Towards a precise picture of electroweak precision observables

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1. Electroweak precision observables

2. Techniques for ≥ 2-loop corrections

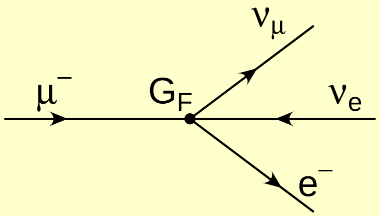
3. Current status of SM predictions

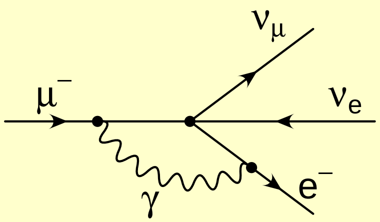
4. Summary

Electroweak precision observables

W mass

μ decay in Fermi Model



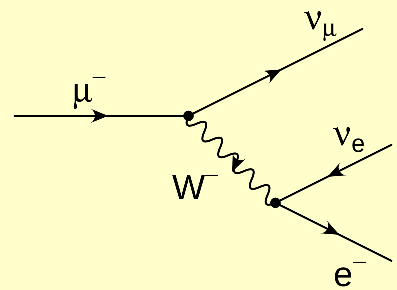


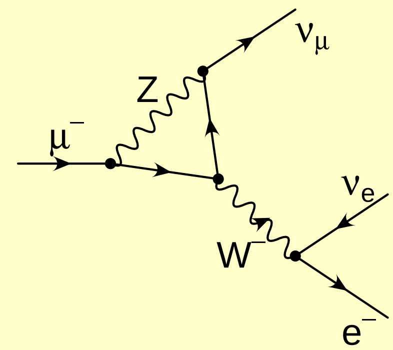
← QED corr. (2-loop)

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right) (1 + \Delta q)$$

Ritbergen, Stuart '98

μ decay in Standard Model



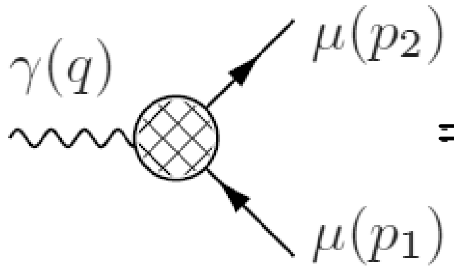


$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1 + \Delta r)$$

electroweak corrections

Experiment: $M_W = 80.385 \pm 0.015$ GeV PDG '12

Muon anomalous magnetic moment

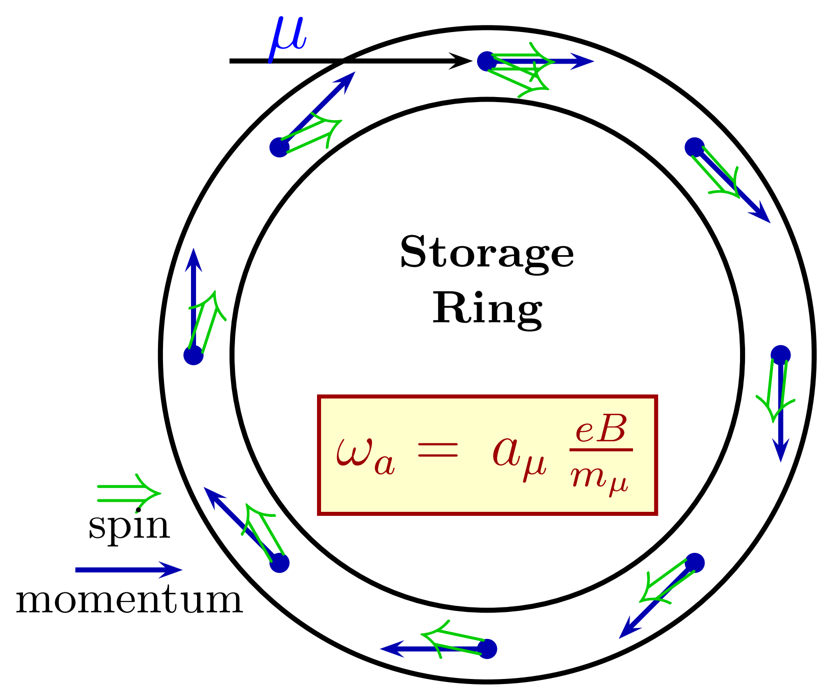


$$= (-ie)\,\bar{u}(p_2)\left[\gamma^\mu F_{\mathsf{E}}(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu}F_{\mathsf{M}}(q^2)\right]u(p_1)$$

$a_\mu = F_{\mathsf{M}}(0)$

Measured at NBL g–2 experiment:

$a_\mu = (11\,659\,208.0 \pm 6.3) \times 10^{-10}$



Jegerlehner, Nyfeller '09

actual precession $\times$ 2

Z-pole observables

$e^+e^- \to f\bar{f}$ for $\sqrt{s} \sim m_Z$:

$$\frac{d\sigma}{d\cos\theta} = \mathcal{R}_{\rm ini}\left[\frac{9}{2}\pi \frac{\Gamma_{ee}\Gamma_{ff}[(1-\mathcal{P}_e\mathcal{A}_e)(1+\cos^2\theta) + 2(\mathcal{A}_e - \mathcal{P}_e)\mathcal{A}_f \cos\theta]}{(s-m_Z^2)^2 - m_Z^2\Gamma_Z^2} + \sigma_{\rm non-res}\right],$$

$$\Gamma_{ff} = \mathcal{R}_V^f g_{Vf}^2 + \mathcal{R}_A^f g_{Af}^2, \qquad \Gamma_Z = \sum_f \Gamma_{ff},$$

$$\mathcal{A}_f = 2\frac{g_{Vf}/g_{Af}}{1+(g_{Vf}/g_{Af})^2} = \frac{1-4|Q_f|\sin^2\theta_{\rm eff}^f}{1-4|Q_f|\sin^2\theta_{\rm eff}^f + 8(|Q_f|\sin^2\theta_{\rm eff}^f)^2}.$$

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QED/QCD corrections on ext. fermions

Chetyrkin, Kataev, Tkachov '79
Dine, Saphirstein '79
Celmaster, Gonsalves '80
Gorishnii, Kataev, Larin '88,91
Chetyrkin, Kühn '90
Surguladze, Samuel '91
Kataev '92
Chetyrkin '93
etc...

Z-pole observables

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additional initial-state QED corrections

Kuraev, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '89,91
Montagna, Nicrosini, Piccinini '97
etc...

