
Saturation model in diffraction

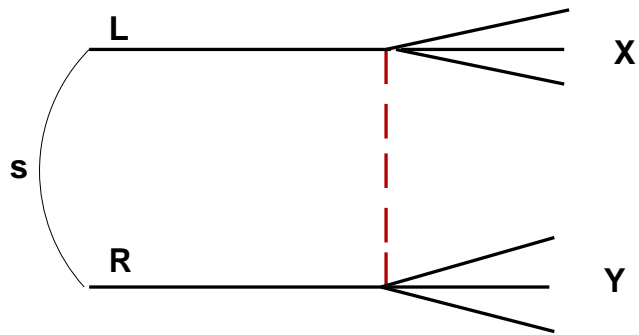
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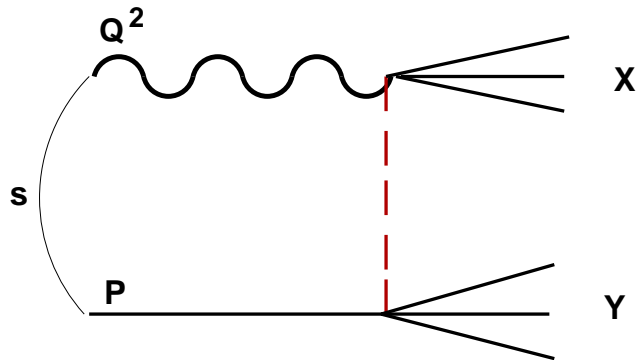
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Diffraction at the LHC, Kraków, 18-19 October 2007

Diffractive processes



- $s \gg -t, m_X^2, m_Y^2$
- vacuum quantum number exchange
- QCD: two gluons in color singlet state

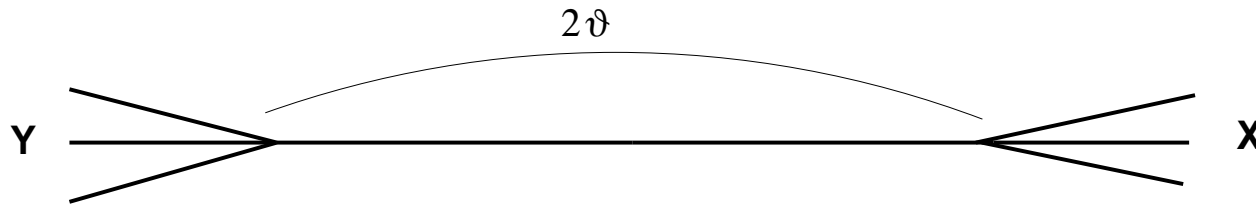


- DIS: $s \gg Q^2 \gg \Lambda_{QCD}^2, -t, m_p^2$
- semihard processes: $x = Q^2/s \ll 1$
- perturbative QCD applicable

Soft and hard diffraction

Two basic features of diffraction:

- rising cross sections with energy s
- large rapidity gaps: $\eta = -\log \tan(\theta/2)$ (color singlet exchange)



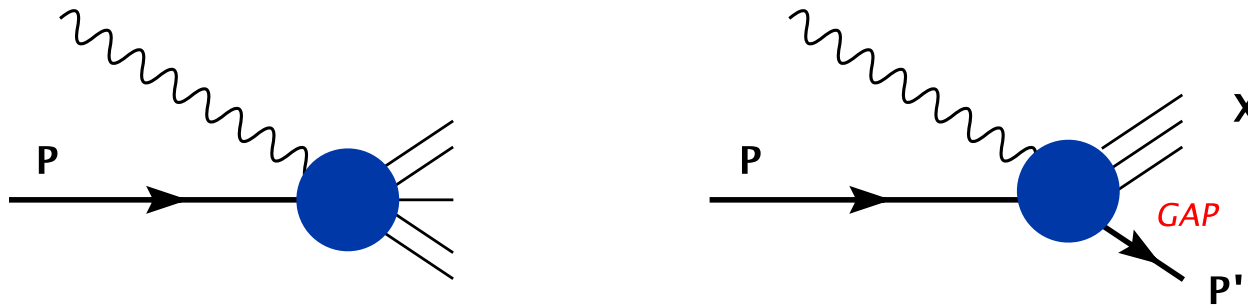
- Soft diffraction (no hard scale): $\sigma \sim s^{\alpha_P(0)}$ $\alpha_P(t) = 1.08 + 0.25 \cdot t$

