

AGK RULES
&
INCLUSIVE JET PRODUCTION
@ LHC

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WORKSHOP ON HARD DIFFRACTION AT LHC - 18-19 OCT 200.
KRAKOW, POLAND

OUTLINE

1. MOTIVATION
2. INCLUSIVE JET PRODUCTION AND
 K_F -FACTORIZATION
3. AGK RULES

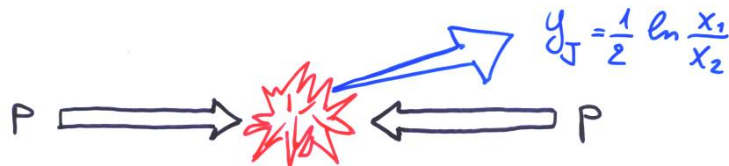
ON THE IMPORTANCE OF MULTIPLE INTERACTIONS

MULTIPLE INTERACTIONS ARE VERY IMPORTANT FOR PHENOMENOLOGY :

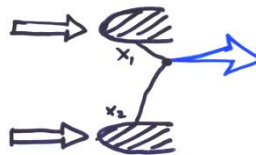
DIFFRACTION AT HERA :



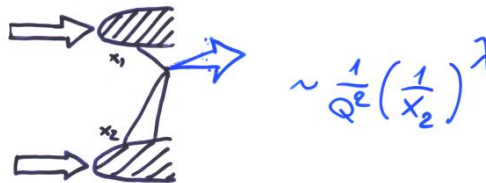
FORWARD JETS AT LHC :



LEADING ORDER :



HIGHER TWIST :



SINCE $\lambda > 0$, FOR x_2 SMALL ENOUGH

$$\frac{1}{Q^2} \left(\frac{1}{x_2}\right)^\lambda \sim \mathcal{O}(1)$$

MOTIVATION FROM THEORY

FROISSART BOUND (UNITARITY):

$$\sigma_{\text{TOT}} \lesssim C(\ln^2 s)$$

SINGLE POMERON EXCHANGE:

$$\sigma_{\text{TOT}} (1-P) \propto s^{\chi_0}$$

$$\chi_0 \sim 0.08 \quad \text{SOFT IP}$$

$$\chi_0 \sim 0.5 \quad \text{BFKL IP}$$

VIOLATES UNITARITY!

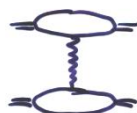
CORRECTIONS:

NLLA: MORE PRECISE VALUE OF χ_0 :

$$\sigma_{\text{TOT}} \propto s^{\chi_0 + \alpha_s \chi_1}$$

STILL VIOLATES F.B.

MULTIPLE INTERACTIONS : DIAGRAMS WITH MORE PARTICLES
EXCHANGED COME WITH ALTERNATING SIGN



(+1)



(-1)

MULTIPLE INTERACTIONS SOFTEN THE RISE OF THE CROSS-SECTION WITH ENERGY - SUMMING THEM UP WOULD EVENTUALLY UNITARIZE THE CROSS-SECTION.

QUICK REVIEW ON AQK RULES

$$I_{m_s} \text{ (circle)} = \sum_{n=1}^{\infty} I_{m_s} \text{ (cylinder)} = A_m$$

[ABRAMOSKI]
GRIBOV
KANCHALI]

$$I_{m_s} \text{ (cylinder)} = \sum_{k=0}^m \text{ (cut cylinder)} = A_m^k$$

IN A_m^k m REGGEON ARE EXCHANGED, k OF WHICH ARE CUT
FOR A GIVEN m , A_m^k FOR DIFFERENT k ARE RELATED BY
SIMPLE INTEGER COEFFICIENTS.

EXAMPLE IN ϕ^3 -THEORY

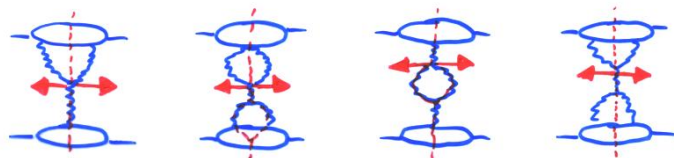
$$\text{Cylinder} : \text{Cut Cylinder} : \text{Cut Cylinder} = 1 : -4 : 2$$

NOTE THAT THEIR SUM IS NEGATIVE COMPARED TO $\text{Cylinder} :$
 $1 - 4 + 2 = \textcircled{-1} \Rightarrow \text{UNITARIZATION}$

