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*Measuring Spin/Parity of the Higgs Boson:  
Theoretical Basis*

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*in collaboration with D. J. Miller and P. M. Zerwas*

**Light Higgs Mass Meeting**

**CERN**

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# Program

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## Necessary and sufficient conditions for $J^P = 0^+$ [vis-a-vis SM]

- **Extension of**

- \* S.Y. Choi, D.J. Miller, M.M. Muhlleitner and P.M. Zerwas  
“Identifying the Higgs spin and parity in decays to  $Z$  pairs”  
Phys. Lett. **B553** (2003) 61 [hep-ph/0210077]

- \* S.Y. Choi, M.M. Muhlleitner and P.M. Zerwas  
“Theoretical Basis of Higgs-Spin Analysis in  $H \rightarrow \gamma\gamma$  and  $Z\gamma$  Decays”  
Phys. Lett. **B718** (2013) 1031 [arXiv:1209.5268 [hep-ph]]

- **Other publications:** Hagiwara eal; De Rujula eal; Y. Gao eal; S. Bolognesi eal; J. Ellis eal

- **Theoretical Tools:**

- \* helicity analyses

- \* operator expansions

→ two typical examples

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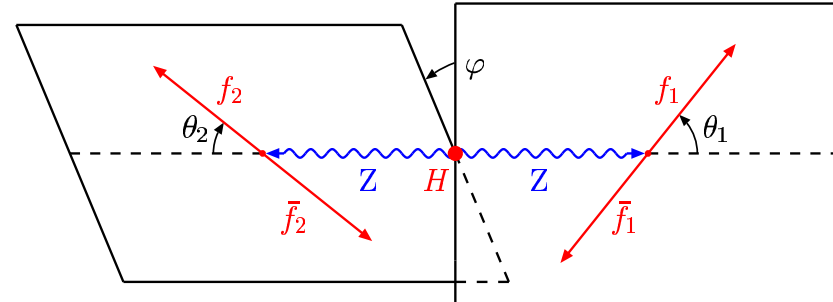
# (I) Angular Distributions/Thresholds in $H \rightarrow VV^* \rightarrow 4\ell$

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◇ Determination of spin and parity in

$$gg \rightarrow H \rightarrow ZZ^{(*)} \rightarrow (f_1\bar{f}_1)(f_2\bar{f}_2)$$

in  $H$  c.m. frame



◇ Helicity methods to generalize to arbitrary spin and parity

$$\langle Z(\lambda_1)Z(\lambda_2)|H_{\mathcal{J}}(m)\rangle = \frac{g_W M_Z}{\cos\theta_W} \mathcal{T}_{\lambda_1\lambda_2} d_{m,\lambda_1-\lambda_2}^{\mathcal{J}}(\Theta) e^{-i(m-\lambda_1+\lambda_2)\varphi}$$

◇ General tensor for  $HZZ$  vertex for each  $\mathcal{J}^{\mathcal{P}}$   $\Rightarrow$  Expressions for reduced vertices  $\mathcal{T}_{ij}$

$$\mathcal{J} = \frac{g_W M_Z}{\cos\theta_W} T_{\mu\nu\beta_1\dots\beta_{\mathcal{J}}} \epsilon^*(Z_1)^\mu \epsilon^*(Z_2)^\nu \epsilon(H)^{\beta_1\dots\beta_{\mathcal{J}}}$$

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## Differential Distributions

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◇ Double polar angular distribution ( $\mathcal{CP}$  invariant theory)

$$\begin{aligned} \frac{d\Gamma_H}{d\cos\theta_1 d\cos\theta_2} &\sim \sin^2\theta_1 \sin^2\theta_2 |\mathcal{T}_{00}|^2 + \frac{1}{2}(1 + \cos^2\theta_1)(1 + \cos^2\theta_2) [|\mathcal{T}_{11}|^2 + |\mathcal{T}_{1,-1}|^2] \\ &\quad + (1 + \cos^2\theta_1) \sin^2\theta_2 |\mathcal{T}_{10}|^2 + \sin^2\theta_1 (1 + \cos^2\theta_2) |\mathcal{T}_{01}|^2 \\ &\quad + 2\eta_1\eta_2 \cos\theta_1 \cos\theta_2 [|\mathcal{T}_{11}|^2 - |\mathcal{T}_{1,-1}|^2] \end{aligned}$$