$$pp \rightarrow p + p \qquad \gamma p \rightarrow V(\rho, \omega, \phi) + p$$

- Hard diffraction (with hard scale): $1.1 < \alpha_P(0) < 1.4$

$$pp \rightarrow jj + pp \qquad \gamma^* p \rightarrow X + p \qquad \gamma^* p \rightarrow V + p \qquad \gamma p \rightarrow J/\psi + p$$

Deep inelastic scattering



- Infinite momentum frame ($P \rightarrow \infty$):
 pointlike γ^* resolves partonic constituents of the proton
- Proton rest frame ($P \approx 0$):
 γ^* develops partonic fluctuations long before the proton target



Coherence length: $l_c = \frac{1}{m_p x} \gg \frac{1}{m_p} \approx 0.2 \text{ fm}$

Dipole states

- Hadron wave function in light-cone quantization approach

$$|\Psi\rangle = \sum \Psi_n |n\rangle \quad |n\rangle = |z_1 \dots z_n; k_{T1} \dots k_{Tn}\rangle$$

$|n\rangle$ are **partonic states** – eigenstates of free Hamiltonian

- **Eigenstates of diffraction** are Fourier transformed partonic states with **transverse positions**

$$|z_1 \dots z_n; k_{T1} \dots k_{Tn}\rangle \rightarrow |z_1 \dots z_n; \vec{r}_1 \dots \vec{r}_n\rangle$$

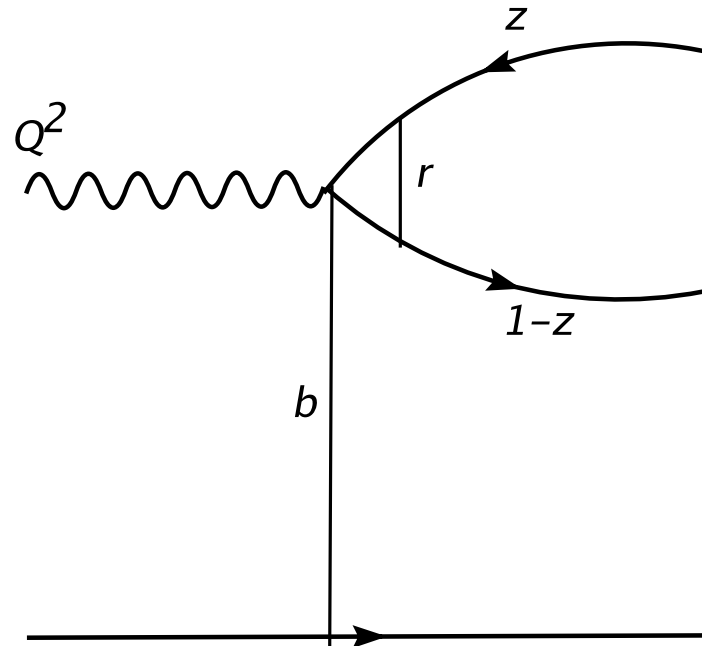
- Virtual photon wave function

$$|\gamma^*\rangle = \sum \Psi_{q\bar{q}} |q\bar{q}\rangle + \sum \Psi_{q\bar{q}g} |q\bar{q}g\rangle + \dots$$

$q\bar{q}$ dipole of size $\vec{r} = \vec{r}_1 - \vec{r}_2$ at impact parameter $\vec{b} = (\vec{r}_1 + \vec{r}_2)/2$.

