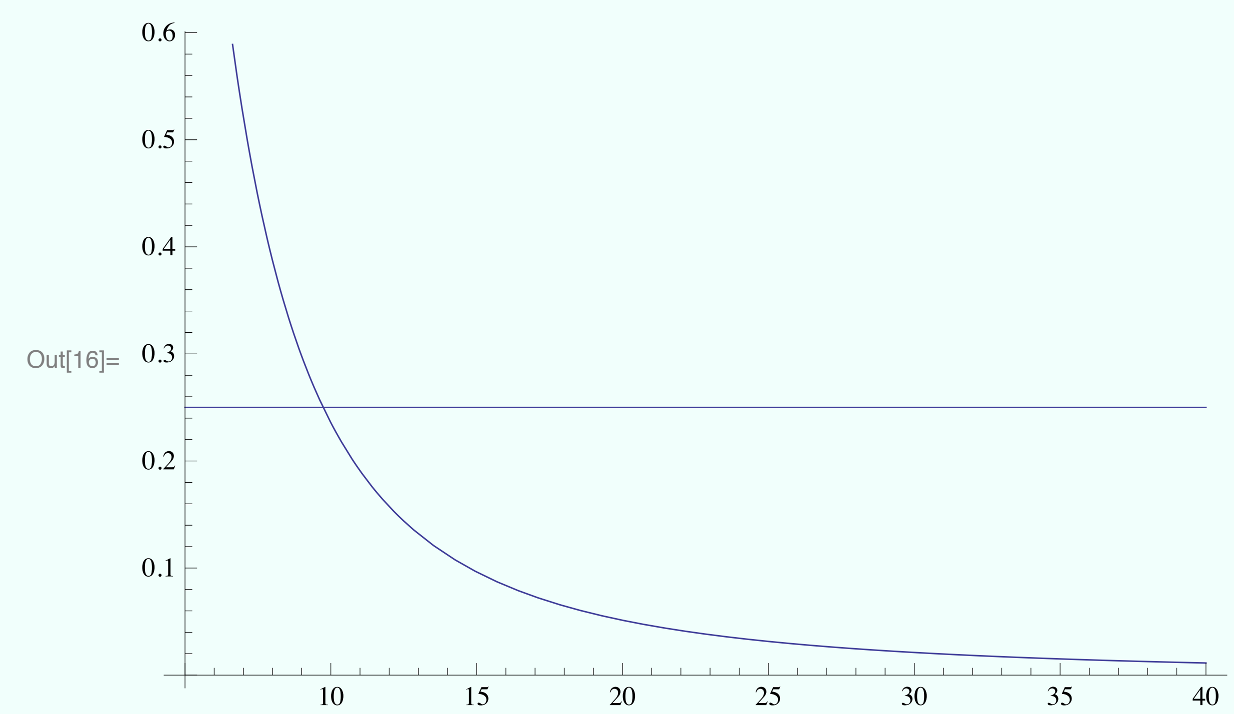
Example: J/ψ production, when is NLO in $\frac{(2m_c)}{p_\perp}$ important?

Ratio of NLO in $\frac{(2m_c)}{p_\perp}$ to LO in $\frac{(2m_c)}{p_\perp}$

$$\sim \frac{\alpha_s(p_\perp)(2m_c)^2}{v^4 p_\perp^2}$$

For what values of $p_\perp$ is this order one?



