

# **New developments in leptonic flavor**

Daniel Hernandez

ICTP



**Normal physicists  
worried about strings  
and other regular stuff**

**Physicist that thought too  
much about the Flavour  
Puzzle**

# Hints or handles to leptonic flavor

- $U_{CKM}$  vs  $U_{PMNS}$ . Small vs Large angles

Related??

- Neutrinos may be Majorana

- Lack of horizontal (flavor) symmetry

maybe continuous or discrete

# Flavor symmetries

.....the discrete path...

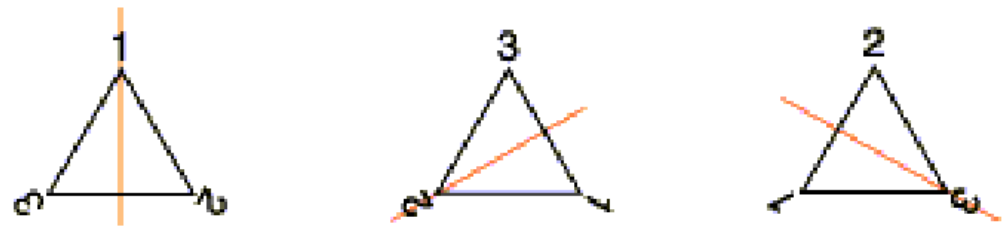
D. H. and A. Yu. Smirnov, ;  
[arXiv:1204.0445](#), [arXiv:1212.2149](#),  
[arXiv:1304.7778](#)

# What are (nonabelian) discrete symmetries?

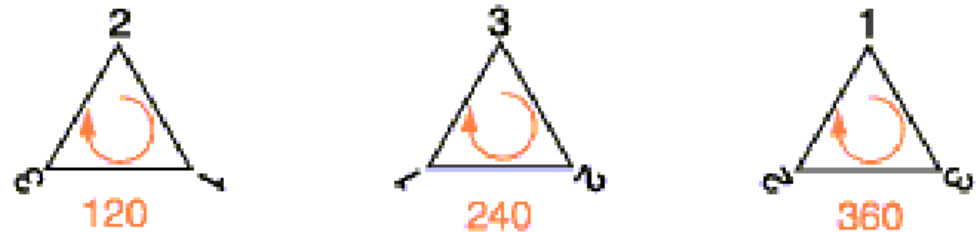
\* Images taken from <http://www.olympus.net/personal/mortenson/preview/definitions/symmetrytransform.html>

## Generators

Reflections( $P$ ) →



120° Rotations( $O$ ) →



Do not commute.

$$PO \neq OP$$

Raised to some power, equal the identity

FOR INSTANCE  $P^2 = O^3 = 1$

**A4:** symmetry group of the tetrahedron

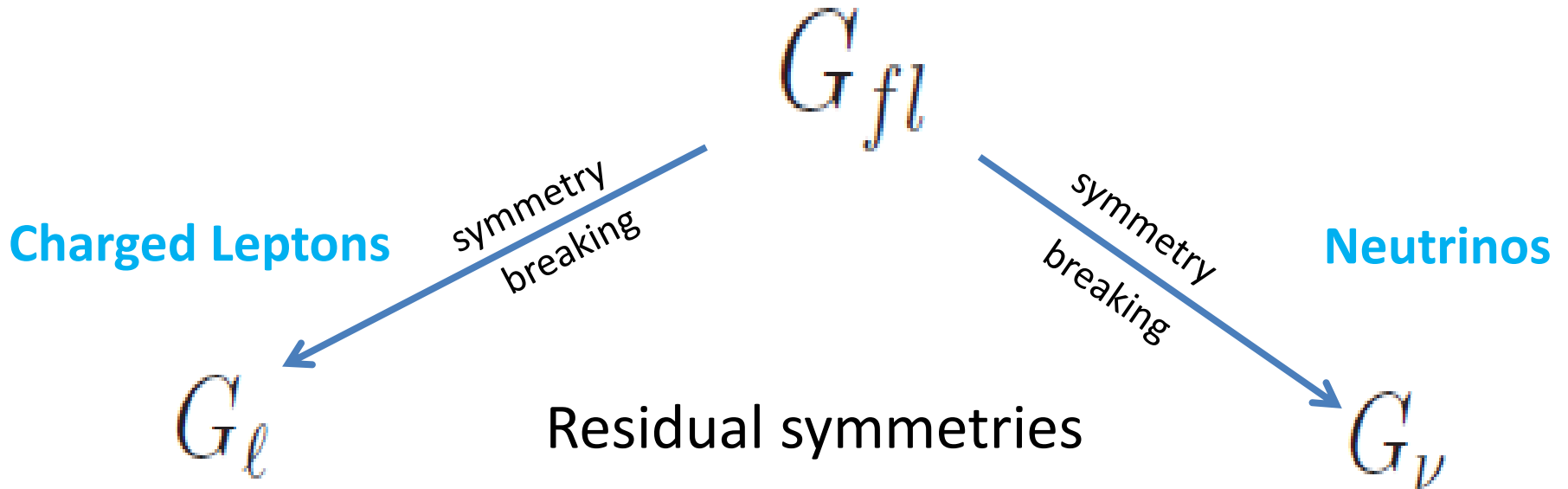
**S4:** symmetry group of the cube

**A5:** symmetry group of the icosahedron

.....