$q\bar{q}$ dipole

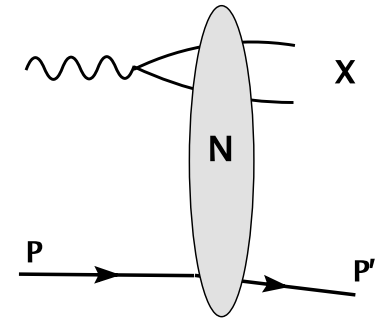
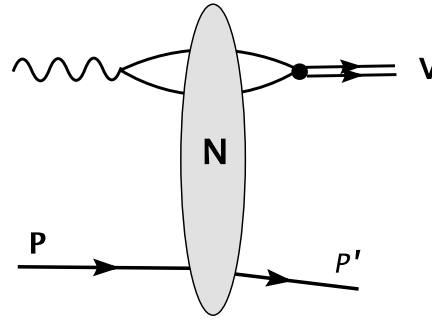
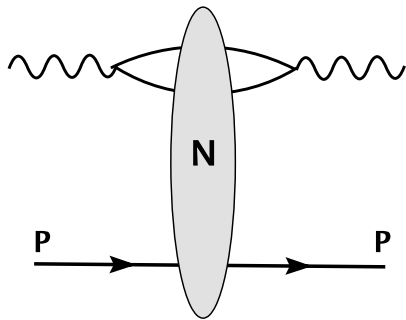
- the lowest component



- light-cone photon wave function: $\Psi_{h,h'}^\lambda(r, z, Q^2)$, known from QED
- dipole states are eigenstates of scattering operator
- \vec{r} and z are conserved in the high energy scattering

Color dipole models

Universal description in the leading order in $\log(s)$ (*N. N. Nikolaev, B. G. Zakharov*)



- Scattering amplitude $(A = \gamma^*, \gamma, V)$

$$\mathcal{A}(\gamma^* + p \rightarrow A + p) = \int d^2 r dz \Psi_A^* N_{q\bar{q}} \Psi_\gamma$$

- $N_{q\bar{q}}(r, b, Y = \ln(1/x))$ is the **dipole** scattering amplitude

- **Find** $N_{q\bar{q}}$ from

$$\sigma_{tot} \sim \text{Im} \mathcal{A}(\gamma^* + p \rightarrow \gamma^* + p)$$

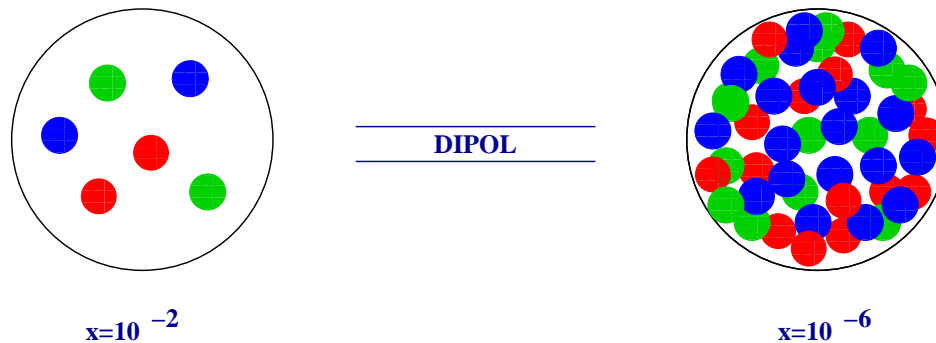
and **test** in DDIS processes.

