New features of the Abramovsky-Gribov-Kancheli unitarity rules in QCD

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Outline:

The principal issue: how to relate miltipolmeron exchanges in the total cross section, diffraction and topological cross sections?

- Nuclear targets: topological cross sections are well defined by hadronic activity in the nucleus hemisphere
- One needs a separation of color excitation (cut pomeron) and color-diagonal (uncut pomeron) interactions
- Non-Abelian evolution of color dipoles in a nuclear medium
- Nonlinear $k_\perp$ factorization: new paradigm for hard pQCD in saturation regime
- Two kinds of unitarity-cut pomerons
- AGK rules from nonlinear $k_\perp$ factorization
- Multipomeron coupling from nonlinear $k_\perp$ factorization

$\star$ Principal reference: NNN & W. Schäfer, Phys. Rev. D74 (2006) 074021

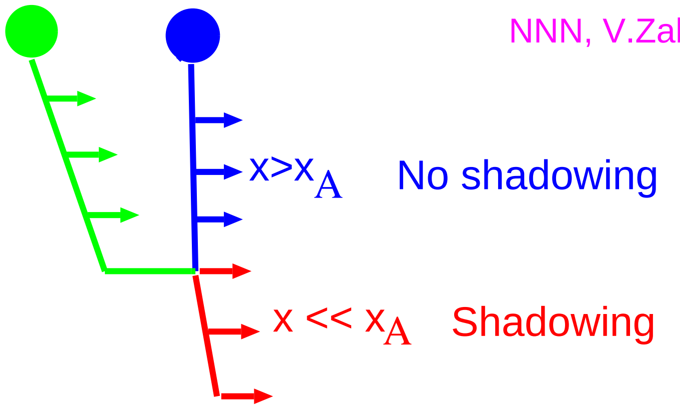
Collective nuclear glue at $x \ll 1$

⋆ Spatial overlap of partons from many nucleons in a Lorentz-contracted ultrarelativistic nucleus at

$$x \lesssim x_A = 1/R_A m_N$$

$\Longrightarrow$ FUSION & NUCLEAR SHADOWING.

NNN, V.Zakharov 1975



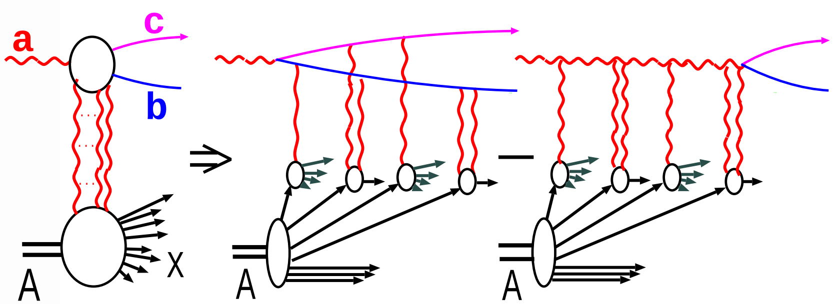
⋆ Nuclear parton density (if it can be meaningfully defined!) is a nonlinear functional of the free nucleon parton density: the same sea is shared by many nucleons.

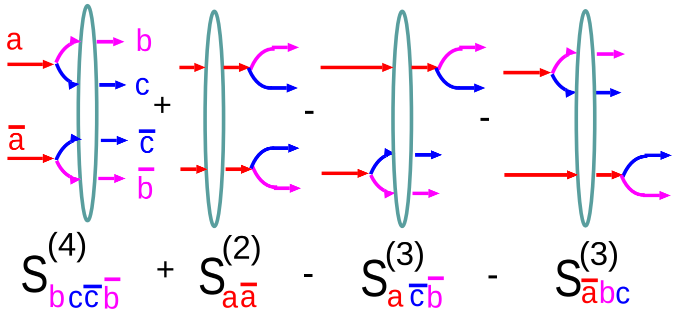
⋆ Must describe all nuclear observables!

⋆ Major strategy of this talk: shadowing from unitarity for dipole amplitudes.