**Important fact: These symmetries have 3-dimensional representations that account for the presence of 3 families**

# Flavor Symmetry

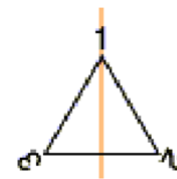


**Example:**  $G_{fl}$  the symmetry of the equilateral triangle

$$G_\ell \equiv O$$



$$G_\nu \equiv P$$



WHAT ARE THE CONSTRAINTS DISCRETE  
SYMMETRIES IMPOSE IN **LEPTON**  
**MIXING?**

ARE THEY **MODEL INDEPENDENT?**

CAN **MASSES** BE INCLUDED IN THE  
GAME?



$$\mathcal{L} = \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma^\mu \nu_L W_\mu^+ + \bar{E}_R M_\ell \ell_L + \frac{1}{2} \bar{\nu}^c_L M_\nu \nu_L + \dots + \text{h.c.}$$

The mixing matrix is the **mismatch between the directions, in flavor space, defined by the mass matrices**

$$M_\nu = U_\nu^T M_{\nu D} U_\nu$$

$$M_\ell = V_E^\dagger M_{\ell D} V_\ell$$

$$U_{PMNS} = V_\ell U_\nu^\dagger$$

***S*** : symmetry preserved by the neutrino mass matrix

***T*** : symmetry preserved by the charged lepton mass matrix

For neutrinos  $S M_\nu S^T = M_\nu$

For charged leptons  $T M_\ell T^\dagger = M_\ell$

# BUILDING THE DISCRETE GROUP

$$S^n = \mathbb{I}$$

$$T^m = \mathbb{I}$$

$U_{PMNS}$  is also the mismatch between  $S$  and  $T$  !

$$(ST)^p = (U_{PMNS} S_D U_{PMNS}^\dagger T_D)^p = \mathbb{I}$$

$$S_D^n = T_D^m = \mathbb{I}$$

symmetry in the  
mass basis

## Constraints on the mixing matrix

$$(ST)^p = (U_{PMNS} S_D U_{PMNS}^\dagger T_D)^p = \mathbb{I}$$



$$\text{Det}[ST - \lambda \mathbb{I}] = 0 \quad \text{cubic equation with} \quad \lambda_i^p = 1$$



$$\lambda^3 + a\lambda^2 - a^*\lambda - 1 = 0 \quad \text{with} \quad a = -\text{Tr}[U_{PMNS} S_D U_{PMNS}^\dagger T_D]$$

Two equations, one for the real and one for the imaginary part of  $a$



**TWO CONSTRAINTS ON THE MIXING MATRIX**

**THUS, IN GENERAL, DISCRETE SYMMETRIES  
IMPOSE TWO CONDITIONS ON THE  
PARAMETERS OF THE MIXING MATRIX**

$$a = -\text{Tr}[U_{PMNS} S_D U_{PMNS}^\dagger T_D]$$

So, the constraints on the entries of the mixing matrix depend on:

$$a = -\text{Tr}[ST]$$

$$T_D = \begin{pmatrix} e^{2\pi i \frac{k_1}{m}} & & \\ & e^{2\pi i \frac{k_2}{m}} & \\ & & e^{-2\pi i \frac{k_1+k_2}{m}} \end{pmatrix}$$

$$S_D^n = T_D^m = \mathbb{I}$$

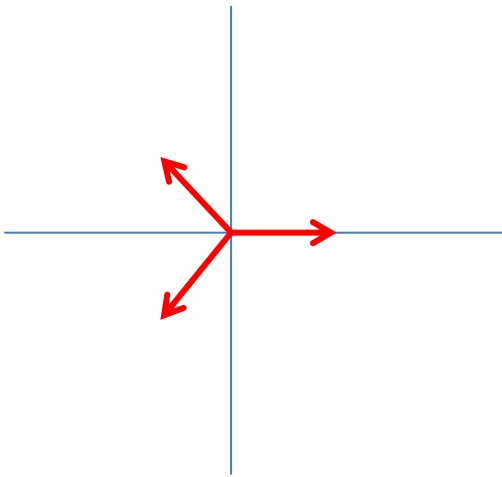
and define  $S_{Di}$  with  $n=2$  so that  $S_{Di}^2 = I$

$$S_{D1} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}, \quad S_{D2} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix}, \quad S_{D3} = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix}$$

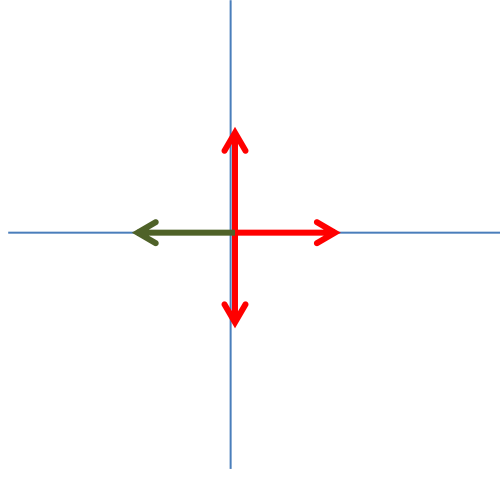
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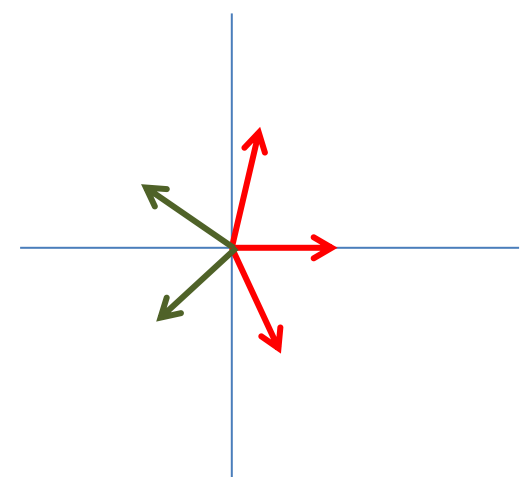
$p=3$