Phenomenological parameterizations

- GB–Wüsthoff parameterization with saturation scale $Q_s(x) = Q_0 x^{-\lambda}$

$$N_{q\bar{q}}(r, b, x) = \theta(b < b_0) \left(1 - e^{-r^2 Q_s^2(x)} \right)$$

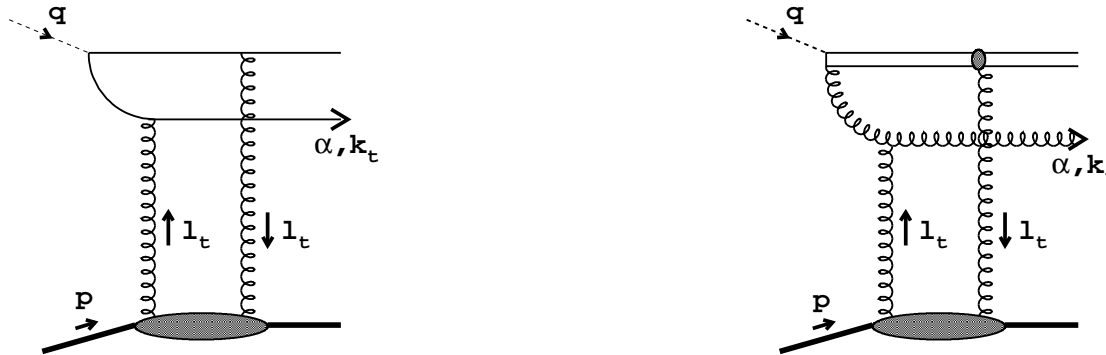
- idea of parton saturation



- unitarity fulfilled: $N_{q\bar{q}}(b) \leq 1$
- geometric scaling: $N_{q\bar{q}}(rQ_s(x), b)$

DIS diffraction and saturation

- Parameters b_0, λ, Q_0 from $\sigma_{tot} \sim F_2$
- Successful description of DDIS with two component diffractive state

