Strings on the Lightcone Worldsheet Lattice

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LoopFest XII, Florida State University
May 15, 2013

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GP & C. Thorn

Outline

Motivation

Lattice for Strings in the Lightcone Gauge

String Field Theory-based Approach

Worldsheet Quantum Field Theory Approach

Conclusions & Future Directions

Introduction

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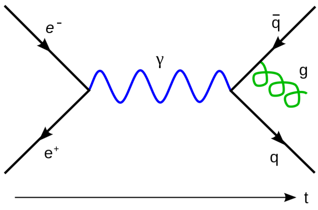

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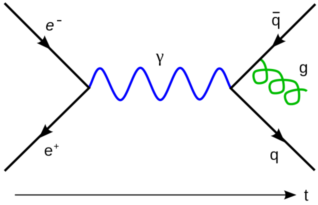

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Alternative approaches: lattice gauge theory OR Large $N \supset AdS/CFT$, however quite challenging to combine both. N fixed at simulations.

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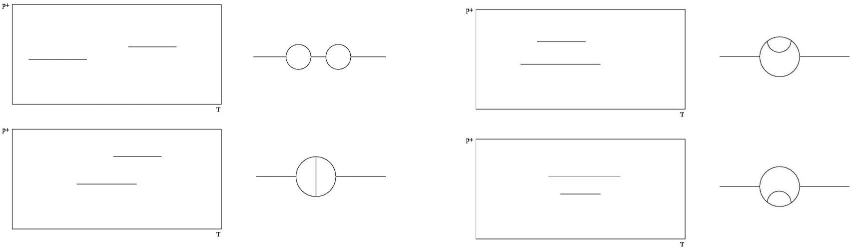

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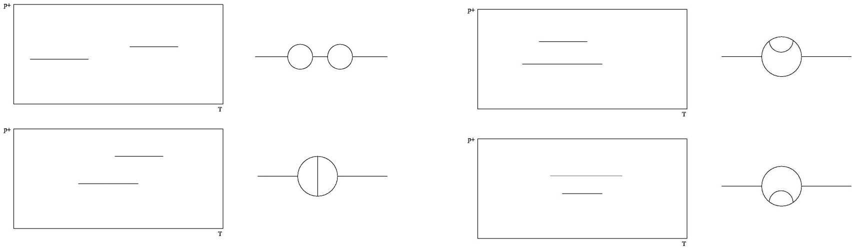

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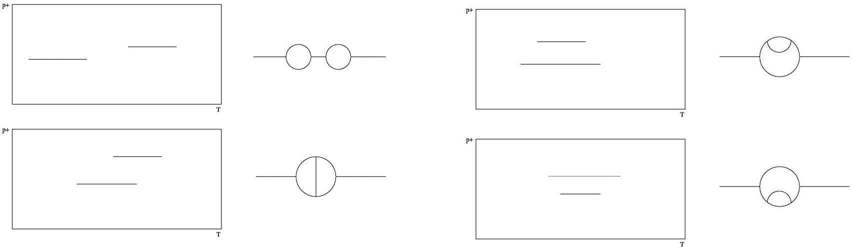

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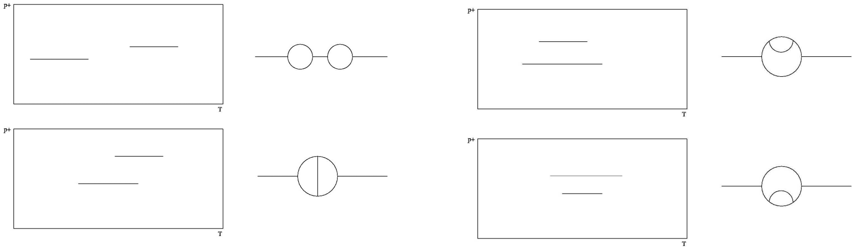

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Aim: Use lattice methods to sum planar multiloop string diagrams, and obtain information about large N QCD by taking $\alpha' \to 0$ at the end.

