Conversion of gravitons into photons in primordial magnetic fields

Damian Ejlli

INFN, Ferrara
APC, Université de Paris Diderot (Paris 7)

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A. D. Dolgov and D. Ejlli, "Relic gravitational waves from light primordial black holes," Phys. Rev. D **84**, 024028(2011)

$g - \gamma$ mixing

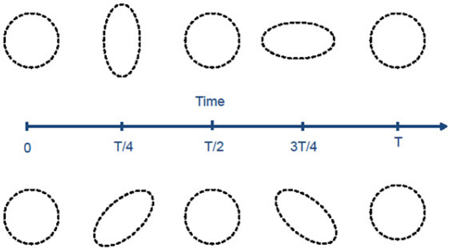
- electromagnetic waves(photons) can transforms into gravitational waves(gravitons) in the presence of a constant external magnetic field, **Gertsenshtein (1962), Lupanov (1967).**
- the reverse process $g \to \gamma$ was considered by **Mitskevich (1969), Boccaletti, De Sabbata, Fortini and Gualdi (1970), Zel'dovich (1973)** etc.
- for an extended region of a magnetic field there are oscillations of gravitons into photons and vice-verse in complete analogy with neutrino oscillations.

$g - \gamma$ mixing

- an electromagnetic wave with vectors **E** and **B** when crosses a static external magnetic field $\mathbf{B}_{ext}$, generates a time varying energy momentum tensor

$$T_{ij} \propto B_i B_j^{ext}$$

- a gravitational wave, h_{ij}, transversing a static external magnetic field $\mathbf{B}_{ext}$ generates distortion in space which stretches the external field $|h_{ij}|\mathbf{B}_{ext}$



How things work: quantitative description

- The starting point is the action of the graviton-photon system

$$\mathcal{S} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}R - \frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- we expand the true metric tensor around the flat Minkowsky space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + kh_{\mu\nu}(\mathbf{x}, t), \quad k = (32\pi G)^{1/2}$$

- after we find the linearised Lagrangian density

$$\mathcal{L} = \frac{1}{4}\partial_\mu h\partial^\mu h - \frac{1}{2}\partial_\alpha h_{\mu\nu}\partial^\alpha h^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{k}{2}h_{\mu\nu}T^{\mu\nu}_{\mathrm{em}}$$

- the equation of motion for the fields $h_{\mu\nu}$ and A^ν are

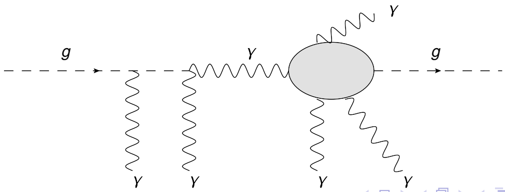
$$\begin{aligned}\Box h_{\mu\nu} &= -\kappa T^{\rm em}_{\mu\nu} \\ \Box A^\nu &= \kappa\partial_\mu h^{\mu\beta}(F^\nu_\beta - h^{\nu\beta}F^\mu_\beta)\end{aligned}$$

- the QED effects are included in the Euler-Heisenberg Lagrangian density

$$\mathcal{L}_{EH} = \frac{\alpha^2}{90 m_e^4}[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(\tilde{F}_{\mu\nu}F^{\mu\nu})^2]$$

- the medium gives an effective mass to the photon

$$\omega^2_{pl} = 4\pi\alpha n_e/m_e$$



Matrix formulation of the equations of motions

- consider a gravitational wave propagating along the z axis, $h_{ij}(z,t) = \epsilon_{ij}(\mathbf{k})e^{i(\omega t - k_{1,2}z)}$ with $m_{1,2} = k_{1,2} - \omega$. In the WKB approximation $\lambda_p \ll \lambda_B$

$$\left[(\omega + i\partial_z) + \begin{bmatrix} m_1 & m_{g\gamma} & 0 & 0 \\ m_{g\gamma} & 0 & 0 & 0 \\ 0 & 0 & m_2 & m_{g\gamma} \\ 0 & 0 & m_{g\gamma} & 0 \end{bmatrix}\right] \begin{bmatrix} A_+ \\ h_+ \\ A_\times \\ h_\times \end{bmatrix} = 0$$

- the solution of the equation of motion can be obtained by an unitary transformation of the matrix, $\mathcal{M}, \mathcal{M}' = U\mathcal{M}U^{-1}$,

$$\mathcal{M} = \begin{bmatrix} m_{1,2} & m_{g\gamma} \\ m_{g\gamma} & 0 \end{bmatrix} \qquad U = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Mixing regimes