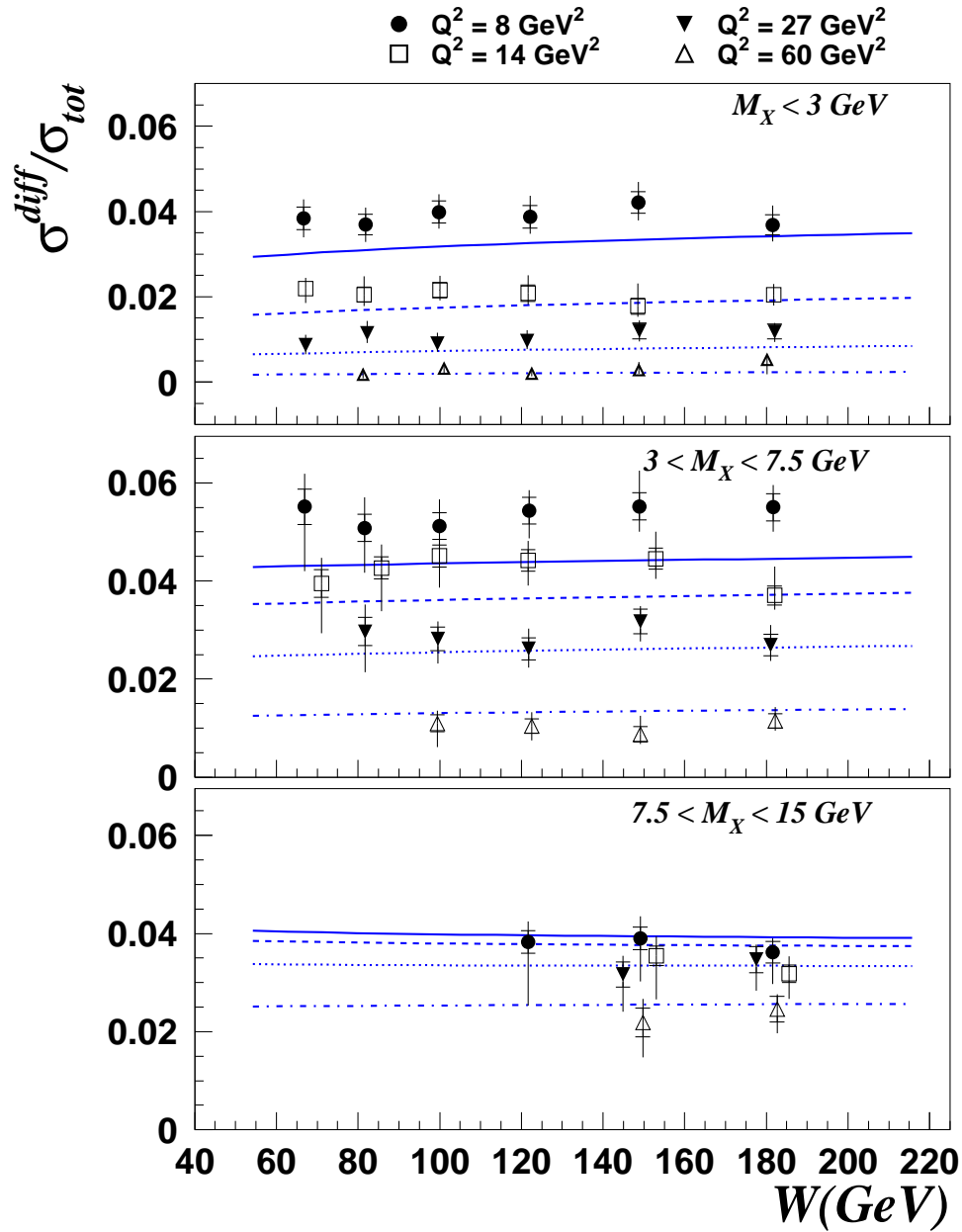


- The only approach which explains **constant ratio** with energy

$$\frac{\sigma_{diff}}{\sigma_{tot}} \sim \frac{1}{\log(Q^2/Q_s^2(x))}$$

DDIS sensitive to **semihard dipoles**: $r \sim 1/Q_s$

Constant ratio



Important improvements

- Small dipole corrections – to match perturbative QCD results
(Frankfurt, Strikman, McDermott, Bartels, GB, Kowalski, Iancu, Itakura, Munier)

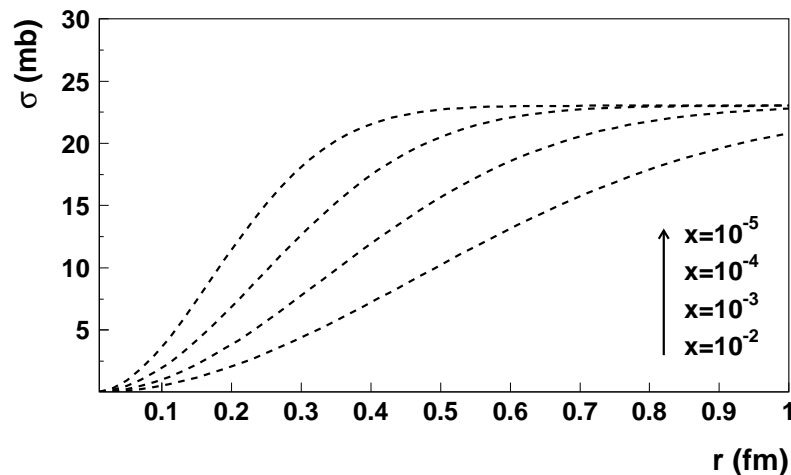
$$N_{q\bar{q}}(r, b, x) \sim r^2 \alpha_s G(x, 1/r^2) T(b)$$

- Realistic b -profile – t -dependence of exclusive diffractive processes
(Kowalski, Teaney, Motyka, Watt)

$$T(b) = \exp(-b^2/2B)$$

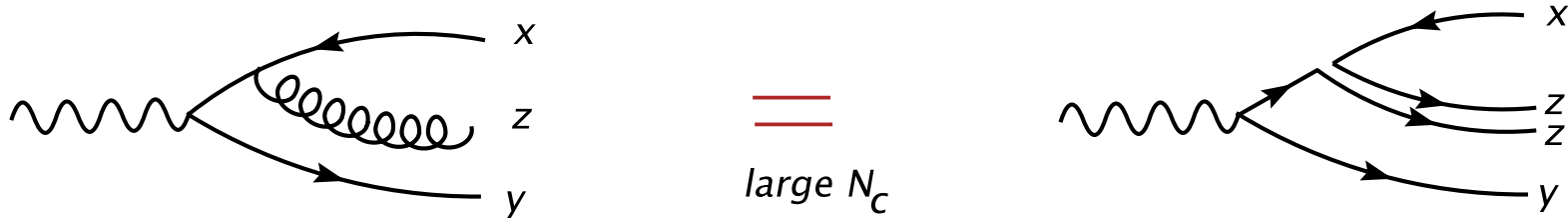
$$Q_s(x, b) = Q_0 x^{-\lambda} T(b)$$

- Theoretical justification?



