# NLO merging with parton showers

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LoopFest XII

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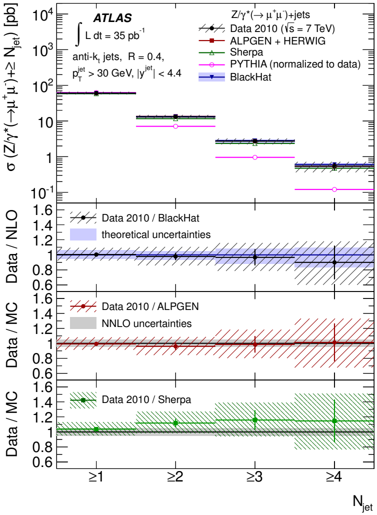
# Status of Z+Jets

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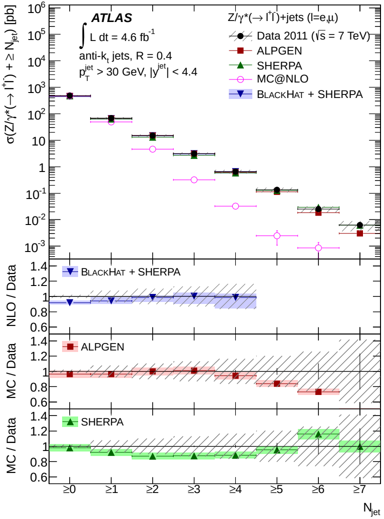


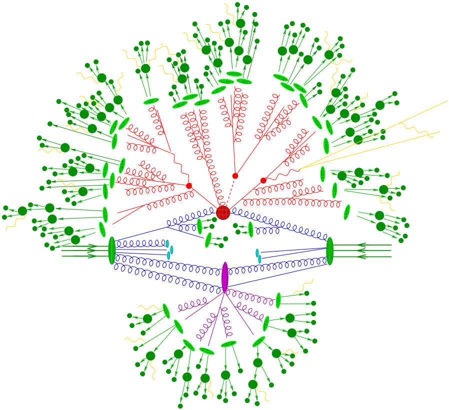

[ATLAS] PRD85(2012)032009

[ATLAS] arXiv:1304.7098

# Outline

- Matching NLO QCD calculations and parton showers with MC@NLO
- Merging matched calculations for varying jet multiplicity

- Aim: NLO-accurate, inclusive MC simulation of X+jets



# Evolution of matching and merging methods

Incomplete and personally biased, just to show the time scales

- Matrix-element corrections to parton showers
  [Bengtsson,Sjöstrand] PLB185(1987)435, [Seymour] CPC90(1995)95
- ME↔PS correspondence, truncated PS [André,Sjöstrand] PRD57(1998)5767
- CKKW merging [Catani,Krauss,Kuhn,Webber] JHEP11(2001)063 JHEP08(2002)015
  Combines LO calculations of varying parton multiplicity
- MC@NLO [Frixione,Webber] JHEP06(2002)029
  Modified subtraction to remove overlap between ME & PS at NLO
- POWHEG [Nason] JHEP11(2004)040, [Frixione,Nason,Oleari] JHEP11(2007)070
  Combination of MC@NLO and ME-corrected parton shower
- $NL^3$ merging [Lavesson,Lönnblad] JHEP12(2008)070
  Combines NLO calculations of varying jet multiplicity, explicit subtraction
- MENLOPS [Hamilton,Nason] JHEP06(2010)039 [Krauss,Schönherr,Siegert,SH] JHEP08(2011)123
  Combines lowest multiplicity NLO with higher-multiplicity LO

# Recent developments

- in SHERPA [Gehrmann,Krauss,Schönherr,Siegert,SH] arXiv:1207.5031 arXiv:1207.5030
  Implicit subtraction technique with truncated pseudo-showers
- in PYTHIA [Lönnblad,Prestel] arXiv:1211.4827 arXiv:1211.7278
  Explicit subtraction with truncated pseudo-showers (& unitarization)
- in MADGRAPH [Frederix,Frixione] arXiv:1209.6215
  CKKW-like approach with analytic Sudakov factors and no truncated showers

