Flavour physics from an approximate $U(2)^3$ symmetry

Dario Buttazzo

Scuola Normale Superiore and INFN, Pisa
CERN PH-TH (UNILHC)

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in collaboration with R. Barbieri, F. Sala, D. Straub, A. Tesi

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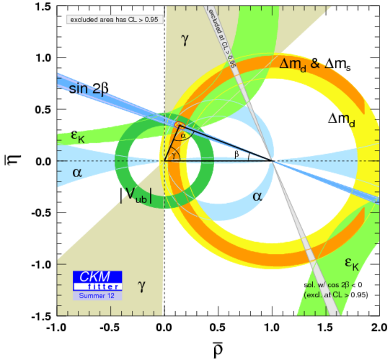
Flavour and CP: the CKM picture

$$\mathcal{L}_{\text{Yuk}} = Y_u^{ij} \bar{q}_L^i H u_R^j + Y_d^{ij} \bar{q}_L^i H^\dagger d_R^j$$

- Excellent agreement between experiments and SM predictions.



The SM flavour puzzle

$$(y_u, y_c, y_t) \sim (10^{-6}, 10^{-2}, 1) \qquad (y_d, y_s, y_b) \sim (10^{-5}, 10^{-3}, 10^{-2})$$

$$V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 4\times10^{-3} \\ 0.2 & 1 & 4\times10^{-2} \\ 9\times10^{-3} & 4\times10^{-2} & 1 \end{pmatrix}$$

The flavour and hierarchy problems

Flavour: excellent agreement between experiments and CKM picture constrains new physics.
If $\Delta\mathcal{L}_{\rm NP} = \sum_i \frac{1}{\Lambda_i^2}\mathcal{O}_i$, in some cases $\Lambda_i \gtrsim 10^3 \div 10^4$ TeV.
Hierarchy: the scalar masses $m_h \approx \Lambda$, where Λ is the highest scale coupled to h. For a natural Higgs $\Lambda \lesssim 4\pi v \approx 3$ TeV.

A SM extension at the weak scale cannot have a generic flavour structure!

Ideally one would have

$$\Delta\mathcal{L} = \frac{1}{\Lambda^2}\sum_i c_i \xi_i \mathcal{O}_i,$$

with $\Lambda \approx 3$ TeV, ξ_i **controlled by some symmetry**, and $c_i \sim 1$.

Minimal Flavour Violation

Full flavour group of the SM: $U(3)^3 = U(3)_q \times U(3)_u \times U(3)_d$.

$$\mathcal{L}_Y = Y_u \bar{q}_L H^\dagger u_R + Y_d \bar{q}_L H d_R \quad \text{is not invariant.}$$

Assume:

- $Y_{u,d}$ have fictitious transformation properties under $U(3)^3$

$$Y_u \sim (3, \bar{3}, 1), \qquad Y_d \sim (3, 1, \bar{3})$$

- No other sources of breaking: Lagrangian is formally invariant.

Example

$$\mathcal{L} \supset \frac{1}{\Lambda^2}\big(\bar{d}_L^i \gamma_\mu Y_u^{ik} (Y_u^{kj})^\dagger d_L^j\big)^2 \xrightarrow[\text{mass basis}]{\text{physical}} \frac{(V_{ti}V_{tj}^*)^2}{\Lambda^2}(\bar{d}_L^i \gamma_\mu d_L^j)^2 + \cdots$$

- Explains smallness of flavour-changing effects
- Does not explain the CKM hierarchies
- y_t is large (not a good expansion parameter)

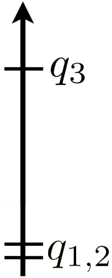
An approximate $U(2)^3$ flavour symmetry

$$\mathcal{L} \approx \sum_{i=1,2,3} \left(\bar{q}_L^i \not{D} q_L^i + \bar{u}_R^i \not{D} u_R^i + \bar{d}_R^i \not{D} d_R^i \right) + y_t H_u \bar{t}_L t_R + y_b H_d \bar{b}_L b_R$$

