Soft Triple-Real Radiation for Higgs Production at N3LO

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in collaboration with
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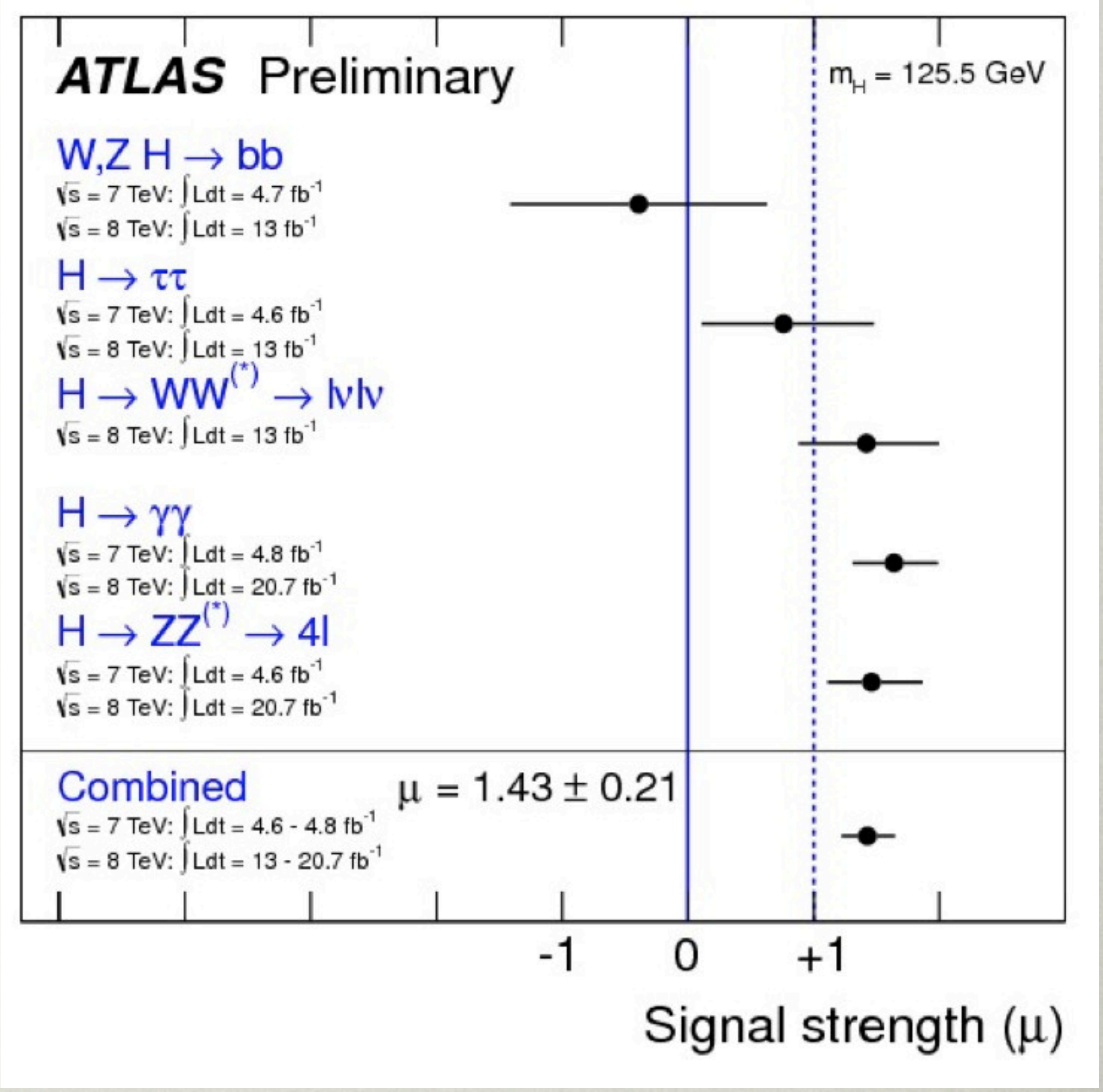
Motivation

- Experimental uncertainty already down to ~30%
- Will go below theory uncertainty with more data
- Higgs cross-section is a test of the Standard Model
- Need to reduce uncertainty of theoretical prediction



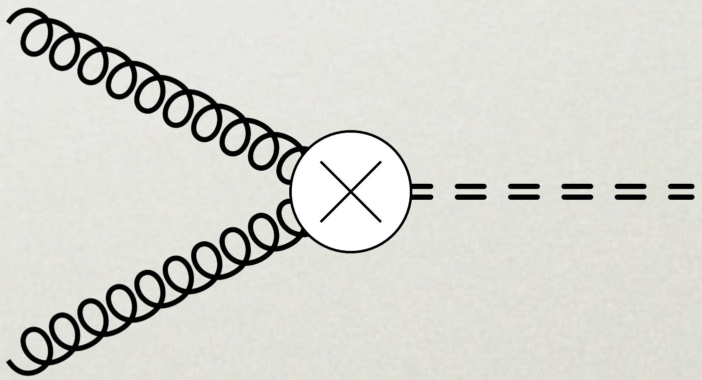
$\Rightarrow \mu = 1.43 \pm 0.16\ (\text{stat}) \pm 0.14\ (\text{sys})$

Challenges

- No previous N3LO calculation for a hadron collider process
- We still struggle with open problems at NNLO
- Factorial growth of the number of Feynman diagrams
- Challenges everything we have learned at NNLO
- We need new ideas and new technology

