

Hard rescattering corrections to exclusive Higgs boson production at LHC

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Based on analysis done with J. Bartels, S. Bondarenko and K. Kutak

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Overview

Motivation

The Khoze-Martin-Ryskin approach

Hard rescattering corrections

Gloun extrapolation and Results

Discussion and Outlook

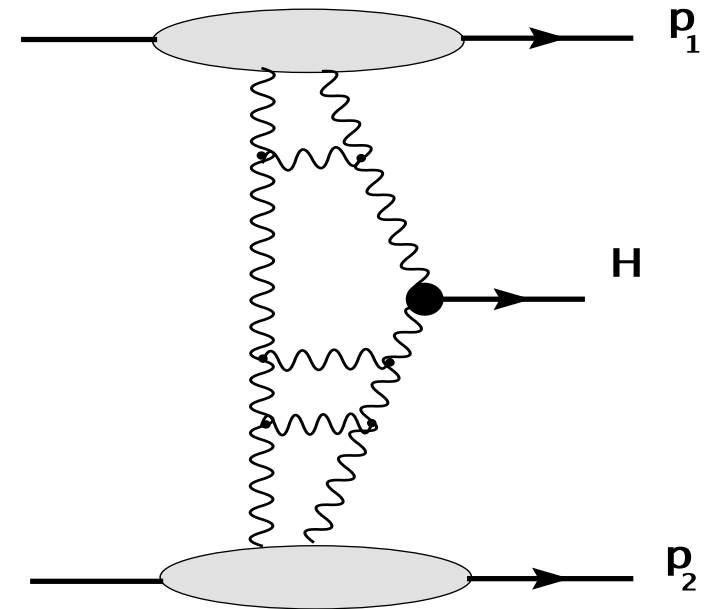
Exclusive Higgs boson production

[O. Nachtmann et al. ; A. Białas and P. Landshoff early 1990s]

→ The exclusive Higgs boson production in pp : $pp \rightarrow pHp$

→ The protons stay intact and are detected by forward detectors

→ Precise determination of kinematics and mass of the produced system, $\Delta M \sim 1$ GeV and of the resonance width



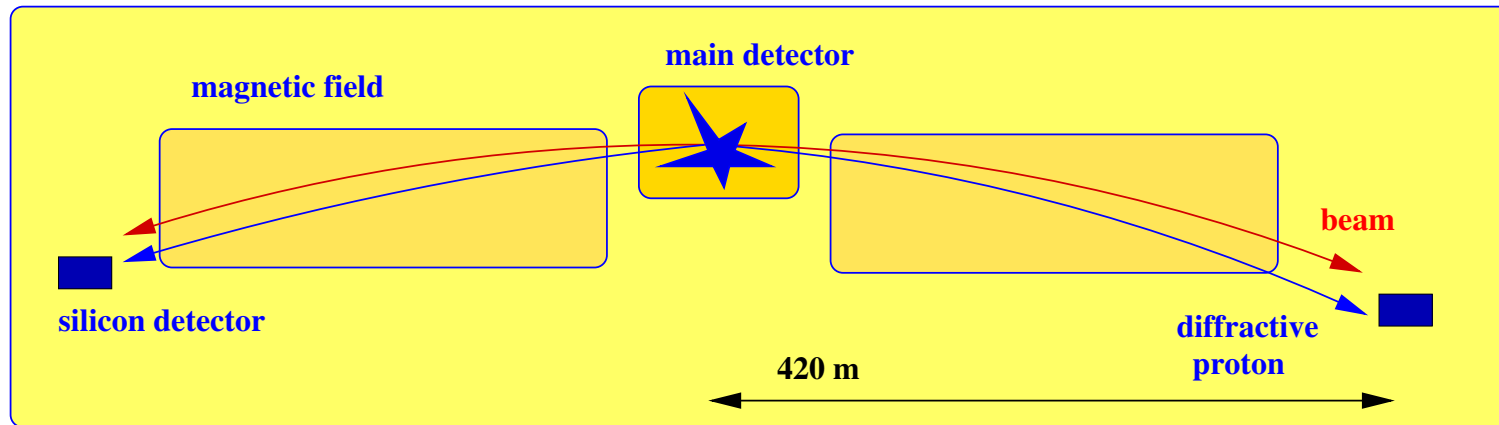
→ Possibility to investigate quantum numbers of the produced state e.g. by observing angular correlations of the protons filtering the scalar from the pseudoscalar

→ Potential to resolve almost degenerate resonances (eg. in *tri-mixing CPX scenario*)

→ Numerous papers by V. Khoze, A. Martin, M. Ryskin, A. Kaidalov, J. Stirling (KMR), by R. Peschanski, C. Royon, et. al. , and B. Cox, J. Forshaw, M. Tasevsky,...