- There are two mixing regimes, weak and strong mixing
- In the weak mixing case ($\theta \ll 1$) and the probability reads

$$P_{g\gamma} = 4\theta^2 \sin^2(\pi z / l_{osc})$$

- For the strong mixing case we have ($\theta = \pi/4$) and the probability is

$$P_{g\gamma} = \sin^2(m_{g\gamma} z)$$

- the oscillation probability depends on the value of magnetic field B, the plasma electronic density n_e and graviton energy ω
- In the case of gravitons with energy $\omega \ll m_e$ the resonance is never crossed! However, for gravitons with energy $\omega \gg m_e$ the resonance is crossed. We here consider both cases

Limits on primordial magnetic field

- Theoretical models and some observational effects suggest the presence of magnetic fields in intergalactic space, extragalactic space and on large scales i.e. Hubble horizon.
- We are interested on large scale homogeneous magnetic fields, namely comparable with horizon scale $z \simeq H_0^{-1} = 1.32 \cdot 10^{28}$ cm
- there are different ways to constrain the primordial magnetic field CMB anisotropies, Faraday rotation on the CMB and BBN bound on gravitational waves.
- From CMB observations $B \lesssim 3 \cdot 10^{-9}$ G (**[D. Paoletti and F. Finelli arXiv:1208.2625 2012]**)
- From Faraday rotation on CMB $B \lesssim 10^{-8} - 10^{-6}$ G **[T. Kahniashvili, Y. Maravin and A. Kosowsky, Phys. Rev. D 80 (2009)]**

Probability estimate in the non resonant case

- After recombination 1+z=1090 the Universe is mostly in the neutral form with a small ionization fraction, $X_e \ll 1$. With an electronic density $n_e(t_i) = 320$ cm^{-3}, $\omega_i = 10^5$ eV and $B \sim 10^{-3}$ G the value of oscillation probability reads

$$P_{g\gamma}(t_i) \simeq 10^{-15}$$

- At present the electronic density is $n_e(t_0) \simeq n_B(t_0) = 2.47 \cdot 10^{-7}$ cm^{-3}, $\omega = 10^3$ eV and $B \simeq 10^{-9}$ G, the probability

$$P_{g\gamma}(t_0) \simeq 2 \cdot 10^{-15}$$

- Interaction of photons with medium has been considered in the wave function approximation!

Damping effects in the non-resonant case

- During the graviton-photon oscillation, photons can scatter with the surrounding medium. For photons with energies $\omega < m_e$ the only dominant process that causes a damping in the oscillation process is due to Thompson scattering

$$\Gamma_\gamma = \sigma_T n_e, \qquad \sigma_T = 6.65 \cdot 10^{-25}\,\mathrm{cm}^2$$

- the wave function approximation is not accurate on describing the oscillation process in the case when the oscillation length is greater than the mean free path, $l_{osc} \gg l_{free}$
- the system becomes open and the total Hamiltonian is not hermitian.
- when there is a loss of coherence a density matrix description is needed

Density matrix formulation (DE) (2012)

- the density matrix operator $\hat{\rho}$ is constructed by 4 complex fields

$$\hat{\rho} \equiv |A_+, h_+, A_\times, h_\times\rangle \otimes \langle A_+, h_+, A_\times, h_\times|,$$

- $\hat{\rho}$ obeys to the Liuville-von Neumann equation

$$i\frac{d\hat{\rho}}{dt} = [\hat{\mathcal{H}}, \hat{\rho}]$$

- For a non-Hermitian Hamiltonian, $H = M - i\Gamma$ the von Neumann equation reads

$$i\frac{d\rho}{dz} = [M, \rho] - i\{\Gamma, (\rho - \rho_{ext})\}$$

Equation of motions of the density matrix

- We need also to take into account the expansion of the Universe on $g - \gamma$ oscillation
- We need to write equations of motions in the FRW metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}x_i\mathrm{d}x_j$$