Only 3rd generation masses, and no quark mixing:

$$U(2)^3 = U(2)_q \times U(2)_u \times U(2)_d$$

$$q_L = \begin{pmatrix} \boldsymbol{q_L} \\ q_{3L} \end{pmatrix}, \qquad u_R = \begin{pmatrix} \boldsymbol{u_R} \\ t_R \end{pmatrix}, \qquad d_R = \begin{pmatrix} \boldsymbol{d_R} \\ b_R \end{pmatrix}$$



- Explains (at least partially) the Yukawa hierarchies
- Explains smallness of flavour-changing effects (as MFV)
- Symmetry weakly broken, at most by $\mathcal{O}(V_{cb}, m_2/m_3)$

Breaking of $U(2)^3$

Exact $U(2)^3$: $m_u = m_d = m_c = m_s = 0$, $V_{CKM} = \mathbb{I}$.

Minimal breaking

$$Y_u = y_t \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & 1 \end{array} \right), \qquad Y_d = y_b \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & 1 \end{array} \right).$$

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- $\Delta Y_u \sim (2, 2, 1)$, $\Delta Y_d \sim (2, 1, 2)$ to explain light quark masses

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Minimal breaking

$$Y_u = y_t \left(\begin{array}{c|c} \Delta Y_u & x_t \boldsymbol{V} \\ \hline 0 & 1 \end{array} \right), \qquad Y_d = y_b \left(\begin{array}{c|c} \Delta Y_d & x_b \boldsymbol{V} \\ \hline 0 & 1 \end{array} \right).$$

- $\Delta Y_u \sim (2,2,1)$, $\Delta Y_d \sim (2,1,2)$ to explain light quark masses
- One doublet $\boldsymbol{V} \sim (2,1,1)$ to explain CKM (3rd gen. mixing)

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General breaking

$$Y_u = y_t \left(\begin{array}{c|c} \Delta Y_u & x_t \boldsymbol{V} \\ \hline \boldsymbol{V_u} & 1 \end{array} \right), \qquad Y_d = y_b \left(\begin{array}{c|c} \Delta Y_d & x_b \boldsymbol{V} \\ \hline \boldsymbol{V_d} & 1 \end{array} \right).$$

- $\Delta Y_u \sim (2,2,1)$, $\Delta Y_d \sim (2,1,2)$ to explain light quark masses
- One doublet $\boldsymbol{V} \sim (2,1,1)$ to explain CKM (3rd gen. mixing)
- Most general breaking $\boldsymbol{V_u} \sim (1,2,1), \boldsymbol{V_d} \sim (1,1,2)$

Breaking of $U(2)^3$

Exact $U(2)^3$: $m_u = m_d = m_c = m_s = 0$, $V_{CKM} = \mathbb{I}$.

$$Y_u = y_t \left(\begin{array}{c|c} \Delta Y_u & x_t \boldsymbol{V} \\ \hline \boldsymbol{V_u} & 1 \end{array} \right), \qquad Y_d = y_b \left(\begin{array}{c|c} \Delta Y_d & x_b \boldsymbol{V} \\ \hline \boldsymbol{V_d} & 1 \end{array} \right).$$

- $\Delta Y_u \sim (2,2,1)$, $\Delta Y_d \sim (2,1,2)$ to explain light quark masses
- One doublet $\boldsymbol{V} \sim (2,1,1)$ to explain CKM (3rd gen. mixing)
- Most general breaking $\boldsymbol{V_u} \sim (1,2,1), \boldsymbol{V_d} \sim (1,1,2)$

Assume: all flavour violation controlled by $\Delta Y_{u,d}$, $\boldsymbol{V}$ (and $\boldsymbol{V_{u,d}}$)

i.e. $\Delta\mathcal{L}$ built with bilinears like $\boldsymbol{\bar{q}_L V} \gamma_\mu q_{3L}$, $\boldsymbol{\bar{q}_L} \Delta Y_d \boldsymbol{d_R}$.

Example:

$$\Delta\mathcal{L} \supset \sum_{i=1,2} (\boldsymbol{\bar{d}_L^i V^i} \gamma_\mu b_L)^2 \quad \xrightarrow[\text{mass basis}]{\text{physical}} \quad \sum_{i=d,s} (V_{tb} V_{ti})^2 (\bar{d}_L^i \gamma_\mu b_L)^2$$

Physical parameters

The unphysical parameters are removed by $U(2)^3$ transformations.