Production as excitation of beam states $a \to bc$

Zakharov (87), NNN, Zakharov, Piller (95), NNN, Schäfer, Zakharov, Zoller (03)





⋆ Interactions with the nucleus after and before the virtual decay interfere destructively.

⋆ Elastic (absorption, uncut pomeron ℙ) and color-excitation (cut pomeron ℙ̸) multiple scatterings

⋆ Hermitian conjugated S-matrix = S-matrix for an antiparticle: $S_a S_b{}^\dagger = S_{a\bar{b}}$

⋆ Apply closure over the nucleon & nucleus excitations

⋆ Glauber-Gribov multiple scattering theory

Non-Abelian evolution and master formula for dijets

NNN, Zakharov, Piller (95), NNN, Schäfer, Zakharov, Zoller (03)

$$\frac{d\sigma(a^* \to bc)}{dz_b d^2\boldsymbol{p}_b d^2\boldsymbol{p}_c} = \frac{1}{(2\pi)^4} \int d^2\boldsymbol{b}_b d^2\boldsymbol{b}_c d^2\boldsymbol{b}'_b d^2\boldsymbol{b}'_c \times \exp[-i\boldsymbol{p}_b(\boldsymbol{b}_b - \boldsymbol{b}'_b) - i\boldsymbol{p}_c(\boldsymbol{b}_c - \boldsymbol{b}'_c)]$$

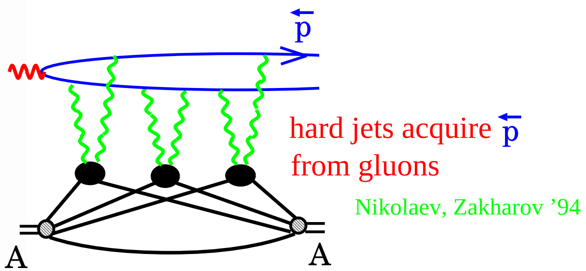
$$\Psi(z_b, \boldsymbol{b}_b - \boldsymbol{b}_c) \times \Psi^*(z_b, \boldsymbol{b}'_b - \boldsymbol{b}'_c)$$

$$\left\{ S^{(4)}_{\bar{b}\bar{c}cb}(\boldsymbol{b}'_b, \boldsymbol{b}'_c, \boldsymbol{b}_b, \boldsymbol{b}_c) + S^{(2)}_{\bar{a}a}(\boldsymbol{b}', \boldsymbol{b}) - S^{(3)}_{\bar{b}\bar{c}a}(\boldsymbol{b}, \boldsymbol{b}'_b, \boldsymbol{b}'_c) - S^{(3)}_{\bar{a}bc}(\boldsymbol{b}', \boldsymbol{b}_b, \boldsymbol{b}_c) \right\}.$$

- DIS: $\gamma^* \to q\bar{q}$: $\Longrightarrow \underbrace{1}_{1} + \underbrace{8}_{N_c^2}$
- Open charm: $g \to c\bar{c}$: $\Longrightarrow \underbrace{1}_{1\ (N_c\ suppressed)} + \underbrace{8}_{N_c^2}$
- Forward dijets: $q \to qg$: $\Longrightarrow \underbrace{3}_{N_c} + \underbrace{6+15}_{N_c \times N_c^2}$
- Central dijets: $g \to gg$: $\Longrightarrow \underbrace{1}_{1\ (N_c\ suppressed)} + \underbrace{8_A + 8_S}_{N_c^2} + \underbrace{10 + \overline{10} + 27 + R_7}_{N_c^2 \times N_c^2}$

⋆ Universality classes depending on color excitation

Coherent diffraction defines coherent nuclear glue



Diffractive DIS off nuclei defines collective nuclear glue

- ⋆ Diffractive hard dijets from pions: $\pi A \to Jet_1 + Jet_2$: $\boldsymbol{p}_{Jet_2} = -\boldsymbol{p}_{Jet_1} \gg 1/R_A$
- ⋆ Diffraction off nuclei (NNN,Shäfer,Schwiete'01): $M_A(\boldsymbol{p}) \propto \int d^2\boldsymbol{r}\Gamma_A(\boldsymbol{b},\boldsymbol{r})\exp(i\boldsymbol{p}\cdot\boldsymbol{r})$

$$\Gamma_A(\boldsymbol{b},\boldsymbol{r}) = 1 - \exp[-\frac{1}{2}\sigma(\boldsymbol{r})T(\boldsymbol{b})] = \int d^2\boldsymbol{\kappa}\phi(\boldsymbol{b},\boldsymbol{\kappa})\{1-\exp[i\boldsymbol{\kappa}\boldsymbol{r}]\}$$