- in the FRW metric the von Neumann equation reads

$$iHa\frac{d\rho}{da} = [M, \rho] - i\{\Gamma, \rho\}$$

- the equations of motions are

$$\begin{aligned}
\rho'_{\gamma\gamma} &= (-2m_{g\gamma}I - \Gamma_\gamma\,\rho_{\gamma\gamma})/(Ha), \\
\rho'_{gg} &= 2m_{g\gamma}I/(Ha), \\
R' &= (mI - \Gamma_\gamma R/2)/(Ha), \\
I' &= (-mR - \Gamma_\gamma I/2 - m_{g\gamma}(\rho_{gg} - \rho_{\gamma\gamma}))/(Ha)
\end{aligned}$$

Equation of motions of density matrix

- In order to solve the system of differential equations one needs $m_{g\gamma}(a)$, $m(a)$ and $\Gamma_\gamma(a)$. Complicated expressions! Also are needed the initial conditions, $\rho_{\gamma\gamma}(t_i) = R(t_i) = I(t_i) = 0$ and $\rho_{gg}(t_i) = 1/2$
- Other parameter to introduce in equations of motions is the ionization fraction $X_e(a)$ which obeys

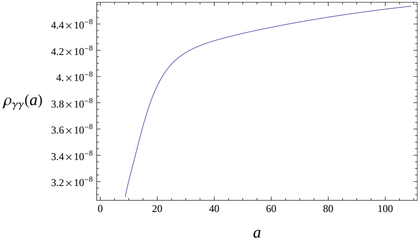
$$\frac{\mathrm{d}X_e}{\mathrm{d}a} = -\frac{\alpha n_B}{Ha}\left(1 + \frac{\beta}{\Gamma_{2s} + 8\pi/\lambda_\alpha^3 n_B(1 - X_e)}\right)^{-1}\left(\frac{SX_e^2 + X_e - 1}{S}\right),$$

- The only free parameter is the magnetic field B. Analytic solutions of equations of motions are extremely difficult without requiring any approximation. Everything has been calculated numerically!

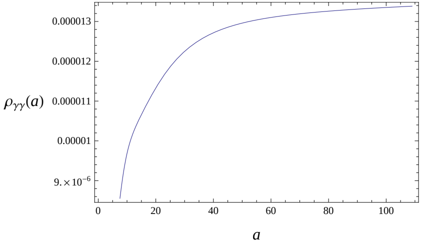
Production probability in the non-resonant case

- The Universe is re-ionized at $z \sim 10$ by the first generation of stars and the medium became ionized again! $X_e(z \simeq 10) = 1$
- Near present epoch $z \sim 1$ the vacuum energy density contributes on the expansion of the Universe ($\Lambda \simeq 0.7$ and $\Omega_M \simeq 0.3$). We took into account all these details and got

- $B_i \simeq 10^{-2}$ G, $\omega_i = 10^5$ eV

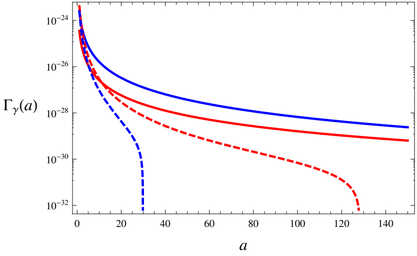


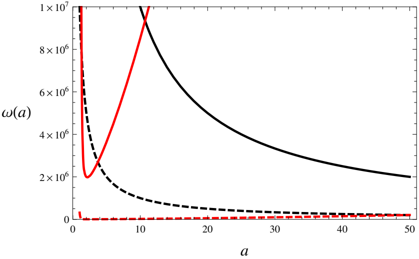
- $B_i \simeq 1.2$ G, $\omega_i = 10^5$ eV



Damping effects in the resonant case

- For $\omega \gg m_e$ the damping terms arises due to Compton scattering with electrons and pair production in atomic nuclei.
- Γ_γ^C (red) and Γ_γ^{pp} (red dashed) for $\omega_i = 10^9$ eV. Γ_γ^C (blue) and Γ_γ^{pp} (blue dashed) for $\omega_i = 10^8$ eV
- ω vs. a for $\omega_i = 10^8$ eV (black) and $\omega_i = 10^9$ eV (black dashed). ω_{res} vs. a for $B_i = 3$ mG (red) and $B_i = 1.2$ G (red dashed)





Gravitational wave production (DE) PRD (2011)

- BHs emit gravitons as well as other particles with, $m < T_{BH}$ (**S. Hawking, 1974**)
- they do emit gravitons at a rate