Experimental conditions

FP-420



Forward Proton measurement – 420 meters downstream

Magnetic spectrometer: small loss of energy \longrightarrow drift outside the beam

Silicon trackers – millimeters from the beam

Measurement of the mass may reach $O(1 \text{ GeV})$ accuracy

Tagging of protons \longrightarrow glue factory

Standard and SUSY Higgs at LHC

[V. Khoze, A. Martin, M. Ryskin]

About **10 Higgs boson events** in $b\bar{b}$ or W^+W^- channels with $S/B \simeq 1$
for mass between **120 GeV** and **140 GeV** with $\mathcal{L} = 30 \text{ fb}^{-1}$ at LHC

Exclusive production is particularly useful in some SUSY scenarios (intense coupling)

Sensitivity to the difficult $b\bar{b}$ channel — 0^{++} selection rule

$$M_A = 130 \text{ GeV}, \quad \tan \beta = 50, \quad \mathcal{L} = 30 \text{ fb}^{-1}$$

	Signal	Background
$M_h = 124.4 \text{ GeV}$	71	3
$M_H = 135.5 \text{ GeV}$	124	2
$M_A = 130 \text{ GeV}$	1	2

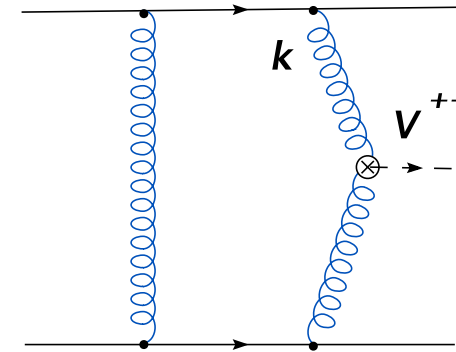
Detailed study of $(M_A - \tan \beta)$ plane — almost full coverage at 3σ level

[S. Heinemeyer et. al. , [hep-ph:0708.3052](#)] — M. Tasevsky's talk

Theoretical description

Consider the lowest order diagram $qq \rightarrow qHq$
 The colour flow requires the exchange of two gluons

High energy kinematics imposes eikonal couplings γ^+
 and γ^- and this leads to $k^2 \sim \mathbf{k}^2$



The Higgs production vertex can be obtained in the effective theory, $m_t \rightarrow \infty$:

$$V^{\mu\nu}(k_1, k_2) = V_0[k_1^\nu k_2^\mu - g^{\mu\nu} k_1 \cdot k_2]$$

Convolution with eikonal couplings: $V^{+-} \sim V_0 \mathbf{k}_1 \cdot \mathbf{k}_2$

In the forward direction:

$$M \sim \int_{\mu_0} \frac{d^2 k k^2}{k^6} \Phi_q(\mathbf{k}) \Phi_q(-\mathbf{k})$$

The lowest order result is strongly infra-red sensitive $\sim 1/\mu_0^2$

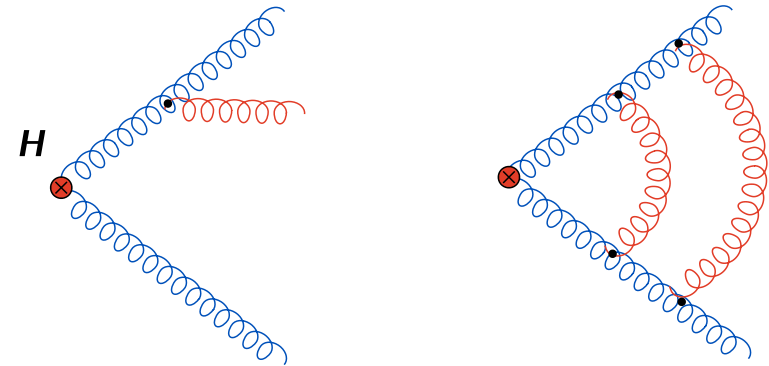
Sudakov form-factor

Beyond two-gluon exchange:

- 1) Higher order QCD corrections should be included
- 2) The gluon radiation is forbidden from the system

The gHg vertex defines the invariant mass of the gg system

Screening of radiated gluons is efficient only if $q < k$



Probability of the radiation of a single gluon

$$P_1 = \int_{k^2}^{M_H^2} \frac{dq^2}{q^2} \frac{C_A \alpha_s(q^2)}{\pi} \int_q^{M_H} \frac{d\omega}{\omega} \simeq \frac{C_A \alpha_s}{4\pi} \log^2(M_H^2/k^2)$$

Classical calculation of the Sudakov form-factor:

$$P(\text{no radiation}) \sim \exp(-P_1)$$

Equivalently – the Sudakov form factor follows from resummation of QCD virtual QCD corrections

$$S(k, \mu) = \exp\left(- \int_{k^2}^{\mu^2} \frac{dq^2}{q^2} \frac{N_c \alpha_s}{\pi} \int_q^{\mu} \frac{d\omega}{\omega}\right)$$

More on Sudakov form-factor

At single logarithmic accuracy:

$$T_g(\mathbf{k}, \mu) = \exp \left(- \int_{k^2}^{\mu^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^{1-q/\mu} dz z \left[P_{gg}(z) + \sum_q P_{qg}(z) \right] \right), \quad \mu \simeq M_H/2$$

Saddle point analysis:

$$\int \frac{d^2k}{k^4} k^{4\gamma} \exp \left[\frac{-N_c \alpha_s}{4\pi} \log^2 \left(\mu^2 / k^2 \right) \right]$$

$$k_s^2 = \mu^2 \exp \left[\frac{-2(1 - \gamma)}{\bar{\alpha}_s} \right]$$

Typical virtuality $k^2 \sim 2 \text{ GeV}^2 \longrightarrow$ Perturbative treatment makes sense

Two Pomeron Fusion amplitude

Amplitudes to find gluon pair in the proton:

→ two-scale off-diagonal unintegrated gluon distributions are introduced:

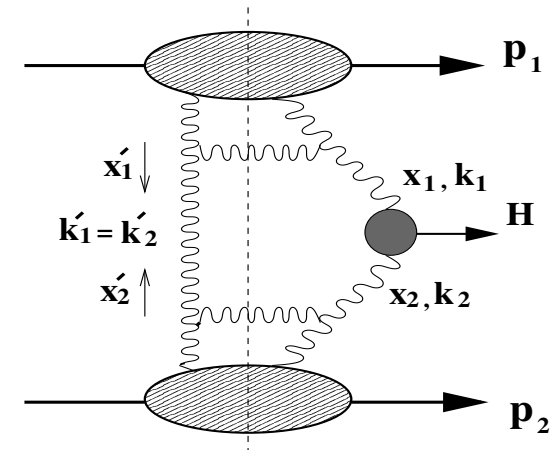
$$f_g(x, x', k, \mu), \quad xg(x, Q^2) = \int^{Q^2} \frac{dk^2}{k^2} f_g(x, k^2, Q)$$

Sudakov form factor is naturally incorporated in f_g : [Kimber, Martin, Ryskin]

$$f_g(x, k^2; \mu) = Q^2 \frac{\partial}{\partial Q^2} \left[xg(x, Q^2) \cdot T_g(Q, \mu) \right]_{Q^2=k^2}$$

$$f_g^{\text{off}}(x, k^2; \mu) = R_\xi Q^2 \frac{\partial}{\partial Q^2} \left[xg(x, Q^2) \cdot \sqrt{T_g(Q, \mu)} \right]_{Q^2=k^2}$$

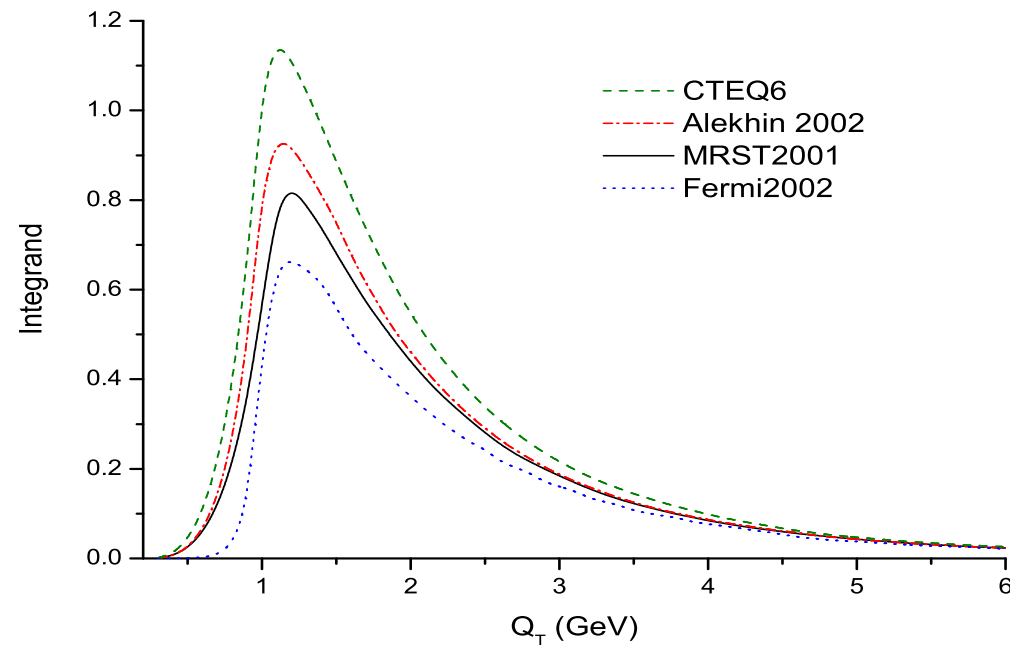
$$\text{Im } M_0(y) \sim \int \frac{dk^2}{k^4} f_g^{\text{off}}(x_1, k^2; \mu) f_g^{\text{off}}(x_2, k^2; \mu)$$



Behaviour of the integrand:

$$\text{Im } M_0(y) \sim \int \frac{k dk}{k^4} f_g^{\text{off}}(x_1, k^2; \mu) f_g^{\text{off}}(x_2, k^2; \mu)$$

[J. Forshaw]



The integrand is dominated by momenta $Q_T \sim 1 - 2 \text{ GeV}$

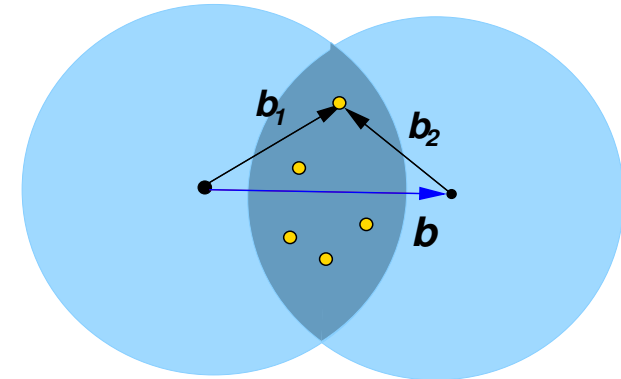
Sudakov form factor reduces the exclusive cross section by two orders of magnitude

Soft Rescattering

The exclusive character of the process may be spoiled by soft rescattering. In the standard approach **one assumes that rescattering does not affect the hard matrix element describing the production**

Simplest assumption: uncorrelated, independent acts of soft rescattering characterised by the amplitude

$$M_1(b) = \Omega(b)/2$$



Unitarised scattering amplitude:

$$\Omega(b)/2 - \frac{(\Omega(b)/2)^2}{2!} + \frac{(\Omega(b)/2)^3}{3!} + \dots = 1 - \exp(-\Omega(b)/2)$$

