Power Corrections to Event-Shape Distributions at e^+e^- Colliders using Eikonal Approximation

Anurag Tripathi

BERGISCHE UNIVERSITAET WUPPERTAL

$25^{\rm th}$ **Rencontres de Blois Particle Physics and Cosmology**
Blois 26 – 31 May, 2013

with L. Magnea

Plan of Talk

- **E**vent Shape Variables
- **P**ower Corrections
- **E**ikonal Dressed Gluon Exponentiation
- **R**esults
  - Independent unpublished work in same direction by Einan Gardi
- **S**ummary

History

1977

Infrared (IR) Safe

Sterman-Weinberg **jet definition** introduced

History

Immediately after Sterman Weinberg jet several IR safe observables were proposed and continue to be proposed..

- Thrust (1977)
- Sphericity
- C-parameter

⋮

- Jet Mass
- Jet Broadening

⋮

- Angularity (2003)

⋮

- planar flow (2008)

⋮

What is Infrared Safety ?

Feynman diagrams are singular at particle production thresholds.

An IR safe observable $\mathcal{O}$ should satisfy, thus

$$\mathcal{O}(p_1^\mu, p_2^\mu, \cdots, (1-\lambda)p_n^\mu, \lambda p_n^\mu) = \mathcal{O}(p_1^\mu, p_2^\mu, \cdots, p_n^\mu)$$

IR Safe Event Shapes

$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i E_i} \qquad \text{Thrust}$$

$$C = \frac{1}{Q} \sum_i \frac{3|\mathbf{p}_i^{\perp}|}{\cosh \eta_i} \qquad \text{C – parameter}$$

$$\tau_a = e^{-|y|(1-a)} \qquad \text{Angularity}$$

Event Shapes

Event Shapes

- Measure geometrical properties of energy flow.
- Among the first observables proposed to test QCD.
- Crucial in α_s extraction. Gehrmann et. al EPJ C73 (2013) 2265
- Essential in parton shower tuning
  &
  non-perturbative components of MC event generators.
- Used in modelling and testing hadronization process.

Renormalons and Power Corrections

Power Corrections

Parameterization of a physical distribution

$$\sigma(Q) = \sigma_{pert}(Q) + \sum_{n=n_0}^{\infty} \frac{\sigma_n}{Q^n}$$

Power Corrections

Parameterization of a physical distribution

$$\sigma(Q) = \sigma_{pert}(Q) + \sum_{n=n_0}^{\infty} \frac{\sigma_n}{Q^n}$$

- This is applicable to total cross-section also.
- Power corrections are small for inclusive variables
  - $1/Q^4$ for total cross-section.
- Differential cross-sections have larger corrections because the scale is much smaller.
  - $1/(1-T)Q$ for Thrust

- Extraction of Power corrections (non-perturbative) from perturbation series !!

How is it possible ?

- Extraction of Power corrections (non-perturbative) from perturbation series !!

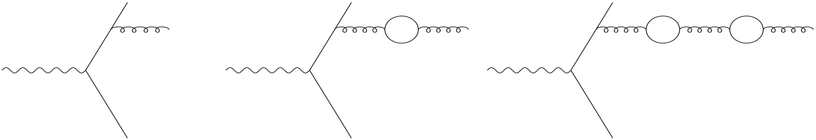
How is it possible ?

- Ambiguity in definition of σ_n (F. David: NPB234,237)
- σ_{pert} not well defined: factorial growth

Ambiguities of σ_{pert} compensated by σ_n

Power Corrections

- **Factorial** divergence made manifest by running coupling.