Minimal $U(2)^3$

$$\boldsymbol{V} = \begin{pmatrix} 0 \\ \epsilon_L \end{pmatrix}, \qquad \Delta Y_u = L_{12}^u \Delta Y_u^{\text{diag}}, \qquad \Delta Y_d = \Phi_L L_{12}^d \Delta Y_d^{\text{diag}},$$

- $L_{12}^{u,d}$ are left-handed rotations of $\theta_L^{u,d}$ in the (1,2) sector,
- $\Phi_L = \text{diag}(e^{i\phi}, 1)$.

3 angles + 1 phase: ϵ_L, $\theta_L^{u,d}$, ϕ

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Generic $U(2)^3$

$$\boldsymbol{V} = \begin{pmatrix} 0 \\ \epsilon_L \end{pmatrix}, \quad \boldsymbol{V_{u,d}} = \begin{pmatrix} 0 \\ \epsilon_R^{u,d} \end{pmatrix}, \qquad \begin{aligned} \Delta Y_u &= L_{12}^u \Delta Y_u^{\text{diag}} \Phi_R^u R_{12}^u, \\ \Delta Y_d &= \Phi_L L_{12}^d \Delta Y_d^{\text{diag}} \Phi_R^d R_{12}^d, \end{aligned}$$

- $L_{12}^{u,d}, R_{12}^{u,d}$ are left- and right-handed rotations of $\theta_{L,R}^{u,d}$,
- $\Phi_L = \text{diag}(e^{i\phi}, 1)$, $\Phi_R^{u,d} = \text{diag}\big(e^{i\phi_1^{u,d}}, e^{i\phi_2^{u,d}}\big)$.

ϵ_L, $\theta_L^{u,d}$, ϕ + additional parameters $\epsilon_R^{u,d}$, $\theta_R^{u,d}$, $\phi_1^{u,d}$, $\phi_2^{u,d}$

The CKM matrix

If $\epsilon_L < \epsilon_R^{u,d}$ (required by data),

$$V_{CKM} = \begin{pmatrix} c_u c_d & \lambda & s_u s e^{-i\delta} \\ -\lambda & c_u c_d & c_u s \\ -s_d d e^{i(\delta-\phi)} & -c_d s & 1 \end{pmatrix},$$

where $s = \epsilon_L$, $s_{u,d} = \sin\theta_L^{u,d}$, $\lambda e^{i\delta} = s_u c_d - s_d c_u e^{i\phi}$.

Fit of tree-level observables

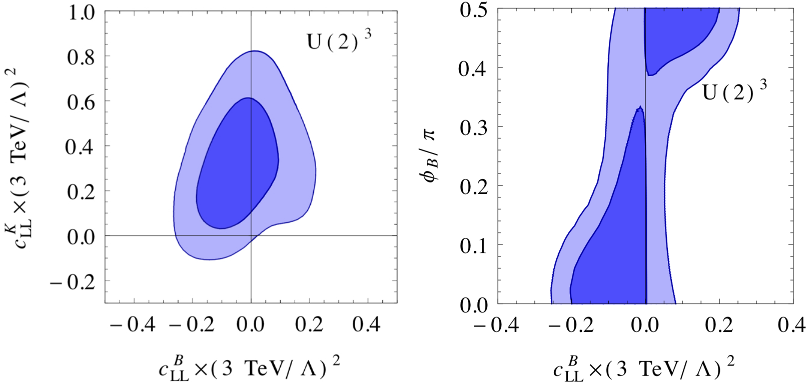
$$s_u = 0.086 \pm 0.003 \qquad s_d = -0.22 \pm 0.01$$
$$s = 0.0411 \pm 0.0005 \qquad \phi = (-97 \pm 9)^\circ$$

- All the parameters determined in Minimal $U(2)^3$,
- "Right-handed" angles undetermined in Generic $U(2)^3$.