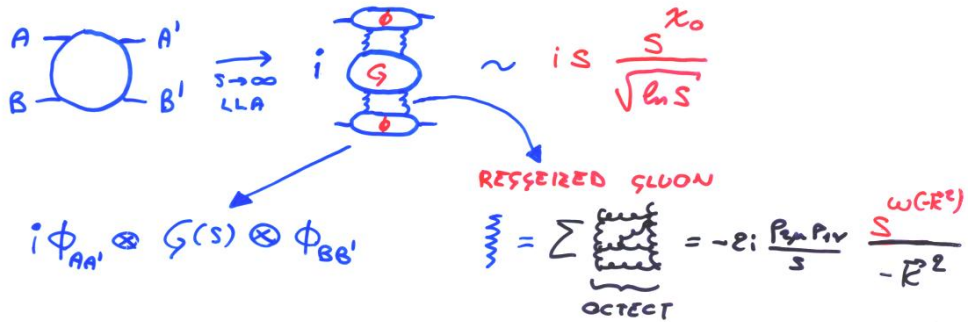
ANOTHER REMARKABLE OBSERVATION IS THE CANCELLATION OF
RESCATTERING ACROSS THE PRODUCTION OF A JET :

$$\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} = 0$$

SOME EXAMPLE OF THE DIAGRAMS WHICH ARE LEFT :



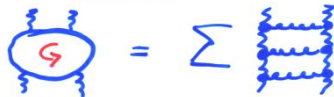
K_t - FACTORIZATION



⊗ = CONVOLUTION IN TRANSVERSE MOMENTUM

IS A D.O.F. LIVING IN THE TRANSVERSE PLANE

THE HIGH ENERGY LIMIT IS DETERMINED AT LLA BY THE BFKL GREEN'S FUNCTION G , WHICH REPRESENT A BOUND STATE OF 2 REGGEIZED GLUONS IN COLOR-SINGLET STATE:



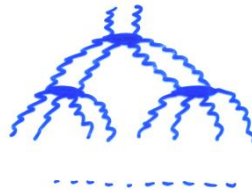
FOR SUFFICIENTLY HIGH RAPIDITY CORRECTIONS DUE TO MULTIPLE INTERACTIONS BECOME RELEVANT!



⇒ BEGINNING OF A POMOON STRUCTURE

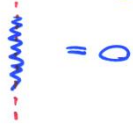
THE POMOON STRUCTURE IS RESUMMED IN THE SOLUTION OF THE BK-EQUATION:

$$\underbrace{\partial_y N}_{\text{BFKL}} = \underbrace{\chi N - N^2}_{\text{POMERON SPLITTING}} \Rightarrow$$

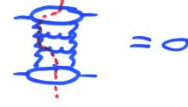
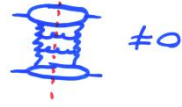


AGK RULES IN PQCD

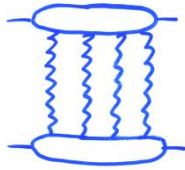
IR-GLUON IS REAL IN LLA



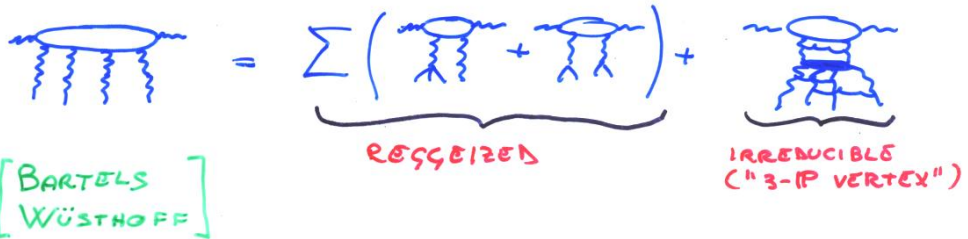
BFKL POTTERON IS IMAGINARY IN LLA AND CANNOT BE PARTIALLY CUT



TWO POTTERONS CORRESPOND TO FOUR IR-GLUONS:



FOR A VIRTUAL PHOTON ONE GETS:



THE IRREDUCIBLE PART SATISFIES THE AGK-LIKE RULES:

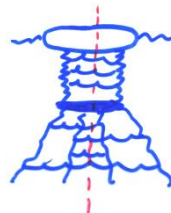
DIFFRACTIVE



1-MULTIPLICITY



2-MULTIPLICITY



[BARTELS
RYSKIN]

