NNLO corrections to (gluonic) jet production

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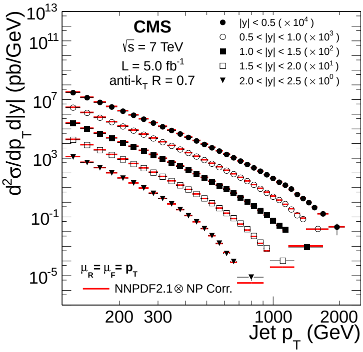
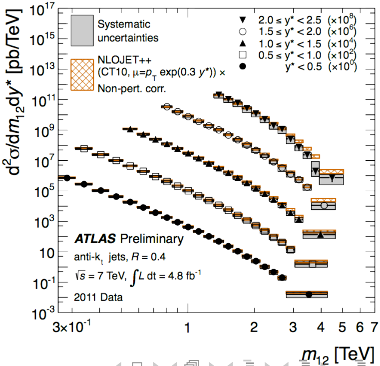
Outline

- Motivation for jet cross sections at NNLO
- Features of the NNLO calculation
- Antenna subtraction method
    - the double-virtual contribution
- Numerical results
- Conclusions and future work

INCLUSIVE JET AND DIJET CROSS SECTIONS

- look at the production of jets of hadrons with large transverse energy in
  - inclusive jet events $pp \to j + X$
  - exclusive dijet events $pp \to 2j$
- cross sections measured as a function of the jet p_T, rapidity y and dijet invariant mass m_{jj} in double differential form





INCLUSIVE JET AND DIJET CROSS SECTIONS

state of the art:

- dijet production is completely known in NLO QCD [Ellis, Kunszt, Soper '92], [Giele, Glover, Kosower '94], [Nagy '02]
- NLO+Parton shower [Alioli, Hamilton, Nason, Oleari, Re '11]
- threshold corrections [Kidonakis, Owens '00]

Goal:

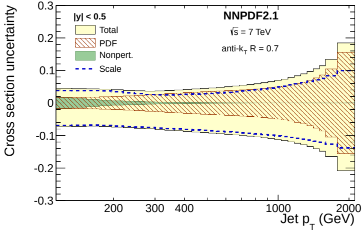
- obtain the jet cross sections at NNLO accuracy in double differential form

$$\frac{d^2\sigma}{dp_T d|y|} \qquad \frac{d^2\sigma}{dm_{jj} dy^*}$$

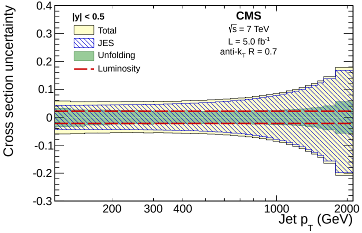
this talk:

- NNLO inclusive jet and dijet cross section (gluons only, leading colour)

THEORETICAL VS EXPERIMENTAL UNCERTAINTIES



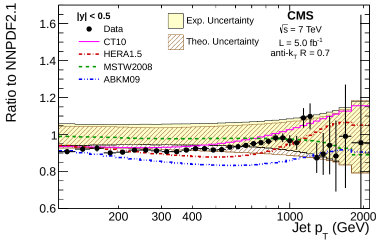
relative theoretical uncertainties for the inclusive jet production (NLO theory input) [CMS, arXiv:1212.6660]



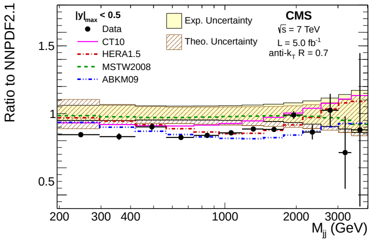
relative experimental uncertainties for the inclusive jet production [CMS, arXiv:1212.6660]

- residual uncertainty due to scale choice at NNLO expected at $\approx$ few percent level
- jet energy scale uncertainty has been determined to less than 5% for central jets $\rightarrow$ expect steady improvement with higher statistics
- theoretical prediction with the same precision as the experimental data $\rightarrow$ need pQCD predictions at NNLO accuracy

INCLUSIVE JET AND DIJET CROSS SECTIONS



(NLO theory input)
[CMS, arXiv:1212.6660]