Relevant $\Delta F = 2$ effects

Observables: ϵ_K, $B_d^0 - \bar{B}_d^0$, $B_s^0 - \bar{B}_s^0$.

$$\mathcal{H}_{\text{eff}}^{\Delta F=2} = \frac{c_{LL}^K}{2\Lambda^2}(V_{td}^* V_{ts})^2(\bar{d}_L\gamma_\mu s_L)^2 + \sum_{i=d,s}\frac{c_{LL}^B e^{i\phi_B}}{2\Lambda^2}(V_{ti}^* V_{tb})^2(\bar{d}_L^i\gamma_\mu b_L)^2 + \text{h.c.}$$

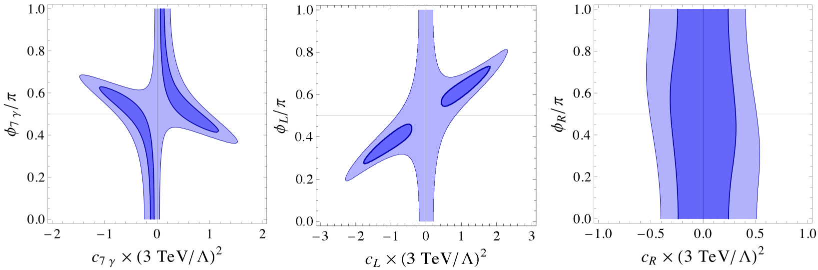


Consistent with $\Lambda \simeq 4\pi v$ and $|c_i| \simeq 0.2 \div 1$.

Relevant $\Delta F = 1$ effects

Observables: $K \to \pi\nu\bar{\nu}$, ϵ'_K, $b \to s(d)\gamma$, $b \to s(d)\ell\bar{\ell}, \nu\bar{\nu}$.
$D \to \pi\pi, KK$ only in Generic $U(2)^3$.

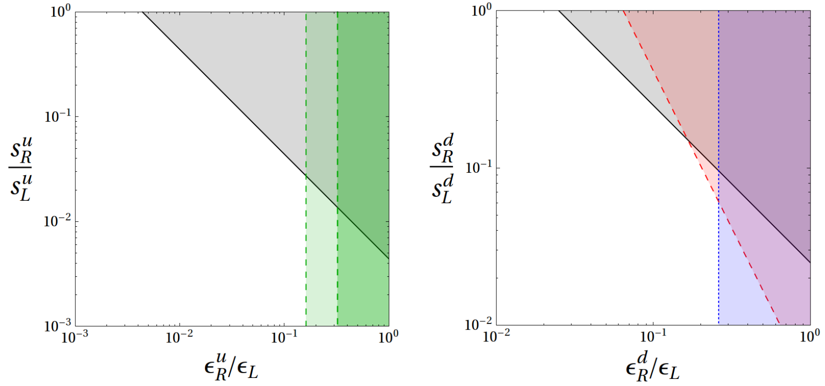
$$\Delta\mathcal{L}_{\text{eff}} = \underbrace{\Delta\mathcal{L}_L^{4f} + \Delta\mathcal{L}^{\text{mag}}}_{\text{Minimal } U(2)^3} + \underbrace{\Delta\mathcal{L}_R^{4f} + \Delta\mathcal{L}_{LR}^{4f}}_{\text{Generic } U(2)^3}$$



Consistent with $\Lambda \simeq 4\pi v$ and $|c_i| \simeq 0.2 \div 1$.

Bounds in Generic $U(2)^3$

Flavour effects in Generic $U(2)^3$ constrain the right-handed parameters, if one wants Wilson coefficients $c_i \approx \mathcal{O}(1)$.



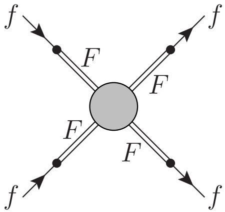
Consistent with $\epsilon_R^{u,d}$ less than one order of magnitude smaller than ϵ_L.