Z-pole observables

$e^+e^- \to f\bar{f}$ for $\sqrt{s} \sim m_Z$:

$$\frac{d\sigma}{d\cos\theta} = \mathcal{R}_{\text{ini}}\left[\frac{9}{2}\pi\,\frac{\Gamma_{ee}\Gamma_{ff}[(1-\mathcal{P}_e\mathcal{A}_e)(1+\cos^2\theta)+2(\mathcal{A}_e-\mathcal{P}_e)\mathcal{A}_f\cos\theta]}{(s-m_Z^2)^2-m_Z^2\Gamma_Z^2} + \sigma_{\text{non-res}}\right],$$

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electroweak corrections

Z-pole observables

Measured quantities:

- $\sigma^0_{\text{had}} = \dfrac{12\pi}{m_Z^2}\dfrac{\Gamma_{ee}\Gamma_{\text{had}}}{\Gamma_Z^2}$ $\qquad$ $\sigma^0_{\text{had}} = 41.540 \pm 0.037$ nb
- $R_f = \Gamma_{ff}/\Gamma_{\text{had}}$ $\qquad$ $R_b = 0.21629 \pm 0.00066$
- Γ_Z $\qquad$ $\Gamma_Z = 2.4952 \pm 0.0023$ GeV
- $A^f_{\text{FB}} \equiv \dfrac{\sigma(\theta < \frac{\pi}{2}) - \sigma(\theta > \frac{\pi}{2})}{\sigma(\theta < \frac{\pi}{2}) + \sigma(\theta > \frac{\pi}{2})} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$ $\qquad$ $A^b_{\text{FB}} = 0.0992 \pm 0.0016$
- $A_{\text{LR}} \equiv \dfrac{\sigma(\mathcal{P}_e > 0) - \sigma(\mathcal{P}_e < 0)}{\sigma(\mathcal{P}_e > 0) + \sigma(\mathcal{P}_e < 0)} = \mathcal{A}_e$ $\qquad$ $A_{\text{LR}} = 0.1513 \pm 0.0021$

$\longrightarrow$ Each observable involves several theoretical building blocks, in different combinations

Techniques for $\geq$ 2-loop corrections

Challenges:

- Large number of diagrams and tensor integrals, $\mathcal{O}(100) - \mathcal{O}(10000)$
- Many different scales (masses and ext. momenta)

Computer algebra methods:

- Generation of diagrams with *FeynArts*, *QGraf*, ...
  Küblbeck, Eck, Mertig '92, Hahn '01
  Nogueira '93
- Dirac/Lorentz algebra with *Form*, *FeynCalc*, ... Vermaseren '89,00
  Mertig '93

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods must be automizable, stable, fastly converging
- Need procedure for isolating divergent pieces

Analytic calculations

- Useful for diagrams with up to two scales
  ($e.g.$ M_W & m_t or M_W & M_Z)
- Reduce to master integrals with integration-by-parts and Lorentz-invariance identities
  Chetyrkin, Tkachov '81
  Gehrmann, Remiddi '00
  Laporta '00
- Evaluate master integrals with differential equations or Mellin-Barnes representations
  Kotikov '91
  Remiddi '97
  Smirnov '00,01

Example applications:

$Zf\bar{f}$ QED vertex corrections
Gorishnii, Kataev, Larin '88,91
Bonciani, Mastrolia, Remiddi '03

$Zf\bar{f}$ electroweak 2-loop vertex diagrams with massless fermions
Awramik, Czakon, Freitas, Weiglein '04

Asymptotic expansions

- Exploit large mass ratios, *e. g.* $M_Z^2/m_t^2 \approx 1/4$
- Simplifies diagrams to 2-loop tadpoles and 1-loop vertices
- Fast numerical evaluation

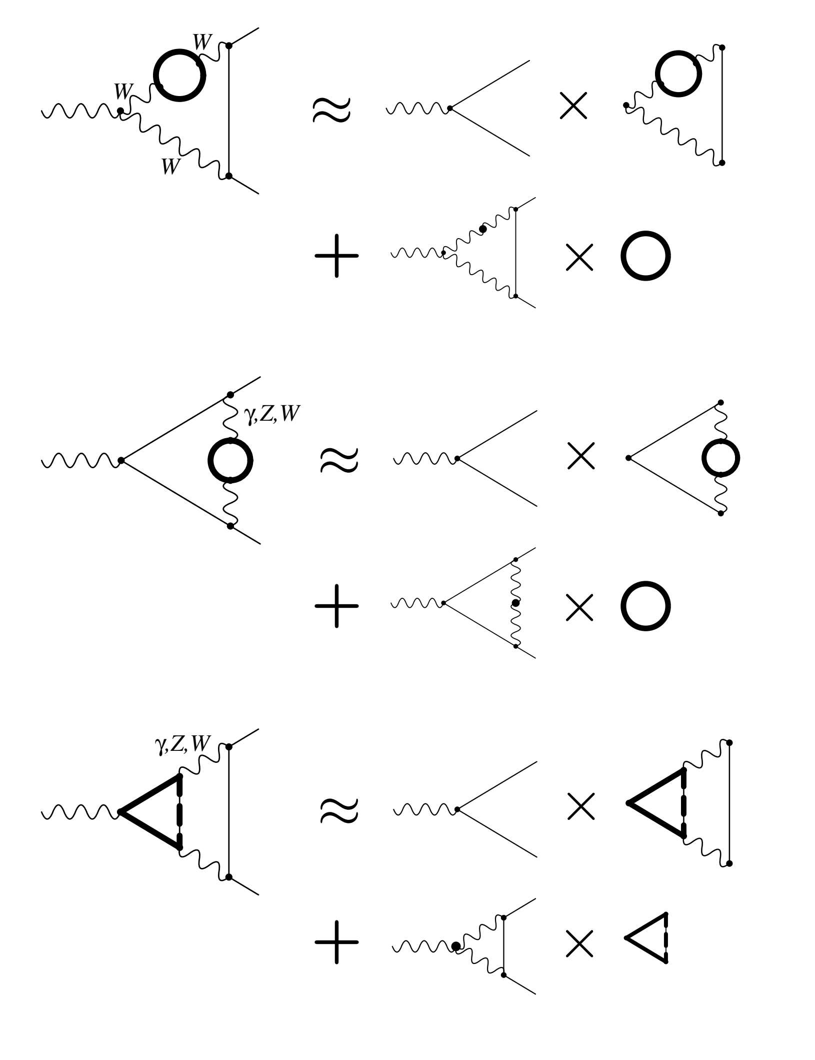
Example applications:

$Zf\bar{f}$ ew. 2-loop vertex corrections

Barbieri et al. '92,93
Fleischer, Tarasov, Jegerlehner '93,95
Degrassi, Gambino, Sirlin '97
Awramik, Czakon, Freitas, Weiglein '04

$\mathcal{O}(\alpha\alpha_s^n)$ corr. to $\Delta\rho$, Δr, ...

Djouadi, Verzegnassi '87
Bardin, Chizhov '88
Chetyrkin, Kühn, Steinhauser '95
Faisst, Kühn, Seidensticker Veretin '03
...