$p=4$



$p=5$



For instance, for  $p = 3 \longrightarrow (\lambda - 1)(\lambda - \omega)(\lambda - \omega^2) = \lambda^3 - 1 \longrightarrow a = 0$

or  $p = 4 \longrightarrow (\lambda - 1)(\lambda + i)(\lambda - i) = \lambda^3 - \lambda^2 + \lambda - 1 \longrightarrow a = -1$

The absolute values squared of one column are determined  
(two constraints plus unitarity)

$$|U_{l\nu}|^2 = \begin{pmatrix} \mathbf{S}_1 & \mathbf{S}_2 & \mathbf{S}_3 \\ |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2 \\ |U_{\tau1}|^2 & |U_{\tau3}|^2 & |U_{\tau3}|^2 \end{pmatrix} \quad \begin{aligned} R_i &= \text{Re}[a_i + \text{Tr}[T]] \\ I_i &= \text{Im}[a_i + \text{Tr}[T]] \end{aligned}$$

$$|U_{ei}|^2 = - \frac{R_i \cos\left(\pi \frac{k_1}{m}\right) - 2 \cos\left(\pi \frac{k_1+2k_2}{m}\right) - I_i \sin\left(\pi \frac{k_1}{m}\right)}{4 \sin\left(\pi \frac{k_1-k_2}{m}\right) \sin\left(\pi \frac{2k_1+k_2}{m}\right)}$$

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**Recapitulating:** What I have shown (under some - mostly harmless - assumptions)

After a number of choices have been made

1. The **T-charge** of the charged leptons ( $k_1$  and  $k_2$  value)
2. The **order of T** ( $m$  value)
3. The **S-charges** of the neutrinos
4. The **eigenvalues of ST** ( $a$  value)

**A two-dimensional surface is cut in the parameter space of the mixing matrix.**

**THIS IS ALL DISCRETE SYMMETRIES CAN TELL YOU FOR SURE ABOUT MIXING!**

# Choose $\alpha = e$ in the 'lazy' case

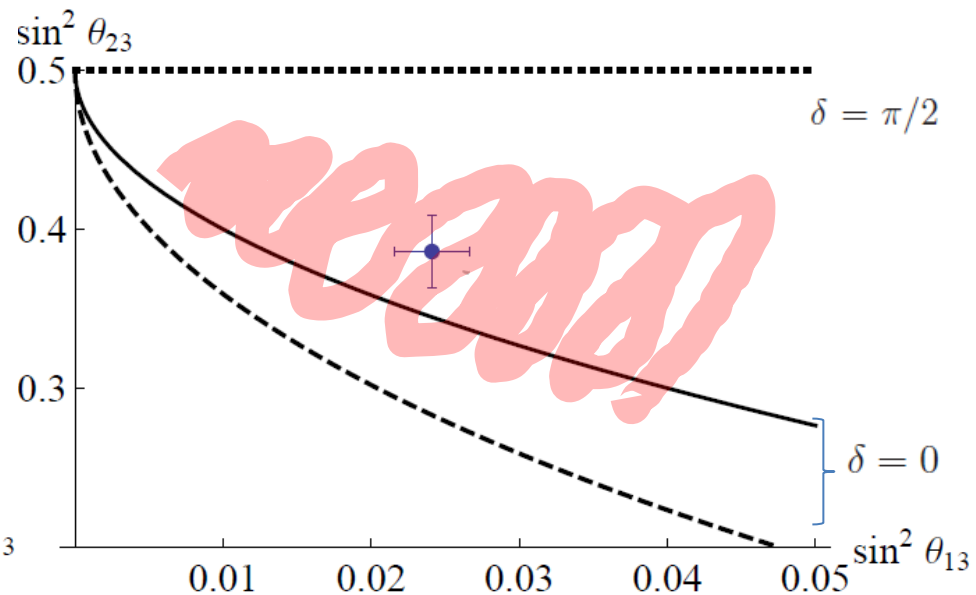
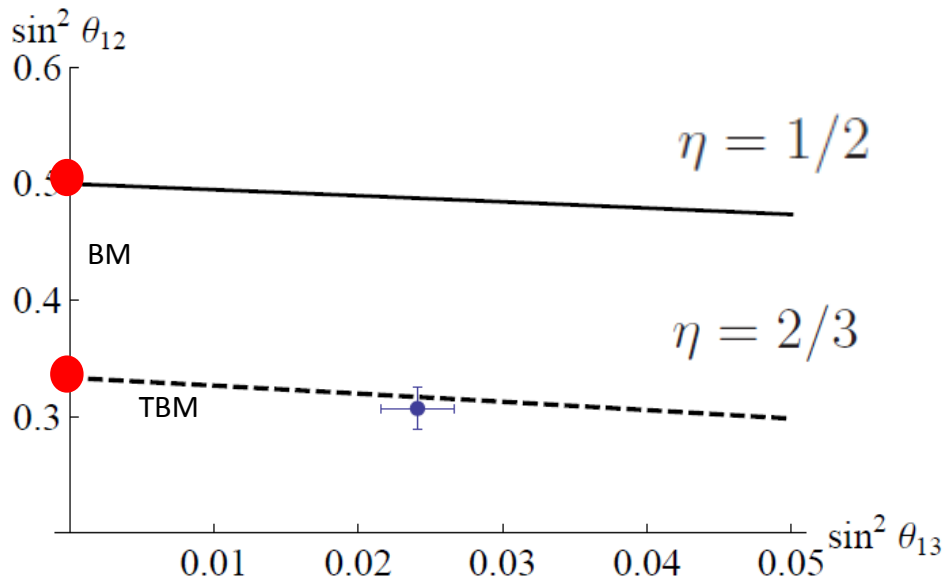
Taking  $i = 1$