Dipole evolution

- Soft gluon emissions with $z_g \ll z_q$



- Splitting probability $(xy) \rightarrow (xz) + (zy)$

$$\frac{dP}{dY} = \frac{N_c \alpha_s}{2\pi^2} \frac{(\vec{x} - \vec{y})^2}{(\vec{x} - \vec{z})^2 (\vec{z} - \vec{y})^2} d^2 z \equiv K(x, y, z) d^2 z$$

- Classical branching process (A. H. Mueller)

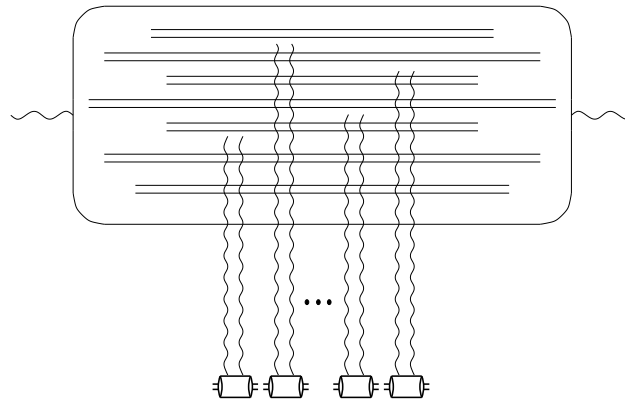
$$\frac{dZ(x, y; u)}{dY} = \int d^2 z K(x, y, z) \{ Z(x, z; u) Z(z, y; u) - Z(x, y; u) \}$$

Multi-dipole distributions: $n_k = \delta^k Z / \delta u^k \big|_{u=0}$ $k = 1, 2, \dots$

Balitsky-Kovchegov evolution equation

- BFKL growth of dipole density: $n_1 \sim e^{4 \ln 2 \bar{\alpha}_s Y}$. **Unitarity violated?**
- **No**, if **simultaneous, uncorrelated** scattering of several dipoles

$$N_{q\bar{q}} = (-\gamma) n_1 + (-\gamma)^2 n_2 + (-\gamma)^3 n_3 + \dots$$



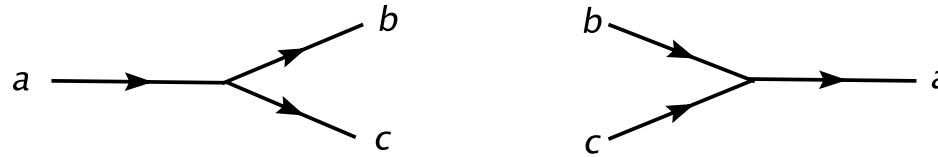
- Nonlinear equation justified in case of $\gamma^* A$ scattering ($S = 1 - N_{q\bar{q}}$)

$$\frac{\partial S(x, y)}{\partial Y} = \int d^2 z K(x, y, z) \{S(x, z) S(z, y) - S(x, y)\}$$

- **Local** unitarity: $S(b) \leq 1$.
- **Global** unitarity: Froissart bound violated.

Parton saturation

- No saturation in dipole number density in BK equation.
- Missing dipole merging: $2 \rightarrow 1$ (Pomeron loops)



- Necessary for symmetric description in pp scattering.

Summary of studies:

- Lot of efforts in Color Glass Condensate approach.
- Negative transition probabilities prevent probabilistic formulation in terms of color dipoles only.
- Formation of color quadrupoles and higher multipoles.
- More intuitive approach ?