- ⋆ Optical thickness $T(\boldsymbol{b}) = \int dz n_A(\boldsymbol{b},z)$ - defines a new large dimensional scale.
- ⋆ Collective glue is a physical observable: $M_{diff,A}(\boldsymbol{p}) \propto \phi(\boldsymbol{b},\boldsymbol{p})$
- ⋆ $\phi(\boldsymbol{b},\boldsymbol{p})$ = nuclear pomeron exchange

Nuclear glue of overlapping nucleons

- Nuclear coherent glue per unit area in the impact parameter space

$$\phi(\boldsymbol{b},\boldsymbol{\kappa}) = \frac{1}{\sigma_0}\sum_{j=1}^{\infty} w_j(\boldsymbol{b}) f^{(j)}(\boldsymbol{\kappa})$$

- Probability to find j overlapping nucleons and their collective glue

$$w_j(\boldsymbol{b}) = \frac{\nu_A^j(\boldsymbol{b})}{j!}\exp\left[-\nu_A(\boldsymbol{b})\right], \quad \nu_A(\boldsymbol{b}) = \frac{1}{2}\sigma_0 T(\boldsymbol{b}), \quad \sigma_0 = \sigma(r\to\infty)$$

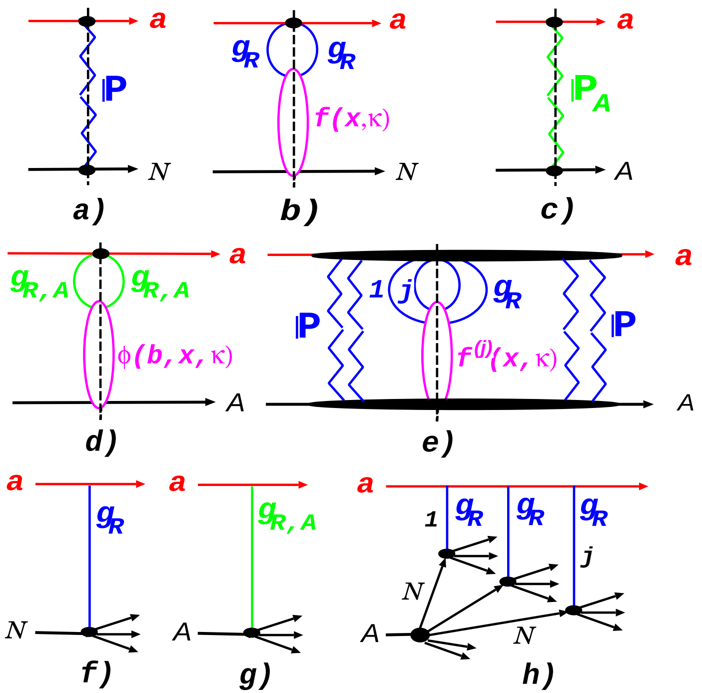
$$f^{(j)}(\boldsymbol{\kappa}) = \frac{1}{\sigma_0^{j-1}}\int\prod_i^j d^2\boldsymbol{\kappa}_i f(\boldsymbol{\kappa}_i)\delta(\boldsymbol{\kappa}-\sum_i^j\boldsymbol{\kappa}_i), \quad f^{(0)}(\boldsymbol{\kappa})\equiv\delta(\boldsymbol{\kappa})$$