Gluon Fusion

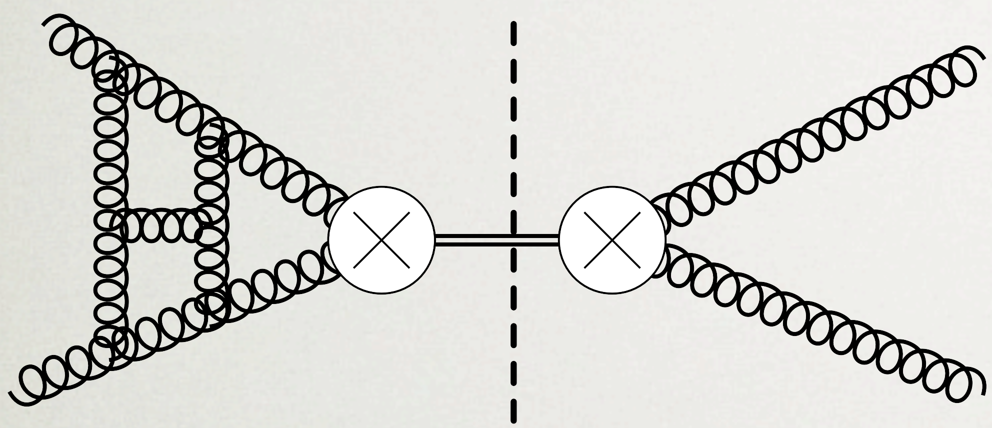
- Dominant production mode at the LHC
- Loop mediated process $m^2_{Higgs} \ll 4m^2_{top}$
- Effective theory
- Massless QCD +



Ingredients of NNNLO

- Renormalization and regularization of initial state collinear divergences
- Requires splitting functions up to three loops [Moch, Vermaseren, Vogt]
- Requires NNLO cross-section at higher order in the dimensional regulator [Pak, Rogal, Steinhauser; Anastasiou, Buehler, Duhr, Herzog]
- Convolution with splitting functions was recently performed, see next talk [Hoeschele, Hoff, Pak, Steinhauser, Ueda]

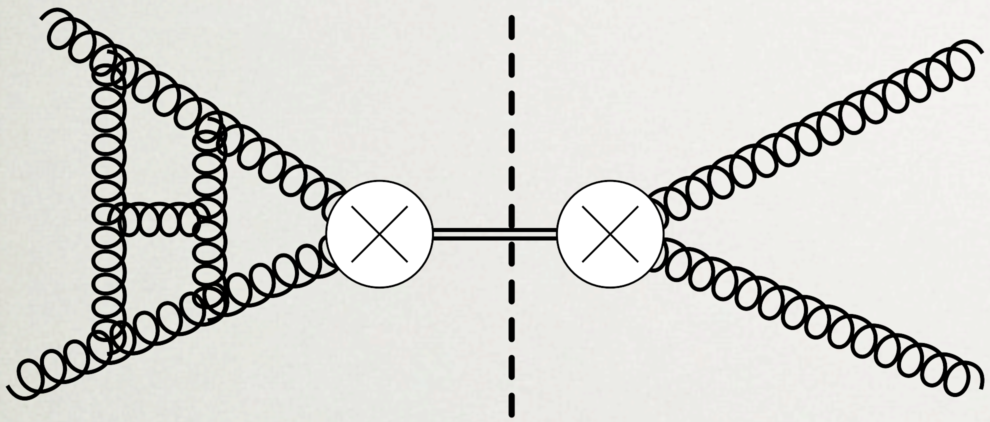
Ingredients of NNNLO

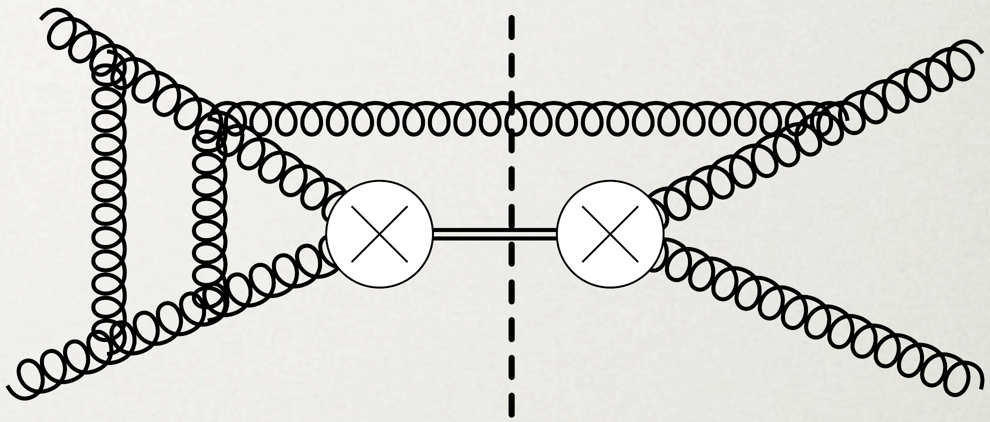


triple-virtual

- Purely virtual contributions are known: 3-loop QCD form factor [Baikov, Chetyrkin, Smirnov, Smirnov Steinhauser, Gehrmann, Glover, Huber, Ikizlerli, Studerus]
- Trivial phase-space