# Basics of NLO+PS matching

- Leading-order calculation for observable $O$

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\,O(\Phi_B)$$

- NLO calculation for same observable

$$\langle O\rangle = \int \mathrm{d}\Phi_B\left\{\mathrm{B}(\Phi_B)+\tilde{V}(\Phi_B)\right\}O(\Phi_B)+\int \mathrm{d}\Phi_R\,\mathrm{R}(\Phi_R)\,O(\Phi_R)$$

- Parton-shower result until first emission

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\left[\Delta^{(\mathrm{K})}(t_c)\,O(\Phi_B)+\int_{t_c}\mathrm{d}\Phi_1\,\mathrm{K}(\Phi_1)\,\Delta^{(\mathrm{K})}(t(\Phi_1))\,O(\Phi_R)\right]$$

$$\overset{\mathcal{O}(\alpha_s)}{\rightarrow}\int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\left\{1-\int_{t_c}\mathrm{d}\Phi_1\mathrm{K}(\Phi_1)\right\}O(\Phi_B)+\int_{t_c}\mathrm{d}\Phi_B\mathrm{d}\Phi_1\,\mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)\,O(\Phi_R)$$

Phase space: $\mathrm{d}\Phi_1 = \mathrm{d}t\,\mathrm{d}z\,\mathrm{d}\phi\,J(t,z,\phi)$
Splitting functions: $\mathrm{K}(t,z)\rightarrow\alpha_s/(2\pi t)\sum\mathrm{P}(z)\,\Theta(\mu_Q^2-t)$
Sudakov factors: $\Delta^{(\mathrm{K})}(t) = \exp\left\{-\int_t \mathrm{d}\Phi_1\mathrm{K}(\Phi_1)\right\}$

# Basics of NLO+PS matching

- Subtract $\mathcal{O}(\alpha_s)$ PS terms from NLO result

$$\int \mathrm{d}\Phi_B \left\{ \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{B}(\Phi_B) \int \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1) \right\} \ldots$$

$$+ \int \mathrm{d}\Phi_R \left\{ \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) \right\} \ldots$$

- In DLL approximation both terms finite $\rightarrow$
  MC events in two categories, $\mathbb{S}$tandard and $\mathbb{H}$ard

$$\mathbb{S} \;\rightarrow\; \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{B}(\Phi_B) \int \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)$$

$$\mathbb{H} \;\rightarrow\; \mathrm{H}^{(\mathrm{K})} = \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\mathrm{K}(\Phi_1)$$

- Color & spin correlations $\rightarrow$ NLO subtraction needed
  $1/N_c$ corrections can be faded out in soft region by smoothing function

$$\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \left[ \mathrm{S}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) \right] f(\Phi_1)$$

$$\mathrm{H}^{(\mathrm{K})}(\Phi_R) = \left[ \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) \right] f(\Phi_1)$$

# MC@NLO

[Frixione,Webber] JHEP06(2002)029

- Add parton shower, described by generating functional $\mathcal{F}_{\rm MC}$

$$\langle O\rangle = \int {\rm d}\Phi_B\,\bar{\rm B}^{\rm (K)}(\Phi_B)\,\mathcal{F}^{(0)}_{\rm MC}(\mu_Q^2,O) + \int {\rm d}\Phi_R\,{\rm H}^{\rm (K)}(\Phi_R)\,\mathcal{F}^{(1)}_{\rm MC}(t(\Phi_R),O)$$

  Probability conservation $\leftrightarrow \mathcal{F}_{\rm MC}(t,1)=1$
- Expansion of matched result until first emission

$$\langle O\rangle = \int {\rm d}\Phi_B\bar{\rm B}^{\rm (K)}(\Phi_B)\bigg[\Delta^{\rm (K)}(t_c)\,O(\Phi_B) + \int_{t_c}{\rm d}\Phi_1{\rm K}(\Phi_1)\Delta^{\rm (K)}(t(\Phi_1))\,O(\Phi_r)\bigg] + \int {\rm d}\Phi_R\,{\rm H}^{\rm (K)}(\Phi_{n+1})\,O(\Phi_R)$$