Dipole swing

(E. Avsar, G. Gustafson, L. Lönnblad)

- Scattering of two dipoles with the same color – change of color flow



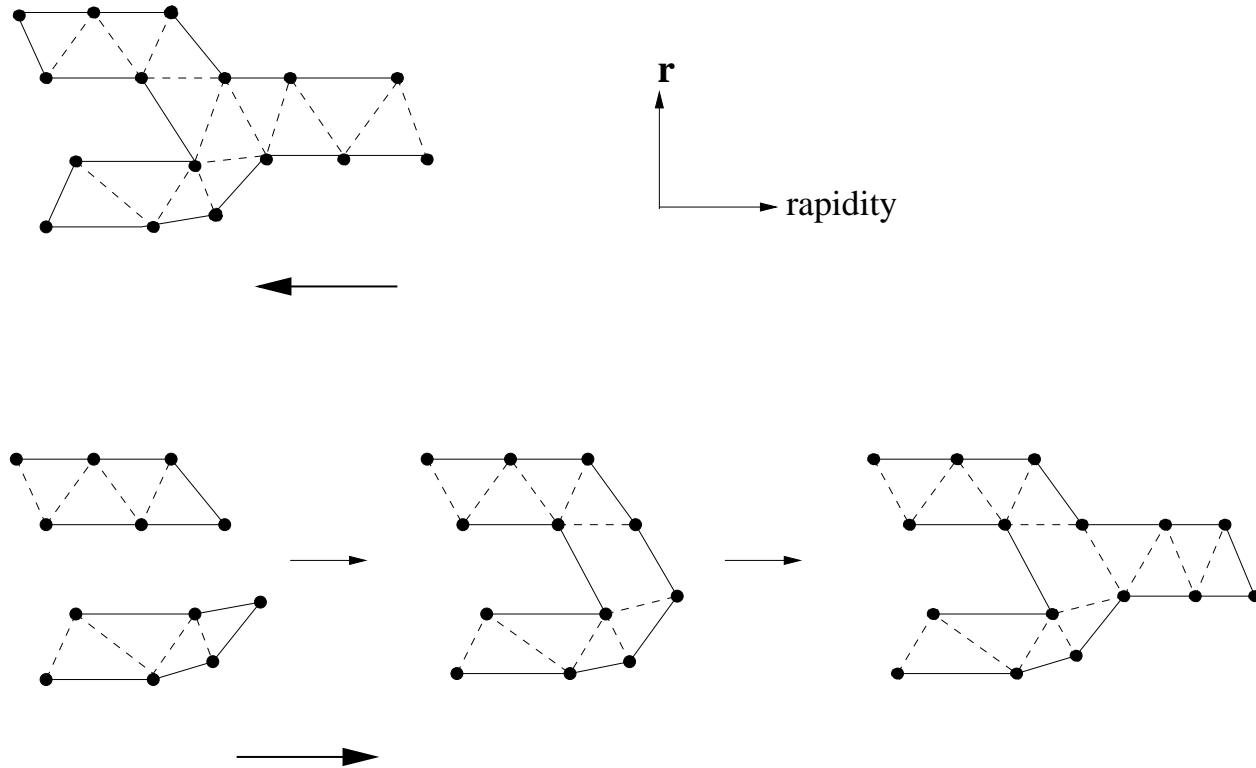
- Weight prefers the formation of smaller dipoles.

$$\frac{(x_1 - y_1)^2 (x_2 - y_2)^2}{(x_1 - y_2)^2 (x_2 - y_1)^2}$$

- Smaller dipoles have smaller cross sections.
- Suppression of the total cross section due to dipole swing.

Dipole chain scattering

● Dipole chain splittings and mergings



● Monte Carlo implementation.