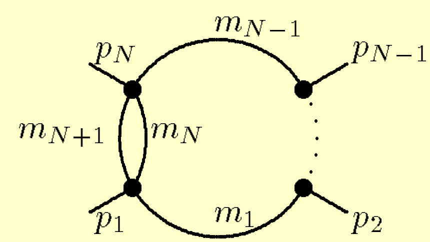
Dispersion relations

Topologies with **self-energy sub-loop** can easily be integrated by using dispsersion relation for B_0 function: S. Bauberger et al. '95

$$B_0(p^2, m_1^2, m_2^2) = -\int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s \, \frac{\Delta B_0(s, m_1^2, m_2^2)}{s - p^2}$$

with
$$\Delta B_0(s, m_1^2, m_2^2) = (4\pi\mu^2)^{4-D} \frac{\Gamma(D/2-1)}{\Gamma(D-2)} \frac{\lambda^{(D-3)/2}(s, m_1^2, m_2^2)}{s^{D/2-1}},$$

$\lambda(a, b, c) = (a - b - c)^2 - 4bc$



$$T_{N+1}(p_i; m_i^2) = -\int_{s_0}^{\infty} \mathrm{d}s \, \Delta B_0(s, m_N^2, m_{N+1}^2)$$
$$\times \int \mathrm{d}^4 q \, \frac{1}{q^2 - s} \frac{1}{(q+p_1)^2 - m_1^2} \cdots \frac{1}{(q+p_1+\cdots+p_{N-1})^2 - m_{N-1}^2}$$

Application: M_W at 2-loop

Freitas, Hollik, Walter, Weiglein '00
Awramik, Czakon '02, Onishchenko, Veretin '02

Dispersion relations

Dispersion relations for **triangle subloops** possible, but difficult

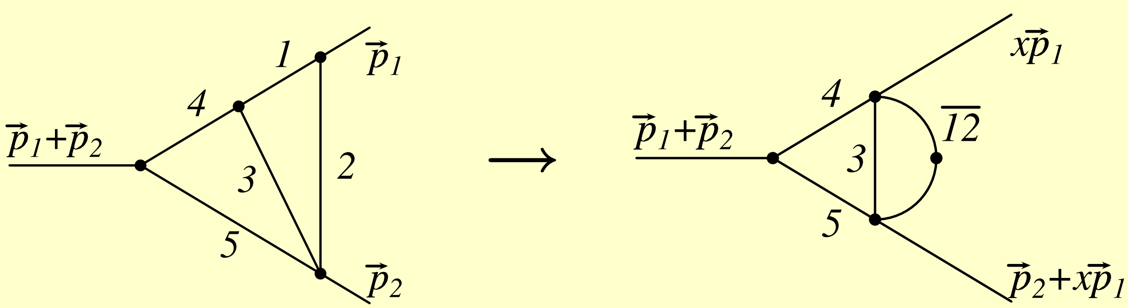
$\longrightarrow$ Alternative approach: Introduce Feynman parameters

J. v.d.Bij, A. Ghinculov '94

$$\frac{1}{(q+p_1)^2 - m_1^2}\,\frac{1}{(q+p_2)^2 - m_2^2} = \int_0^1 \mathrm{d}x\,\frac{1}{[(q+\bar{p})^2 - \overline{m}^2]^2}$$

$$\bar{p} = x\,p_1 + (1-x)p_2, \qquad \overline{m} = x\,m_1 + (1-x)m_2 - x(1-x)(p_1-p_2)^2$$

Reduces triangle to self-energy sub-loops:



Application: $Zf\bar{f}$ vertex corrections

Awramik, Czakon, Freitas '04

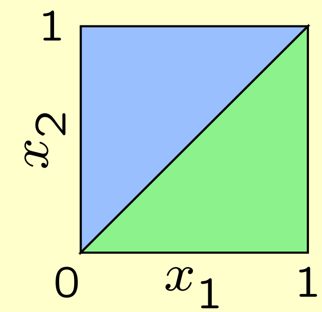
Feynman parameter integration

General form of Feynman integral:

$$I = \int_0^1 dx_1 \ldots dx_n \, \delta(1 - \sum_i x_i) \, \frac{N(x_i)}{D(x_i)^{r+\varepsilon}}$$

$\longrightarrow$ Can be integrated numerically (if finite)

Treatment of divergencies:



- **Sector decomposition:** Sub-divide integration space such that divergent terms factorize

  Binoth, Heinrich '00,03

$\longrightarrow$ many applications (see talks by F. Caola, S. Borowka)

- **Subtraction terms:** Remove divergencies with simple terms that can be integrated analytically

  Nagy, Soper '03
  Becker, Reuschle, Weinzierl '10
  Freitas '12

Mellin-Barnes representations

Transform Feynman integral with Mellin-Barnes representation

$$\frac{1}{(A_0 + \ldots + A_m)^Z} = \frac{1}{(2\pi i)^m} \int_{\mathcal{C}_1} dz_1 \cdots \int_{\mathcal{C}_m} dz_m$$

$$\times A_1^{z_1} \cdots A_m^{z_m} A_0^{-Z-z_1-\ldots-z_m}$$

$$\times \frac{\Gamma(-z_1) \cdots \Gamma(-z_m) \Gamma(Z + z_1 + \ldots + z_m)}{\Gamma(Z)},$$

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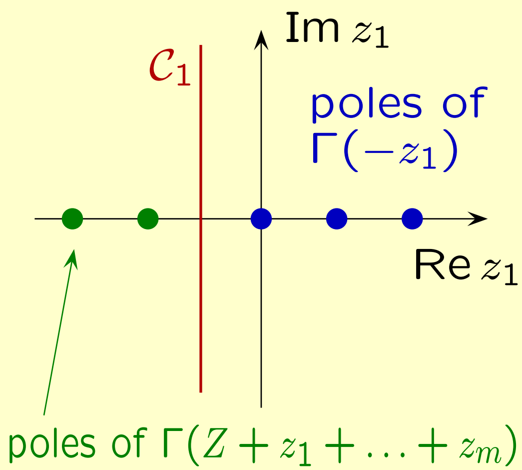

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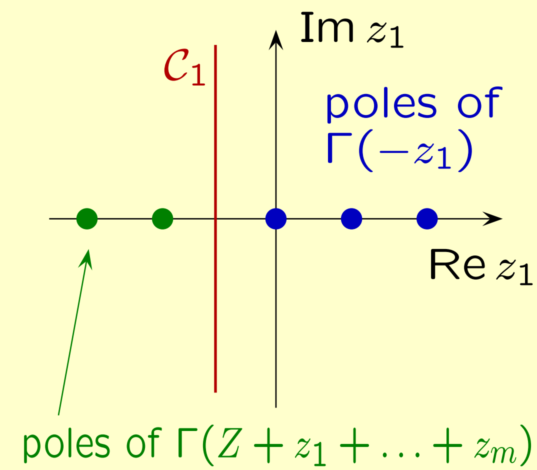
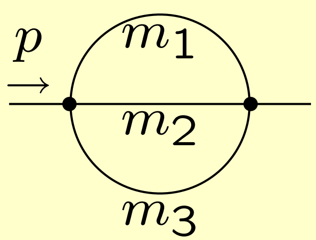