- Parametrically $\mathcal{O}(\alpha_s)$ correct
- Preserves logarithmic accuracy of PS

# Handling soft singularities in MC@NLO

Method 1 [Frixione,Webber] JHEP06(2002)029

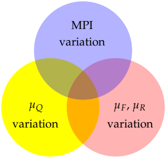
- $f(\Phi_1) \to 0$ in soft-gluon limit
- Full NLO only in hard / collinear region
  Missing subleading color terms in soft domain
- Only affects unresolved gluons $\to$ no need to correct

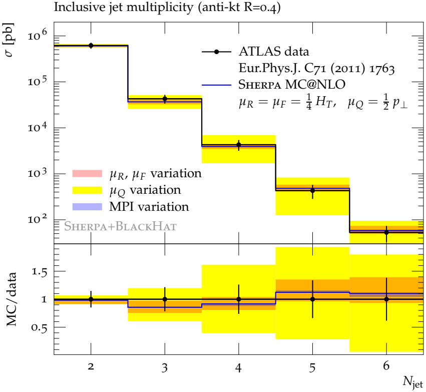
Method 2 [Krauss,Schönherr,Siegert,SH] JHEP09(2012)049

- Replace $\mathrm{B}(\Phi_B)\mathrm{K}(\Phi_1) \to \mathrm{S}(\Phi_R)$, i.e. include color & spin correlations
- May lead to non-probabilistic $\Delta^{(\mathrm{S})}(t)$
  Requires modification of veto algorithm
- Exact cancellation of all divergences without additional smoothing
  Equivalent to one-step full color parton shower algorithm

# MC@NLO in Sherpa

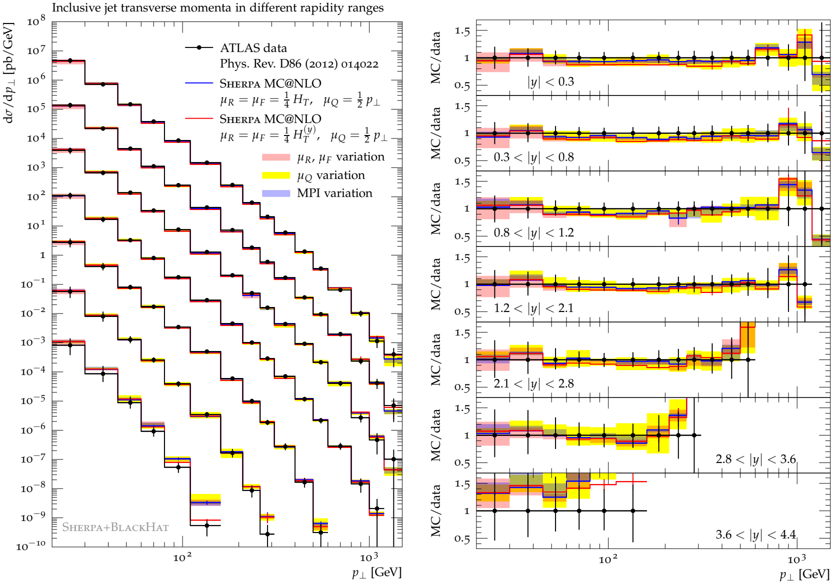
- Automated using CS subtraction
  [Catani,Seymour] NPB485(1997)291
  [Gleisberg,Krauss] EPJC53(2008)501
  [Schumann,Krauss] JHEP03(2008)038
- Validated in QCD jets production
  - CT10, $\alpha_s(M_Z) = 0.118$
  - Full hadron level, incl. MPI
  - Virtual corrections $\rightarrow$ BlackHat
    [Berger et al.] PRD78(2008)036003
    [Giele,Glover,Kosower] NPB403(1993)633
  - $p_{T,j1} > 20$ GeV, $p_{T,j2} > 10$ GeV
  - $\mu_{R/F} = H_T/4$, $\mu_Q = p_T/2$
- Implementation allows to assess renormalization/factorization and resummation scale uncertainty