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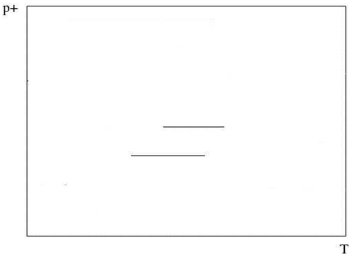
- In path integral formalism with $\tau \to -i\tau$, add interactions as discontinuities of $\boldsymbol{x}$ in σ = slits on the same rectangular worldsheet. [Mandelstam'73]



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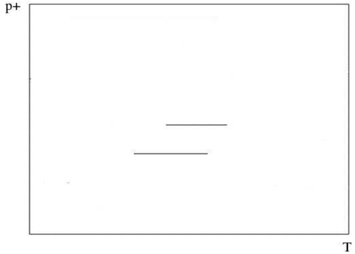

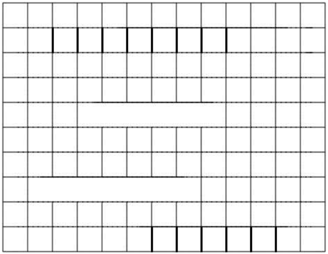

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- Extract string state energies by identifying exponential behaviors $e^{-a(N+1)E_\lambda(M)}$ of different P^- eigenvalues $E_\lambda(M)$. Typically

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- First two terms lead to divergences in $m^2 = P^+P^-$, but at tree level can be canceled by two geometrical counterterms (bulk/boundary).

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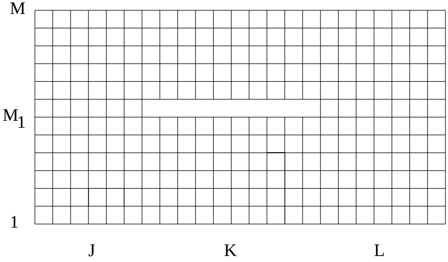

Self-energy diagram with

$$J + K + L = N + 1 \to \infty$$

and $J, L \sim N/2$

Hence characterized by K, M_1, M.

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$$-\frac{a\Delta P^-_{\rm tach}}{M} \equiv \sum_{K=1}^{\infty} \delta P^-_K = \sum_{K=1}^{\infty} \left[\left(\frac{\coth(M \sinh^{-1} 1)}{M\sqrt{2}} \right)^{1/2} \frac{e^{K\sum_{m=1}^{M-1}(\lambda^c_m - \lambda^o_m) - (K-1)(B_0+\epsilon)}}{\det A' \det B' \prod_{m=1,odd}^{M-1}(1-e^{-2K\lambda^o_m})} \right]^{12}$$

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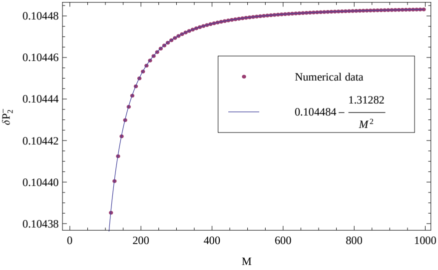
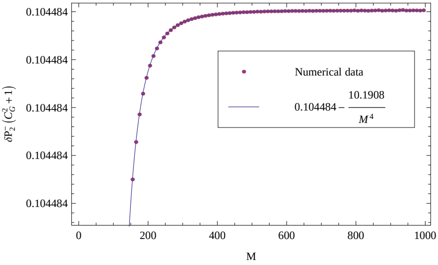
- Obtained complicated formula involving M-dimensional determinants
- We numerically evaluated summand for wide range of M, K and found dependence by fitting with the help of `Mathematica`.