After Feynman parameter integration: Γ functions and exponentials

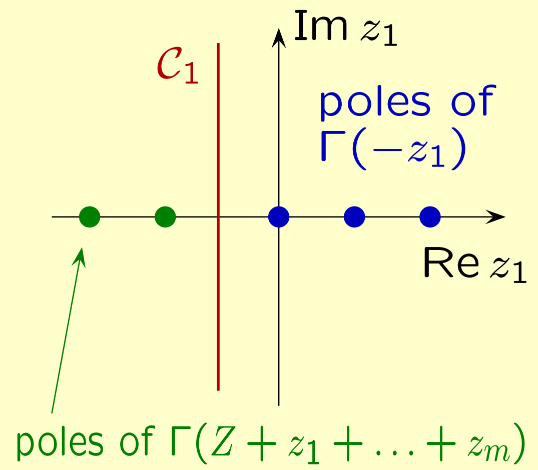
Example:



$$= \frac{-1}{(2\pi i)^3} \int dz_1 dz_2 dz_3\, (m_1^2)^{-\varepsilon-z_1-z_2} (m_2^2)^{z_2} (m_3^2)^{1-\varepsilon+z_1-z_3} (-p^2)^{z_3}$$
$$\times\, \Gamma(-z_2)\Gamma(-z_3)\Gamma(1+z_1+z_2)\Gamma(z_3-z_1)$$
$$\times\, \frac{\Gamma(1-\varepsilon-z_2)\Gamma(\varepsilon+z_1+z_2)\Gamma(\varepsilon-1-z_1+z_3)}{\Gamma(2-\varepsilon+z_3)}$$

Mellin-Barnes representations

- Consistent choice of all $\mathcal{C}_i$ often requires $\varepsilon \neq 0$ $(Z = n + \epsilon)$



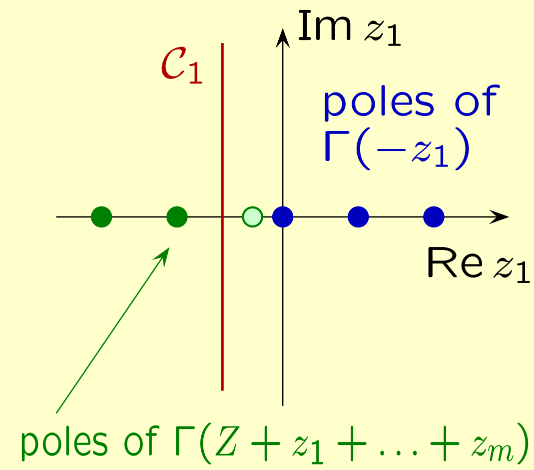
$\varepsilon \neq 0$

Mellin-Barnes representations

- Consistent choice of all $\mathcal{C}_i$ often requires $\varepsilon \neq 0$ $(Z = n + \epsilon)$
- For $\varepsilon \longrightarrow 0$: residues from pole crossings $\longrightarrow 1/\varepsilon^k$ terms

  Czakon '06

  Anastasiou, Daleo '06
- Do remaining $\mathcal{C}_i$ integrations numerically



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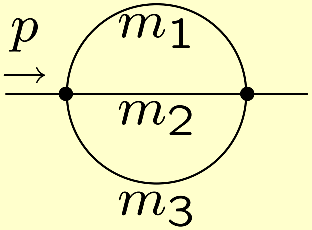
Mellin-Barnes representations

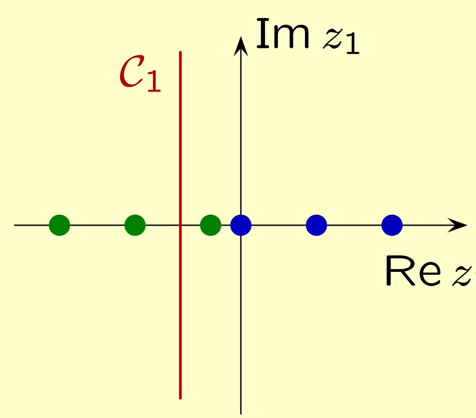


$$= \frac{-1}{(2\pi i)^3} \int dz_1 dz_2 dz_3 \, (m_1^2)^{-\varepsilon - z_1 - z_2} (m_2^2)^{z_2} (m_3^2)^{1-\varepsilon+z_1-z_3} (-p^2)^{z_3}$$
$$\times \Gamma(-z_2)\Gamma(-z_3)\Gamma(1+z_1+z_2)\Gamma(z_3-z_1)$$
$$\times \frac{\Gamma(1-\varepsilon-z_2)\Gamma(\varepsilon+z_1+z_2)\Gamma(\varepsilon-1-z_1+z_3)}{\Gamma(2-\varepsilon+z_3)}$$

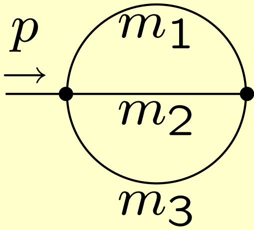
$z_3 = c_3 + iy_3, \qquad y_i \in (-\infty, \infty)$

$$(-p^2)^{z_3} = \underbrace{(p^2)^{c_3+iy_3} e^{-i\pi c_3}}_{\text{oscillating}} \underbrace{e^{\pi y_3}}$$

div. for $y_3 \to \infty$, eventually overcome by Γ funct.