- Nuclear S-matrix for the dipole: $S_A(\boldsymbol{b},\boldsymbol{r}) = \exp[-\frac{1}{2}\sigma(\boldsymbol{r})T(\boldsymbol{b})]$

$$\Phi(\boldsymbol{b},\boldsymbol{\kappa}) = \frac{1}{(2\pi)^2}\int d^2\boldsymbol{r}\, S_A(\boldsymbol{b},\boldsymbol{r})\exp(-i\boldsymbol{r}\boldsymbol{\kappa}) = \phi(\boldsymbol{b},\boldsymbol{\kappa}) + w_0(\boldsymbol{b})\delta(\boldsymbol{\kappa})$$

Unitarity content of the collective glue

⋆ Leading quark spectrum probes collective glue



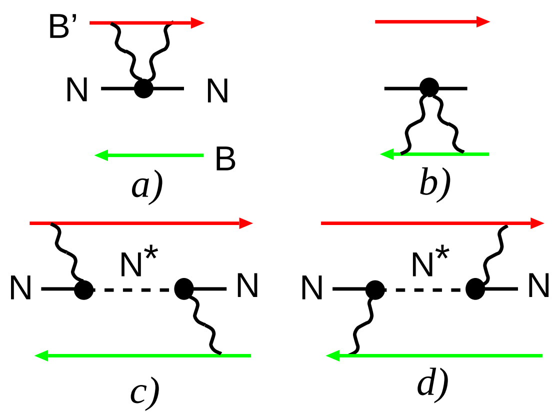
$$\frac{d\sigma_{Qel}}{d^2\boldsymbol{q}} = \frac{1}{2} f(x, \boldsymbol{q})\,, \qquad \frac{d\sigma_{Qel}^{(\nu)}(\boldsymbol{q})}{d^2\boldsymbol{q}} = \frac{1}{2} f^{(\nu)}(x, \boldsymbol{q}), \qquad \frac{d\sigma_{Qel,A}}{d^2\boldsymbol{b}d^2\boldsymbol{q}} = \sum_{j=1} w_j \Big(\nu_A(\boldsymbol{b})\Big) \frac{d\sigma_{Qel}^{(j)}(\boldsymbol{q})}{\sigma_{Qel} d^2\boldsymbol{q}}$$

⋆ Expansion of the cut nuclear pomeron in cut free-nucleon pomerons

⋆ Screening by uncut pomerons in the expansion coefficients

Deivation of unitarity cuts and AGK rules

⋆ Elastic and excitation interactions of (multi)parton-antiparton states:



$$\hat{\Sigma}_{ex}(C) + \hat{\Sigma}_{el}(C) = \hat{\Sigma}(C)$$

$$\mathcal{S}_A^{(n)}(C) = \mathrm{S}[\boldsymbol{b}, \hat{\Sigma}_{ex}(C) + \hat{\Sigma}_{el}(C)]$$

⋆ Expansion in powers of $\hat{\Sigma}_{ex}(C)$ is an expansion in cut pomerons

⋆ Expansion in powers of $\hat{\Sigma}_{el}(C)$ gives multipomeron absorption corrections

Inclusive single-jet DIS

$$\frac{d\sigma_{in}}{d^2\mathbf{b}d^2\mathbf{p}dz} = \frac{1}{(2\pi)^2} \times \Big\{ \int d^2\boldsymbol{\kappa}\phi(\boldsymbol{\kappa})\,|\Psi(z,\mathbf{p}) - \Psi(z,\mathbf{p}-\boldsymbol{\kappa})|^2 - \underbrace{\Big| \int d^2\boldsymbol{\kappa}\phi(\boldsymbol{\kappa})\,(\Psi(z,\mathbf{p}) - \Psi(z,\mathbf{p}-\boldsymbol{\kappa}))\Big|^2 \Big\}}_{Nonlinear}$$

$$\frac{d\sigma_{D}}{d^2\mathbf{b}d^2\mathbf{p}dz} = \frac{1}{(2\pi)^2} \times \underbrace{\Big| \int d^2\boldsymbol{\kappa}\phi(\boldsymbol{\kappa})\,(\Psi(z,\mathbf{p}) - \Psi(z,\mathbf{p}-\boldsymbol{\kappa}))\Big|^2}_{Nonlinear} .$$

$$\frac{d[\sigma_D + \sigma_{in}]}{d^2\mathbf{b}d^2\mathbf{p}dz} = \frac{1}{(2\pi)^2} \int d^2\boldsymbol{\kappa}\phi(\boldsymbol{\kappa})\,|\Psi(z,\mathbf{p}) - \Psi(z,\mathbf{p}-\boldsymbol{\kappa})|^2$$

⋆ Exceptional case of linear $k_\perp$-factorization: cancellation of nonlinearities of inelastic and coherent diffractive DIS, FSI and ISI are fully reabsorbed into collective nuclear glue!