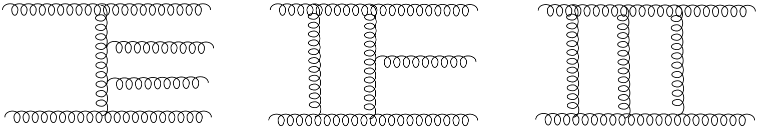


(NLO theory input)
[CMS, arXiv:1212.6660]

Phenomenological applications with NNLO:

- data can be used to constrain parton distribution functions
- size of NNLO correction important for precise determination of PDF's
- inclusion of jet data in NNLO parton distribution fits requires NNLO corrections to jet cross sections
- α_s determination from hadronic jet observables limited by the unknown higher order corrections

$pp \to 2j$ AT NNLO: GLUONIC CONTRIBUTIONS



$A_6^{(0)}(gg \to gggg)$ $A_5^{(1)}(gg \to ggg)$ $A_4^{(2)}(gg \to gg)$

[Berends, Giele '87], [Mangano, Parke, Xu '87], [Britto, Cachazo, Feng '06]
[Bern, Dixon, Kosower '93]
[Anastasiou, Glover, Oleari, Tejeda-Yeomans '01],[Bern, De Freitas, Dixon '02]

$$d\hat{\sigma}_{NNLO} = \int_{d\Phi_4} d\hat{\sigma}^{RR}_{NNLO} + \int_{d\Phi_3} d\hat{\sigma}^{RV}_{NNLO} + \int_{d\Phi_2} d\hat{\sigma}^{VV}_{NNLO}$$

- explicit infrared poles from loop integrations
- implicit poles in phase space regions for single and double unresolved gluon emission
- procedure to extract the infrared singularities and assemble all the parts in a parton-level generator

NNLO IR subtraction schemes

- sector decomposition: expansions in distributions, numerical integration [Binoth, Heinrich '02], [Anastasiou, Melnikov, Petriello '03]
  - $pp \to H$ [Anastasiou, Melnikov, Petriello '04]
  - $pp \to V$ [Melnikov, Petriello '06]
- q_T-subtraction for colorless high-mass systems [Catani, Grazzini '07]
  - $pp \to H$ [Catani, Grazzini '07]
  - $pp \to V$ [Catani, Cieri, Ferrera, de Florian, Grazzini '09]
  - $pp \to VH$ [Ferrera, Grazzini, Tramontano '11]
  - $pp \to \gamma\gamma$ [Catani, Cieri, de Florian, Grazzini '11]
- sector decomposition combined with subtraction [Czakon' 11], [Boughezal, Melnikov, Petriello '11]
  - $pp \to t\bar{t}$ [Baernreuther, Czakon, Fiedler, Mitov '13]
  - $pp \to Hj$ (gluons only) [Boughezal, Caola, Melnikov, Petriello, Schulze '13]
- antenna subtraction [Gehrmann-De Ridder, Gehrmann, Glover '05]
  - $e\bar{e} \to 3j$ [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07], [Weinzierl 08]
  - $pp \to 2j$ (gluons only) [Gehrmann-De Ridder, Gehrmann, Glover, JP '13]

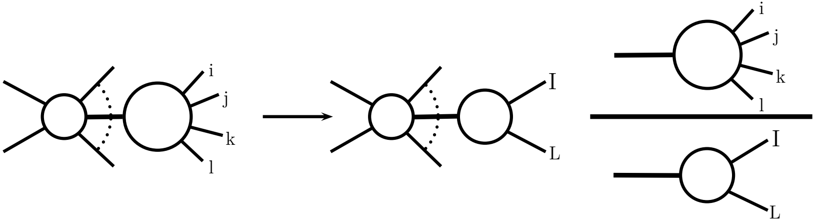
NNLO ANTENNA SUBTRACTION

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{NNLO} &= \int_{\mathrm{d}\Phi_4} \left(\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right) \\ &+ \int_{\mathrm{d}\Phi_3} \left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right) \\ &+ \int_{\mathrm{d}\Phi_2} \left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right) \end{aligned}$$

- $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$: real radiation subtraction term for $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$
- $\mathrm{d}\hat{\sigma}^{T}_{NNLO}$: one-loop virtual subtraction term for $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$
- $\mathrm{d}\hat{\sigma}^{U}_{NNLO}$: two-loop virtual subtraction term for $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$
- subtraction terms constructed using the antenna subtraction method at NNLO for hadron colliders $\rightarrow$ presence of initial state partons to take into account
- contribution in each of the round brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically

NNLO antenna subtraction

- universal factorisation of both colour ordered matrix elements and the $(m+2)$-particle phase space $\to$ colour connected unresolved particles



$$|M_{m+4}(\ldots,i,j,k,l,\ldots)|^2\, J(\{p_{m+4}\}) \longrightarrow |M_{m+2}(\ldots,I,L,\ldots)|^2\, J(\{p_{m+2}\}) \cdot X_4^0(i,j,k,l)$$

- phase-space factorisation

$$\mathrm{d}\Phi_{m+2}(p_a,\ldots,p_i,p_j,p_k,p_l,\ldots,p_{m+2}) = \mathrm{d}\Phi_m(p_a,\ldots,p_I,p_L,\ldots,p_{m+2})\, \mathrm{d}\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l)$$

- integrated antennae is the inclusive integral

$$\mathcal{X}^0_{ijkl}(s_{ijkl}) = \frac{1}{C(\epsilon)^2}\int \mathrm{d}\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l) X_4^0(i,j,k,l)$$

INTEGRATED ANTENNAE

- antennae integrals are performed once and for all to become universal building blocks for subtraction of IR singularities at NNLO
- massless antennae ($m = 0$)

	NLO	NNLO
final-final	✓[1]	✓[1]
initial-final	✓[2]	✓[3]
initial-initial	✓[2]	✓[4,5,6]

[1] A. Gehrmann-De Ridder, T. Gehrmann and E. W. N. Glover, *JHEP* **09** (2005) 056 [hep-ph/0505111];

[2] A. Daleo, T. Gehrmann and D. Maître, *JHEP* **04** (2007) 016 [hep-ph/0612257];

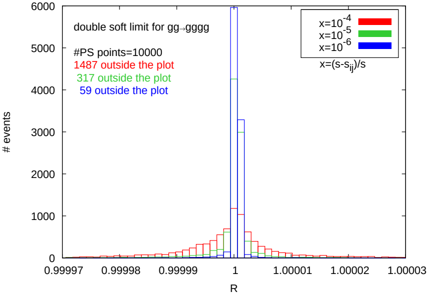
[3] A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann and G. Luisoni, *JHEP* **01** (2010) 118 [0912.0374];

[4] R. Boughezal, A. Gehrmann-De Ridder and M. Ritzmann, *JHEP* **02** (2011) 098 [1011.6631];

[5] T. Gehrmann, P.F. Monni, *JHEP* **12** (2011) 049 [1107.4037];

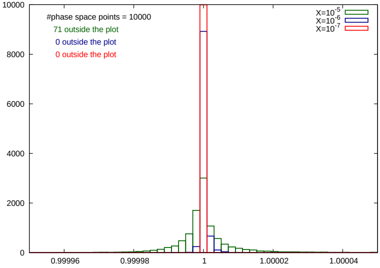
[6] A. Gehrmann-De Ridder, T. Gehrmann and M. Ritzmann, *JHEP* **10** (2012) 047 [1207.5779];

NNLO CORRECTIONS TO $pp \to 2j$



Double-real contribution

- $d\sigma^{RR}_{NNLO}/d\sigma^{S}_{NNLO}$ (gluons only)
- numerical convergence between double-real matrix element $d\sigma^{RR}_{NNLO}$ and antenna subtraction term $d\sigma^{S}_{NNLO}$ tested in all soft and collinear phase space regions [N.Glover, JP]



Real-virtual contribution

- $d\sigma^{RV}_{NNLO}/d\sigma^{T}_{NNLO}$ (gluons only)
- local (pointwise) analytic cancellation of explicit ϵ-poles
- numerical convergence tested in all soft or collinear phase space regions [A.Gehrmann-De Ridder, N.Glover, JP]

DOUBLE-VIRTUAL CONTRIBUTION

$$\int_{\mathrm{d}\Phi_m} \left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO} \right)$$

- renormalized $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$ contains explicit infrared ϵ-poles
- $\mathrm{d}\hat{\sigma}^{U}_{NNLO}$ is made up of integrated subtraction terms from the double-real radiation and real-virtual radiation
- initial-state collinear singularities absorbed by the mass factorization counterterm $\mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}$

$$\mathrm{d}\sigma^{U}_{NNLO} = -\int_2 \mathrm{d}\hat{\sigma}^{S}_{NNLO} - \int_1 \mathrm{d}\hat{\sigma}^{VS}_{NNLO} - \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}$$