The total cross section: $\sigma_{tot} = 2 \int d^2b [1 - \exp(-\Omega(b)/2)]$

The elastic cross section $\sigma_{el} = \int d^2b [1 - \exp(-\Omega(b)/2)]^2$

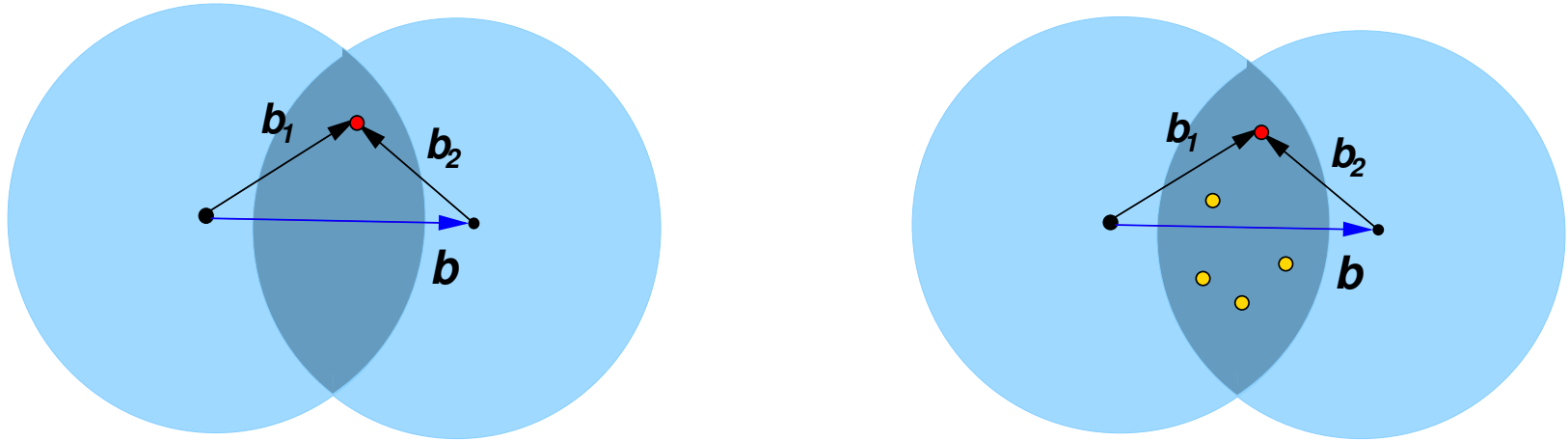
The inelastic cross section $\sigma_{inel} = \int d^2b [1 - \exp(-\Omega(b))]$

Soft gap survival

Soft rescattering corrections to a hard exclusive scattering process \longrightarrow opacity $\Omega(b)$

Independence of hard production and rescattering is assumed

$$M_{corr}(b) = M_{hard}(b) [1 - \Omega(b)/2 + (\Omega(b)/2)^2/2! - (\Omega(b)/2)^3/3! + \dots] = M_0(b) \exp(-\Omega(b)/2)$$



Amplitude of matter distribution in the proton

$$S(b_1) \sim \exp(-b_1^2/R^2), \quad R^2 \sim 8 \text{ GeV}^{-2}$$

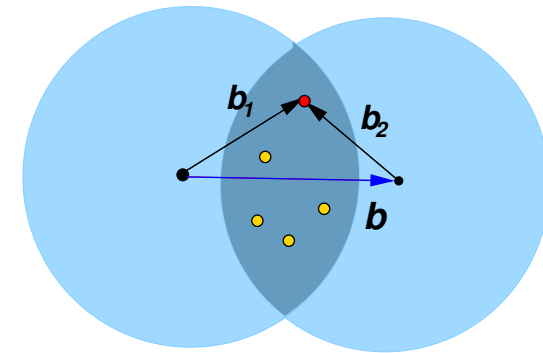
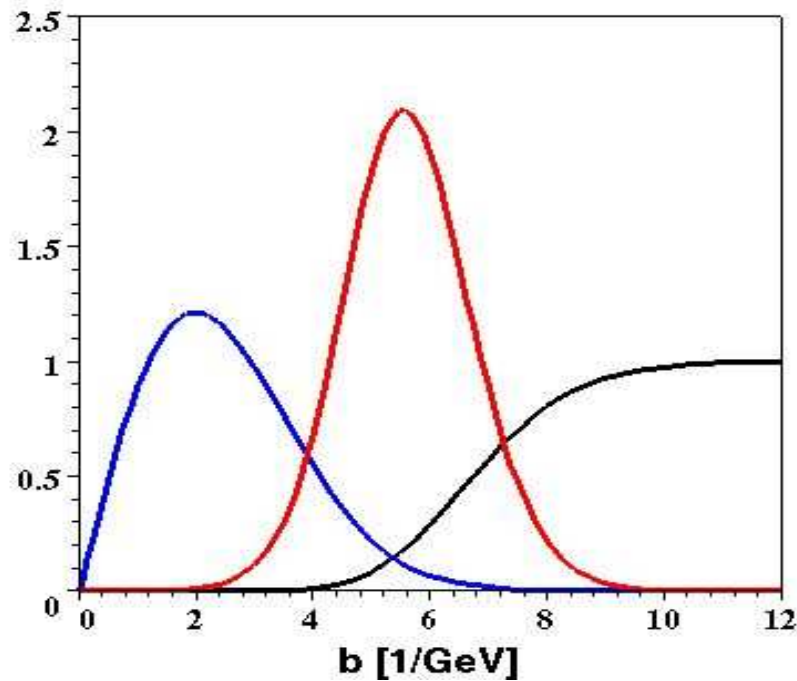
$$M_{hard}(b) \sim M_0 \int d^2b_1 S(\mathbf{b}_1) S(\mathbf{b}_1 - \mathbf{b})$$

$$\sigma_{excl} = \int d^2b \int d^2b_1 |M_0 S(\mathbf{b}_1) S(\mathbf{b}_1 - \mathbf{b})|^2 \exp(-\Omega(b))$$