Mellin-Barnes representations

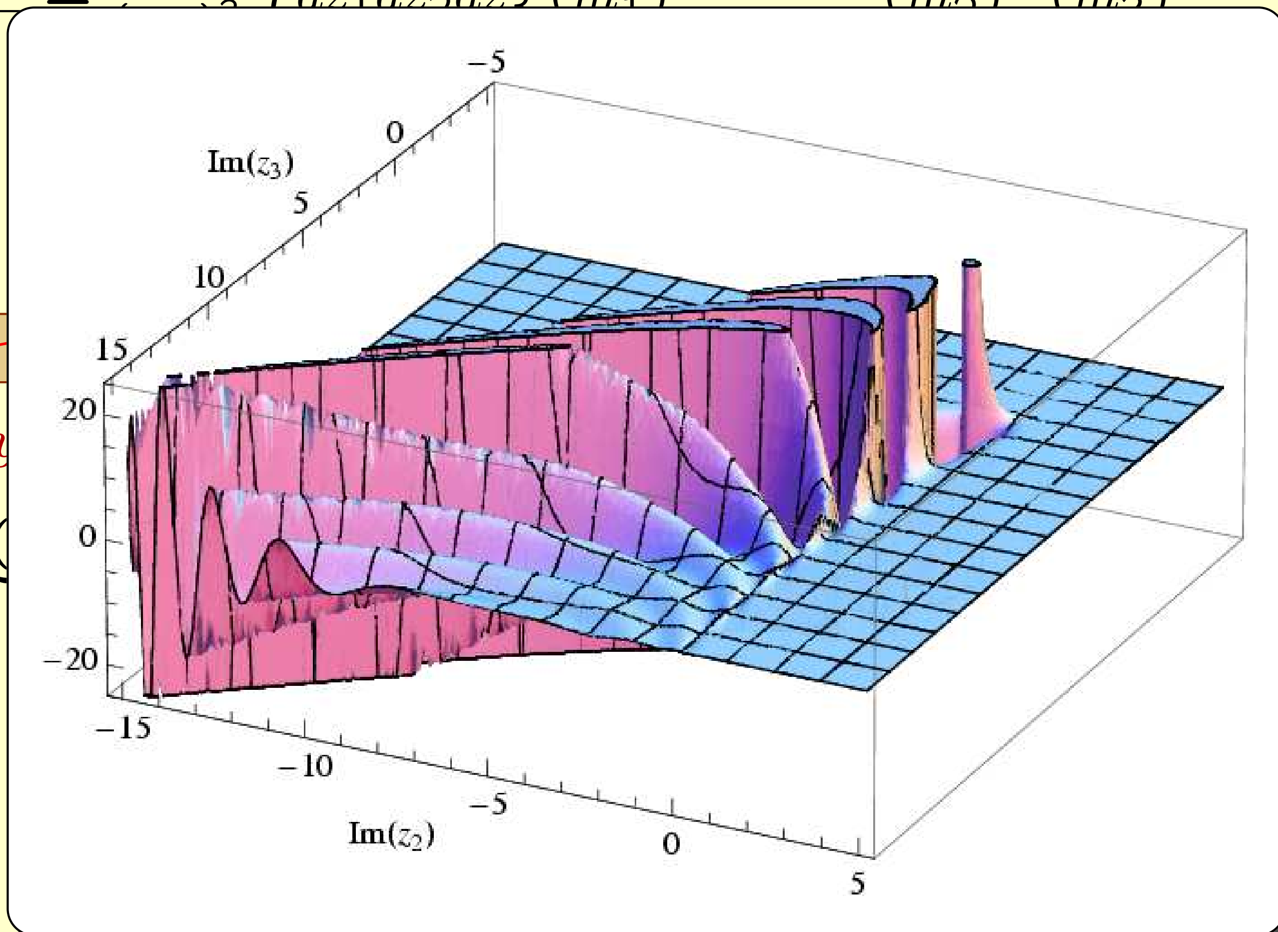


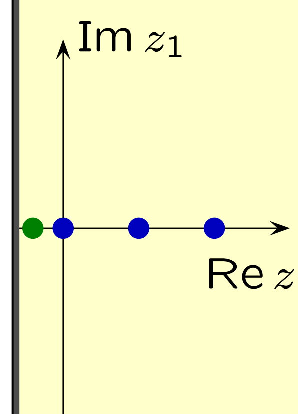
$$= \frac{-1}{\ldots} \int dz_1 dz_2 dz_3 \, (m_1^2)^{-\varepsilon-z_1-z_2}(m_2^2)^{z_2}(m_3^2)^{1-\varepsilon+z_1-z_3}(-p^2)^{z_3}$$

$\ldots z_1 + z_3)$

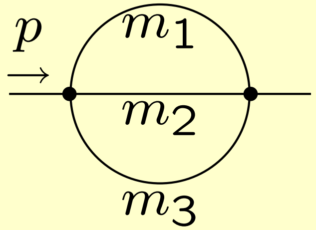
$z_3 = c_3 + iy \ldots$

$(-p^2)^{z_3} = (\ldots$





Mellin-Barnes representations



$$= \frac{-1}{(2\pi i)^3}\int dz_1 dz_2 dz_3\, (m_1^2)^{-\varepsilon-z_1-z_2}(m_2^2)^{z_2}(m_3^2)^{1-\varepsilon+z_1-z_3}(-p^2)^{z_3}$$
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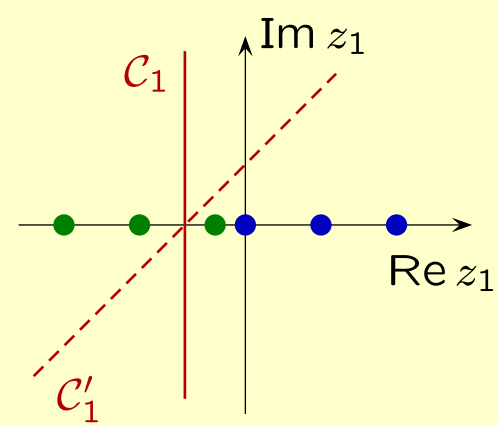
div. for $y_3 \to \infty$

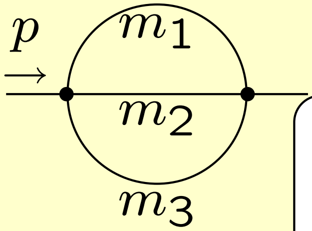
$y_i \to y_i - i\theta$

$$(-p^2)^{z_3} = (p^2)^{c_3+iy_3}e^{-i\pi(c_3+\theta y_i)}e^{(\pi+\theta \log p^2)y_3}$$