Embedding in composite Higgs models

$\mathcal{L}_\mathrm{c}$ has some approx. global symmetry (H pseudo-Goldstone?)

$$\mathcal{L} \supset \mathcal{L}_\mathrm{el} + \underbrace{M_F \bar{F} F + Y_F \bar{F} H F}_{\mathcal{L}_\mathrm{c}} + \underbrace{\lambda_L \bar{F} f_L + \lambda_R \bar{F} f_R}_{\mathcal{L}_\mathrm{mix}}$$

Flavour is communicated to the elementary sector through the bilinear mixings: $Y_f = \lambda_L^\dagger \cdot Y_F \cdot \lambda_R$.



- Flavour effects suppressed by λ: RS GIM (tensions with some observable)
- Flavour symmetry in the strong sector

- Left-compositeness: $\lambda_L = \mathrm{diag}(\lambda_1, \lambda_1, \lambda_3)$, $\lambda_R = \lambda_R(\boldsymbol{V}, \Delta Y)$.
- Right-compositeness: $\lambda_L = \lambda_L(\boldsymbol{V}, \Delta Y)$, $\lambda_R = \mathrm{diag}(\lambda_1, \lambda_1, \lambda_3)$.

$\lambda_1 \ll \lambda_3$ allowed by $U(2)^3$ solves many tensions (light gen. elem.).

Summary of effects: EFT vs. composite Higgs

	$b_L \leftrightarrow q_L$	$s_L \leftrightarrow d_L$	$b_R \leftrightarrow q_L$	$c_R \leftrightarrow u_L$
$U(3)^3$, R-compositeness	$\mathbb{R}$	$\mathbb{R}$	$\emptyset$	$\emptyset$
$U(3)^3$, L-compositeness	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
Minimal $U(2)^3$, R-comp.	$\mathbb{C}$	$\mathbb{R}$	$\emptyset$	$\emptyset$
Minimal $U(2)^3$, L-comp.	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{C}$	$\emptyset$
$U(3)^3$ moderate t_β	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{C}$	$\emptyset$
Minimal $U(2)^3$, $U(3)^3$ large t_β	$\mathbb{C}$	$\mathbb{R}$	$\mathbb{C}$	$\emptyset$
Generic $U(2)^3$	$\mathbb{C}$	$\mathbb{C}$	$\mathbb{C}$	$\mathbb{C}$
Relevant processes	B_q^0-$\bar{B}_q^0$ $b \to s l^+ l^-, s\nu\bar{\nu}$	K^0-$\bar{K}^0$ $K \to \pi\pi, \pi\nu\bar{\nu}$	$b \to s\gamma, s l^+ l^-$	$D^0 \to K^+K^-, \pi^+\pi^-$

$\mathbb{C}$ = possible effects with new CP phase,

= correlated effects,

$\mathbb{R}$ = possible effects, aligned in phase with the SM,

$\emptyset$ = no or negligible effect.

Flavour vs. compositeness bounds

The size of the parameters λ, Y_F, M_F, etc. is constrained not only by flavour observables, but also by Higgs physics, EWPT and collider constraints.

Examples

$$m_h \sim Y_F \quad \delta\hat{T} \sim Y_F^2/M_F^2, \quad \delta g_{Zbb} \sim 1/M_F^2 Y_F, \quad \epsilon_K \sim \lambda_L/\lambda_R$$

$\times$ some model dependent factor...

Different representations of $SU(2)_L \times SU(2)_R$ for fermions:

- doublets: $Q \sim (2,1)$, $(U,D) \sim (1,2)$ (MHCM4)
- bidoublets: $Q_u, Q_d \sim (2,2)$, $U, D \sim (1,1)$ (MHCM5)
- triplets: $Q \sim (2,2)$, $T \sim (3,1) \oplus (1,3)$ (MHCM10)

Bounds on fermion resonance masses

Minimal fermion resonance mass M_F in TeV compatible with all the bounds, assuming $\mathcal{O}(1)$ parameters.

	doublet	triplet	bidoublet
$U(2)_{\mathrm{LC}}$	4.9	0.5	0.4
$U(2)_{\mathrm{RC}}$	-	-	$1.2*$
$U(3)_{\mathrm{LC}}$	3.8	5.3	4.3
$U(3)_{\mathrm{RC}}$	-	-	3.1
Ⓐ	4.9	1.7	$1.2*$