Impact parameter profile of exclusive process

Gap survival factor:
$$S^2 = \frac{\int b db \exp(-\Omega(b)) |M_{\text{hard}}(b)|^2}{\int b db |M_{\text{hard}}(b)|^2}$$

Exclusive Production = **Hard matrix element** \times Amplitude of no rescattering
 Production profile (red) for LHC is magnified by factor of 100



Production dominated by $b \simeq 1.2$ fm and $b_1 \simeq 0.6$ fm

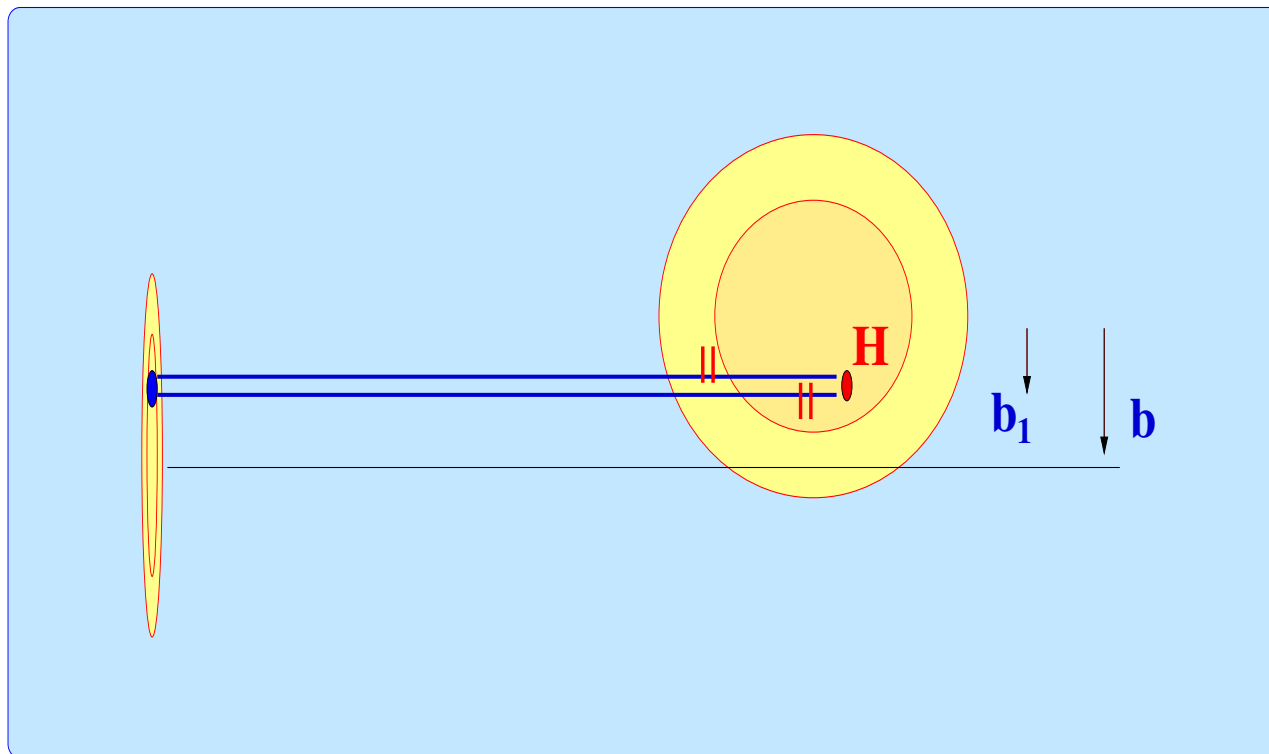
Two-channel eikonal model of gap survival is used that incorporates low-mass diffractive intermediate states. Typically: $S^2 = 0.02 - 0.03$ for exclusive processes at the LHC

Hard rescattering correction – the main idea

Exclusive $pp \rightarrow pHp$ \longrightarrow Higgs mass gives hard scale \longrightarrow Sudakov form-factor
 \longrightarrow fusion of two hard ladders into Higgs