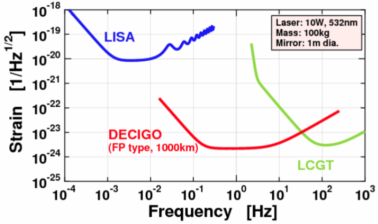
$$\frac{dE}{dtd\omega} = \frac{2N_{eff}}{\pi}\frac{M^2}{m_{Pl}^4}\frac{\omega^3}{e^{\omega/T_{BH}} - 1}$$

- only about 10% of emitted particles are in form of gravitons
- the density parameter today would be

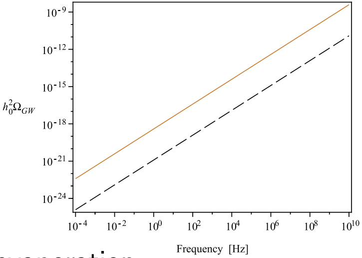
$$h_0^2\Omega_{GW}(f, t_0) \simeq 1.36\cdot 10^{-27}\left(\frac{f}{\mathrm{GHz}}\right)^4\left(\frac{10^5 g}{M}\right)^2\left(\frac{N_{eff}}{100}\right)^2 \cdot I(\frac{2\pi \cdot f}{T_0})$$

Gravitational wave production: Results

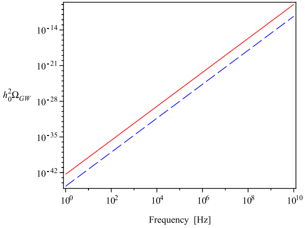
- DECIGO



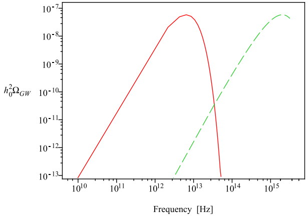
- scattering



- binaries

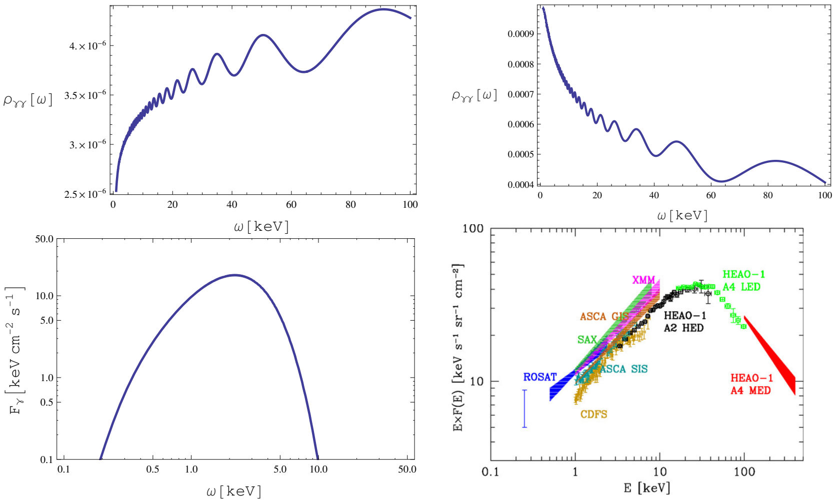


- evaporation



- DECIGO/BBO can detect the lower part of the spectrum from PBH scattering in the frequency range of 0.1 up to 10 Hz.
- A matter dominated regime by PBHs would change the standard picture where in general the universe is thought to be radiation dominated after the inflationary epoch.
- Matter domination by PBHs would dilute the background of GWs from inflation for those wavelengths within horizon during PBH domination!
- The high frequency part of GWs from PBHs evaporation is not detectable by any actual or planned GW detector. Moreover, in the high energy part the density parameter is larger than any other GW generation mechanism!
- What can be done in order to detect it? Is there any alternative way to detect it?

Resonant case, (DE) PRD (2013)



Conclusions

- For the first time in cosmology the conversion of gravitons into photons has been studied
- Gravitons transform efficiently into photons in presence of large scale magnetic field with a probability up to $P_{g\gamma} \simeq 10^{-5}$ in the non resonant case and $P_{g\gamma} \simeq 10^{-3}$ in the resonant case.
- A new stage of cosmological evolution with a matter dominated regime by PBHs has been proposed with a rather large emission of GWs
- We predict that the low frequency part of the spectrum can be observed by the future space interferometer DECIGO/BBO **Dolgov, Ejlli, PRD(2011)**
- the produced background can explain the origin of soft to hard of CXB **Dolgov, Ejlli, JCAP (2012)** and **Dolgov, Ejlli PRD (2013)**