Ingredients of NNNLO

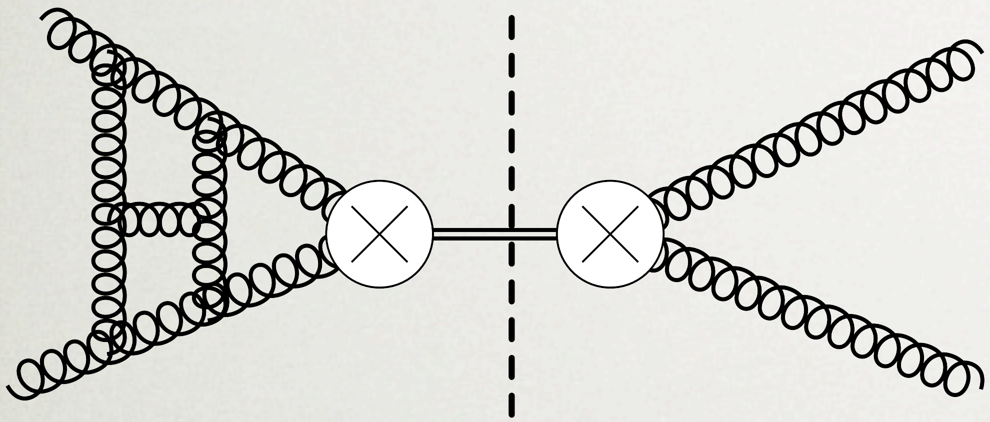




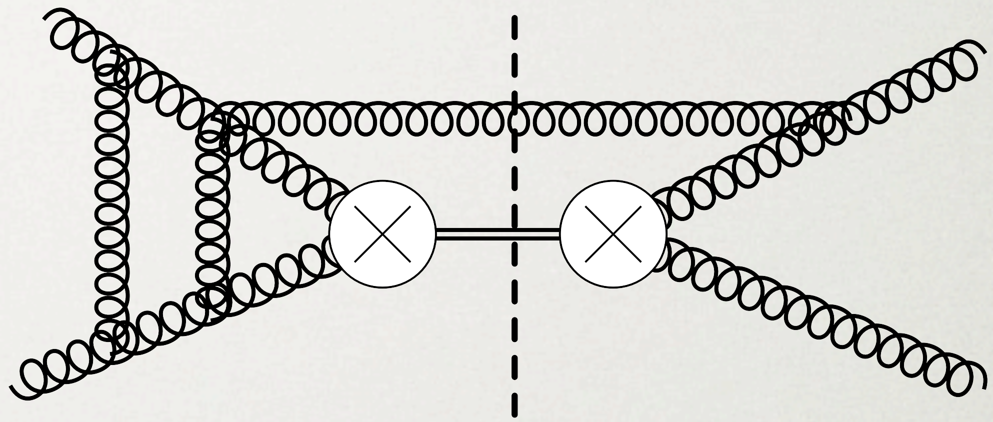
triple-virtual double-virtual real

- Loop contributions are known:
  2-loop master integrals [Gonsalves; Kramer, Lamp; Gehrmann, Huber, Maître]
- Phase-space integration needs to be done
- Introduces additional singularities