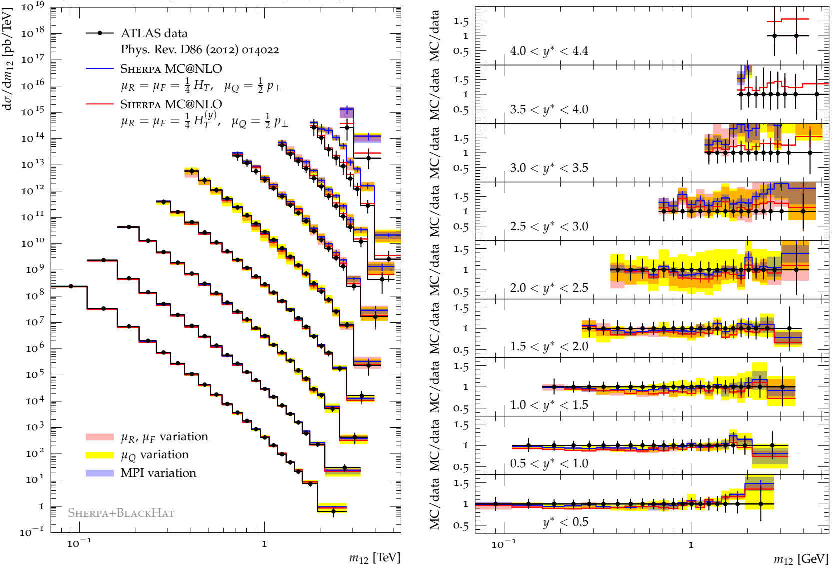

[Schönherr,SH] PRD86(2012)094042

# Inclusive jet production at the LHC



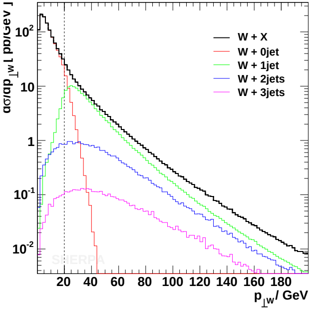
# Inclusive jet production at the LHC

Dijet invariant mass spectra in different rapidity ranges



# Basic idea of ME⊕PS merging

- Separate phase space into "hard" and "soft" region
- Matrix elements populate hard domain
- Parton shower populates soft domain



# Constructing ME⊕PS@NLO

- ME⊕PS for 0+1-jet in MC@NLO notation

$$\langle O\rangle = \int \mathrm{d}\Phi_B \mathrm{B}(\Phi_B)\left[\Delta^{(\mathrm{K})}(t_c)\,O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\,\Delta^{(\mathrm{K})}(t)\,\Theta(Q_{\mathrm{cut}} - Q)\,O(\Phi_R)\right]$$
$$+ \int \mathrm{d}\Phi_R\,\mathrm{R}(\Phi_R)\,\Delta^{(\mathrm{K})}(t(\Phi_R); > Q_{\mathrm{cut}})\,\Theta(Q - Q_{\mathrm{cut}})\,O(\Phi_R) + \ldots$$

- Reorder by parton multiplicity $k$, change notation $\mathrm{R}_k \to \mathrm{B}_{k+1}$
- Analyze exclusive contribution from $k$ hard partons only $(t_0 = \mu_Q^2)$

$$\langle O\rangle_k^{\mathrm{excl}} = \int \mathrm{d}\Phi_k\,\mathrm{B}_k \prod_{i=0}^{k-1} \Delta_i^{(\mathrm{K})}(t_{i+1}, t_i; > Q_{\mathrm{cut}})\,\Theta(Q_k - Q_{\mathrm{cut}})$$
$$\times \left[\Delta_k^{(\mathrm{K})}(t_c, t_k)\,O_k + \int_{t_c}^{t_k} \mathrm{d}\Phi_1\,\mathrm{K}_k\,\Delta_k^{(\mathrm{K})}(t_{k+1}, t_k)\,\Theta(Q_{\mathrm{cut}} - Q_{k+1})\,O_{k+1}\right]$$