◇ Azimuthal angular distribution ( $\mathcal{CP}$  invariant theory)

$$\begin{aligned} \frac{d\Gamma_H}{d\varphi} &\sim |\mathcal{T}_{11}|^2 + |\mathcal{T}_{10}|^2 + |\mathcal{T}_{1,-1}|^2 + |\mathcal{T}_{01}|^2 + |\mathcal{T}_{00}|^2/2 \\ &\quad + \eta_1\eta_2 \left(\frac{3\pi}{8}\right)^2 \Re(\mathcal{T}_{11}\mathcal{T}_{00}^* + \mathcal{T}_{10}\mathcal{T}_{0,-1}^*)\cos\varphi + \frac{1}{4}\Re(\mathcal{T}_{11}\mathcal{T}_{-1,-1}^*)\cos 2\varphi \end{aligned}$$

$$\eta_i = 2v_i a_i / (v_i^2 + a_i^2), \quad v_i, a_i \text{ electroweak fermion } f_i \text{ charges}$$

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## Determination of Spin and Parity, Necessary Conditions

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- **Standard Model:** Necessary conditions:

- ◇ Double polar angular distribution

$$\frac{d\Gamma_H}{d\cos\theta_1 d\cos\theta_2} \sim \sin^2\theta_1 \sin^2\theta_2 + \frac{1}{2\gamma_1^2\gamma_2^2(1+\beta_1\beta_2)^2} [(1+\cos^2\theta_1)(1+\cos^2\theta_2) + 4\eta_1\eta_2\cos\theta_1\cos\theta_2]$$

- ◇ Azimuthal angular distribution

$$\frac{d\Gamma_H}{d\varphi} \sim 1 - \eta_1\eta_2 \frac{1}{2} \left(\frac{3\pi}{4}\right)^2 \frac{\gamma_1\gamma_2(1+\beta_1\beta_2)}{\gamma_1^2\gamma_2^2(1+\beta_1\beta_2)^2 + 2} \cos\varphi + \frac{1}{2} \frac{1}{\gamma_1^2\gamma_2^2(1+\beta_1\beta_2)^2 + 2} \cos 2\varphi$$

$\beta_i, \gamma_i$  ( $i = 1, 2$ ): velocity and boost factors

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## Determination of Spin and Parity, Sufficient Conditions