Mining Factorial Divergence

Let R be a physical quantity.

$$R \sim \sum_{n=0}^{\infty} r_n \alpha^{n+1}$$

The Borel transform

$$B[R](u) = \sum_{n=0}^{\infty} r_n \frac{u^n}{n!}$$

The Borel sum

$$R = \int_0^{\infty} du e^{-u/\alpha} B[R](u)$$

If B[R] has pole at $u = x/\beta_0$ then ambiguity is $\delta R \sim \left(\frac{\Lambda_{QCD}^2}{Q^2}\right)^x$

$$\text{where} \quad \alpha_s \sim 1/\beta_0 \ln(Q^2/\Lambda_{QCD}^2)$$

Dressed Gluon

Renormalon-resummed differential x-section

$$\frac{1}{\sigma}\frac{d\sigma}{dt}(t, Q^2)\Big|_{SDG} = -\frac{C_F}{2\beta_0}\int_0^1 d\xi \frac{d\mathcal{F}(\xi, t)}{d\xi} A(\xi Q^2)$$

$$t = 1 - T$$

(Ball, Beneke, Braun NPB 452 (1995) 563)
(Beneke, Braun NPB 426 (1994) 301)
(Dokshitzer, Marchesini, Webber NPB 469 (1996) 9)

- Characteristic f^n with non-zero gluon mass
- $A(\xi Q^2)$ is running coupling with a Borel representation.

$$A(\xi Q^2) = \int_0^\infty du \left(\frac{Q^2}{\Lambda^2}\right)^{-u} \frac{\sin \pi u}{\pi u} e^{\frac{5}{3}u} \xi^{-u}$$

Borel Function

$$\frac{1}{\sigma}\frac{d\sigma}{dt}(t,Q^2)\Big|_{SDG} = \frac{C_F}{2\beta_0}\int_0^\infty du\left(\frac{Q^2}{\Lambda^2}\right)^{-u} B(t,u)$$

$$B(t,u)\Big|_{logs} = 2e^{\frac{5}{3}u}\frac{\sin\pi u}{\pi u}\left[\frac{2}{u}t^{-1-2u} - t^{-1-u}\left(\frac{2}{u}+\frac{1}{1-u}+\frac{1}{2-u}\right)\right]$$

- $u = 0$ poles cancel: perturbative coefficients defined.
- Because of $\sin\pi u$ factor there are no singularities in u.
- The first term originates from soft gluons.
- Second term originates from collinear gluons.

Dressed Gluon Exponentiation

- QCD matrix elements factorize for soft and collinear radiation
- Log enhanced terms exponentiate in Laplace space

$$\frac{1}{\sigma}\frac{d\sigma}{dt}(t, Q^2)\Big|_{DGE} = \int \frac{d\nu}{2\pi i} \quad e^{\nu t} \; exp\,[S(\nu, Q^2)]$$

- S contains renormalons

(Gardi NPB 622 (2002) 365)
(Gardi and Rathsman, NPB 609 (2001) 123)

Dressed Gluon Exponentiation

$$S(\nu, Q^2) = \frac{C_F}{2\beta_0} \int_0^\infty du \left(\frac{Q^2}{\Lambda^2}\right)^{-u} B_\nu^t(u)$$

and

$$B_\nu^t(u) = 2e^{\frac{5}{3}u} \frac{\sin \pi u}{\pi u} \left[\Gamma(-2u)(\nu^{2u} - 1)\frac{2}{u} \right.$$

$$\left. -\Gamma(-u)(\nu^u - 1)\left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u}\right)\right]$$

- Renormalon singularities at odd $2u$ $(= m)$
- Non-Perturbative correction of $\mathcal{O}(\Lambda\nu/Q)^m$ in $S(\nu, Q^2)$ necessary to compensate ambiguity

Dressed Gluon Exponentiation

$$B(t,u)\Big|_{log} \sim \frac{2}{u}t^{-1-2u} - t^{-1-u}\left(\frac{2}{u}+\frac{1}{1-u}+\frac{1}{2-u}\right)$$

Gardi and Rathsman : Nucl.Phys.B609 : 123, 2001.

$$B(c,u)\Big|_{log} \sim 4(2c)^{-1-2u}\frac{\sqrt{\pi}\Gamma(u)}{\Gamma(\frac{1}{2}+u)} - c^{-1-u}\left(\frac{2}{u}+\frac{1}{1-u}+\frac{1}{2-u}\right)$$

Gardi and Magnea : JHEP0308 : 030, 2003

Aim: Leading power corrections without doing the full painful calculation of the characteristic function.