# Constructing ME⊕PS@NLO

- Analyze exclusive contribution from $k$ hard partons

$$\langle O\rangle_k^{\rm excl} = \int {\rm d}\Phi_k\,{\rm B}_k\,\prod_{i=0}^{k-1}\Delta_i^{\rm (K)}(t_{i+1},t_i;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut})$$

$$\times\left[\Delta_k^{\rm (K)}(t_c,t_k)\,O_k+\int_{t_c}^{t_k}{\rm d}\Phi_1\,{\rm K}_k\,\Delta_k^{\rm (K)}(t_{k+1},t_k)\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}\right]$$

# Constructing ME⊕PS@NLO

- Analyze exclusive contribution from $k$ hard partons

$$\langle O\rangle_k^{\rm excl} = \int {\rm d}\Phi_k\,{\rm B}_k\,\prod_{i=0}^{k-1}\Delta_i^{\rm (K)}(t_{i+1},t_i;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut})$$

$$\times\left[\Delta_k^{\rm (D)}(t_c,t_k)\,O_k+\int_{t_c}^{t_k}{\rm d}\Phi_1\,\frac{\rm D_k}{\rm B_k}\,\Delta_k^{\rm (D)}(t_{k+1},t_k)\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}\right]$$

- PS evolution kernels → dipole terms

# Constructing ME⊕PS@NLO

- Analyze exclusive contribution from $k$ hard partons

$$\langle O\rangle_k^{\rm excl} = \int {\rm d}\Phi_k\,\bar{\rm B}_k^{\rm (K)}\,\prod_{i=0}^{k-1}\Delta_i^{\rm (K)}(t_{i+1},t_i;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut})$$

$$\times\left[\Delta_k^{\rm (D)}(t_c,t_k)\,O_k+\int_{t_c}^{t_k}{\rm d}\Phi_1\,\frac{\rm D_k}{\rm B_k}\,\Delta_k^{\rm (D)}(t_{k+1},t_k)\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}\right]$$

- PS evolution kernels → dipole terms
- Born matrix element → NLO-weighted Born

# Constructing ME⊕PS@NLO

- Analyze exclusive contribution from $k$ hard partons

$$\langle O\rangle_k^{\rm excl} = \int {\rm d}\Phi_k\,\bar{\rm B}_k^{\rm (K)} \prod_{i=0}^{k-1}\Delta_i^{\rm (K)}(t_{i+1},t_i;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut})$$

$$\times\left[\Delta_k^{\rm (D)}(t_c,t_k)\,O_k + \int_{t_c}^{t_k}{\rm d}\Phi_1\,\frac{\rm D_k}{\rm B_k}\,\Delta_k^{\rm (D)}(t_{k+1},t_k)\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}\right]$$

$$+\int{\rm d}\Phi_{k+1}\,{\rm H}_k^{\rm (D)}\,\Delta_k^{\rm (K)}(t_k;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut})\,\Theta(Q_{\rm cut}-Q_{k+1})\;O_{k+1}$$

- PS evolution kernels → dipole terms
- Born matrix element → NLO-weighted Born
- Add hard remainder function

# Constructing ME⊕PS@NLO

- Analyze exclusive contribution from $k$ hard partons

$$\langle O\rangle_k^{\rm excl} = \int {\rm d}\Phi_k\, \bar{\rm B}_k^{\rm (K)} \prod_{i=0}^{k-1} \Delta_i^{\rm (K)}(t_{i+1},t_i;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut})$$

$$\times \prod_{i=0}^{k-1}\left(1+\int_{t_{i+1}}^{t_i}{\rm d}\Phi_1 {\rm K}_i\,\Theta(Q_i-Q_{\rm cut})\right) F_i(t_{i+1},t_i;\mu_F^2)$$

$$\times\left[\Delta_k^{\rm (D)}(t_c,t_k)\,O_k+\int_{t_c}^{t_k}{\rm d}\Phi_1\,\frac{\rm D_k}{\rm B_k}\,\Delta_k^{\rm (D)}(t_{k+1},t_k)\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}\right]$$

$$+\int{\rm d}\Phi_{k+1}\,{\rm H}_k^{\rm (D)}\,\Delta_k^{\rm (K)}(t_k;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut})\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}$$