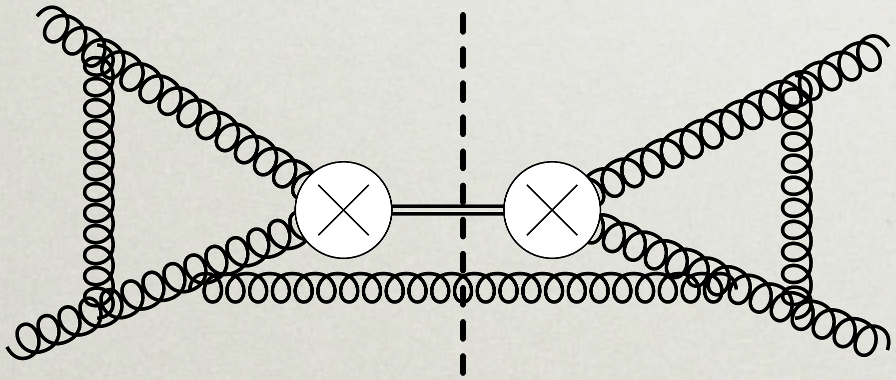
Ingredients of NNNLO



triple-virtual

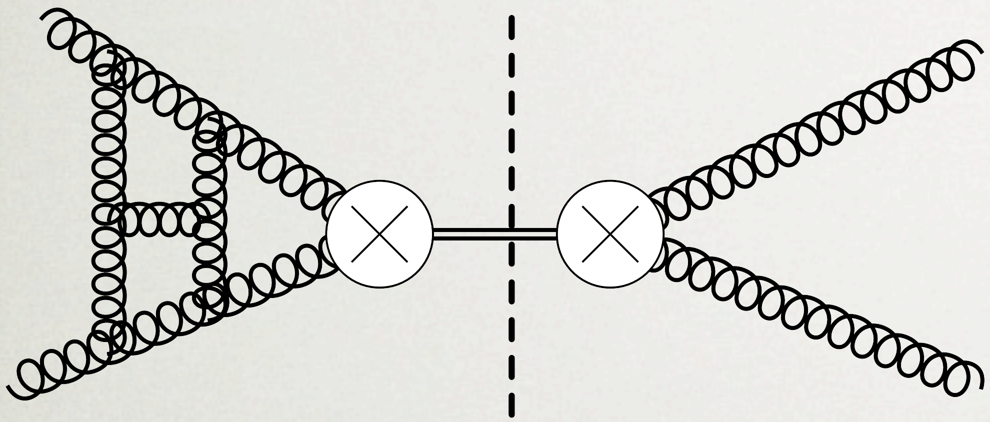


double-virtual real

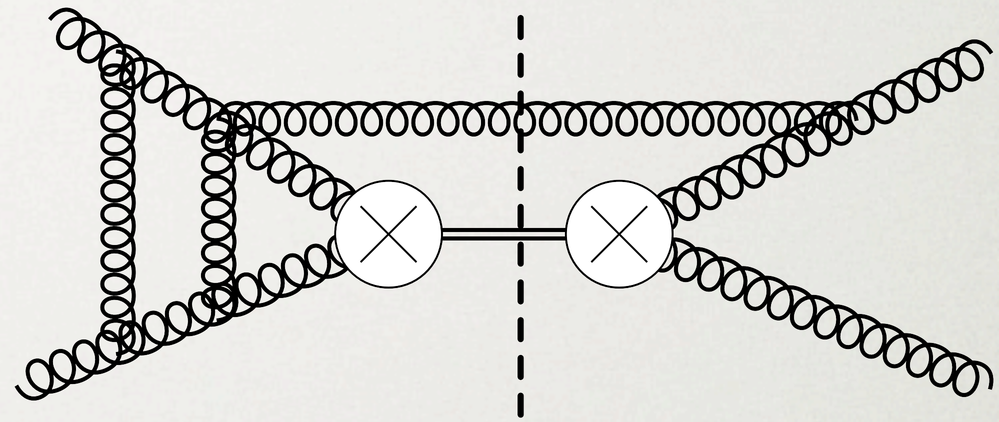


(real-virtual)^2

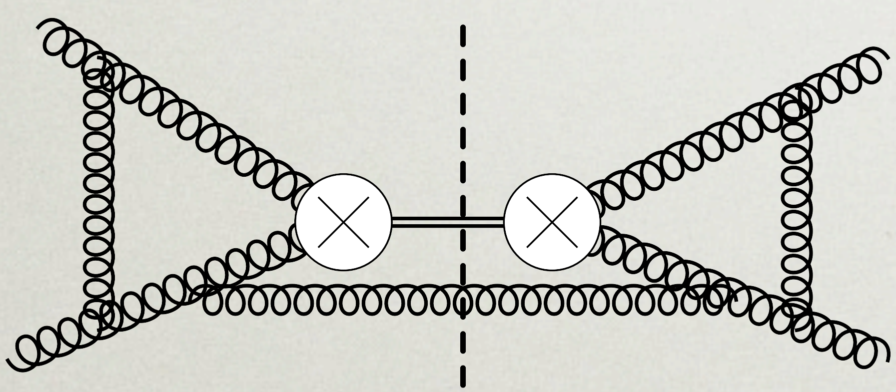
Ingredients of NNNLO



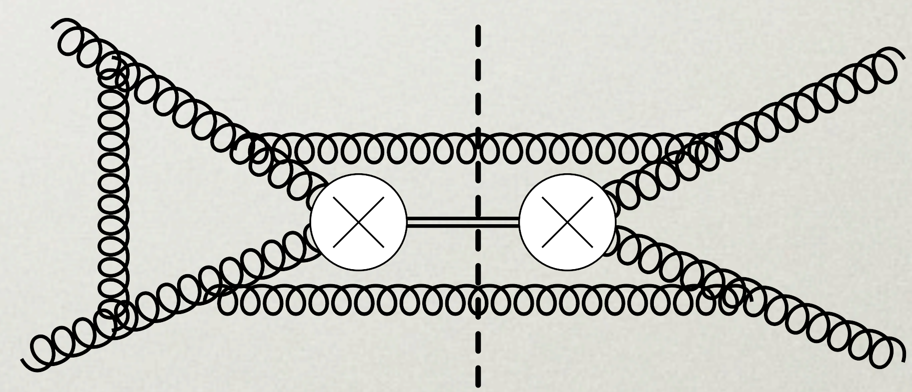
triple-virtual



double-virtual real

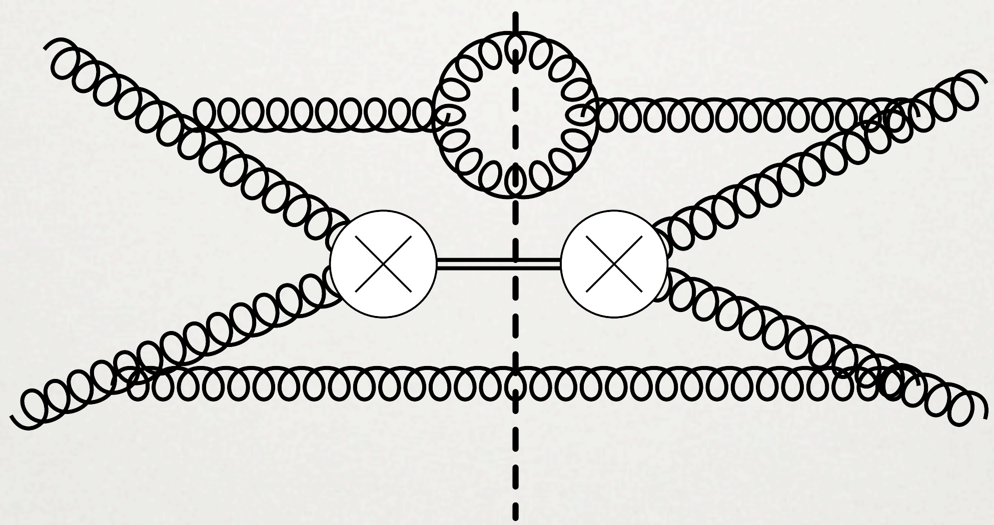


(real-virtual)^2



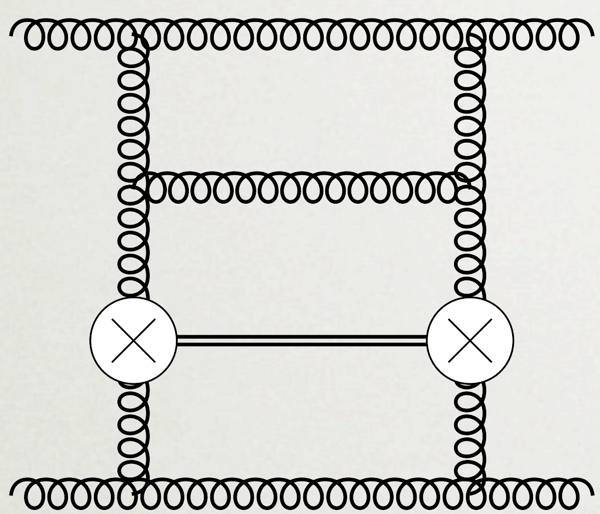
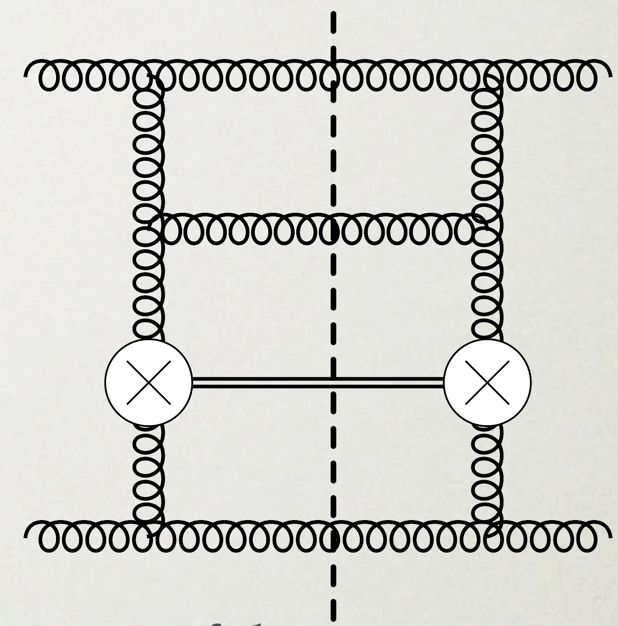
double-real virtual

The Triple-Real



- Real radiation was the most complicated piece at NNLO
- Calculation requires novel techniques