$$S_{iU}^n = T^m = (S_{iU}T)^p = \mathbb{I}$$

$$\lambda^3 + a\lambda^2 - a^*\lambda - 1 = 0$$

$$\eta \equiv \frac{1-a}{4 \sin^2\left(\frac{\pi k}{m}\right)}$$

- Solid:  $m = 4, p = 3, k=1$  and from  $(\lambda - 1)(\lambda - \omega)(\lambda - \omega^2) = \lambda^3 - 1$ ,  $a=0$ . Group is  $\mathbf{S}_4$
- Dashed:  $m = 3, p = 4, k=1, a=-1$ . Group is  $\mathbf{S}_4$



**THIS KIND OF PLOTS CODIFY THE MOST  
GENERAL PREDICTION THAT DISCRETE  
SYMMETRIES CAN MAKE ABOUT THE MIXING  
PARAMETERS**

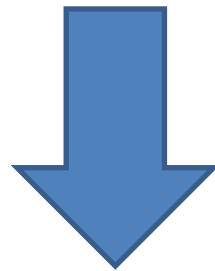
**NO DEPENDENCE ON MODEL SPECIFICS, JUST  
GO AND CHOOSE YOUR FAVORITE DISCRETE  
GROUP AND FIND OUT WHAT MIXINGS YOU  
MAY GET**

**WHAT ABOUT THE MASSES?**

# IN DETAIL

For Majorana neutrinos,  $S_D$  is an orthogonal matrix

$$S^n = \mathbb{I}$$



$$n \geq 3$$

**Neutrinos must be degenerate!**

$$S^n = T^m = \mathbb{I}$$

$$S_D = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \phi = \frac{2\pi}{n}$$
$$m_\nu = S_D m_\nu S_D^T$$

Only for approximately degenerate  $m_1$  AND  $m_2$

$$M_{\nu D} \simeq \begin{pmatrix} m & & \\ & m & \\ & & m' \end{pmatrix}$$

So that the group is finite

$$(ST)^2 = (U_{PMNS} S_D U_{PMNS}^\dagger T_D)^2 = 1$$



**Again, there seem to be 2 constraints on mixing**

$$(|U_{\alpha 3}|^2 \mp x)^2 - 2x \cdot \text{Im}[U_{\beta 1} U_{\beta 2}^* - U_{\gamma 1} U_{\gamma 2}^* \mp 1] = 0$$

$$2\text{Im}[U_{\alpha 1} U_{\alpha 2}^*] + y \cdot (|U_{\gamma 3}|^2 - |U_{\beta 3}|^2) = 0$$

$\beta, \gamma \neq \alpha$

**$x$  and  $y$  fixed by  $T_D$ , for instance**  $x = \cot\left(\frac{\pi k_n}{n}\right) \cot\left(\frac{\pi k_n}{m}\right)$