Huang, Freitas '10



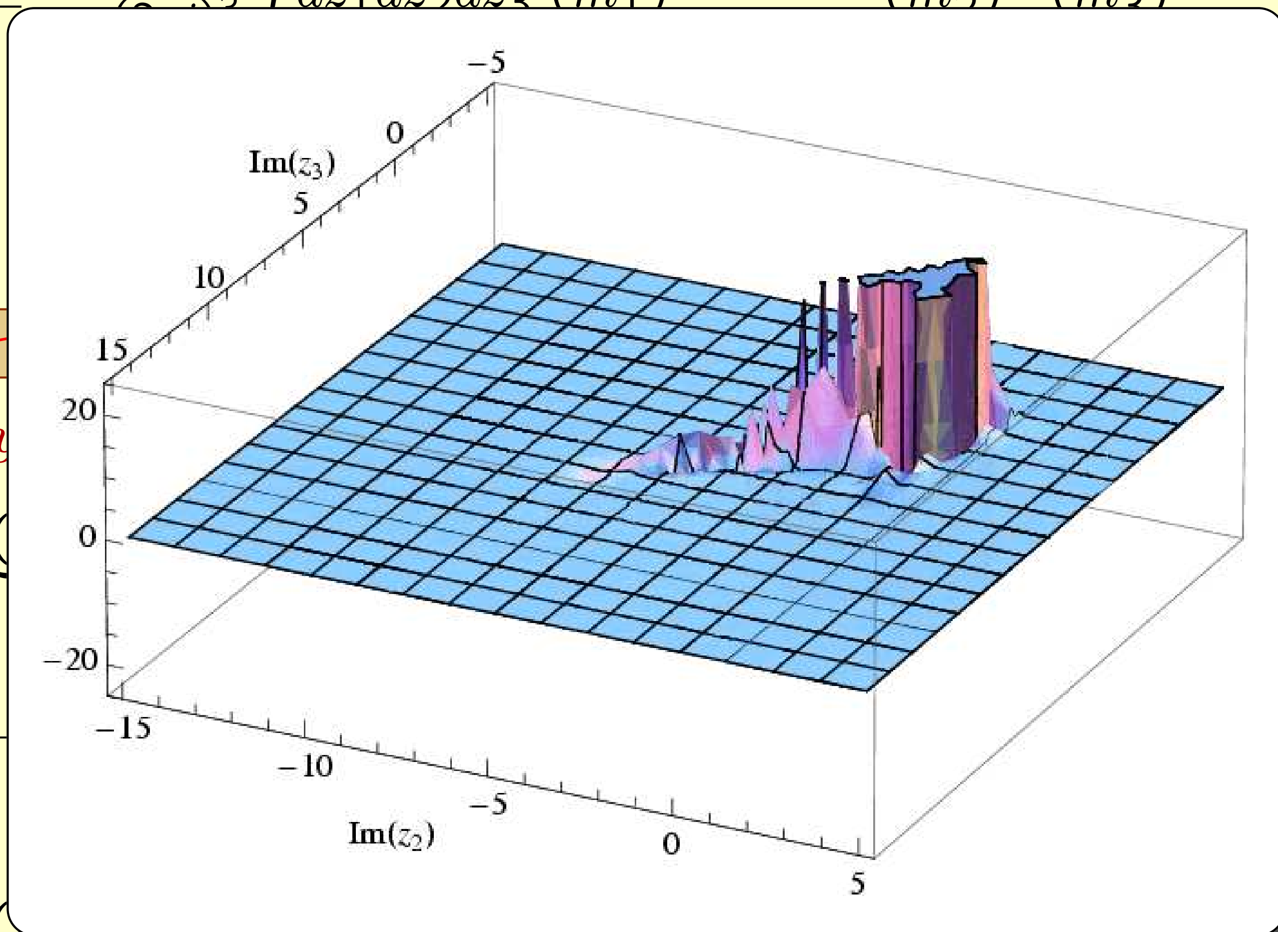
Mellin-Barnes representations



$= \frac{-1}{(2\pi i)^3} \int dz_1 dz_2 dz_3 \, (m_1^2)^{-\varepsilon-z_1-z_2} (m_2^2)^{z_2} (m_3^2)^{1-\varepsilon+z_1-z_3} (-p^2)^{z_3}$

$z_1)$

$z_1 + z_3)$



$z_3 = c_3 + iy$

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$y_i \to y_i - i\theta$

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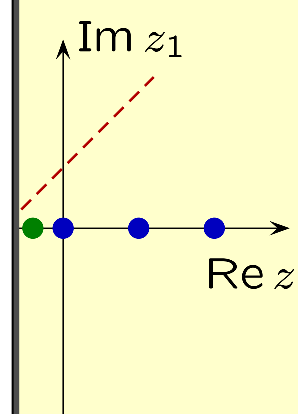
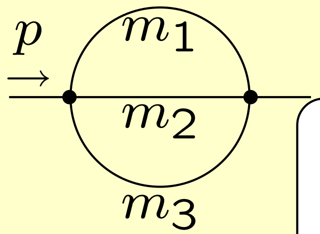
Freitas, Huang '10



Mellin-Barnes representations



$= \frac{-1}{(2\pi i)^3} \int dz_1 dz_2 dz_3 \, (m_1^2)^{-\varepsilon-z_1-z_2}(m_2^2)^{z_2}(m_3^2)^{1-\varepsilon+z_1-z_3}(-p^2)^{z_3}$

$\ldots)$

$\ldots z_1 + z_3)$

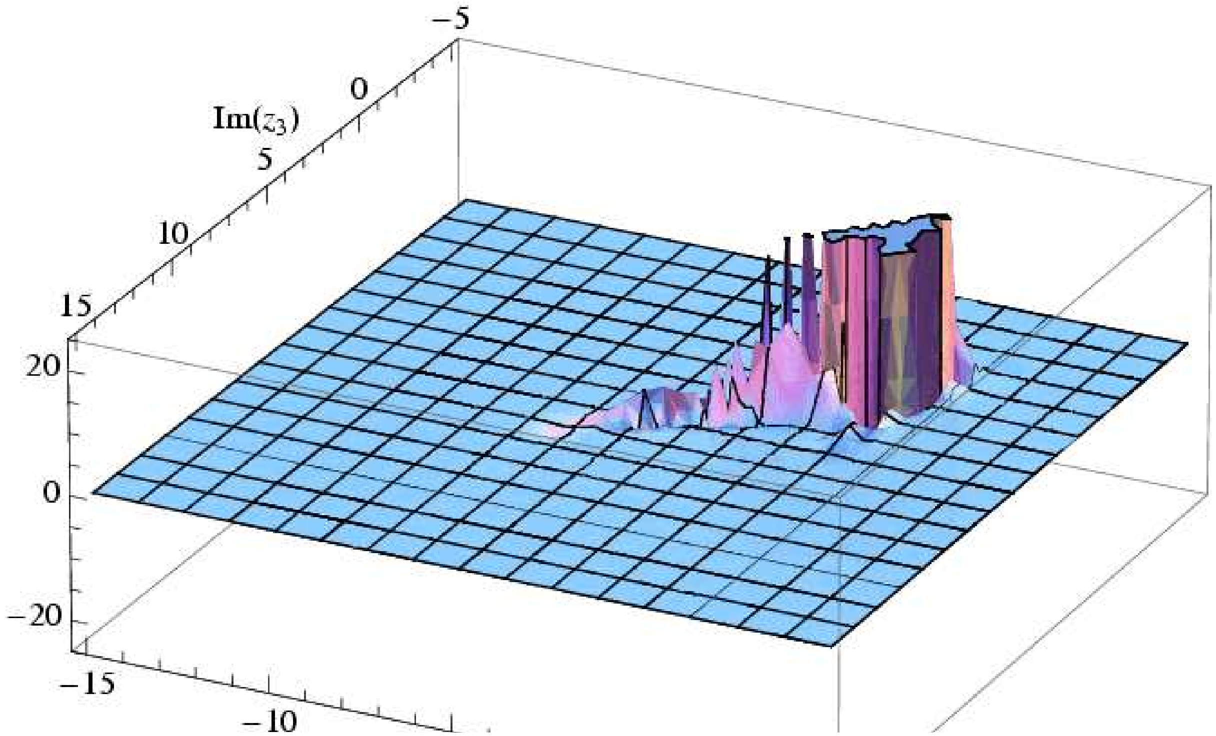
$z_3 = c_3 + iy \ldots$

$(-p^2)^{z_3} = \ldots$

$y_i \to y_i - i\theta \ldots$

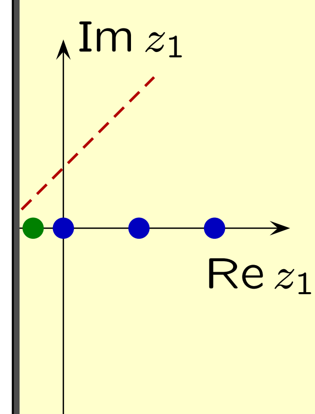
$(-p^2)^{z_3} = (p \ldots$

Freitas, Huang '10



Application: R_b ew. 2-loop corrections

Freitas, Huang '12



Current status of SM predictions

M_W:

- Complete NNLO corrections

  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02

- Partial 3/4-loop corrections

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06

- Theoretical error $\delta_{\rm th} \sim 4$ MeV
  mainly from $\mathcal{O}(\alpha^2\alpha_s)$, $\mathcal{O}(N_f^{\geq 2}\alpha^3)$
  (3-loop self-energies)