- to show explicit pole cancellation at NNLO recast integrated subtraction terms and mass factorization contribution in a form of a convolution integral evaluated analytically

	gg	qg	qq
$\Gamma^1 \otimes \Gamma^1$	✓[1]	✓	✓
$\mathcal{X}_3^0 \otimes \Gamma^1$	✓[1]	✓	✓
$\mathcal{X}_3^0 \otimes \mathcal{X}_3^0$	✓[1]	✓	✓

[1] [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, JP]

Double-virtual contribution

- new structures arise made from the integrated antennae building blocks

$$\mathbb{X}_3^0(\bar{1}_g,\bar{2}_g,i_g,j_g;z_1,z_2) = \mathcal{F}_3^0(s_{\bar{1}\bar{2}},z_1,z_2) + \frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}i},z_1,z_2) + \frac{1}{3}\mathcal{F}_3^0(s_{ij},z_1,z_2) + \frac{1}{2}\mathcal{F}_3^0(s_{j\bar{1}},z_1,z_2)$$

- integrated antennae string with the mass factorization contribution is in direct connection with the I_1 operator of Catani at NLO

$$-2\boldsymbol{I}^{(1)}(\epsilon;\bar{1}_g,\bar{2}_g,i_g,j_g;z_1,z_2) \quad = \quad \mathbb{X}_3^0(\bar{1}_g,\bar{2}_g,i_g,j_g;z_1,z_2) - \Gamma^{(1)}_{gg;gg}(z_1,z_2)$$

- similarly at NNLO the integrated antennae convolution integrals together with the mass factorization contribution yield in the double-virtual channel

$$-2\boldsymbol{I}^{(1)}(\epsilon;\bar{1}_g,\bar{2}_g,i_g,j_g;z_1,z_2)^2 = \left(\mathbb{X}_3^0 - \Gamma^{(1)}_{gg;gg}\right) \otimes \left(\mathbb{X}_3^0 - \Gamma^{(1)}_{gg;gg}\right)(\bar{1}_g,\bar{2}_g,i_g,j_g;z_1,z_2)$$

- double virtual antennae subtraction term $\mathrm{d}\sigma^{U}_{NNLO}$ written compactly rederives the predicted Catani pole structure of the two-loop contribution in the antennae language

$pp \to 2j$ AT NNLO

[A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, JP]

Implementation checks (gluons only channel at leading colour):

- subtraction terms correctly approximate the matrix elements in all unresolved configurations of partons j, k

$$\mathrm{d}\hat{\sigma}^{RR,RV}_{NNLO} \xrightarrow{\forall\{j,k\},\{j\}\to 0} \mathrm{d}\hat{\sigma}^{S,T}_{NNLO}$$

- local (pointwise) analytic cancellation of all infrared explicit ϵ-poles when integrated subtraction terms are combined with one, two-loop matrix elements

$$\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right) = 0$$

$$\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right) = 0$$

- process independent NNLO subtraction scheme
- singularities in intermediate steps cancel analytically
- allows the computation of multiple differential distributions in a single program run

Numerical setup

[A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, JP]

- jets identified with the anti-k_T jet algorithm
- jets accepted at rapidities $|y| < 4.4$
- leading jet with transverse momentum $p_t > 80$ GeV
- subsequent jets required to have at least $p_t > 60$ GeV
- MSTW2008nnlo PDF
- dynamical factorization and renormalization central scale equal to the leading jet p_T ($\mu_R = \mu_F = \mu = p_{T1}$)

NUMERICAL SETUP

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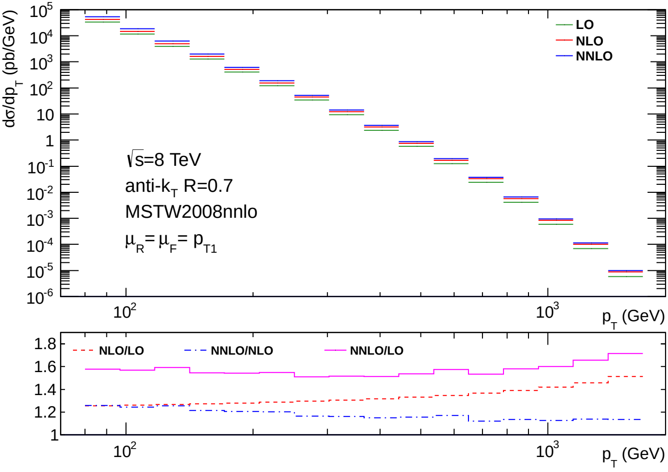
Integrated cross section results (gluons only channel) with scale variations