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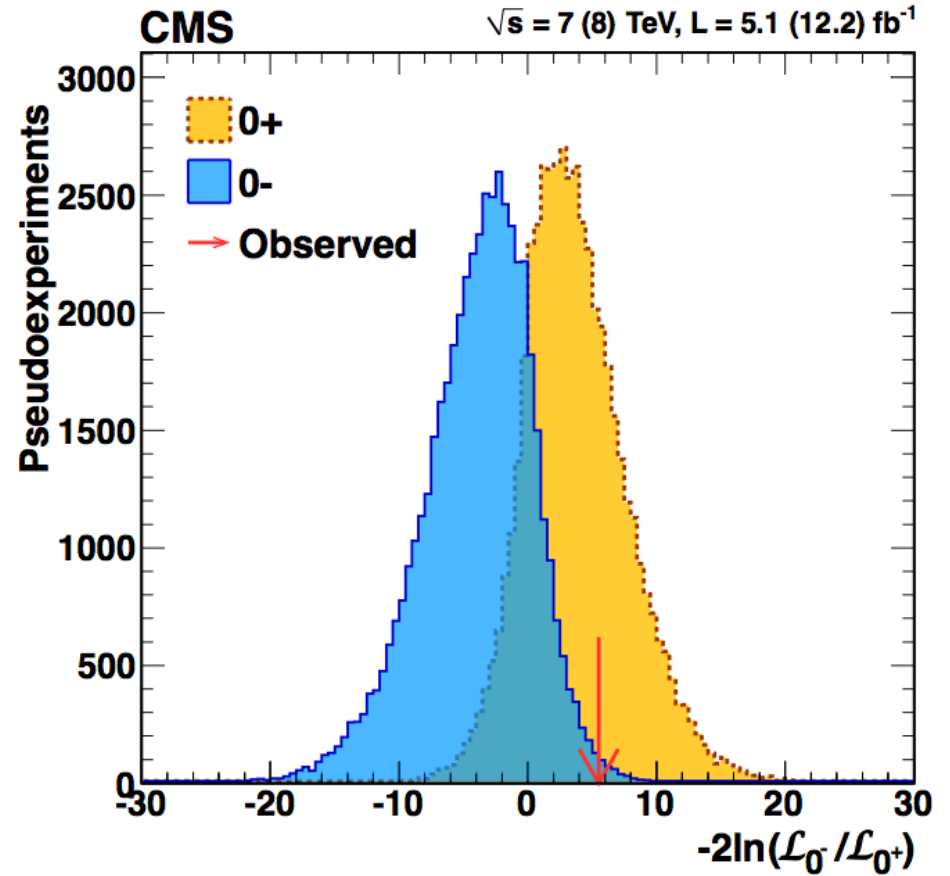
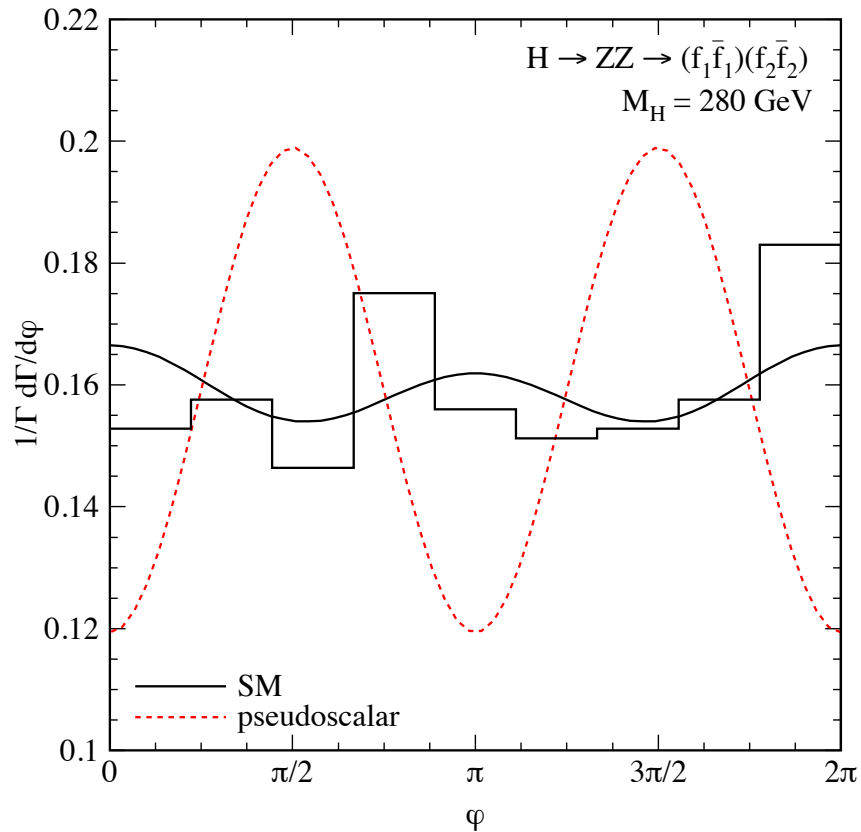
•  $M_H < 2M_Z$ :  $d\Gamma/dM_*^2 \sim \beta$  for  $\mathcal{J}^{\mathcal{P}} = 0^+$