Reverse Unitarity

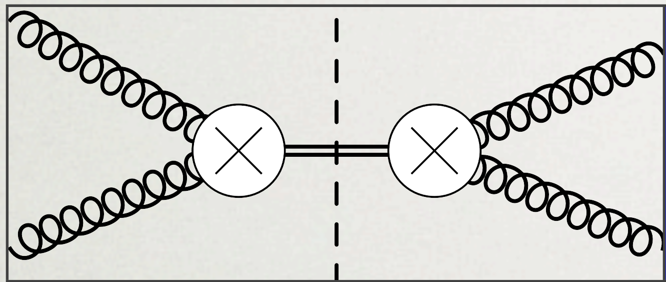
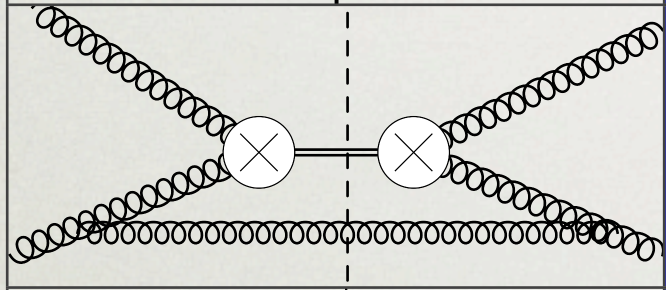
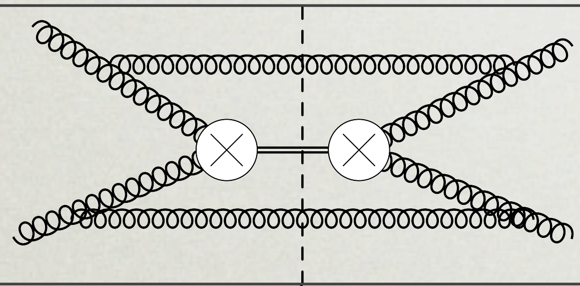
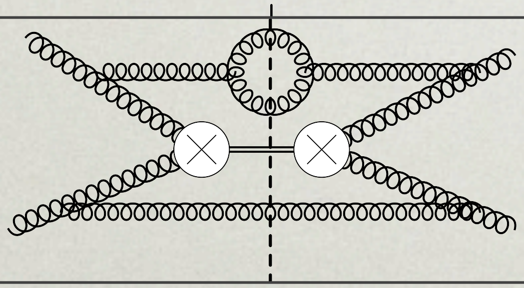
Im  $= \int d\Phi_4$ 

- Rewrite phase-space integrals as cuts of loop integrals [Anastasiou, Melnikov]
- Revert Cutkosky's rule

$$\delta_+(q^2) \to \left(\frac{1}{q^2}\right)_c = \frac{1}{2\pi i}\left(\frac{1}{q + i\epsilon} - \frac{1}{q^2 - i\epsilon}\right)$$

- Exploit loop technology: IBPs, Master integrals, differential equations

Real Complexity

	1 diagram	1 Integral
	10 diagrams	1 Integral
	351 diagrams	18 Integrals
	26 565 diagrams	~200 Integrals