	$\sigma^{8TeV-LO}_{incl.jet}(pb)$	$\sigma^{8TeV-NLO}_{incl.jet}(pb)$	$\sigma^{8TeV-NNLO}_{incl.jet}(pb)$
$\mu = 0.5p_{T1}$	$(12.586 \pm 0.001) \times 10^5$	$(11.299 \pm 0.001) \times 10^5$	$(15.33 \pm 0.03) \times 10^5$
$\mu = p_{T1}$	$(9.6495 \pm 0.001) \times 10^5$	$(12.152 \pm 0.001) \times 10^5$	$(15.20 \pm 0.02) \times 10^5$
$\mu = 2.0p_{T1}$	$(7.5316 \pm 0.001) \times 10^5$	$(11.824 \pm 0.001) \times 10^5$	$(15.21 \pm 0.01) \times 10^5$

- NNLO result increased by about 25% with respect to the NLO cross section

INCLUSIVE JET p_T DISTRIBUTION

[A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, JP]



- NNLO effect stabilizes the NLO k-factor growth with p_T

INCLUSIVE JET p_T DISTRIBUTION

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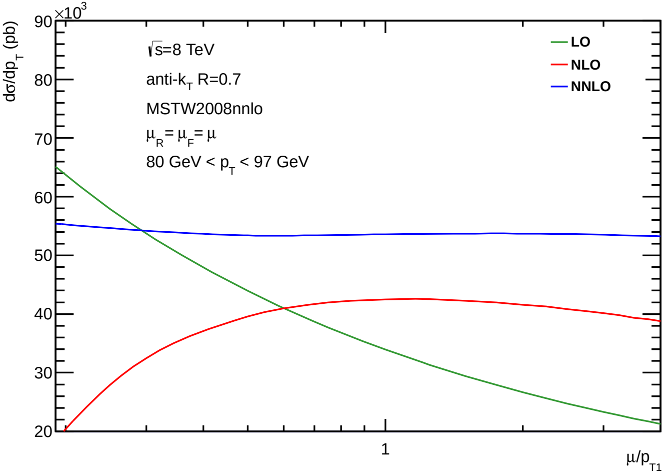
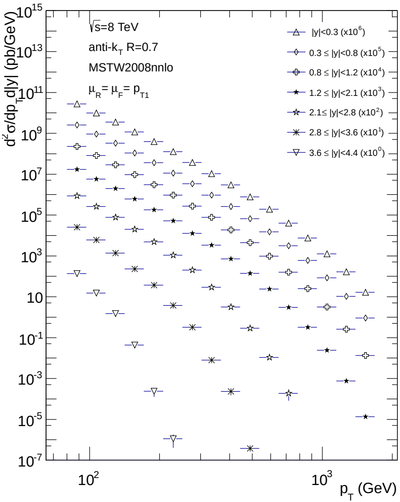
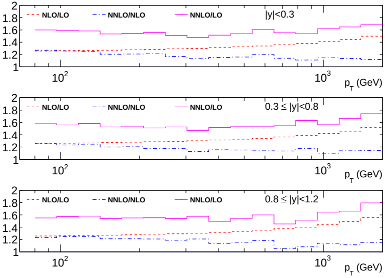


- flat scale dependence at NNLO

INCLUSIVE JET p_T DISTRIBUTION $R = 0.7$

- double differential inclusive jet p_T distribution at NNLO (gluons only)