- PS evolution kernels → dipole terms
- Born matrix element → NLO-weighted Born
- Add hard remainder function
- Subtract $\mathcal{O}(\alpha_s)$ terms from truncated vetoed PS

# Generation of the PS counterterm

- Almost like a normal parton shower expression

$$\to \prod_{i=0}^{k-1}\left(1+\int_{t_{i+1}}^{t_i} \mathrm{d}\Phi_1 \mathrm{K}_i\, \Theta(Q_i - Q_{\mathrm{cut}})\right)\Delta_i^{(\mathrm{K})}(t_{i+1}, t_i; > Q_{\mathrm{cut}})$$

- Modified MC algorithm for NLO-vetoed truncated PS
  - Generate emission, stop if $t < t_{i+1}$
  - If $Q > Q_{\mathrm{cut}}$ in first emission,
    skip emission, i.e. do not modify event
  - Continue as for LO merging

# Strong couplings and PDFs

- PS dictates choice of renormalization scale ($n$ - order $\alpha_s$ in Born)

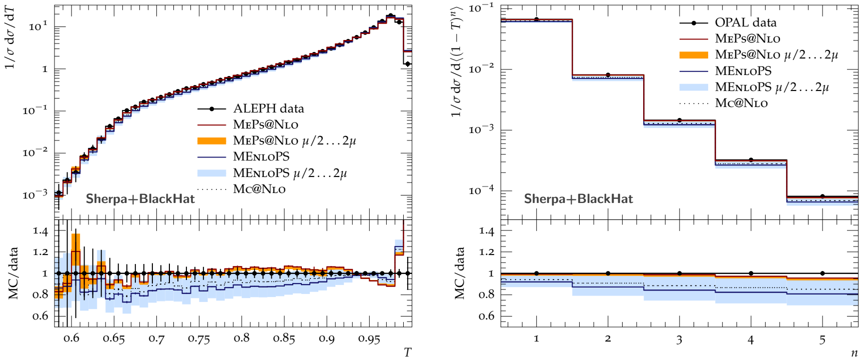
$$\Big[\alpha_s(\mu_R^2)\Big]^{n+k} = \Big[\alpha_s(\mu_{\rm core}^2)\Big]^n \prod_{i=0}^{k} \alpha_s(b\,k_T^2(t_i,z_i))\,, \qquad b = \text{const}$$

- Monotonicity of strong coupling allows to solve for $\mu_R$
- PS also dictates choice of factorization scales (DGLAP evolution)
- Introduces collinear counterterms due to constrained evolution

$$F_i(t,t';\mu_F^2) = 1 - \frac{\alpha_s(\mu_R^2)}{2\pi}\log\frac{t'}{t}\sum_{b=q,g}\int_{x_i}^1 \frac{{\rm d}z}{z}\,P_{ba}(z)\,\frac{f_b(x_i/z,\mu_F^2)}{f_a(x_i,\mu_F^2)}$$

# $e^+e^-\to$hadrons at LEP

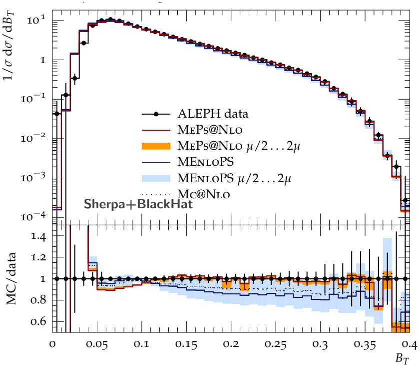
[Gehrmann,Krauss,Schönherr,Siegert,SH] arXiv:1207.5031




- Thrust & its moments
- MEPS@NLO with 2,3&4 jet PL at NLO plus 5&6 jet PL at LO vs MENLOPS with up to 6 jets at LO