$$x = \cot\left(\frac{\pi k_n}{n}\right) \cot\left(\frac{\pi k_n}{m}\right)$$

**Actually, we get 4 constraints!**

$$|U_{\alpha i}|^2 = x$$

$$\text{Im}[U_{\alpha j} U_{\alpha k}^*] = 0, \quad j, k \neq i$$

$$|U_{\beta i}|^2 = |U_{\gamma i}|^2$$


$$\text{Im}[U_{\beta j} U_{\beta k}^* - U_{\gamma j} U_{\gamma k}^*] = \pm 1$$

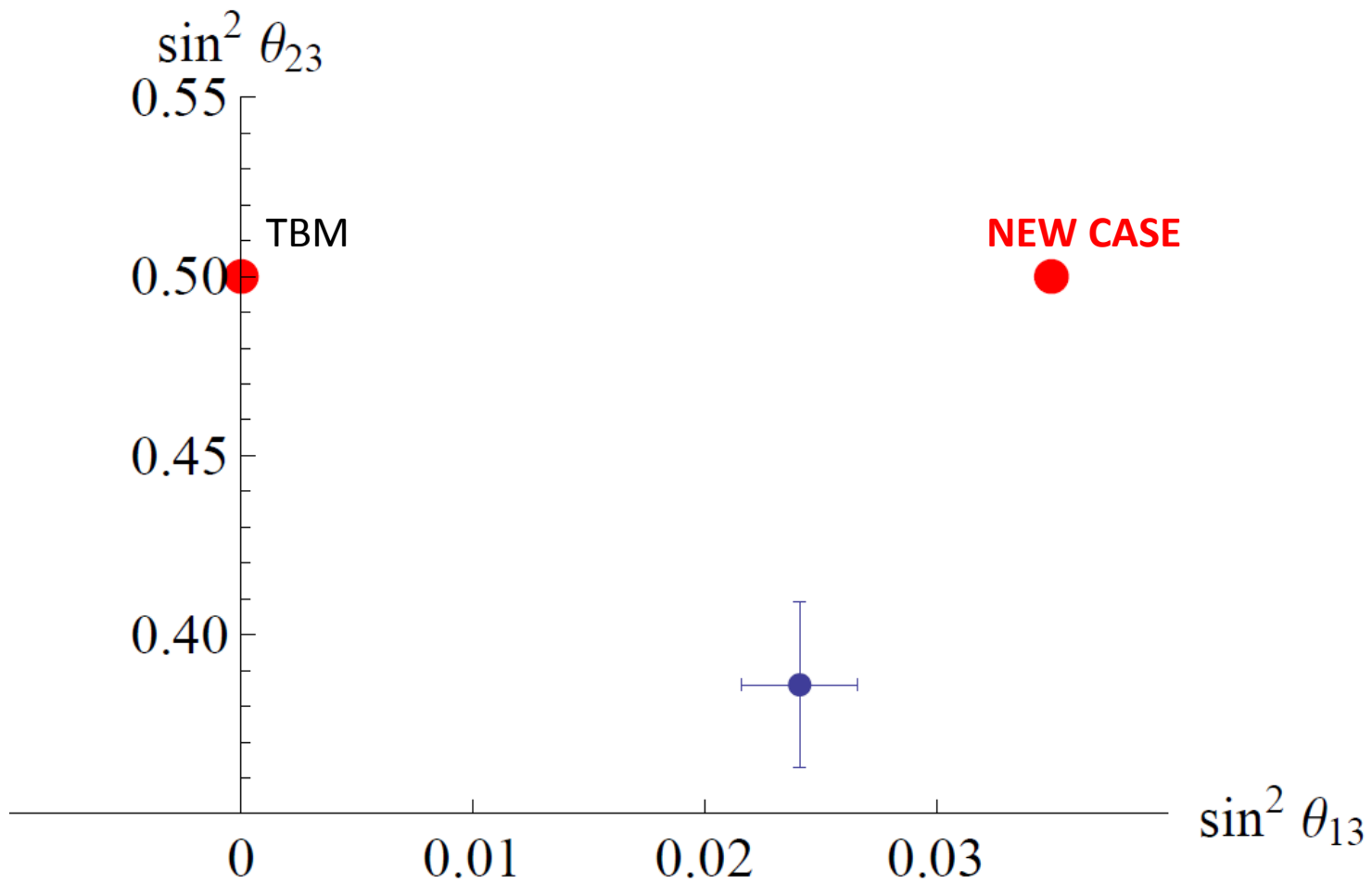


$$\alpha = e$$

$$\sin \theta_{13} = \pm \cot \frac{\pi k_n}{n} \cot \frac{\pi k_m}{m}, \quad \theta_{23} = \frac{\pi}{4}, \quad \delta = \frac{\pi}{2}, \quad \kappa = 0$$

$$m = 3, \quad k_m = 1; \quad n = 5, \quad k_n = 2$$


$$\sin \theta_{13} = \cot \frac{\pi}{3} \cot \frac{2\pi}{5} = \sqrt{\frac{1}{3} \left( 1 - \frac{2}{\sqrt{5}} \right)} \simeq 0.187$$

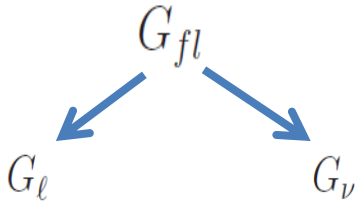


# Conclusions

- Constraints on mixing from discrete symmetries are model independent...
- ... and can be obtained in a systematic, rather simple way
- $\theta_{13}$  is not forced to be zero. In general, two parameters are defined out of 4 in the mixing matrix.
- Many results can be obtained: TBM, BM and analysis of less known groups made easy.

# Conclusions

- IF a **larger residual symmetry** is imposed in the neutrino sector, **up to 4 constraints in the mixing matrix**.
- In this case, there's still **compatibility with measured mixings**. **Masses predicted degenerate**. Corrections expected to account for both the  $\Delta m^2$  mass difference and the exact values for the mixing



## BOTTOM UP

1. Find accidental symmetries of the charged lepton and neutrino mass terms
2. Choose discrete subgroups in both cases
3. Combine them to define  $G_{fl}$

# IN DETAIL

## 1.- Identifying the accidental symmetries

$$\mathcal{L} = \frac{g}{\sqrt{2}} \bar{\ell}_L U_{PMNS} \gamma^\mu \nu_L W_\mu^+ + \bar{E}_R m_\ell \ell_L + \frac{1}{2} \bar{\nu}^c_L m_\nu \nu_L + \dots + \text{h.c.}$$

### Charged Leptons

$\bar{E}_R m_\ell \ell_L$  is invariant under  $U(1)^3$  **accidental**

$$E_R \rightarrow T E_R, \quad \ell_L \rightarrow T \ell_L \quad T = \text{diag}\{e^{i\alpha}, e^{i\beta}, e^{i\gamma}\}$$

# IN DETAIL

## 1.- Identifying the accidental symmetries

$$\mathcal{L} = \frac{g}{\sqrt{2}} \bar{\ell}_L U_{PMNS} \gamma^\mu \nu_L W_\mu^+ + \bar{E}_R m_\ell \ell_L + \frac{1}{2} \bar{\nu}^c_L m_\nu \nu_L + \dots + \text{h.c.}$$

### Neutrinos

$\frac{1}{2} \bar{\nu}^c_L m_\nu \nu_L$  invariant under  $Z_2 \otimes Z_2$  accidental

$$S_1 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}, \quad S_2 = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix}, \quad S_3 = S_1 S_2 = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix}$$