◇  $d\Gamma/dM_*^2$  rules out  $\mathcal{J}^{\mathcal{P}} = 0^-, 1^-, 2^-, 3^\pm, 4^\pm$  [threshold rise  $\sim \beta^{2J+1}$ ]

◇  $d\Gamma/dM_*^2$  and no  $[1 + \cos^2 \theta_1] \sin^2 \theta_2$   
 $[1 + \cos^2 \theta_2] \sin^2 \theta_1$  rules out  $\mathcal{J}^{\mathcal{P}} = 1^+, 2^+$

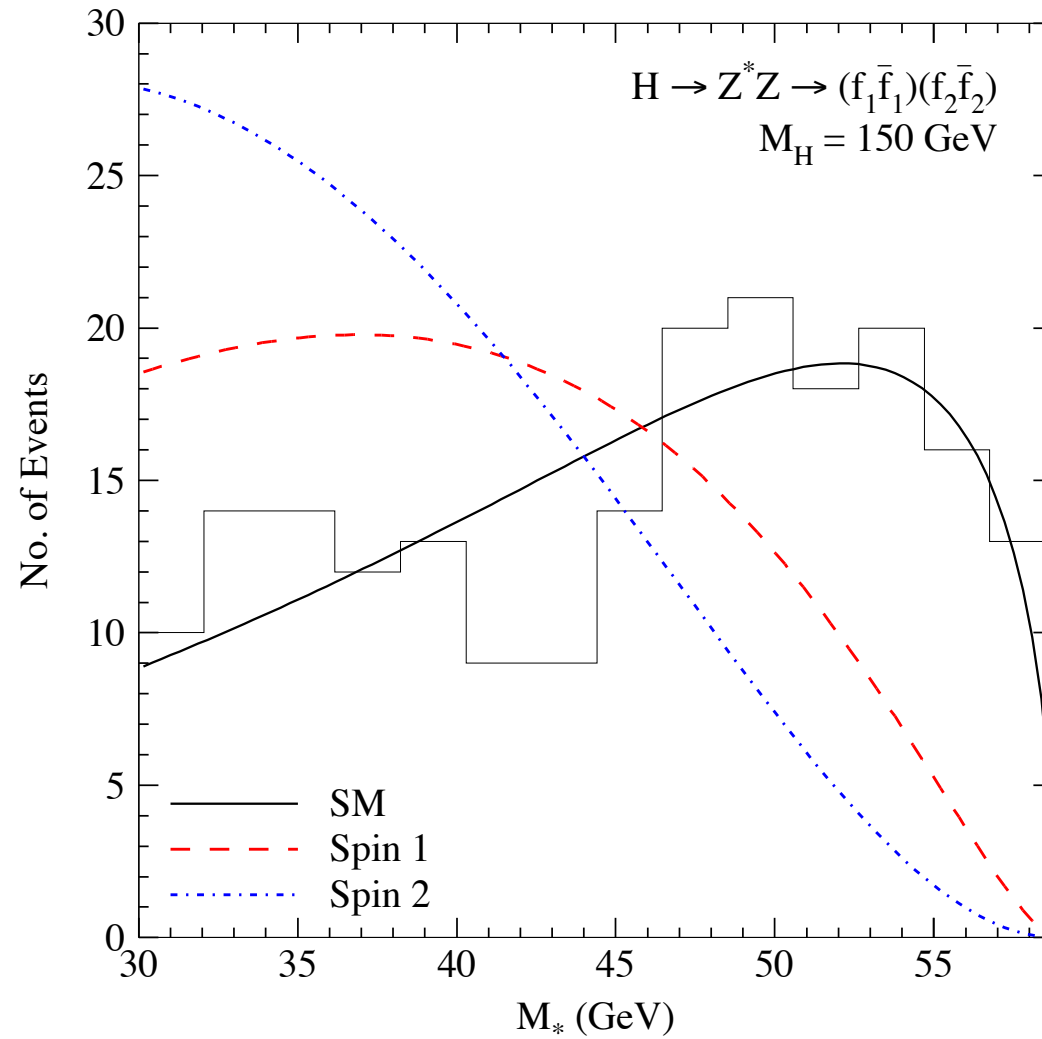
$\Rightarrow$  only  $0^+$  left (sufficient conditions)