⋆ Doesn't hold for the two-particle and all other single-particle spectra

Dijets: Universality class of coherent diffraction

⋆ Coherent distortion of dipole WF by uncut multipomeron exchanges defined by $S[\beta; \boldsymbol{b}, \sigma(\boldsymbol{r})]$ for the slice $[0, \beta]$ of the nucleus:

$$\Psi(\beta; z, \boldsymbol{p}) = \int d^2\boldsymbol{\kappa} \Phi(\beta; \boldsymbol{b}, x, \boldsymbol{\kappa}) \Psi(z, \boldsymbol{p} + \boldsymbol{\kappa})$$

⋆ Diffractive DIS:

$$\frac{(2\pi)^2 d\sigma_A(\gamma^* \to Q\overline{Q})}{d^2\boldsymbol{b} dz d^2\boldsymbol{p} d^2\boldsymbol{\Delta}} = \delta^{(2)}(\boldsymbol{\Delta}) \Big| \Psi(1; z_g, \boldsymbol{p}) - \Psi(z_g, \boldsymbol{p}) \Big|^2 ,$$

⋆ $q \to qg$: Intranuclear attenuation by net color charge of the incident parton (Bjorken's gap survival)

$$\frac{(2\pi)^2 d\sigma_A(q^* \to qg)}{d^2\boldsymbol{b} dz d^2\boldsymbol{p}_g d^2\boldsymbol{\Delta}} = \delta^{(2)}(\boldsymbol{\Delta}) S^2[\boldsymbol{b}, \sigma_0] \Big| \Psi(1; z_g, \boldsymbol{p}_g) - \Psi(z_g, \boldsymbol{p}_g) \Big|^2 .$$

Dijets in higher color representations: $q \to qg\Big|_{6+15}$

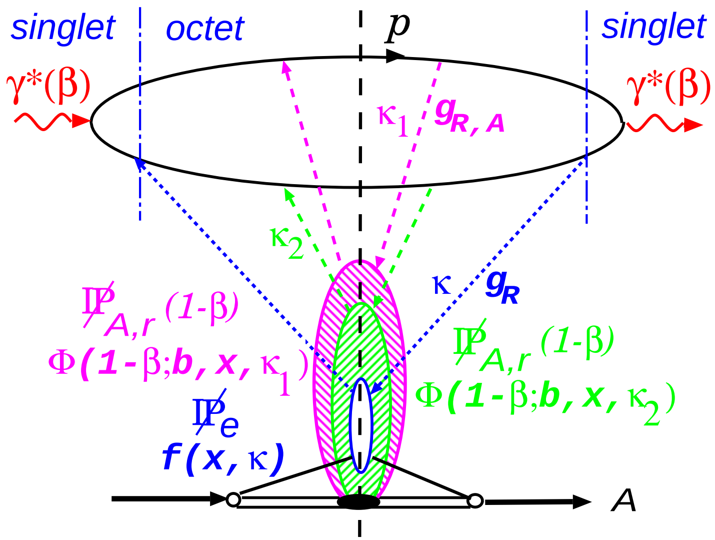
$$\frac{d\sigma(q^* \to qg)}{d^2\boldsymbol{b}dzd^2\boldsymbol{\Delta}d^2\boldsymbol{p}}\Bigg|_{6+15} = \frac{1}{(2\pi)^2}T(\boldsymbol{b})\int_0^1 d\beta \int d^2\boldsymbol{\kappa}d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 d^2\boldsymbol{\kappa}_3 \delta(\boldsymbol{\kappa}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2+\boldsymbol{\kappa}_3-\boldsymbol{\Delta})$$