# 1.- Enter mixing matrix

$$\mathcal{L} = \frac{g}{\sqrt{2}} \bar{\ell}_L U_{PMNS} \gamma^\mu \nu_L W_\mu^+ + \bar{E}_R m_\ell \ell_L + \frac{1}{2} \bar{\nu}^c_L m_\nu \nu_L + \dots + \text{h.c.}$$

Change of basis

$$\mathcal{L} = \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma^\mu \nu_L W_\mu^+ + \bar{E}_R M_\ell \ell_L + \frac{1}{2} \bar{\nu}^c_L M_\nu \nu_L + \dots + \text{h.c.}$$

$$M_\nu = U^* m_\nu U^\dagger$$

$$M_\ell = m_\ell V$$

$$U_{PMNS} = VU$$

Take  $U \equiv U_{PMNS} \quad V \equiv 1$

Invariance of  $M_\nu$   
under  $Z_2 \otimes Z_2$   $\rightarrow$   $S_{iU}^\dagger M_\nu S_{iU} = M_\nu$  with  $S_{iU} = U S_i U^\dagger$

accidental

Still  $S_{iU}^2 = 1$



## 2.- Choosing the flavor subgroups

### For the neutrinos

Simply choose at least one of the  $S_{iU}$

## 2.- Choosing the flavor subgroups

For charged leptons, use a **finite abelian** subgroup of  $U(1)^3$  as the group of flavor

Impose  $T^m = 1$ ,  $T$  unitary

$$T = \begin{pmatrix} e^{2\pi i k_1/m} & & \\ & e^{2\pi i k_2/m} & \\ & & e^{-2\pi i(k_1+k_2)/m} \end{pmatrix}$$

### 3.- Defining the flavor group

- Define a relation between  $S_{iU}$  and  $T$

We had  $T^m = 1$ ,  $S_{iU}^2 = 1$

Add  $(S_{iU}T)^p = (US_iU^\dagger T)^p = \mathbb{I}$

## Constraints on the mixing matrix

$$W_i = S_{iU}T = US_iU^\dagger T, \quad W_i^P = 1$$



$$\text{Det}[W_i - \lambda\mathbb{I}] = 0 \quad \text{cubic equation with} \quad \lambda_i^P = 1$$



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Two equations, one for the real and one for the imaginary part of  $a$



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So, the constraints on the entries of the mixing matrix depend on

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and which  $S_i$  is chosen

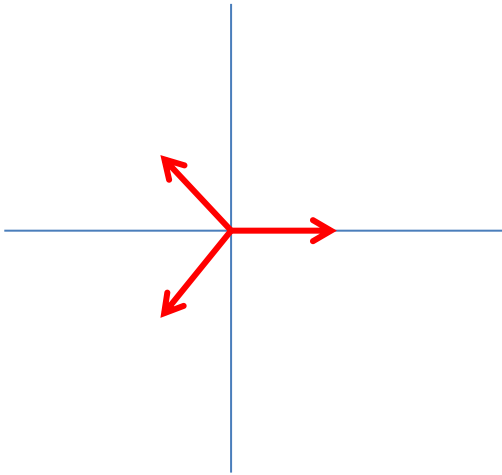
$$S_1 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix} \quad S_2 = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \quad S_3 = S_1 S_2 = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix}$$

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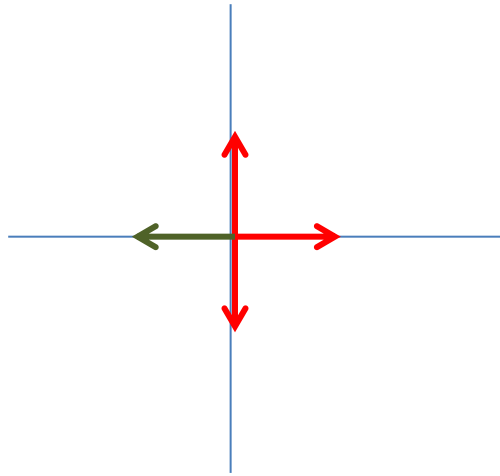
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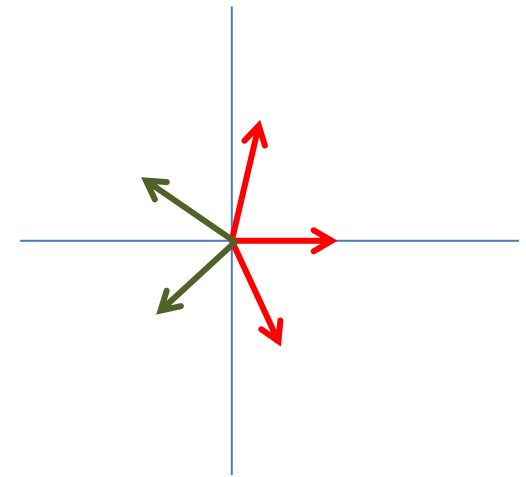
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(two constraints plus unitarity)

$$|U_{l\nu}|^2 = \begin{pmatrix} \mathbf{S}_1 & \mathbf{S}_2 & \mathbf{S}_3 \\ |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2 \\ |U_{\tau1}|^2 & |U_{\tau3}|^2 & |U_{\tau3}|^2 \end{pmatrix} \quad \begin{aligned} R_i &= \text{Re}\{\text{Tr}[W_i + T]\} \\ I_i &= \text{Im}\{\text{Tr}[W_i + T]\} \end{aligned}$$

$$|U_{ei}|^2 = - \frac{R_i \cos\left(\pi \frac{k_1}{m}\right) - 2 \cos\left(\pi \frac{k_1+2k_2}{m}\right) - I_i \sin\left(\pi \frac{k_1}{m}\right)}{4 \sin\left(\pi \frac{k_1-k_2}{m}\right) \sin\left(\pi \frac{2k_1+k_2}{m}\right)}$$