Diffraction in pp scattering

- Good-Walker picture with **dipole states** as eigenstates of diffraction

$$|\psi_I\rangle = |L, R\rangle = \sum_{n,m} c_n^L c_m^R |n, m\rangle$$

- The scattered state

$$|\psi_S\rangle = \text{Im T} |\psi_I\rangle = \sum_{n,m} c_n^L c_m^R t_{nm} |n, m\rangle$$

- Diffractive cross section (measure of fluctuations)

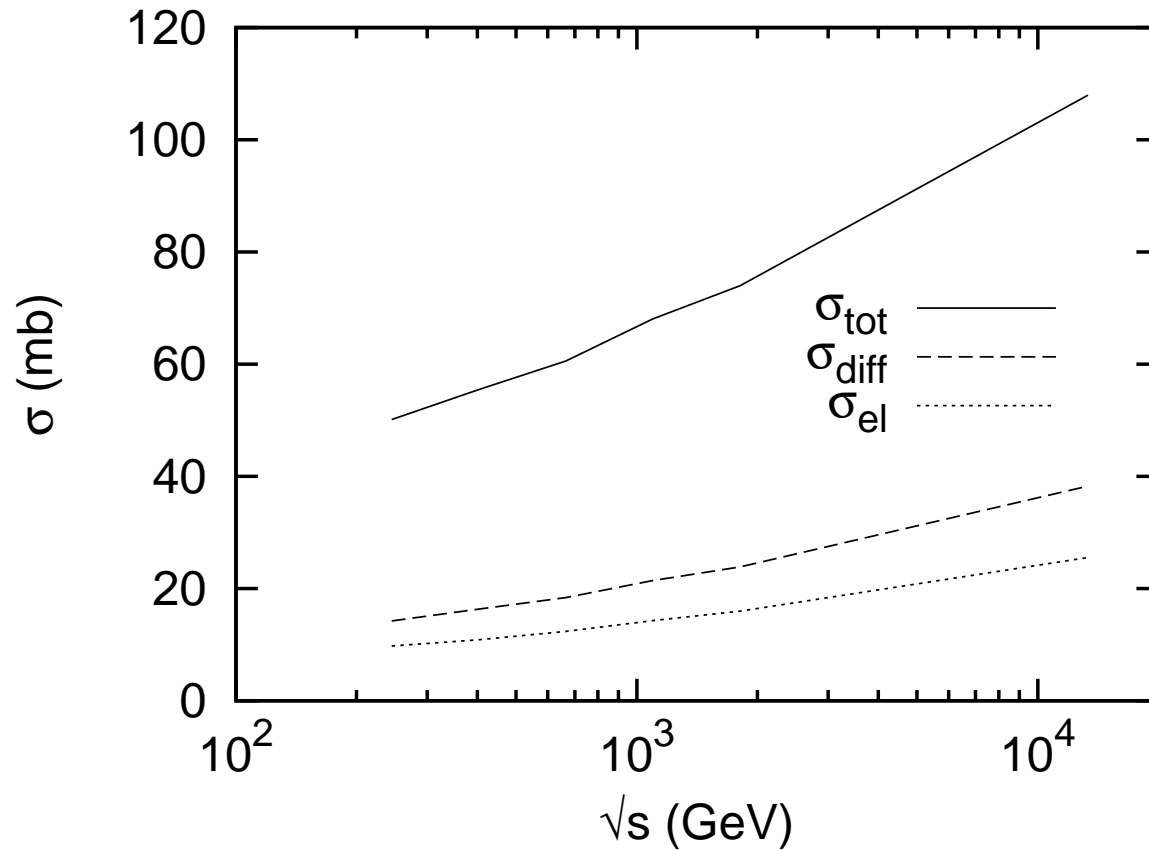
$$\frac{d\sigma_{diff}}{d^2b} = \langle \psi_S | \psi_S \rangle = \sum_{n,m} P_n^L P_m^R t_{nm}^2$$

- Eikonal form of the scattering amplitudes

$$t_{nm} = 1 - e^{-F_{nm}} \quad F_{nm} = \sum_{i \in n, j \in m} F_{ij}$$

Predictions for the LHC

(E. Avsar, G. Gustafson, L. Lönnblad, 0709.1368 [hep-ph])



● $\sigma_{tot} = 108$ mb $\sigma_{el} = 26$ mb $\sigma_{diff} = 12$ mb

Conclusions

- Saturation models are very successful in describing diffractive data in ep scattering.
- Dipole approach with saturation can be applied to diffractive pp scattering.
- Let's listen and discuss.