$$\times \quad \underbrace{\Phi(\beta;\boldsymbol{b},\boldsymbol{\kappa}_3)}_{Quark\ \ ISI=\not{\mathbb{P}}_{A,r}} \ \underbrace{f(\boldsymbol{\kappa})\left|\Psi(\beta;z,\boldsymbol{p}-\boldsymbol{\kappa}_1)-\Psi(\beta;z,\boldsymbol{p}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa})\right|^2}_{Hard\ \ Excitation=\not{\mathbb{P}}_e}$$

$$\times \quad \underbrace{\Phi(1-\beta;\boldsymbol{b},\boldsymbol{\kappa}_1)}_{Quark\ \ FSI=\not{\mathbb{P}}_{A,r}}\underbrace{\Phi(\frac{C_A}{C_F}(1-\beta);\boldsymbol{b},\boldsymbol{\kappa}_2)}_{Gluon\ \ FSI=\not{\mathbb{P}}_{A,r}}$$

- $\star$ $\gamma^* \to q\bar{q}\Big|_8$: the same as $q \to qg\Big|_{6+15}$ but vanishing ISI;
- $\star$ $g \to gg\Big|_{10+\overline{10}+27+R_7}$: the same as $q \to qg$ subject to (i) Quark FSI/ISI $\Longrightarrow$ Gluon FSI/ISI, C_A/C_F: different glue!
- $\star$ β-dependent nuclear multipomeron vertex

Pomeron diagrams for inleastic DIS



⋆ Manifest dependence of collective glue on the partial nuclear thickness β — a footprint of the non-Abelian evolution of dipoles

⋆ Multiple-scattering expansion of $\Phi's$ gives topological cross sections

⋆ Cheshire Cat grin: integrating out the antiquark jet does not eliminate the antiquark contribution to the multiplicity of cut pomerons, inherent to QCD

⋆ ISI uncut pomeron exchanges in color-singlet $|\gamma^*(\beta)\rangle(1-\mathbb{P}_A(\beta))\otimes|\gamma\rangle$

Dijets in the beam color representations: $q \to qg\Big|_3$

$$\left.\frac{d\sigma\left(q^*A \to qg\right)}{d^2\boldsymbol{b}\,dz\,d^2\boldsymbol{\Delta}\,d^2\boldsymbol{p}}\right|_3 = \frac{1}{(2\pi)^2}\phi(\boldsymbol{b},\boldsymbol{\Delta})\left|\Psi(1;z,\boldsymbol{p}-\boldsymbol{\Delta}) - \Psi(z,\boldsymbol{p}-z\boldsymbol{\Delta})\right|^2$$

⋆ $\Psi(z,\boldsymbol{p}-z\boldsymbol{\Delta})$ = probability amplitude for the qg state in physical quark - driving term of the quark jet fragmentation