$*$ = also $f \gtrsim 0.5$ TeV taken into account, with $M_F = Y_F f \Rightarrow Y_F \approx 2.5$

Ⓐ = anarchic generation of flavour via RS-GIM, bound from ϵ_K excluded

Summary of observable effects: composite Higgs

	Ⓐ	$U(3)_{\mathrm{LC}}$	$U(3)_{\mathrm{RC}}$	$U(2)_{\mathrm{LC}}$	$U(2)_{\mathrm{RC}}$
ϵ_K, $\Delta M_{d,s}$	★	○	★	★	★
$\Delta M_s/\Delta M_d$	★	○	○	○	○
$\phi_{d,s}$	★	○	○	★	○
$\phi_s - \phi_d$	★	○	○	○	○
C_{10}	★	○	○	★	○
C_{10}'	★	○	○	○	○
$pp \to jj$	○	★	★	○	○
$pp \to q'q'$	★	○	○	★	★

Observables where NP effects could show up with realistic experimental and/or lattice improvements in the most favourable cases.

Summary and conclusions

- A weakly broken $U(2)^3$ flavour symmetry is consistent with current data and

$$\Delta\mathcal{L} = \sum_i \frac{c_i}{(4\pi v)^2} \xi_i \mathcal{O}_i, \quad \text{with } |c_i| \sim 0.2 \div 1.$$

- Several observables to watch:

$$S_{\psi\phi}, \quad b \to s(d)\gamma, \quad b \to s(d)\ell\bar{\ell}, \nu\bar{\nu}, \quad K \to \pi\nu\bar{\nu}.$$

- If new signals are observed, best signature of $U(2)^3$ is s–d universality as in SM in b decays (as in MFV, but without K-B correlation).
- Concrete realization in composite Higgs models possible. Many competing bounds - not only from flavour physics.
- A $U(2)^3$ flavour symmetry of the composite can accomodate a 125 GeV Higgs with fermionic resonances below ~ 1 TeV in agreement with all bounds.

Thank you!

Backup slides

Lepton sector within $U(2)^3$ $(\times U(2)^2)$

$U(2)_\ell \times U(2)_e$ flavour symmetry in the charged-lepton sector: ignore effects due to neutrino masses.

	Chirality conserving			Chirality breaking		
	$\tau \leftrightarrow \mu$	$\tau \leftrightarrow e$	$\mu \leftrightarrow e$	$\tau \leftrightarrow \mu$	$\tau \leftrightarrow e$	$\mu \leftrightarrow e$
R-compositeness	✓	✓	✓	✗	✗	m_μ
L-compositeness	✓	✓	✓	✓	✓	m_μ
Relevant processes	$\tau \to 3\mu, 3e$; $\mu \to 3e$, e(Ti)			$\tau \to \mu\gamma, e\gamma$; $\mu \to e\gamma$		

✓ = new effect,

✗ = no effect,

m_μ = effect subleading in spurion expansion.

$\Delta S = 1$: a digression on ϵ'

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \frac{1}{\Lambda^2}\xi_{ds}\left(\bar{d}_L^\alpha \gamma_\mu s_L^\beta\right)\left[c_K^d\left(\bar{d}_R^\beta\gamma_\mu d_R^\alpha\right) + c_K^u\left(\bar{u}_R^\beta\gamma_\mu u_R^\alpha\right)\right]$$

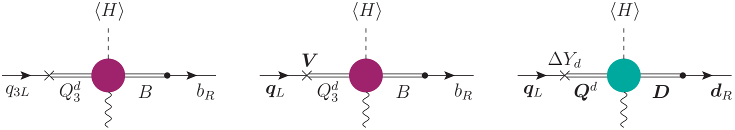
- $\langle(\pi\pi)_{I=2}|Q_{LR}^d|K\rangle = -\langle(\pi\pi)_{I=2}|Q_{LR}^u|K\rangle \propto \left(\frac{m_K}{m_s}\right)^2$
- $\left|\frac{\epsilon'}{\epsilon}\right| \simeq \frac{|{\rm Im}\mathcal{A}_2|}{\sqrt{2}|\epsilon|{\rm Re}\mathcal{A}_0} \qquad \omega = \frac{{\rm Re}\mathcal{A}_2}{{\rm Re}\mathcal{A}_0} \approx 20$

$$\Longrightarrow \quad \left|\frac{\epsilon'}{\epsilon}\right| \simeq 1.3\times 10^{-2}\left(\frac{3\,{\rm TeV}}{\Lambda}\right)^2 c_K^{u,d}$$