Analyzed ground (tachyon) and 1^{st} excited (graviton) state, left- and right-hand side respectively

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$$\Delta_{ij,kl} = T_0\langle x_i^j x_k^l\rangle = T_0\frac{\int \mathcal{D}x\;\; x_i^j x_k^l\;\; e^{-S}}{\int \mathcal{D}x\;\; e^{-S}}\text{ WS propagator.}$$

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Main result of paper: Closed WS propagator a simple sum

$$\Delta^c_{hj,kl} = \frac{N_T - |l-j|}{2M} + \frac{1}{2M}\sum_{m=1}^{M-1}\frac{e^{-|l-j|\lambda^c_m}}{\sinh \lambda^c_m}\exp\frac{2m(h-k)i\pi}{M}$$

where $2N_T = 2N + l - j$, and similarly for open string.

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where $G \simeq 1.282$, $m = 0$ tachyon, $m = 1$ gluon,
$\Delta P^- = \sum_K e^{-\epsilon(K-1)}\delta P_K^-$

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When we add a Dp-brane however, we find that bulk and boundary divergences are multiplied by factors of $(\log M)^{\frac{p-25}{2}}$. Can no longer be canceled by the counterterms.

Conclusions & Future Directions

In this presentation we talked about

- How lattice-regularized string theory in the lightcone gauge can be used as a numerical tool for summing planar string diagrams, which via $\alpha' \to 0$ limit could teach us about large N QCD.
- How to test the reliability of the lattice as a regulator of divergences in bosonic string perturbation theory at 1-loop level.
- The fact that bosonic open string theory passes this test in the absence of D-branes, but not in their presence.

Next Stage

- Extend lattice model for the case of the superstring, which improves behavior of divergences and hence may restore power dependence on the regulator M in the presence of D-branes.
- Should this be possible, the next natural step would be the numerical evaluation of the full path integral with the help of Monte Carlo methods.

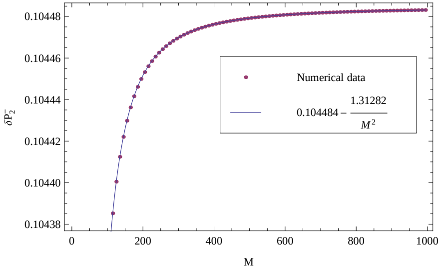
Tachyon Self-Energy in String Field Theory Approach

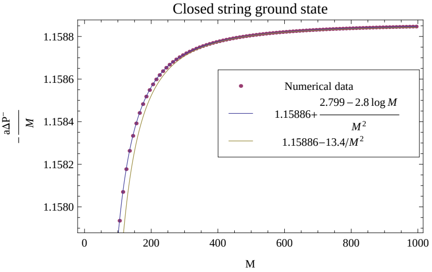
For value of boundary counterterm that makes tree-level Lorentz invariant,

$$\delta P^-_{\text{tach}}(K,M) \simeq c_1^K + \frac{2.8}{KM^2} + \mathcal{O}(1/M^3)$$

$$-\frac{a\Delta P^-_{\text{tach}}}{M} = c_1 + c_2\frac{1}{M^2} + c_3\frac{\log M}{M^2}$$

$$c_1 = 1.158863267 \pm 3\cdot 10^{-9}\,, \quad c_2 = 2.799 \pm 0.011\,, \quad c_3 = -2.800 \pm 0.002$$





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- Boundary counterterm in energy shift $e^{-(K-1)(B_0+\epsilon)}$
- Previous computations for $\epsilon = 0$, namely for value that makes tree-level energy shift Lorentz invariant.

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- Boundary counterterm in energy shift $e^{-(K-1)(B_0+\epsilon)}$
- Previous computations for $\epsilon = 0$, namely for value that makes tree-level energy shift Lorentz invariant.
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- Divergence as $\epsilon \to 0$ simply renormalizes T_0.