⋆ Color triplet dijets: fragments of the multiply-scattered quark

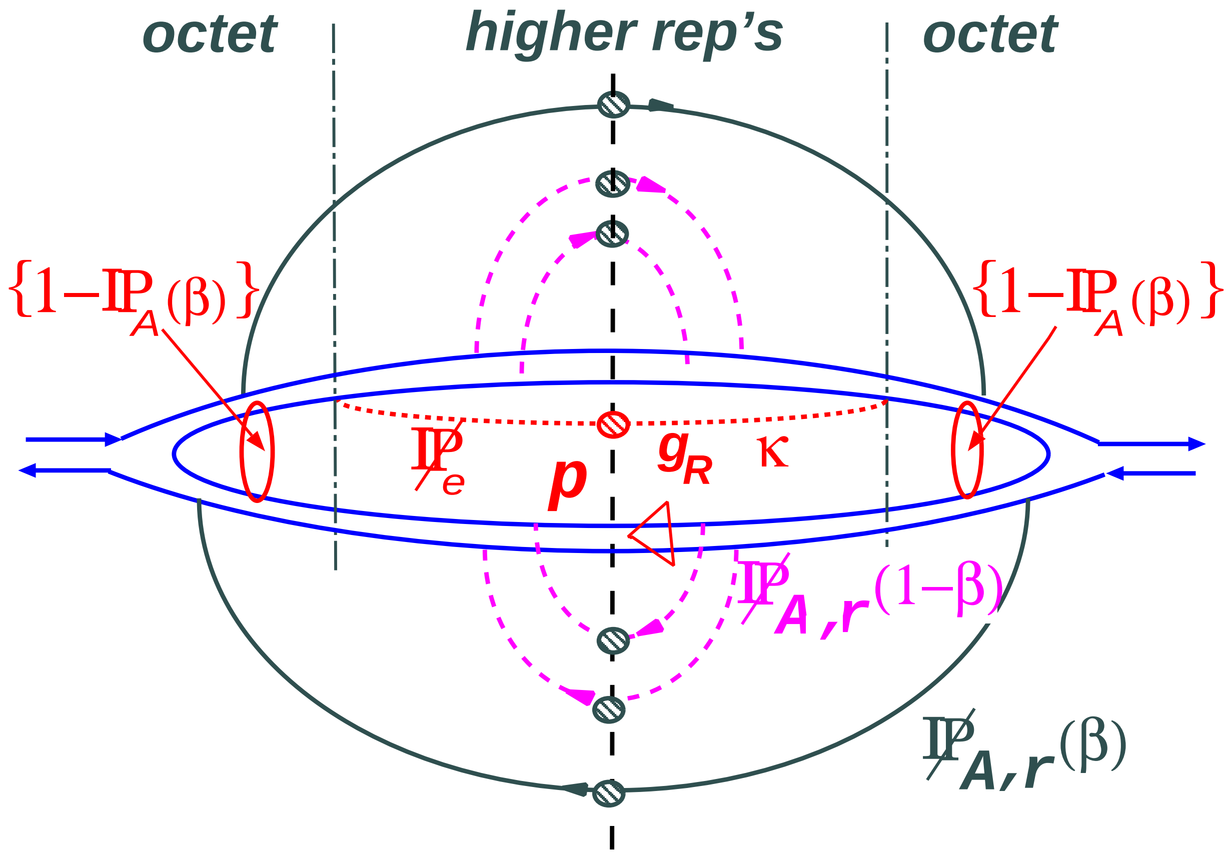
⋆ Coherent nuclear-distorted $\Psi(1;z,\boldsymbol{p}-\boldsymbol{\Delta})$:

$$\Big|\underbrace{\Psi(z,\boldsymbol{p}-\boldsymbol{\Delta})}_{in-vacuum} - \Psi(z,\boldsymbol{p}-z\boldsymbol{\Delta})\Big|^2 \Longrightarrow \Big|\underbrace{\Psi(1;z,\boldsymbol{p}-\boldsymbol{\Delta})}_{in-nucleus\ \ distorted} - \Psi(z,\boldsymbol{p}-z\boldsymbol{\Delta})\Big|^2$$

Interpretation: nuclear modification of the fragmentation function, alias the multipomeron vertex

⋆ In the related universality class: $g \to q\bar{q}\Big|_8$, $g \to gg\Big|_{8_A+8_S}$, $g \to gg\Big|_{8_S}$