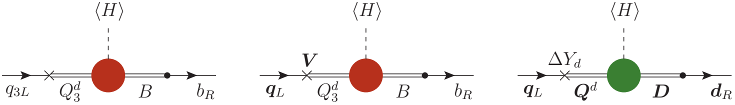
i.e. $c_K^{u,d} \lesssim 0.1 \div 0.2\left(\frac{3\,{\rm TeV}}{\Lambda}\right)^2$

Chirality breaking bilinears: RH compositeness

- Flavour violating dipole operators are generated through the mixings, from the flavour conserving composite-quark dipoles. The dipole moments of the composite 3rd generation are different.

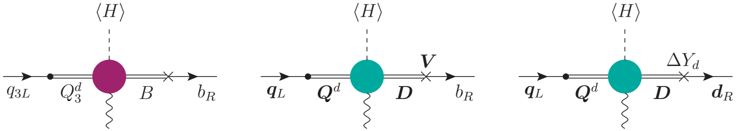


- There is an alignment between the dipole moments and the Yukawa couplings $\Rightarrow$ in the physical quark basis there is no FV effect.

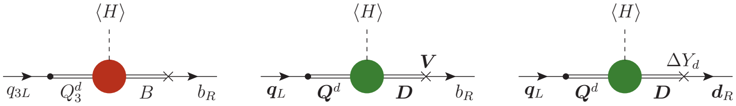


Chirality breaking bilinears: LH compositeness

- As before, FV dipole operators are generated through the mixings.

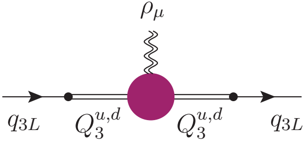


- No alignment: the ratio of the two dipole moments differs from the one of the Yukawa couplings $\Rightarrow$ the mismatch between dipole and Yukawa operators gives a neat flavour-changing effect when rotating to the physical quark basis.



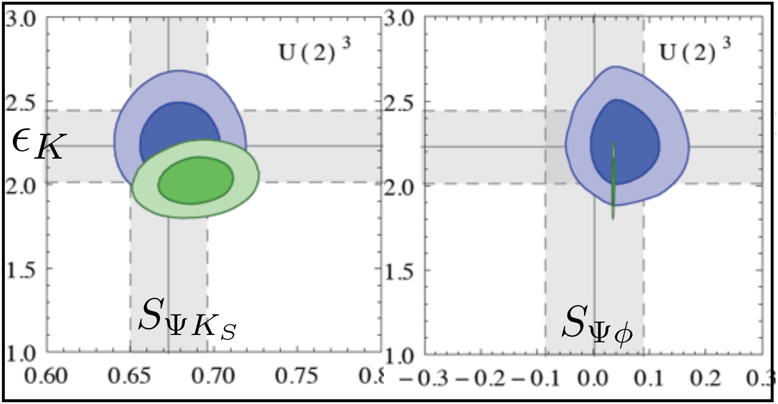
Chirality conserving bilinears

- **Left-handed compositeness:** no flavour-changing LL currents can be constructed with the minimale spurions. Rotating to the mass basis, flavour effects are generated, although aligned in phase with the SM.



- **Right-handed compositeness:** the mismatch between the coefficients of the two composite quark currents give rise to flavour and CP violation when rotating to the physical quark basis.

$U(2)$ vs $U(3)$ fits



U(2)³
U(2)³
ϵ_K
$S_{\Psi K_S}$
$S_{\Psi\phi}$
3.0
2.5
2.0
1.5
1.0
0.60
0.65
0.70
0.75
−0.3
−0.2
−0.1
0.0
0.1
0.2