## # Azimuthal Angular Distributions: Parity



Large masses:  $0^+ : d\Gamma/d\varphi \sim \text{const.}, \quad 0^- : d\Gamma/d\varphi \sim 1 - 1/4 \cos 2\varphi$

# # Threshold Behaviour: Spin





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## (II) Higgs-Spin Analysis in $H \rightarrow \gamma\gamma$ Decays

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- Systematic helicity analyses for angular distributions

$$\frac{1}{\sigma} \frac{d\sigma(\gamma\gamma)}{d\cos\Theta} = (2J + 1)[\mathcal{X}_0^J \mathcal{Y}_0^J \mathcal{D}_{00}^J + \mathcal{X}_0^J \mathcal{Y}_2^J \mathcal{D}_{02}^J + \mathcal{X}_2^J \mathcal{Y}_0^J \mathcal{D}_{20}^J + \mathcal{X}_2^J \mathcal{Y}_2^J \mathcal{D}_{22}^J]$$

- \*  $\mathcal{D}_{m\lambda}^J$  squared Wigner functions,  $m = S_z$  spin component,  $\lambda \equiv \lambda_\gamma - \lambda'_\gamma$
- \*  $\mathcal{X}$  production helicity probability
- \*  $\mathcal{Y}$  decay helicity probability

- Decays

'scalar-type assignment':  $\mathcal{X}_0^J = \mathcal{Y}_0^J = 1$  and  $\mathcal{X}_2^J = \mathcal{Y}_2^J = 0$  [ $J \geq 0$ ]

'tensor-type assignment':  $\mathcal{X}_0^J = \mathcal{Y}_0^J = 0$  and  $\mathcal{X}_2^J = \mathcal{Y}_2^J = 1$  [ $J \geq 2$ ]

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## General Spin/Parity Assignments

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- **Selection rules for Higgs spin/parity** from observing the polar angular distributions of a spin- $J$  Higgs state in  $gg \rightarrow H \rightarrow \gamma\gamma$

$\mathcal{P} \setminus J$	0	1	2, 4, $\dots$	3, 5, $\dots$
even	1	forbidden	$\mathcal{D}_{00}^J$ $\mathcal{D}_{02}^J$ $\mathcal{D}_{20}^J$ $\mathcal{D}_{22}^J$	$\mathcal{D}_{22}^J$
odd	1	forbidden	$\mathcal{D}_{00}^J$	forbidden

- **Squared Wigner functions**  $\mathcal{D}_{m\lambda}^J$  up to  $\sim |\cos^{2J} \Theta|$

$$\mathcal{D}_{00}^0 = 1$$

$$\mathcal{D}_{00}^2 = (3 \cos^2 \Theta - 1)^2/4 \quad \mathcal{D}_{22}^2 = (\cos^4 \Theta + 6 \cos^2 \Theta + 1)/16$$

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odd	1	forbidden	$\mathcal{D}_{00}^J$	forbidden

$0^\pm$  :  $D_{00}^0$  observed, none else  $\rightsquigarrow \pm$  undisc       $1^\pm$  : forbidden by Landau/Yang

$2^+$  :  $D_{00}^2$  and  $D_{22}^2 \neq 0$ , both (KK)       $3^+$  :  $D_{22}^3 \neq 0$ , none else