Soft Expansion

- Even with reverse unitarity: lots of very difficult integrals
- Idea: Perform a soft expansion of the cross-section
- Threshold expansion: $\bar{z} = (1 - z) = \left(1 - \frac{M_H^2}{\hat{s}}\right)$
  - Higgs produced at rest
  - Only soft radiation

A new Method for Expansions

- **Cut propagators can be differentiated and expanded!**

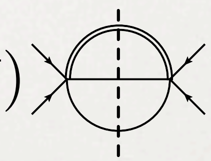
$$\left(\frac{1}{k^2+2\bar{z}(k\cdot q)}\right)_c = \frac{1}{2\pi i}\left(\frac{1}{k^2+2\bar{z}(k\cdot q)+i\epsilon} - \frac{1}{k^2+2\bar{z}(k\cdot q)-i\epsilon}\right)$$

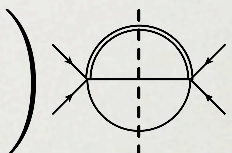
$$= \left(\frac{1}{k^2}\right)_c \sum_{i=0}^{\infty} \bar{z}^i \left(\frac{-(k\cdot q)}{k^2}\right)^i$$

- Expansion at the <u>integrand</u> level
- Expanded integrand has a diagrammatic interpretation

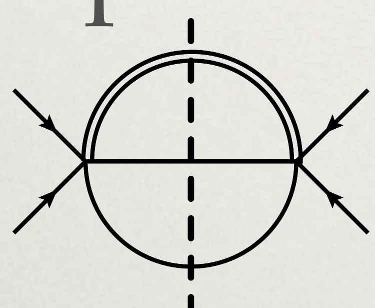
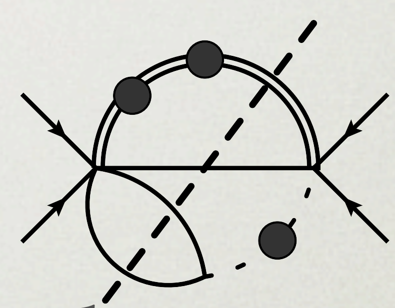
Check at NNLO - Expansion of $\int d\phi_3$

- General kinematics:

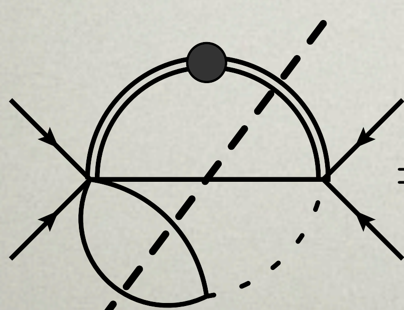
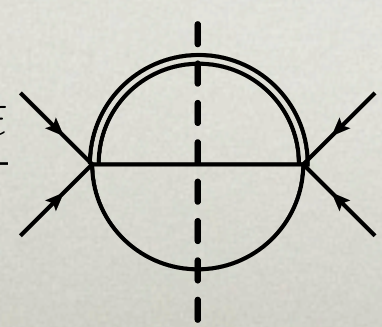
$$\int d\Phi_3 = \bar{z}^{3-4\epsilon} {}_2F_1(1-\epsilon, 2-2\epsilon, 4-4\epsilon, \bar{z})$$ 

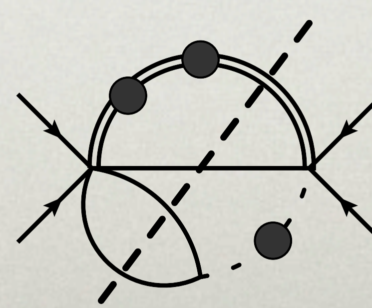
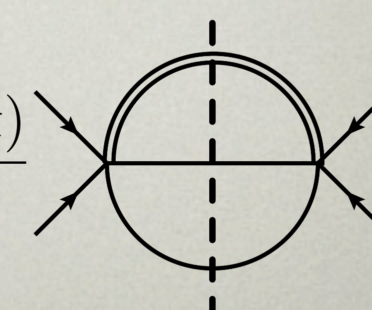
$$= \bar{z}^{3-4\epsilon}\left(1 + \frac{1-\epsilon}{2}\bar{z} + \frac{(1-\epsilon)(2-\epsilon)(3-2\epsilon)}{4(5-4\epsilon)}\bar{z}^2 + \dots\right)$$ 

- Soft expansion using our method

$= \bar{z}^{3-4\epsilon}$ [ $-\bar{z}$  $+\,\bar{z}^2$  $+\dots$]