# $e^+e^-\to$hadrons at LEP

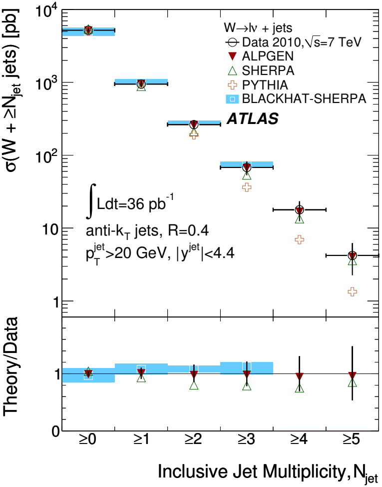
[Gehrmann,Krauss,Schönherr,Siegert,SH] arXiv:1207.5031

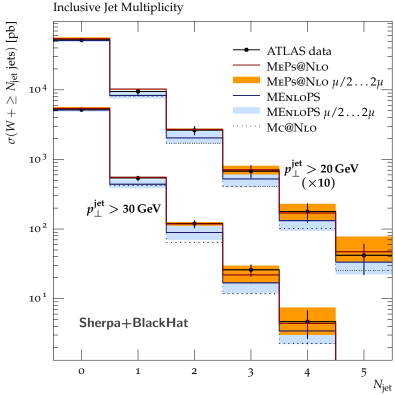



- Total jet broadening & its moments
- MEPS@NLO with 2,3&4 jet PL at NLO plus 5&6 jet PL at LO vs MENLOPS with up to 6 jets at LO

# $W$+jets production at the LHC

[ATLAS] arXiv:1201.1276
[SH,Krauss,Schönherr,Siegert] arXiv:1207.5030




- ME⊕PS@NLO with 0,1&2 jet PL at NLO plus 3&4 jet PL at LO
- MENLOPS with up to 4 jets at LO

# $W$+jets production at the LHC

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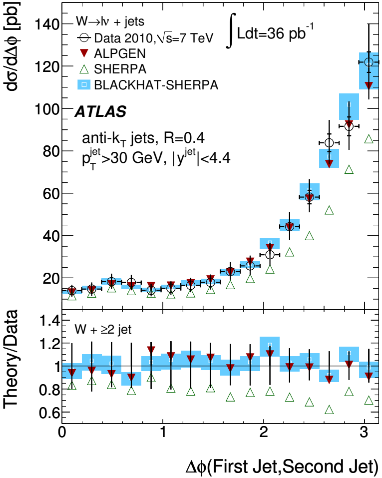
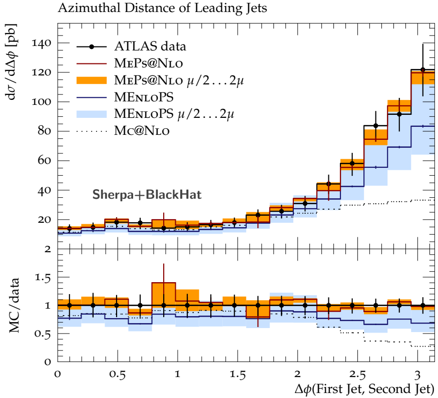



- ME⊕PS@NLO with 0,1&2 jet PL at NLO plus 3&4 jet PL at LO
- MENLOPS with up to 4 jets at LO