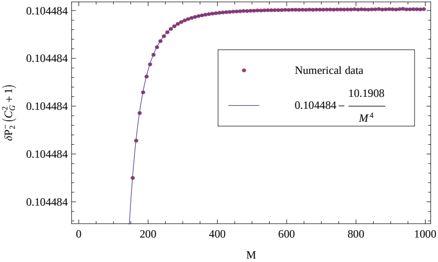
For value of boundary counterterm that makes tree-level Lorentz invariant,

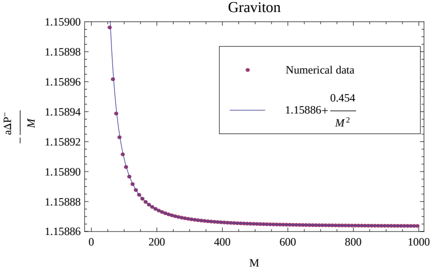
$$\delta P^-_{\mathsf{grav}}(K,M) \simeq c_1^K + \frac{\hat{c}_2 K}{M^4} + \mathcal{O}(1/M^5)$$

$$-\frac{a\Delta P^-_{\mathsf{grav}}}{M} = \tilde{c}_1 + \tilde{c}_2 \frac{1}{M^2}\,,$$

with

$$\tilde{c}_1 = 1.158863276 \pm 1.5\cdot 10^{-8} \qquad \tilde{c}_2 = 0.454 \pm 0.004\,,$$





Worldsheet Propagator Interpretation

When transformed to Fourier space w.r.t. σ, inverse of lattice Laplacian,

$$(-\triangle + 4\sinh^2 \lambda/2)f_j \equiv 2f_j - f_{j+1} - f_{j-1} + 4f_j \sinh^2 \lambda/2\,.$$

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$$\begin{aligned}
2e^{-|l-j|\lambda} - e^{-|l+1-j|\lambda} - e^{-|l-1-j|\lambda} &= \begin{cases} e^{-(l-j)\lambda}\,(2-2\cosh\lambda) & l>j \\ e^{-(j-l)\lambda}\,(2-2\cosh\lambda) & l<j \\ 2-e^{-\lambda}-e^{-\lambda} & l=j \end{cases} \\
&= -4e^{-|l-j|}\sinh^2\frac{\lambda}{2} + 2\delta_{lj}\sinh\lambda\,, \\
\left(-\triangle + 4\sinh^2\frac{\lambda}{2}\right)\frac{e^{-|l-j|\lambda}}{2\sinh\lambda} &= \delta_{lj}\,.
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As we will see, drastically improves calculational efficiency.

Testing Efficiency of Worldsheet Approach

Reobtain 1-loop self-energy corrections to low-lying states of the closed string, and compare with earlier results.

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Analytic formulas far more tractable: For one loop of K time steps

$$\det(I + V\Delta) = \det(h_{lp}), \quad l, p = 1, 2, \ldots K - 1,$$

where $h_{lp} = \delta_{lp} - \frac{1}{M}\sum_{m=1}^{M-1} \frac{\sin(m\pi/M)\left(\sin(m\pi/M)+\sqrt{1+\sin^2(m\pi/M)}\right)^{-2|l-p|}}{\sqrt{1+\sin^2(m\pi/M)}}$.

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Allow for asymptotic expansion in M with Euler-Mclaurin formula, and coefficients can be calculated exactly!

K	$\det(h_{lp}(x))$ up to $\mathcal{O}(x)$, $x = \frac{\pi}{6M^2}$
2	$\frac{1}{2} + x$
3	$-\frac{4}{\pi^2} + \frac{2}{\pi} + \frac{4x}{\pi}$
4	$-2 - \frac{64}{\pi^3} + \frac{16}{\pi^2} + \frac{8}{\pi} + \left(-4 + \frac{16}{\pi}\right) x$
5	$-16 - \frac{8192}{9\pi^4} - \frac{2048}{9\pi^3} + \frac{256}{\pi^2} + \frac{64}{3\pi} + \left(-64 - \frac{16384}{9\pi^3} + \frac{2048}{3\pi^2} + \frac{512}{3\pi}\right) x$

Tachyon Self-Energy Summand

$$-\delta P^-_{\mathsf{tach}} = \frac{e^{-24(K-1)B_0}}{\det^{12}(h_{lp})} = \frac{e^{-24(K-1)B_0}}{\det^{12}(h_{lp}(0))}\left(1-\frac{2\pi}{M^2}\sum_{l,s=1}^{K-1} h^{-1}_{ls}(0)\right)+\mathcal{O}(1/M^4)\,.$$