$2^-$  :  $D_{00}^2 \neq 0$ , none else       $3^-$  : forbidden

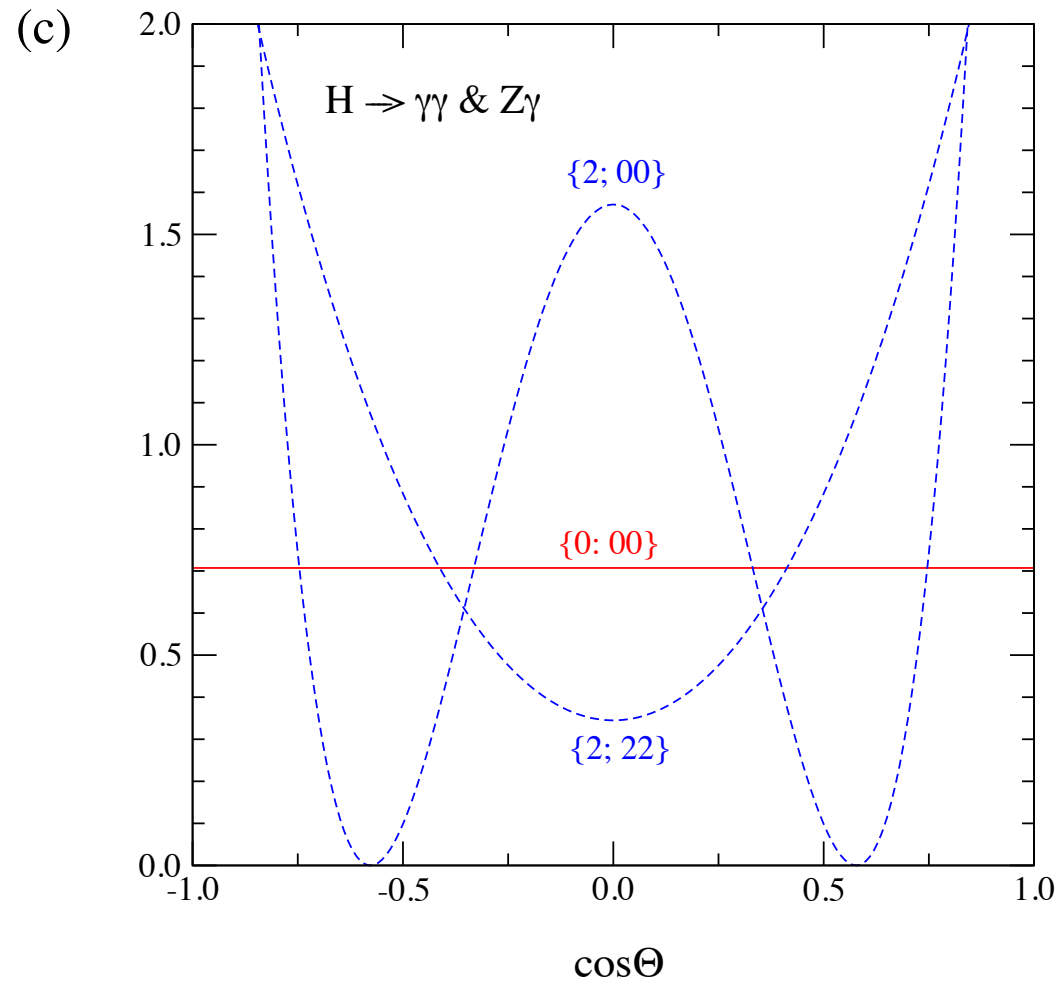
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## Scalar-type, Tensor-type