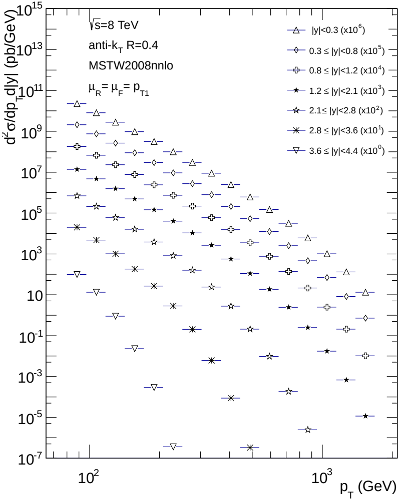


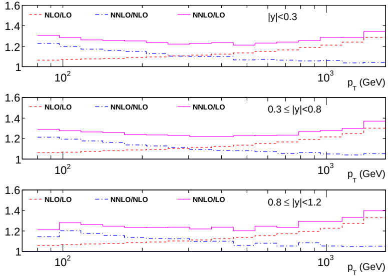
double differential k-factors

- NNLO result varies between 25% to 12% with respect to the NLO cross section
- similar behaviour between the rapidity slices

INCLUSIVE JET p_T DISTRIBUTION $R = 0.4$

- double differential inclusive jet p_T distribution at NNLO (gluons only)



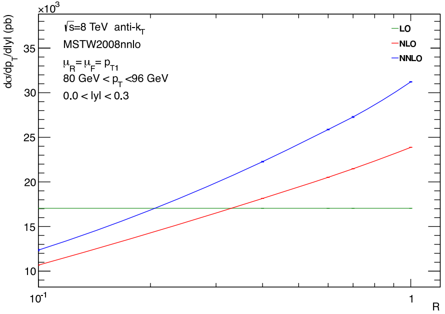


double differential k-factors

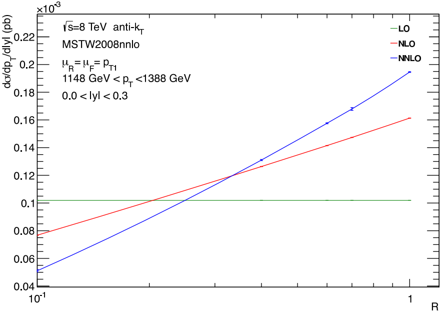
- NNLO result varies between 20% to 5% with respect to the NLO cross section
- similar behaviour between the rapidity slices

INCLUSIVE JET p_T DISTRIBUTION

$80\text{GeV} < p_T < 96\text{GeV}$



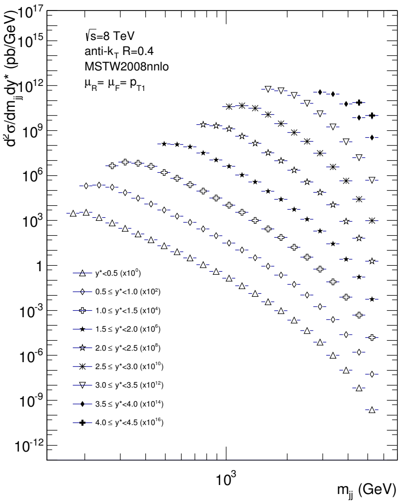
$1148\text{GeV} < p_T < 1388\text{GeV}$

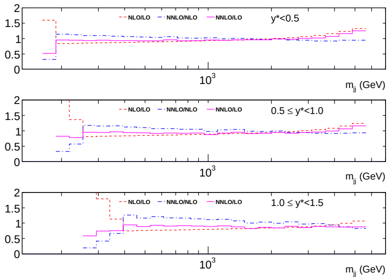


- inclusive jet cross section versus R
- can the NNLO cross section describe data for different values of R simultaneously at low and high jet p_T?

EXCLUSIVE DIJET MASS DISTRIBUTION $R = 0.4$

- double differential dijet mass distribution at NNLO (gluons only)





double differential k-factors

- NNLO corrections up to 20% with respect to the NLO cross section
- corrections increase slightly for large $y^* = 1/2|y_1 - y_2|$

Conclusions

- antenna subtraction method generalised for the calculation of NNLO QCD corrections for exclusive collider observables with partons in the initial-state
- non-trivial check of analytic cancellation of infrared singularities between double-real, real-virtual and double-virtual corrections
- proof-of principle implementation of the $gg \to gg$ contribution to $pp \to 2j$ at NNLO in the new NNLOJET parton-level generator
- computation of multiple differential distributions at NNLO in a single program run $\to$ experimentalists input welcome

Future work:

- go beyond gluons only leading colour approximation
- include remaining partonic subprocesses
  - 4g2q processes [Currie, Glover, Wells '13]
  - 2g4q processes
  - 6q processes