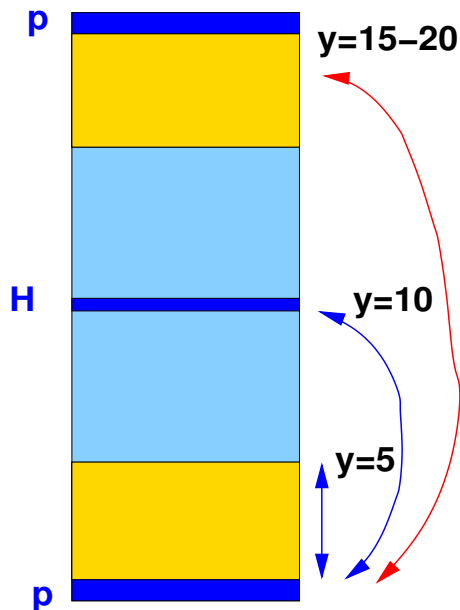
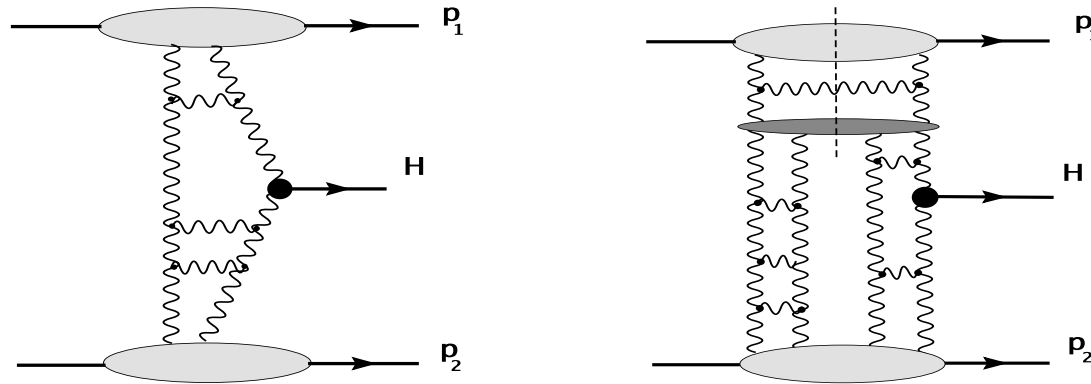
Multiple soft rescattering of the spectator partons (fields) assumed to be independent of the hard production mechanism

Are hard ladders are small enough to see the protons as colour transparent?



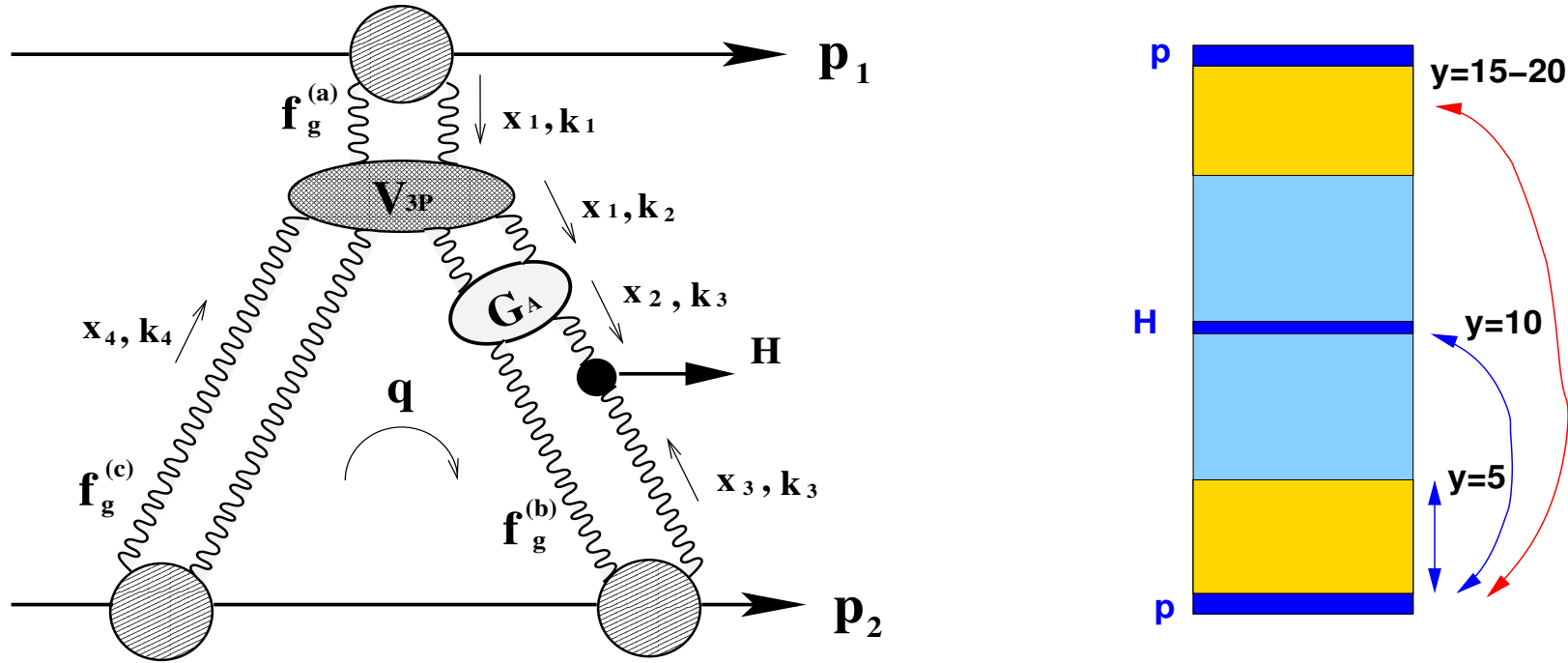
Key issues: hard ladder size, coupling, transverse profiles of matter and impact parameter dependence of scattering \longrightarrow relation of “ladder size” to saturation radius

Kinematics of exclusive production and rescattering



Large rapidity distance $y = 15 - 20$ characterizing the rescattering of the hard ladder \rightarrow long low- x evolution and enhancement