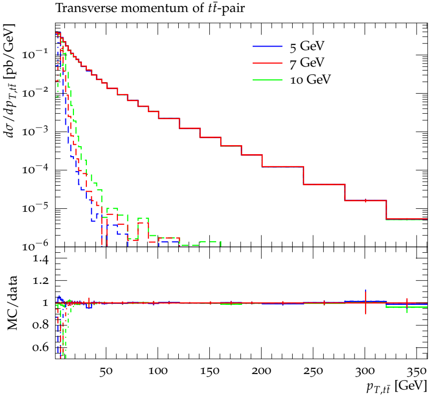
# Extension to massive quarks

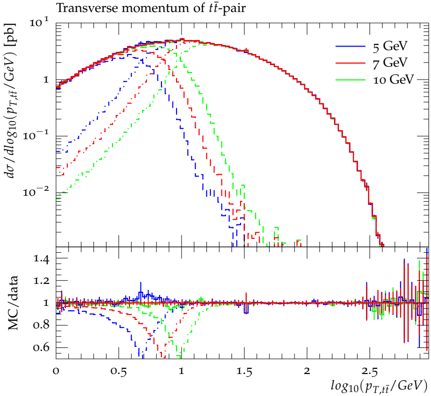
- Massive dipole splitting functions from
  [Catani,Dittmaier,Seymour,Trocsanyi] NPB627(2002)189
- Drop negative values in both PS and MC@NLO
- Extend evolution variables naturally

$$
\begin{aligned}
t^{(\mathrm{FS})} &= 2\,p_i p_j\,\tilde{z}_i(1-\tilde{z}_i) \\
&\to 2\,p_i p_j\,\tilde{z}_i(1-\tilde{z}_i) - (1-\tilde{z}_i)^2\,m_i^2 - \tilde{z}_i^2\,m_j^2 \\
t^{(\mathrm{IS})} &= 2\,p_a p_j\,(1-x_{aj,k})
\end{aligned}
$$

# Top pair production at the Tevatron

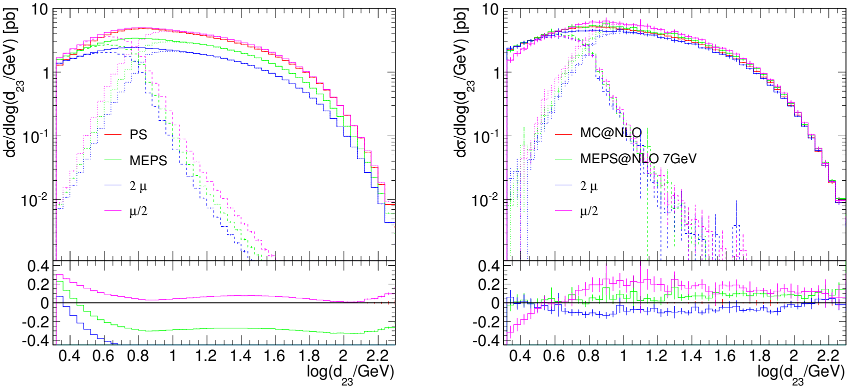
[Huang,Luisoni,Schönherr,Winter,SH] to appear



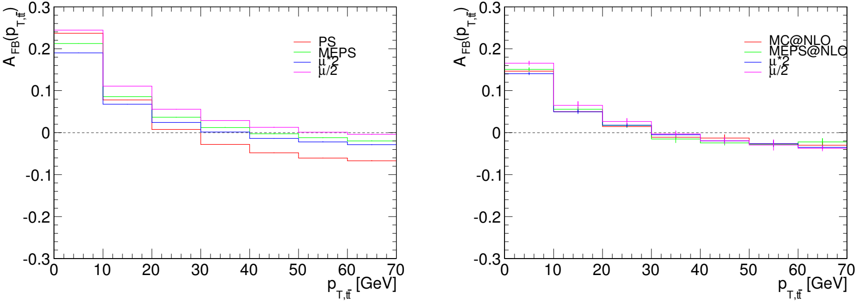

- Consistency check: $Q_{cut}$-variation
- Note: Central value very small, dynamic range $\sqrt{1/2}\ldots\sqrt{2}$

# Top pair production at the Tevatron




- Renormalization/factorization scale variation
- Central scale according to [Marchesini,Webber] NPB310(1988)461
- Virtual corrections from GoSam

# Top pair production at the Tevatron




- Renormalization/factorization scale variation
- Central scale according to [Marchesini,Webber] NPB310(1988)461
- Virtual corrections from GoSam

# Summary

- MC@NLO and ME⊕PS@NLO automated in Sherpa
- To become new standard soon $\nearrow$ v2.0.0 "Annapurna"
- Full exploitation of a wealth of NLO calculations
- Assessment of fixed-order scale uncertainties in MC