- Exp. uncertainty $\delta_{\rm exp} \sim 15$ MeV
  $\delta_{\rm ILC} \sim 7$ MeV

a_μ:

- Complete 5-loop QED

  Aoyama, Hayakawa, Kinoshita, Nio '12

- Complete NNLO

  Czarnecki, Krause, Marciano '96
  Knecht et al. '02
  Czarnecki, Marciano, Vainshtain '03

- Theory error
  $\delta_{\rm th} \sim 6.5 \times 10^{-10}$
  mainly from hadronic contributions

- Exp. error
  $\delta_{\rm exp} \sim 6.3 \times 10^{-10}$

Effective weak mixing angles

$\sin^2\theta^{\ell}_{\text{eff}}$:

- Complete NNLO corrections

  Awramik, Czakon, Freitas, Weiglein '04

  Awramik, Czakon, Freitas '06

  Hollik, Meier, Uccirati '05,07

- Partial 3/4-loop corrections

  Chetyrkin, Kühn, Steinhauser '95

  Faisst, Kühn, Seidensticker, Veretin '03

  Boughezal, Tausk, v. d. Bij '05

  Schröder, Steinhauser '05

  Chetyrkin et al. '06; Boughezal, Czakon '06

- Theoretical error $\delta_{\text{th}} \sim 4.5 \times 10^{-5}$

  mainly from $\mathcal{O}(\alpha^2\alpha_s)$, $\mathcal{O}(N_f^{\geq 2}\alpha^3)$

  (3-loop vertices with sub-bubbles,

  3-loop self-energies)

- Exp. uncertainty $\delta_{\text{exp}} \sim 16 \times 10^{-5}$

  $\delta_{\text{ILC}} \sim 1.3 \times 10^{-5}$

Effective weak mixing angles

$\sin^2\theta^{\ell}_{\rm eff}$:

- Complete NNLO corrections

  Awramik, Czakon, Freitas, Wei

  Awramik, Czakon, Fr

  Hollik, Meier, Uccira

- Partial 3/4-loop corrections

  Chetyrkin, Kühn, Steinha

  Faisst, Kühn, Seidensticker, Ve

  Boughezal, Tausk, v. d

  Schröder, Steinha

  Chetyrkin et al. '06; Boughezal, Cz

- Theoretical error $\delta_{\rm th} \sim 4.5 \times 10$

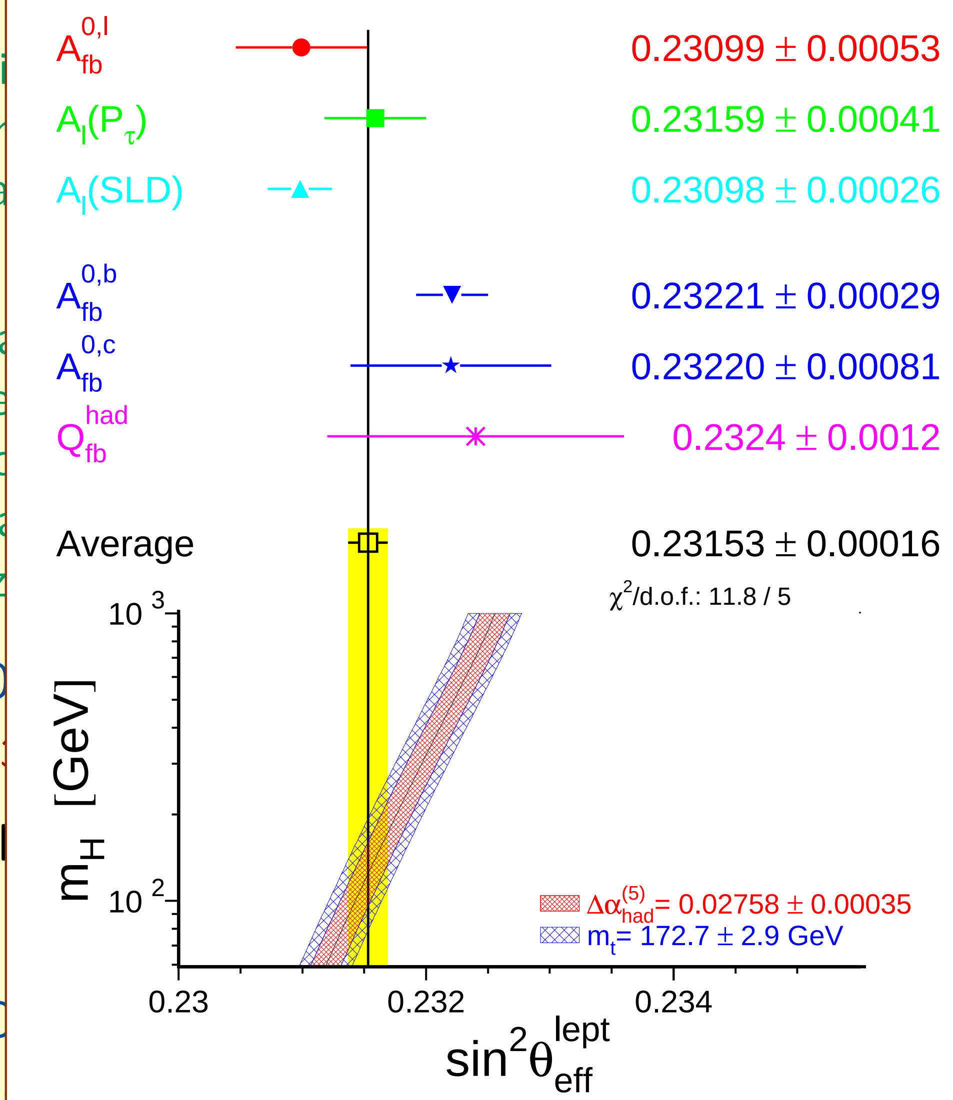
  mainly from $\mathcal{O}(\alpha^2\alpha_s)$, $\mathcal{O}(N_f^{\geq 2}\alpha$

  (3-loop vertices with sub-bubbl

  3-loop self-energies)

- Exp. uncertainty $\delta_{\rm exp} \sim 16 \times 10$

  $\delta_{\rm ILC} \sim 1.3 \times 1$



LEP EWWG '05

Effective weak mixing angles

$\sin^2\theta^q_{\text{eff}}$:

- Complete *fermionic* NNLO corrections — Awramik, Czakon, Freitas '06; Awramik, Czakon, Freitas, Kniehl '08
- Partial 3/4-loop corrections
- Theoretical error $\delta_{\text{th}} \gtrsim 5 \times 10^{-5}$

  mainly from $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(\alpha^2\alpha_s)$ (general 3-loop vertex diagrams)
- Exp. uncertainty $\delta_{\text{exp}} \sim 3 \times 10^{-2}$ $(q = b)$