Eikonal Dressed Gluon Approximation

Eikonal Approximation

- Computation of Power Corrections are involved, contain elliptic integrals, ...
- Traditionally calculations done using energy fractions x_i

However, Leading power corrections can be obtained easily using Eikonal approximation and $p_\perp$ and y variables.

For the approximation to be useful

- Phase space should factorize, and
- Matrix element should factorize

For soft and collinear gluons emitted from nearly on-shell partons, factorization occurs. (Gardi: NPB 622, 365)

Event Shapes In $p_\perp$ And y Variables

Consider the following class of shapes (Gardi)

$$e = \sum_i \sqrt{\frac{p_{i\perp}^2 + p^2}{Q^2}}\, h_e(y)$$

$1 - T$, C-parameter, and angularity fall in this

$$h_e(y) = \begin{cases} e^{-|y|} & \text{1-thrust} \\ \dfrac{3}{\cosh y} & c\text{-parameter} \\ e^{-|y|(1-a)} & \text{angularity} \end{cases}$$

Eikonal Approximation

- Off-shell soft gluon phase space & Matrix element factorize

$$\text{Eikonal current} \quad j^{\mu} = \frac{p^{\mu}}{p.k} \tag{1}$$
$$\text{Matrix Element} \quad |\mathcal{M}|^2 = \frac{2}{k^2 + k_{\perp}^2}|\mathcal{M}_{\text{B}}|^2$$

- gluon phase space

$$\int \frac{d^3k}{(2\pi)^3 2k^0} = \frac{1}{(4\pi)^2}\int dk_{\perp}^2 dy$$

- This yields a simple result for characteristic function.

$$\mathcal{F} = \frac{8}{e}\int_{y_{min}}^{y_{max}} dy$$

- Singularity in $k^2 \to 0$ limit appears from wide angle soft gluons.

Borel Function In Eikonal approximation

$$
\begin{aligned}
t: \qquad & y_{min} = \ln\left(\frac{1}{t}\sqrt{k^2/Q^2}\right) \\
c: \qquad & y_{min} = cosh^{-1}\left(\frac{1}{2c}\sqrt{k^2/Q^2}\right)
\end{aligned} \tag{2}
$$

We immediately obtain the Borel function

$$
B(c,u) = 2\frac{\sin\pi u}{\pi u} e^{\frac{5}{3}u}\left[4(2c)^{-1-2u}\frac{\sqrt{\pi}\Gamma(u)}{\Gamma(u+\frac{1}{2})}\right]
$$

$$
B(t,u) = 2\frac{\sin\pi u}{\pi u} e^{\frac{5}{3}u}\frac{1}{t}\left[\frac{2}{u}t^{-1-2u}\right]
$$

Leading power corrections reproduced !

Angularity in Eikonal Approximation

$$\text{Angularity} \qquad \tau_a = \sum_i \sqrt{\frac{p_{i\perp}^2 + p^2}{Q^2}}\, e^{-|y|(1-a)}$$

IR safety implies $-\infty < a < 2$. We will be restricted to $a \leq 0$

$$\tau_a : \qquad y_{min} = \frac{1}{1-a} \ln\left(\frac{1}{\tau_a}\sqrt{k^2/Q^2}\right)$$

$$B(\tau_a, u) \;=\; 2\frac{\sin \pi u}{\pi u} e^{\frac{5}{3}u} \frac{1}{(1-a)\tau_a}\left[\frac{2}{u}\tau_a^{-1-2u}\right]$$

Scaling property discovered by Berger and Sterman is reproduced.

(Berger, Sterman JHEP 0309 058, EPJ C33 S407)

Summary And Outlook

- Power corrections give information about hadronization.
- These corrections can be estimated from the ambiguity of perturbation series.
- Many shapes have been studied and power corrections estimated .
- We use the method of Dressed Gluon Exponentiation and use Eikonal approx. to obtain Leading Power Corrections in a simple manner.
- Would be useful to get Power corrections if new shapes are defined.
- This method may be extended to Hadronic event shapes.

Thank you !