K	$-\delta P^-_{\mathsf{tach}}$ fit	$-\delta P^-_{\mathsf{tach}}$ actual
2	$0.1044844648 - 1.31291/M^2$	$0.104484465146 - 1.31299/M^2$
3	$0.027700432 - 0.9578/M^2$	$0.0277004334342 - 0.957933/M^2$
4	$0.010959556 - 0.7268/M^2$	$0.0109595576932 - 0.727031/M^2$
5	$0.005388196 - 0.5811/M^2$	$0.00538819758183 - 0.581471/M^2$
6	$0.003032942 - 0.4828/M^2$	$0.00303294412639 - 0.483277/M^2$

Results agree within margins of error, notice however difference increases with K. Due to systematic error from not taking into account $\mathcal{O}(1/M^4)$ term in the fits, whose relative size also increases with K.

Graviton Self-Energy Summand

This is equal to tachyon summand times

$$1 + 2\tilde{U} + 2\tilde{U}^2 \simeq 1 + \frac{2\pi}{M^2} \sum_{l,s=1}^{K-1} h_{ls}^{-1}(0) + \mathcal{O}\left(\frac{1}{M^4}\right) ,$$

$$\tilde{U} = \frac{\sin \frac{\pi}{M}}{M\sqrt{1 + \sin^2 \frac{\pi}{M}}} \sum_{l,s=1}^{K-1} \left(\sin \frac{\pi}{M} + \sqrt{1 + \sin^2 \frac{\pi}{M}} \right)^{2(l-s)} h_{ls}^{-1} .$$

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Able to rigorously prove two important facts, for which we only had strong indications up to now:

1. Leading term in the M-expansion same for tachyon and graviton.
2. Nontrivial cancelation of $\mathcal{O}(1/M^2)$ term! Graviton massless in $K \ll M$ (UV) region.

Euler-Mclaurin Formulas & Graviton Table

$$\frac{1}{M}\sum_{m=0}^{M-1} f\left(\frac{m}{M}\right) = \int_0^1 dx f(x) - \frac{1}{2M}\, f(x)\big|_0^1 + \sum_{k=1}^{\infty}\frac{B_{2k}}{(2k)!}\frac{f^{(2k-1)}(x)\big|_0^1}{M^{2k}}$$

$$\frac{1}{M}\sum_{m=0}^{M-1}\frac{\sin\frac{\pi m}{M}\left(\sin\frac{\pi m}{M}+\sqrt{1+\sin^2\frac{\pi m}{M}}\right)^{-2n}}{\sqrt{1+\sin^2\frac{\pi m}{M}}} \simeq I_n - \frac{\pi}{6M^2} + \frac{(-1+3n^2)\pi^3}{90M^4}$$

$$I_n = \sum_{r=0}^{n}\frac{n}{n+r}\begin{pmatrix} n+r \\ 2r \end{pmatrix} 2^{2r}\frac{\Gamma(\frac{1}{2}+\frac{r}{2})}{2\sqrt{\pi}\Gamma(1+\frac{r}{2})}\, {}_2F_1(1-n,1+n;2;-1)\,.$$

K	$-\delta P^-_{\rm Graviton}$ fit	$-\delta P^-_{\rm Graviton}$ actual
2	$0.104484465145 - 10.19/M^4$	$0.10448446514630 - 10.1905/M^4$
3	$0.027700433434 - 3.85/M^4$	$0.02770043343416 - 3.8499/M^4$
4	$0.010959557693 + 1.87/M^4$	$0.01095955769317 + 1.8837/M^4$
5	$0.0053881975 + 6.82/M^4$	$0.00538819758183 + 6.8571/M^4$
6	$0.003032944127 + 11.28/M^4$	$0.00303294412639 + 11.3355/M^4$