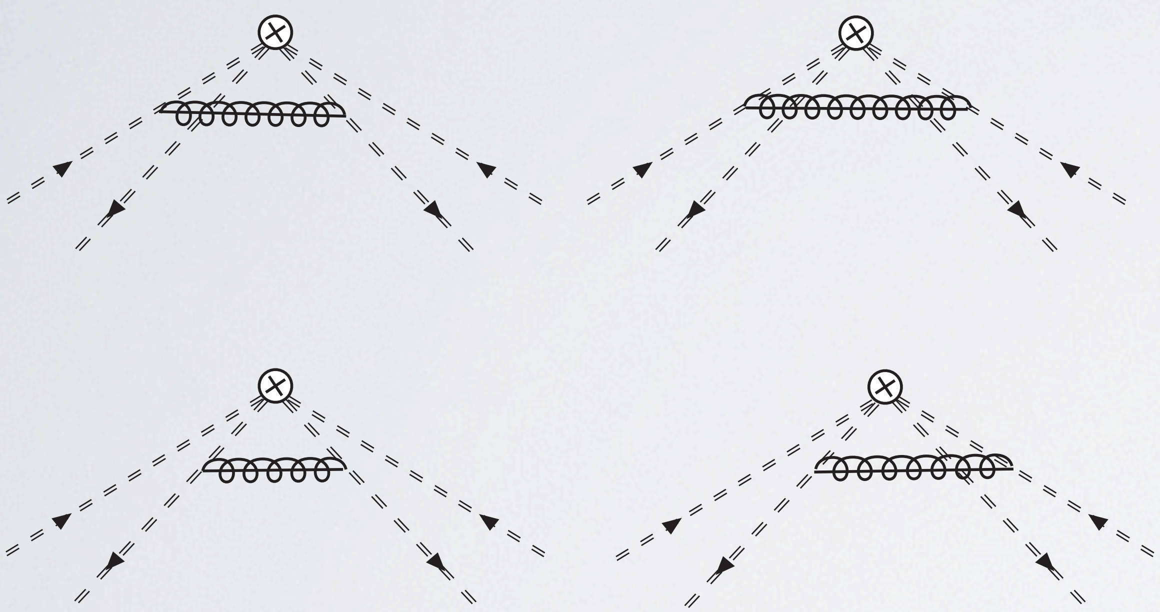


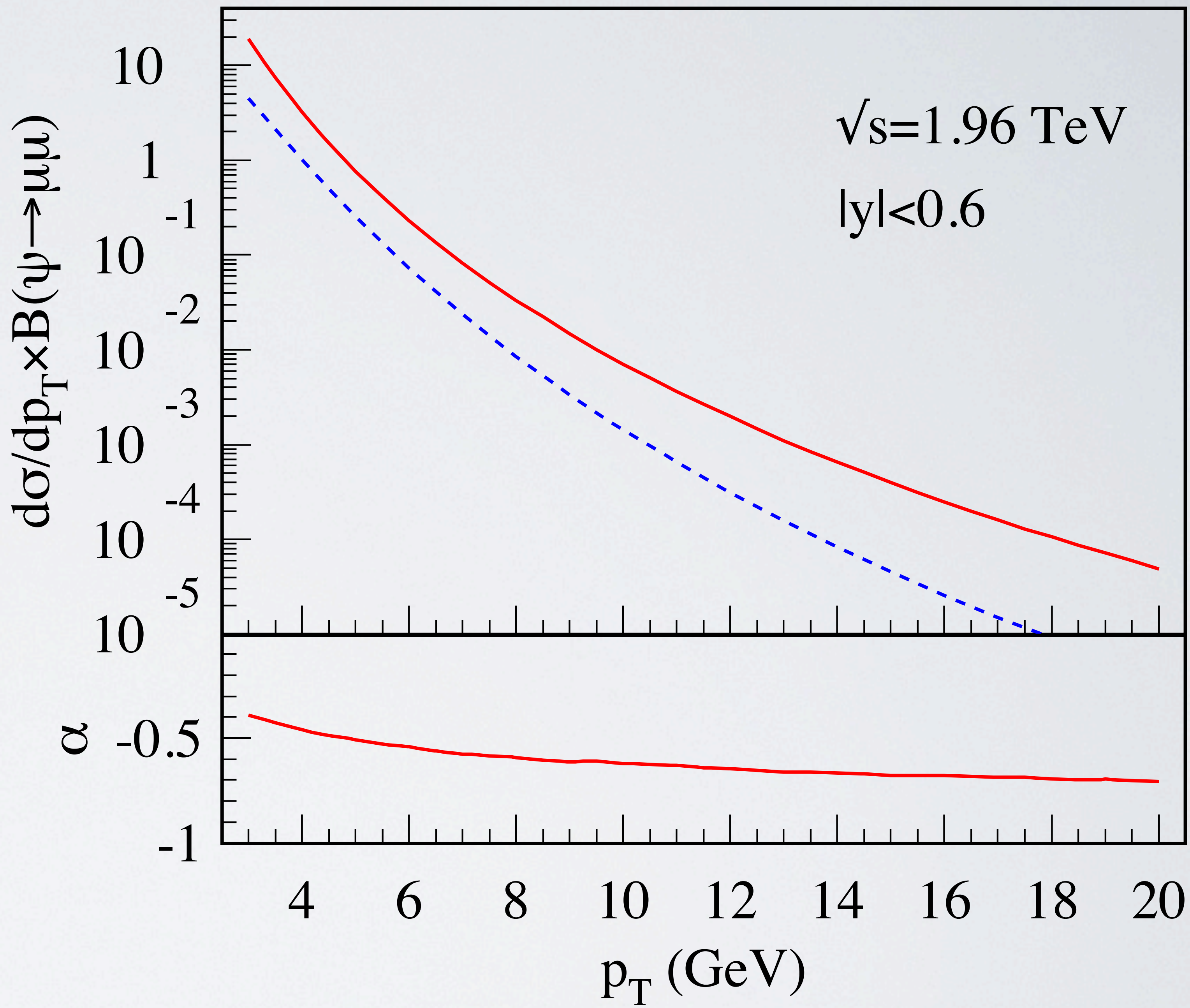
POLARIZATION FIX? MAYBE!

(Kang, Qiu, Sterman)

One-Loop Estimate

$Q\bar{Q}(8) \rightarrow Q\bar{Q}(1)$ Mixing





Still a factor of ~ 5 smaller than Octet Fragmentation

SUMMARY

- Solid NLO analysis of quarkonium production at moderate transverse momentum
- Reorganize calculation at large transverse momentum using improved NRQCD factorization, sum logs of $\frac{(2m_c)}{p_\perp}$
- Polarization is the big question?!?!
  - Are the NRQCD production matrix elements really universal?
  - CDF measurements need to be confirmed. More data please