- IBP reduction yields exactly the expansion of the ${}_2F_1$

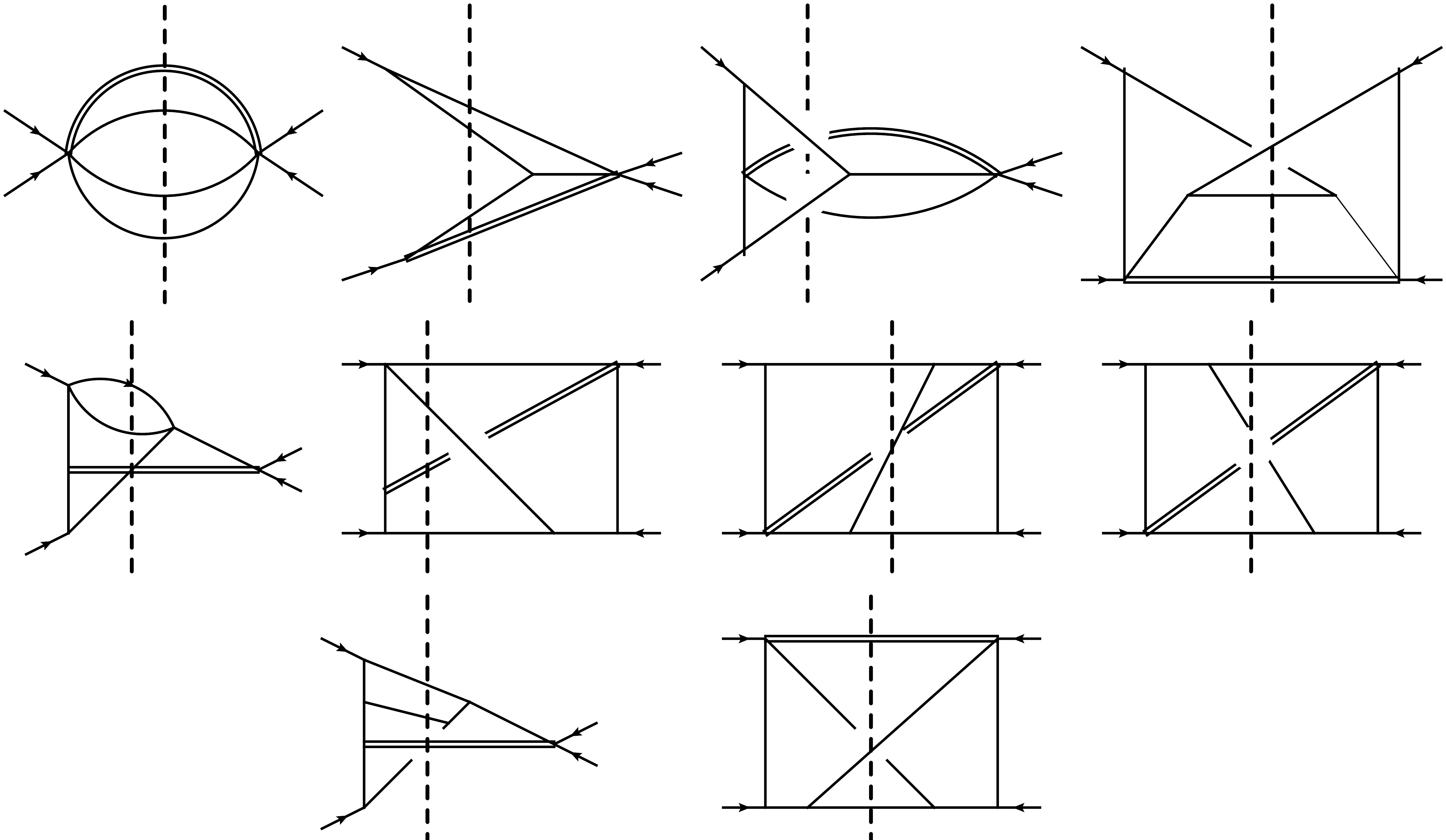
 $= -\dfrac{1-\epsilon}{2}$ 

 $= \dfrac{(1-\epsilon)(2-\epsilon)(3-2\epsilon)}{4(5-4\epsilon)}$ 

Soft Expansion of the Triple-Real

- We can expand amplitude for all 2 to H+3 parton processes to any order
- For the moment, we compute the first two terms in the expansion
- IBP reduce resulting terms
- We find 10 master integrals

Soft Masters

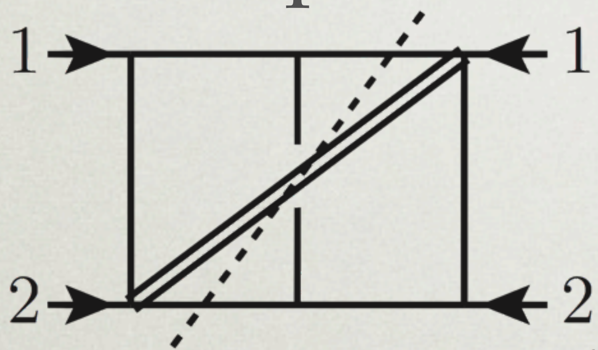


Soft Masters

- The masters are not specific for Higgs production
- The masters are numbers!
- Main challenge of our calculation: fully analytic computation of the masters

Analytic Calculation

- We developed an algorithm to derive Mellin-Barnes representations for phase space integrals
[van Neerven; Somogyi]
- Complicated integrals can be rewritten in a compact form



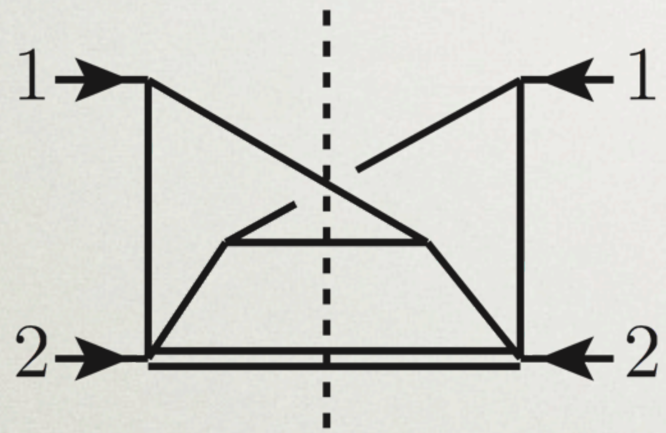
$$= -\frac{6\epsilon\,\Gamma(6-6\epsilon)}{\Gamma(1-\epsilon)^4\Gamma(1-6\epsilon)} \int_{-i\infty}^{+i\infty} \frac{dz_2 dz_3 dz_4}{(2\pi i)^3}\,\Gamma(-z_2)\,\Gamma(-z_3)\,\Gamma(-z_4)$$