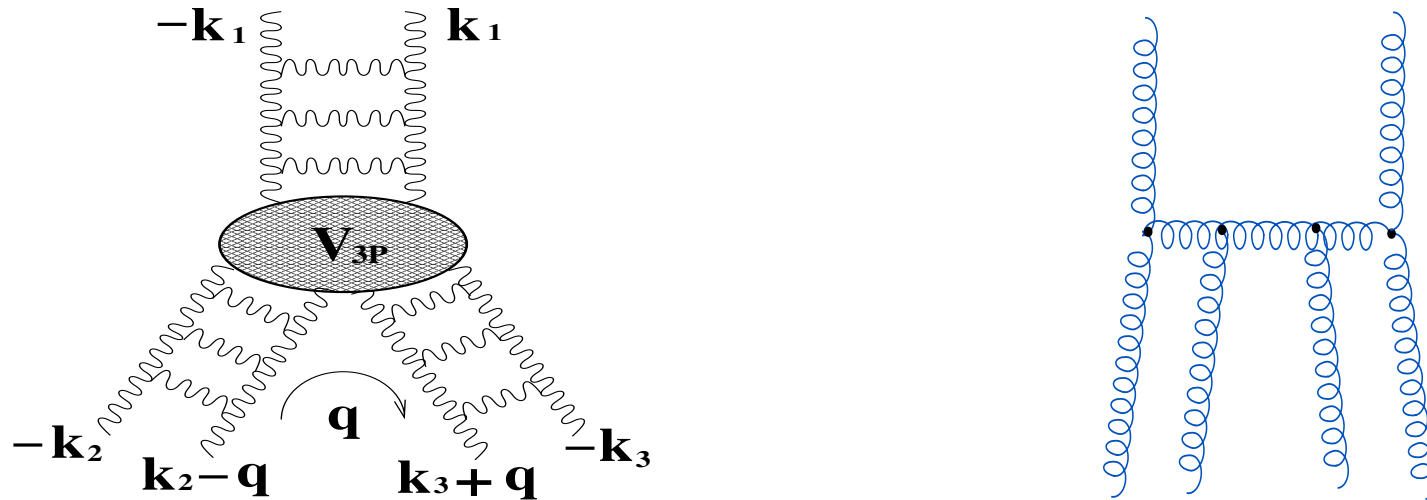
Evaluation of hard rescattering correction



$$\begin{aligned} \text{Im } M_{\text{corr}}^{(1)}(y) &= -\frac{9}{8} 16\pi^2 (2\pi^3 A) \int_{x_a}^{x_b} \frac{dx_4}{x_4} \int \frac{d^2 q}{(2\pi)^2} \int \frac{d^2 k_2}{2\pi k_2^4} \int \frac{d^2 k_3}{2\pi k_3^4} \int \frac{d^2 k_4}{2\pi k_4^4} \\ &\times [V_{3P} \otimes f_g^{(a)}(x_1)](\mathbf{k}_2, \mathbf{k}_4) G_A(\mathbf{k}_2, \mathbf{k}_3; x_1, x_2; M_H/2) \\ &\times f_g^{(b)}(x_3, \mathbf{k}_3, \mathbf{q} - \mathbf{k}_3; M_H/2) f_g^{(c)}(x_4, \mathbf{k}_4, -\mathbf{q} - \mathbf{k}_4) \end{aligned}$$

Triple Pomeron Vertex

[J. Bartels]