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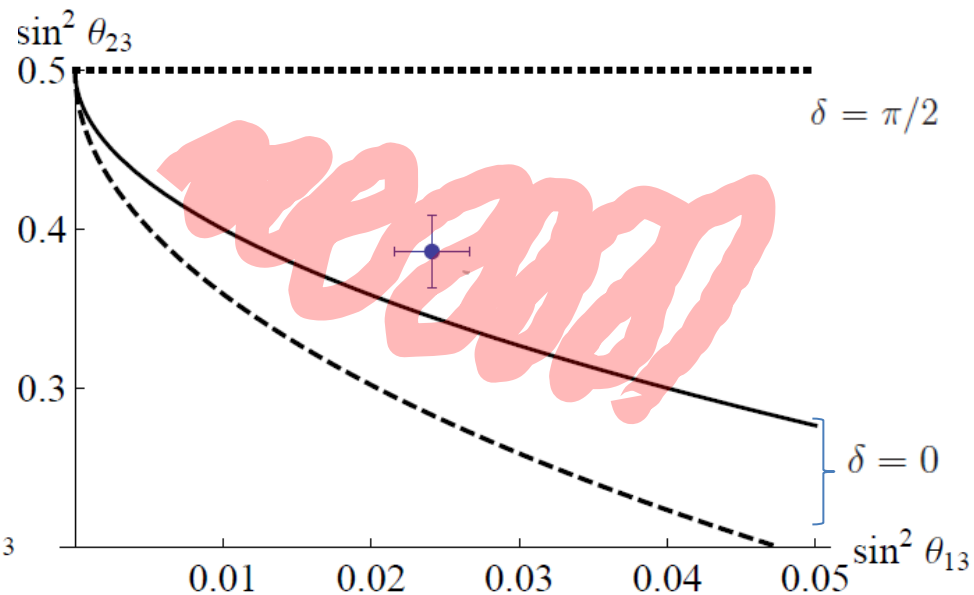
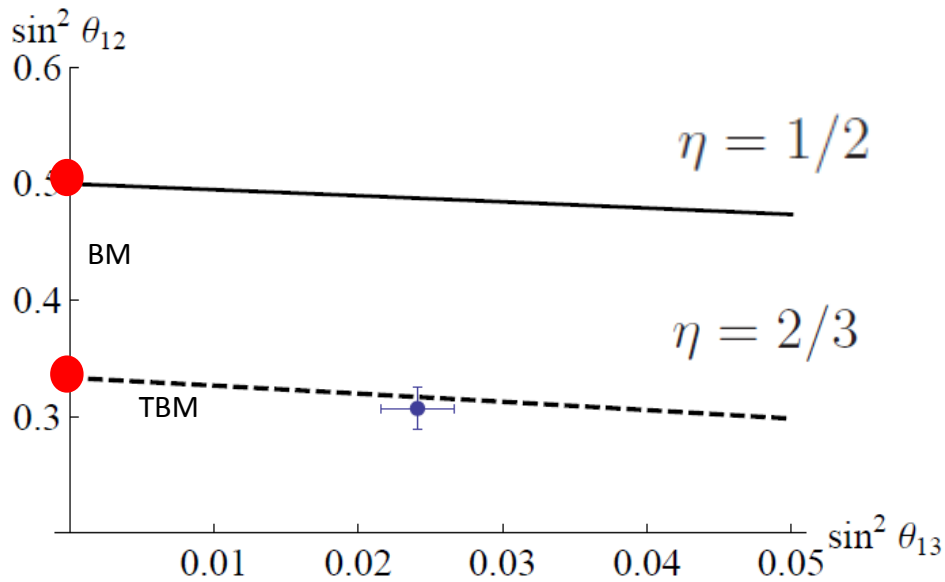
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WHAT ABOUT THE MASSES?

# IN DETAIL

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$$\mathcal{L} = \frac{g}{\sqrt{2}} \bar{\ell}_L U_{PMNS} \gamma^\mu \nu_L W_\mu^+ + \bar{E}_R m_\ell \ell_L + \frac{1}{2} \bar{\nu}^c_L m_\nu \nu_L + \dots + \text{h.c.}$$

### Neutrinos

$$\frac{1}{2} \bar{\nu}^c_L m_\nu \nu_L \text{ invariant under } Z_2 \otimes Z_2 \text{ accidental}$$

**ONLY IF THE THREE NEUTRINO MASSES ARE DIFFERENT!**

# FOR DEGENERATE NEUTRINO MASSES

$$m_\nu = \begin{pmatrix} m & & \\ & m & \\ & & m' \end{pmatrix}$$

$$m_\nu = S m_\nu S \quad S = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S^n = 1, \quad \phi = \frac{2\pi}{n}$$

$$W_i = S_{iU} T = U S_i U^\dagger T$$

So that the group is finite

$$S_{iU}^n = T^m = W_{iU}^2 = 1$$



## Constraints on mixing

$$x = \cot\left(\frac{\pi k_n}{n}\right) \cot\left(\frac{\pi k_n}{m}\right)$$

$$\left(|U_{\alpha i}|^2 \mp x\right)^2 - 2x \cdot \text{Im}[U_{\beta j} U_{\beta k}^* - U_{\gamma j} U_{\gamma k}^* \mp 1] = 0 \quad j, k \neq i$$

$$2\text{Im}[U_{\alpha j} U_{\alpha k}^*] + y \cdot (|U_{\gamma i}|^2 - |U_{\beta i}|^2) = 0 \quad \beta, \gamma \neq \alpha$$