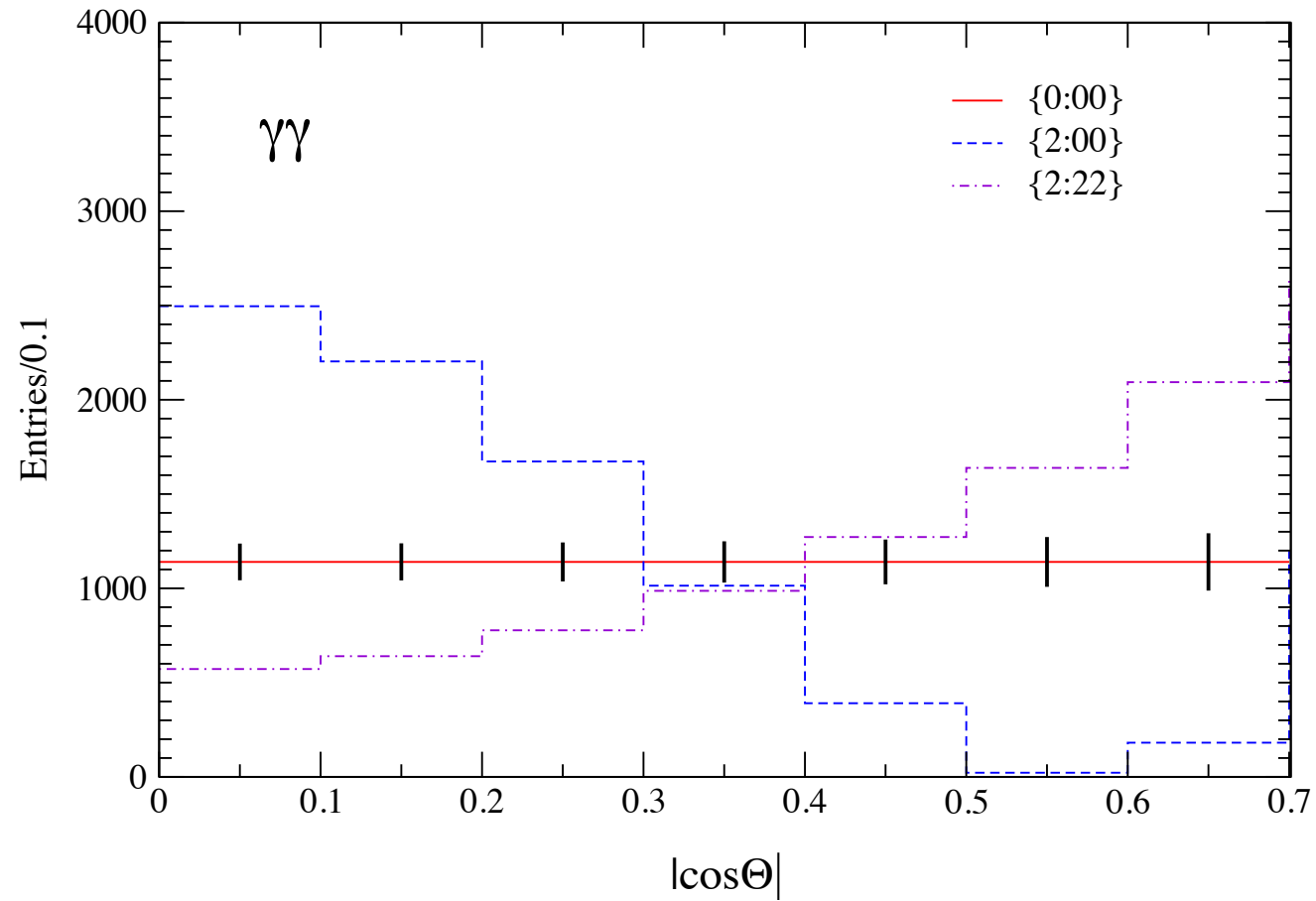
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## Distinction Scalar-type, Tensor-type

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## Future

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Future directions Systematics of spin/parity in vector boson fusion  
 $\tau$  decays

for  $0^+$  in the SM and  $0^-$  in  $A$  [MSSM]

Straightforward strategies identified for proving  $J^P = 0^+$  experimentally  
under necessary and sufficient conditions.