SINGLE JET PRODUCTION IN K_F FACTORIZATION

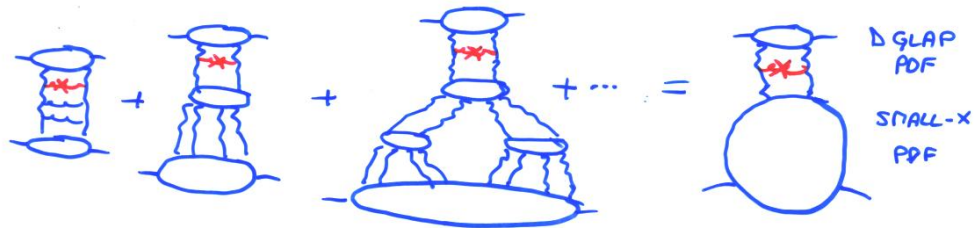
A "RUNG" (GLUON) IN \mathcal{G} IS "FIXED":



BUT IF $y_1 \ll y_2$ (FORWARD DIRECTION AT LHC):



THE NAÏVE EXPECTATION WOULD BE A FAN STRUCTURE:

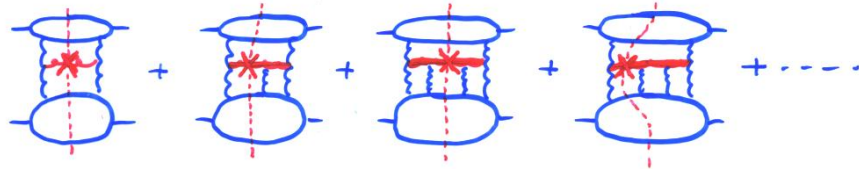


IF SO, IT WOULD BE STANDARD K_F -FACTORIZATION WITH A SUITABLE PDF EVOLVED UP TO VERY SMALL x .

EXPLICIT COMPUTATION SHOWS THAT THIS IS NOT THE CASE!

MORE GENERAL CORRELATORS ARE NEEDED

FROM THE EXPLICIT COMPUTATION IN PERTURBATION THEORY EMERGES THAT ONE HAS TO TAKE INTO ACCOUNT:



UP TO FOUR GLUONS THEY HAVE BEEN COMPUTED

IMMEDIATE CONSEQUENCE: AGK BREAKING

ACCORDING TO AGK RESCATTERING ACROSS A JET SHOULD CANCEL:

$$\sum \text{[Diagram of a jet with a gluon exchange]} \stackrel{?}{=} 0$$

BUT IF WE CONSIDER NOW DOUBLE JET PRODUCTION,

SINCE  WE HAVE

$$\sum \text{[Diagram of double jet production with two gluon exchanges]} \neq 0$$

[BARTELS
SALVADORE
VACCA]

THE COMPUTATION: REVIEW OF INCLUSIVE CASE

IN ORDER TO JUSTIFY THE USE OF PERTURBATION THEORY WE FOCUS ON THE SCATTERING OF VIRTUAL PHOTONS.

ONE IS LEAD TO COMPUTING THE MULTIPLE DISCONTINUITIES OF $\gamma^* \gamma^* - n$ GLUONS AMPLITUDES.

ACCOMPLISHED BY SUMMING UP FEYNMAN DIAGRAMS BY MEANS OF BETHE-SALPETER EQUATIONS.

$$\begin{aligned}
 (\alpha_y - \omega_1 - \omega_2) \text{diagram} &= \sum \text{diagram} \delta(y) + \text{diagram} \quad \text{BFKL} \\
 (\alpha_y - \sum_{i=1}^3 \omega_i) \text{diagram} &= \sum \text{diagram} \delta(y) + \sum \text{diagram} + \text{diagram} \\
 (\alpha_y - \sum_{i=1}^4 \omega_i) \text{diagram} &= \sum \text{diagram} \delta(y) + \\
 &+ \sum \text{diagram} + \sum \text{diagram} + \text{diagram}
 \end{aligned}$$

THE EQUATIONS CAN BE RESHUFFLED (BOOTSTRAP EQ.)