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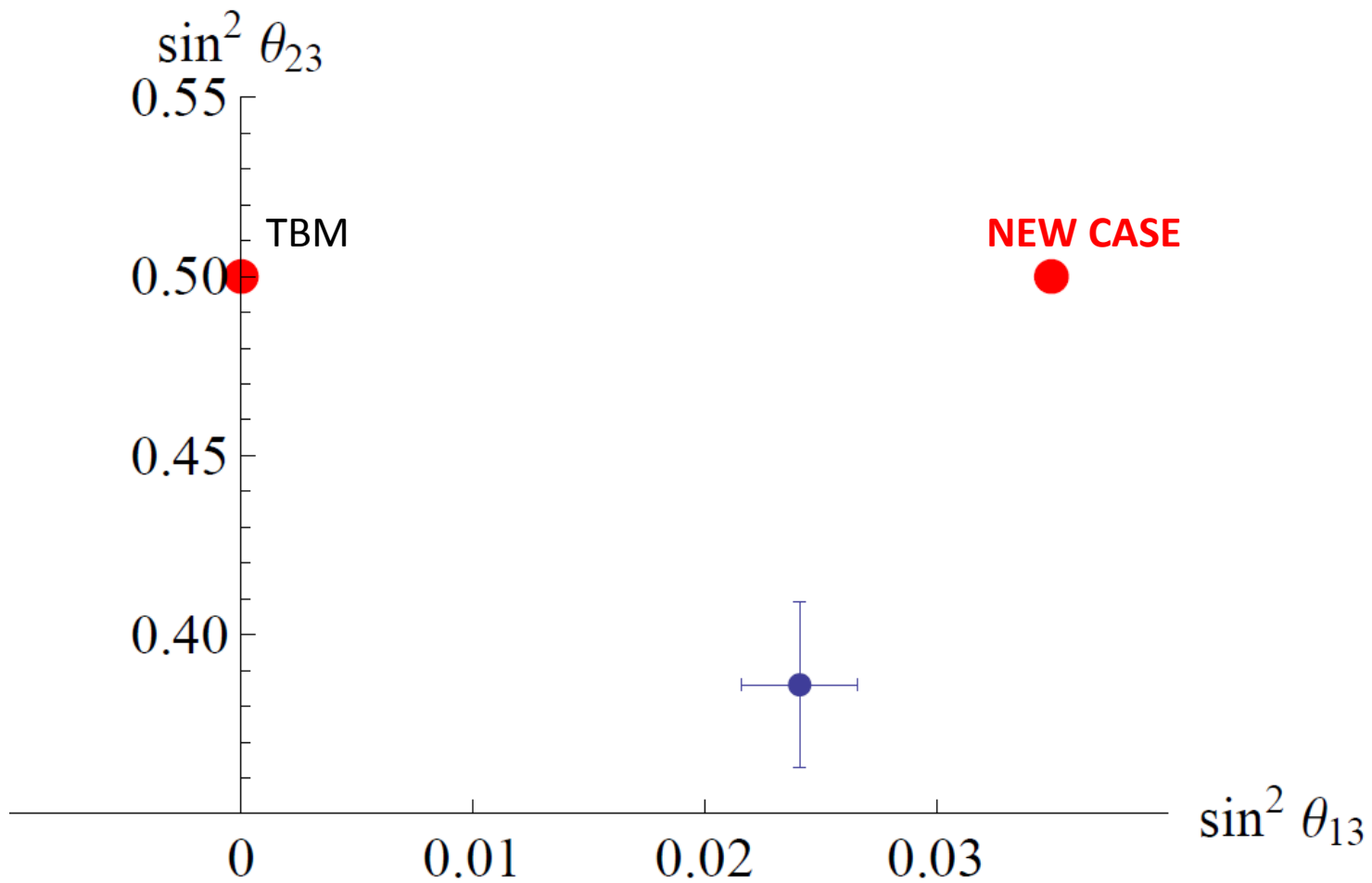
$$\text{Im}[U_{\beta j} U_{\beta k}^* - U_{\gamma j} U_{\gamma k}^*] = \pm 1$$

$$\alpha = e, \quad i = 3$$

$$\sin \theta_{13} = \pm \cot \frac{\pi k_n}{n} \cot \frac{\pi k_m}{m}, \quad \theta_{23} = \frac{\pi}{4}, \quad \delta = \frac{\pi}{2}, \quad \kappa = 0$$

$$m = 3, \quad k_m = 1; \quad n = 5, \quad k_n = 2$$

$$\sin \theta_{13} = \cot \frac{\pi}{3} \cot \frac{2\pi}{5} = \sqrt{\frac{1}{3} \left( 1 - \frac{2}{\sqrt{5}} \right)} \simeq 0.187$$





# Conclusions

- Constraints on mixing from discrete symmetries are model independent...
- ... and can be obtained in a systematic, rather simple way
- $\theta_{13}$  is not forced to be zero. In general, two parameters are defined out of 4 in the mixing matrix.
- Many results can be obtained: TBM, BM and analysis of less known groups made easy.

# Conclusions

- IF a **larger residual symmetry** is imposed in the neutrino sector, **up to 4 constraints in the mixing matrix**.
- In this case, there's still **compatibility with measured mixings**. **Masses predicted degenerate**.