$$\times\,\Gamma(z_3+1)\,\Gamma(z_2-2\epsilon)\,\Gamma(-z_2-z_4)\,\Gamma(z_2+z_4+1)\,\Gamma(-\epsilon-z_3)\,\Gamma(z_3-\epsilon)$$

$$\times\,\frac{\Gamma(-2\epsilon+z_2-z_3)\,\Gamma(-\epsilon-z_4)\,\Gamma(z_4-\epsilon)}{\Gamma(-2\epsilon+z_2+1)\,\Gamma(-2\epsilon-z_3-z_4)}\,.$$

Soft Masters

- Some MB integrals can be solved in terms of harmonic or hypergeometric sums



$$= -\frac{3\,\Gamma(6-6\epsilon)\Gamma(1-2\epsilon)}{2\epsilon^4\Gamma(1-6\epsilon)}$$
$$\times \left[\frac{3\,\Gamma(1-2\epsilon)\Gamma(\epsilon+1)}{(1+3\epsilon)\Gamma(1-3\epsilon)}\,{}_3F_2(-3\epsilon-1,-2\epsilon,-\epsilon;-3\epsilon,-3\epsilon;1)\right.$$
$$\left.+\frac{1}{(1+\epsilon)\Gamma(1-2\epsilon)}\,{}_4F_3(1,1,1-\epsilon,-2\epsilon;1-2\epsilon,1-2\epsilon,2+\epsilon;1)\right]$$

Soft Masters

- Other integrals require more sophisticated techniques
- We developed an algorithm to turn MB integrals into parametric integrals
- Solve in terms of iterated integrals over generalized polylogarithms
- Requires the use of recent results from number theory and modern algebra

Soft Masters

- Surprising result:

$$\mathcal{F}_9(\epsilon) = \frac{160}{\epsilon^5} - \frac{1712}{\epsilon^4} + \frac{1}{\epsilon^3}\Big(-120\,\zeta_2 + 2784\Big) + \frac{1}{\epsilon^2}\Big(-120\,\zeta_3 + 1284\,\zeta_2 + 31968\Big)$$
$$+ \frac{1}{\epsilon}\Big(2520\,\zeta_4 + 1284\,\zeta_3 - 2088\,\zeta_2 - 216864\Big) + 15720\,\zeta_5 + 1920\,\zeta_2\,\zeta_3$$
$$- 26964\,\zeta_4 - 2088\,\zeta_3 - 23976\,\zeta_2 + 795744 + \epsilon\Big(82520\,\zeta_6 + 9600\,\zeta_3^2$$
$$- 168204\,\zeta_5 - 20544\,\zeta_2\,\zeta_3 + 43848\,\zeta_4 - 23976\,\zeta_3 + 162648\,\zeta_2 - 2449440\Big)$$
$$+ \mathcal{O}(\epsilon^2)$$

- Just Zeta values and integers!

Checks

- MB integrals allow for direct numerical evaluation
- Dimensional shift identities

$\mathcal{I}^{D=6-2\epsilon}_{Master} = \mathcal{I}'^{D=4-2\epsilon}$

Reduction

$$\mathcal{I}^{D=6-2\epsilon}_{Master} = \sum_i c_i(D,z)\mathcal{I}^{D=4-2\epsilon}_{Master_i}$$

- Integrals in $D = 6 - 2\epsilon$ are finite
- Poles of the integrals in $D = 4 - 2\epsilon$ have to cancel
- Powerful check of the masters and the reduction method

The Soft Triple-Real Cross-Section

$$\sigma^{S(0)}_{g\,g\to H+g\,g\,g} = \frac{2^5}{3^4}\frac{1}{3!}\frac{1}{8(N_c^2-1)^2}(4\pi\alpha_S)^3\Phi_4^S(\epsilon)C_A^4 C_F c_H^2$$
$$\times\Bigg\{-\frac{218700}{\epsilon^5}+\frac{2554740}{\epsilon^4}+\frac{1}{\epsilon^3}\big(131220\zeta_2-9709605\big)$$
$$+\frac{1}{\epsilon^2}\big(782460\zeta_3-1630854\zeta_2+14950359\big)+\frac{1}{\epsilon}\big(2869830\zeta_4-9687762\zeta_3$$
$$+6810588\zeta_2-8547924\big)+8373780\zeta_5+301320\zeta_2\zeta_3-35377641\zeta_4$$
$$+40216932\zeta_3-11741904\zeta_2+107996+\epsilon\big(24995385\zeta_6+763020\zeta_3^2$$
$$-103032486\zeta_5-3541644\zeta_2\zeta_3+145858644\zeta_4-68849712\zeta_3$$
$$+7687776\zeta_2-455984\big)+\mathcal{O}(\epsilon^2)\Bigg\}.$$

Conclusion

- First result for the triple real contribution to the Higgs cross-section at N3LO
- We analytically computed 10 soft master integrals
- Essential step towards computing the full N3LO Higgs cross-section
- New method for efficient threshold expansion
- New method for deriving parametric integrals from MB integrals

Outlook

- We can use our method to easily compute more terms in the expansions
- An extension of the method to compute the virtual pieces looks promising
- Our method can be used to compute other processes such as Drell-Yan