AND ONE OBTAINS :

$$\text{diagram} = \text{diagram} + \text{diagram} + \text{diagram} \quad \text{(BOOTSTRAP)}$$



D_3 IS "RESGEIZED": IT IS REDUCED TO A SUM OF D_2 'S.

$$\text{diagram} = \underbrace{\sum (\text{diagram} + \text{diagram})}_{\text{RESGEIZED PART}} + \underbrace{\text{diagram}}_{\text{IRREDUCIBLE PART}} \quad \text{[BARTELS WUSTHOFF]}$$

THE IRREDUCIBLE PART IS GAUGE INVARIANT AND BOSE SYMMETRIC. IT SATISFIES AGK RULES. [BARTELS, RYSKIN]

THE COMPUTATION: SINGLE JET PRODUCTION

WE TAKE A SIMILAR APPROACH. THE UNDERLINE ASSUMPTION IS THAT THE MULTIPLE DISCONTINUITIES CONTAIN ALL THE INFORMATION NEEDED TO RECONSTRUCT THE X-SECTION.

$$(2y - \omega_1 - \omega_2) = \text{diagram} = \text{diagram} \delta(y - y_1) + \text{diagram}$$

THE X MEANS THAT A SLOON IS KEPT FIXED.

y_1 IS THE RAPIDITY OF THE JET.

THE SOLUTION IS THEN:



FOR THREE SLOONS THERE ARE TWO POSSIBILITIES:

$$(2y - \sum_{i=1}^3 \omega_i) \text{diagram} = \left(\sum \text{diagram} + \text{diagram} \right) \delta(y - y_1) + \sum \text{diagram} + \text{diagram}$$

$$(2y - \sum_{i=1}^3 \omega_i) \text{diagram} = \left(\sum \text{diagram} + \text{diagram} \right) \delta(y - y_1) + \sum \text{diagram} + \text{diagram}$$

DUE TO THE PRESENCE OF A FIXED SLOON, THE RESUMEZATION IS NOW INCOMPLETE:

$$\text{diagram} = \sum \text{diagram} + \text{diagram}$$

$$\text{diagram} = \sum \text{diagram} + \text{diagram}$$

THE COMPUTATION: FOUR GLUONS

THERE ARE THREE $\gamma^* \gamma^*$ -4GLUON+JET AMPLITUDES:



THE EQUATION FOR THE CENTRAL CUT IS:

$$\left(2g - \sum_{i=1}^4 \omega_i\right) \text{diagram} = \left(\sum \text{diagram} + \sum \text{diagram} + \sum \text{diagram} \right) d(y, y_1) + \sum \text{diagram} + \sum \text{diagram} + \sum \text{diagram}$$

THE EQUATIONS FOR AND ARE SIMILAR.

THE SOLUTION CAN BE FACTORIZED IN DIFFERENT PARTS:

$$\text{diagram} = \sum \left(\text{diagram} + \text{diagram} \right) + (\text{RESGEIZED PART})$$

$$+ \sum \left(\text{diagram} + \text{diagram} \right) + (\text{OK WITH AGK})$$

$$+ \text{diagram} + \text{diagram}$$

NEW $2 \rightarrow 4$
PRODUCTION VERTEX

$$+ \text{diagram}$$

NEW $3 \rightarrow 4$
VERTEX

[BARTELS
SALVADORE
YACCA]

NEW VERTICES

THE NEW PRODUCTION VERTICES WE HAVE COMPUTED ARE



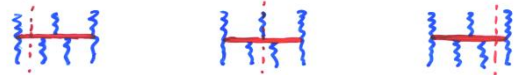
THEY SATISFY THE WARD IDENTITIES :

$$\text{VERTEX}(\vec{k}_i) \xrightarrow[\vec{k}_i \rightarrow 0]{} \emptyset$$

THEY ARE BASE-SYMMETRIC W.R.T. THE GLUONS ON EACH SIDE OF THE CUT.

DIFFERENT CUTS ARE GENUINELY DIFFERENT \rightarrow NO AQK!

FROM THE COMPUTATION ONE OBTAINS AS WELL NEW $3 \rightarrow 4$ VERTICES :



THEY ALSO SATISFY WARD IDENTITIES AND HAVE GOOD SYMMETRY PROPERTIES, BUT SEEM TO DISAGREE WITH GRIBOV'S SIGNATURE CONSERVATION RULE!

THIS ISSUE NEEDS MORE INVESTIGATION...

CONCLUSION

- MULTIPLE INTERACTIONS ARE IMPORTANT :
PHENOMENOLOGY: FORWARD JETS @ LHC
THEORY: UNITARIZATION
- IN JET PRODUCTION THE STANDARD K_f -FACTORIZATION IS NOT SUFFICIENT. ONE HAS TO INCLUDE HIGHER ORDER CORRELATOR.
- THE NEW VERTICES OBTAINED DO NOT FULFIL THE STANDARD AQG RULES.
- MORE INVESTIGATIONS ARE NEEDED...