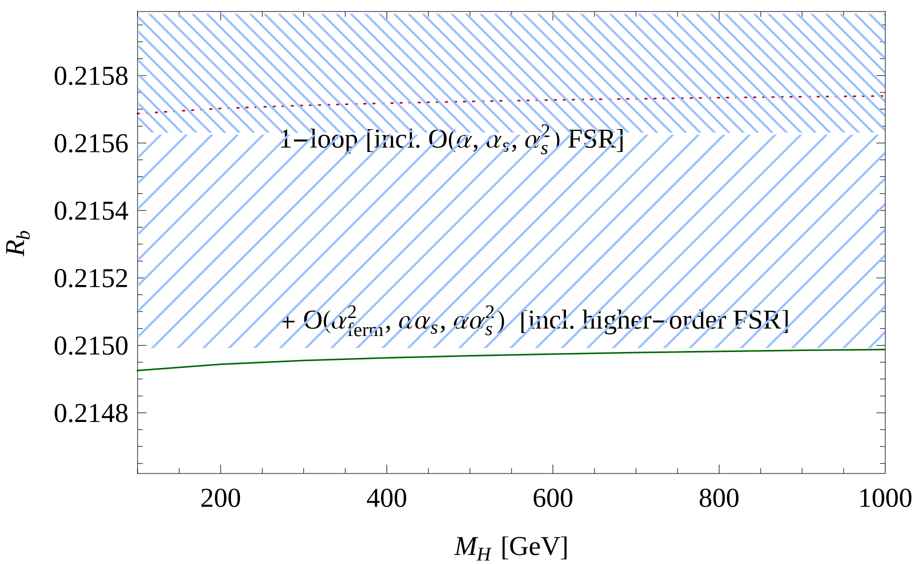
  $\delta_{\text{exp}} \sim 8 \times 10^{-3}$ $(q = c)$

  $\delta_{\text{LHC}} \sim 3 \times 10^{-4}$ $(q = u, d)$

Branching ratios

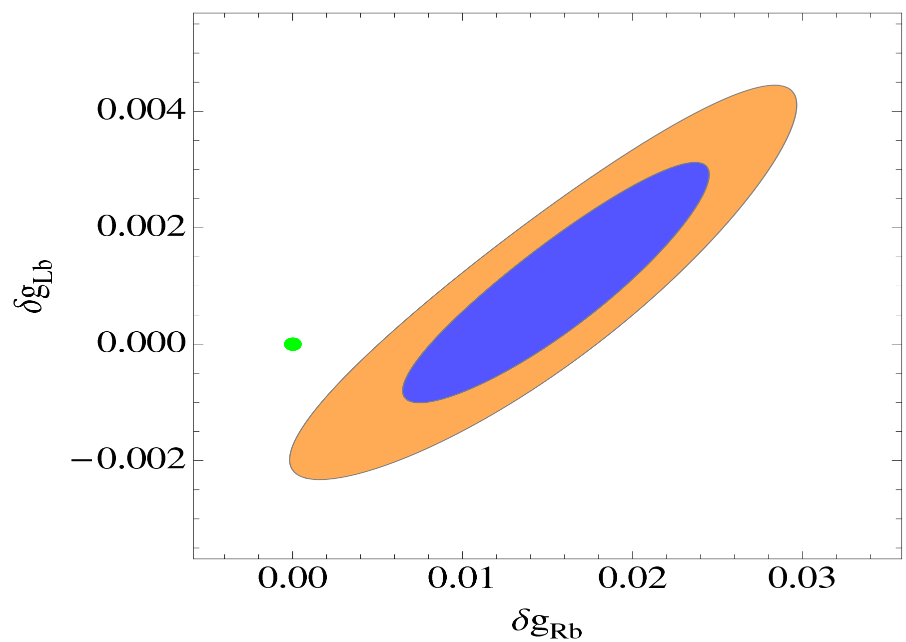
$R_b = \Gamma_b / \Gamma_{\text{had}}$:

- Complete *fermionic* NNLO corrections — Freitas, Huang '12
- Th. uncertainty $\delta_{\text{th}} \sim 2 \times 10^{-4}$
  mainly from $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(\alpha^2\alpha_s)$ (general 3-loop vertex diagrams)
- Exp. uncertainty $\delta_{\text{exp}} \sim 6.6 \times 10^{-4}$
  $\delta_{\text{ILC}} \sim 1.5 \times 10^{-4}$



2.1σ deviation

New physics? — Batell, Gori, Wang '12



Total width and cross section

Γ_Z:

- Approximate NNLO corrections for large m_t — Barbieri et al. '92,93; Fleischer, Tarasov, Jegerlehner '93,95; Degrassi, Gambino '99; Czarnecki, Kühn '96; Harlander, Seidensticker, Steinhauser '98
- Th. uncertainty $\delta_{\text{th}} \sim 7$ MeV

  dominant missing piece: $\mathcal{O}(N_f \alpha^2)$

  (2-loop vertex diagrams)
- Exp. uncertainty $\delta_{\text{exp}} \sim 2$ MeV

σ^0_{had}:

- Status similar to Γ_Z
- Th. uncertainty: ?
- Exp. uncertainty: $\delta_{\text{exp}} \sim 0.037$ nb

  $\delta_{\text{ILC}} \lesssim 0.03$ nb

Summary

- Experimental precision from LEP/SLC demands SM prediction with **complete 2-loop corrections**
- Much progress during last 10–20 years, but still large theoretical uncertainties in some observables
- **LHC** will provide independent results for $\sin^2\theta_{\text{eff}}$ and M_W, but overall precision not improved
- **ILC** with $\sqrt{s} \sim M_Z$ will reduce experimental error by $\mathcal{O}(2-10)$ $\longrightarrow$ Challenge for theorists!
- Need modular and flexible computer program implementation at fully consistent NNLO order

Backup slides

Z-pole observables: theory

Quantity	Current th. error	Est. improvement
$\sin^2\theta^{\ell}_{\rm eff}$	4.5×10^{-5}	factor 3–5
$\sin^2\theta^{q}_{\rm eff}$	5×10^{-5}	factor 1–1.5
R_b	$\sim 2\times10^{-4}$	factor 1–1.5
Γ_Z	~ 7 MeV	factor 3–5

Methods for theory error estimates:

- Parametric factors, $i.e.$ factors of α, N_c, N_f, ...
- Geometric progression, $e.g.$ $\frac{\mathcal{O}(\alpha^3)}{\mathcal{O}(\alpha^2)} \sim \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}$
- Renormalization scheme dependence (use with care!)
- Experience from similar calculations

Implementation

ZFITTER: Bardin et al. '99, Arbuzov et al. '05

- Originially designed for NLO, not NNLO
- Mismatches between NNLO electroweak and final-state QED/QCD corrections (due to approximations)
- Expansion about Z-pole not consistent to NNLO

$\longrightarrow$ Currently no indication for numerically large problems, but fully consistent NNLO treatment is desirable

Other programs:

GFITTER Flächer et al. '08

TOPAZ0 (deprecated) Montagna, Nicrosini, Passarino, Piccinini '98,01

Implementation

Elements of consistent and flexible code:

- Define amplitude a Laurent expansion about complex pole $s = m_Z^2 - i m_Z \Gamma_Z$
- Real corrections: only treat soft/collinear pieces analytically, the rest numerically (allows for flexible cuts)
- Include ISR resummation without double-counting
- Modular (object oriented?) code structure