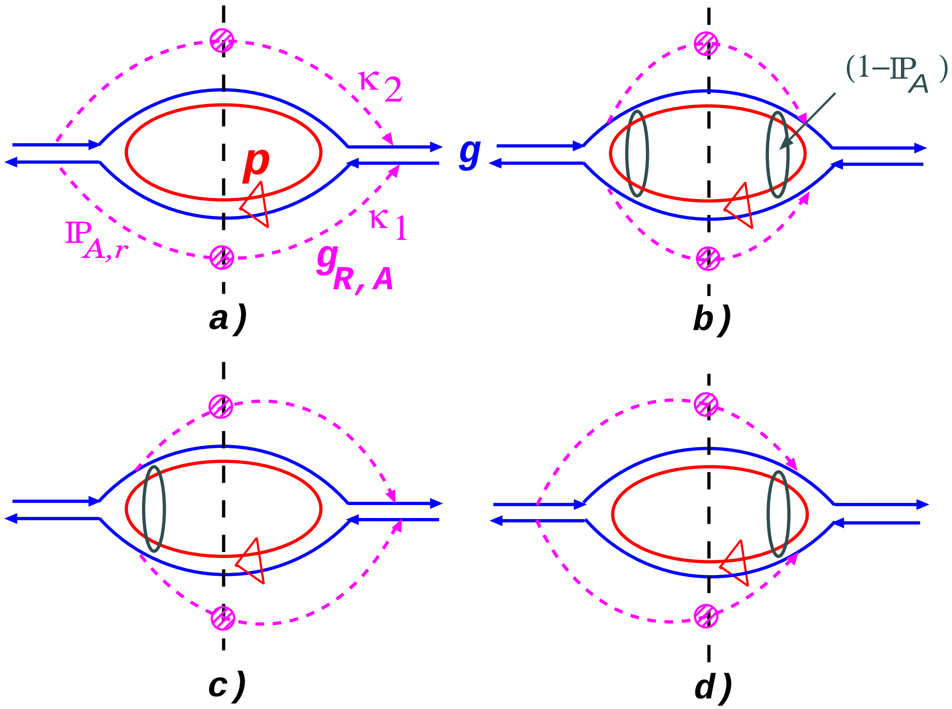
Gluon-gluon dijets in higher representations



⋆ gluons in the quark-antiquatk representation

⋆ Nuclear slice-dependent modification of the multipomeron couplings associated with the color-singlet “quark” loop

Dijets in the beam representation: $g \to gg|_8$



⋆ Nuclear modification of multipomeron couplings is different from the one for dijets in higher representatios

QCD vs. standard AGK : two-$\mathbb{P}$ X-sections

⋆ AGK (Capella-Kaidalov-Bertocchi-Treleani) for DIS off a nucleus:

$$\Delta_2 \Gamma_1^{in}(\not{\mathbb{P}}\not{\mathbb{P}}; \boldsymbol{b}, \boldsymbol{r}) = -[\sigma(x, \boldsymbol{r}) T(\boldsymbol{b})]^2$$

$$\Delta_2 \Gamma_D(\mathbb{P}\mathbb{P}; \boldsymbol{b}, \boldsymbol{r}) \quad : \quad \Delta_2 \Gamma_1^{in}(\not{\mathbb{P}}\mathbb{P}; \boldsymbol{b}, \boldsymbol{r}) \quad : \quad \Delta_2 \Gamma_2^{in}(\not{\mathbb{P}}\not{\mathbb{P}}; \boldsymbol{b}, \boldsymbol{r}) = 1 \quad : -4 \quad : \quad 2.$$

⋆ QCD: cut pomerons couple differently to singlet-to-octet excitation and octet-to-octet rotation:

$$\Delta_2 \Gamma_1^{in}(\not{\mathbb{P}}_e \mathbb{P}; \boldsymbol{b}, \boldsymbol{r}) = -\frac{1}{2} \cdot [\sigma_0(x) T(\boldsymbol{b})] \cdot [\sigma(x, \boldsymbol{r}) T(\boldsymbol{b})] - \frac{1}{2} [\sigma(x, \boldsymbol{r}) T(\boldsymbol{b})]^2.$$

$$\Delta_2 \Gamma_2^{in}(\not{\mathbb{P}}_r \not{\mathbb{P}}_e; \boldsymbol{b}, \boldsymbol{r}) = \frac{1}{2} \cdot [\sigma_0(x) T(\boldsymbol{b})][\sigma(x, \boldsymbol{r}) T(\boldsymbol{b})]$$

⋆ The conventional AGK rules break down in QCD

Conclusions

- ⋆ Novel property of QCD unitarity cutting rules: two kinds of cut pomerons
- ⋆ Topological cross sections follow directly from nonliner $k_\perp$ factorization for inclusive cross sections
- ⋆ Cheshire Cat grin: by QCD gauge invariance, comover/spectator interactions contribute to topological cross sections
- ⋆ Large variety of multipomeron vertices which vary from one universality class to another