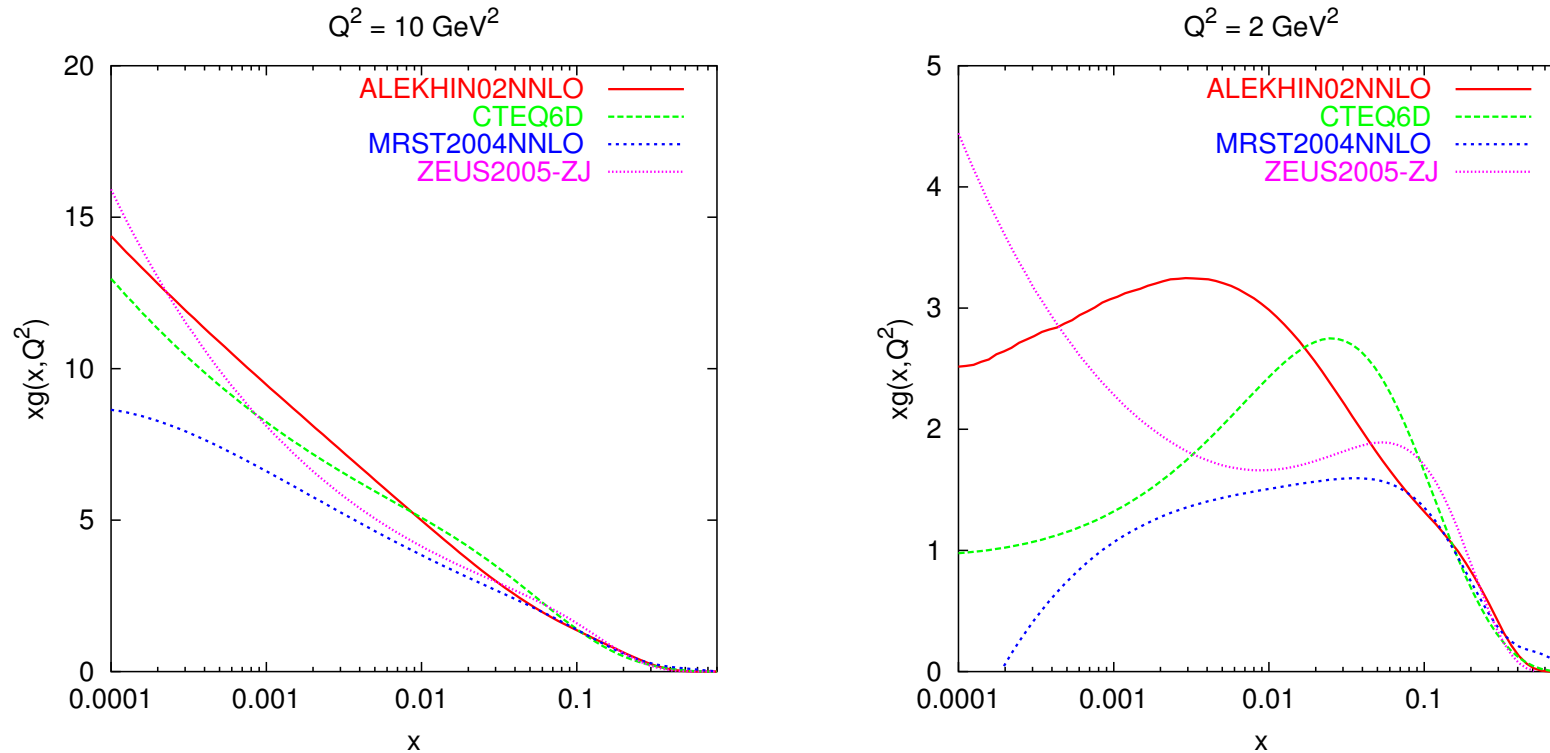


$$\begin{aligned}
 [V_{3P} \otimes D](\mathbf{k}_2, \mathbf{k}_3) = & \\
 \alpha_s^2 c_{3P} \int \frac{d^2 \mathbf{k}_1}{2\pi k_1^2} \left\{ \left(\frac{k_2^2}{(\mathbf{k}_1 - \mathbf{k}_2)^2} + \frac{k_3^2}{(\mathbf{k}_1 + \mathbf{k}_3)^2} - \frac{(\mathbf{k}_2 + \mathbf{k}_3)^2 k_1^2}{(\mathbf{k}_1 - \mathbf{k}_2)^2 (\mathbf{k}_1 + \mathbf{k}_3)^2} \right) D(k_1) \right. & \\
 - \frac{k_1^2}{(\mathbf{k}_1 - \mathbf{k}_2)^2} \left(\frac{k_2^2}{(\mathbf{k}_1 - \mathbf{k}_2)^2 + k_1^2} - \frac{(\mathbf{k}_2 + \mathbf{k}_3)^2}{(\mathbf{k}_1 - \mathbf{k}_2)^2 + (\mathbf{k}_1 + \mathbf{k}_3)^2} \right) D(k_2) & \\
 \left. - \frac{k_1^2}{(\mathbf{k}_1 + \mathbf{k}_3)^2} \left(\frac{k_3^2}{(\mathbf{k}_1 + \mathbf{k}_3)^2 + k_1^2} - \frac{(\mathbf{k}_2 + \mathbf{k}_3)^2}{(\mathbf{k}_1 - \mathbf{k}_2)^2 + (\mathbf{k}_1 + \mathbf{k}_3)^2} \right) D(k_3) \right\} &
 \end{aligned}$$

Problems with DGLAP extrapolations

Overall description of F_2 is satisfactory down to $x = 10^{-5}$ and $Q^2 \sim 1 \text{ GeV}^2$

Much worse picture if one looks inside the parametrisations. For the gluon:



Uncertainties are huge, gluon density may even get negative Consequence of small x logarithms

Higher twist effects

b -dependent Balitsky-Kovchegov equation

Resummation of BFKL pomeron fan diagrams in the LL approximation with rescattering corrections

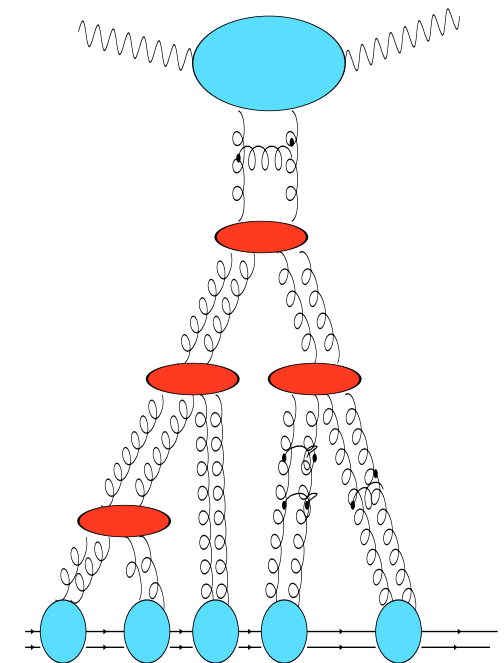
$$\frac{\partial \tilde{f}_g(x, k^2, b)}{\partial \log 1/x} = \frac{\alpha_s N_c}{\pi} k^2 \int_{k_0^2} \frac{dk'^2}{k'^2} \left\{ \frac{\tilde{f}_g(x, k'^2, b) - \tilde{f}_g(x, k^2, b)}{|k'^2 - k^2|} + \frac{\tilde{f}_g(x, k^2, b)}{[4k'^4 + k^4]^{\frac{1}{2}}} \right\} - \pi \alpha_s^2 \left(1 - k^2 \frac{d}{dk^2} \right)^2 k^2 \left[\int_{k^2}^{\infty} \frac{dk'^2}{k'^4} \log \left(\frac{k'^2}{k^2} \right) \tilde{f}_g(x, k'^2, b) \right]^2$$

Improvements in the linear part: **[Kwieciński, Martin, Staśto, Kutak, LM]**

- collinearly improved NLL corrections to the BFKL kernel
- running coupling constant
- non-singular part of the DGLAP splitting function P_{gg} included
- the equation coupled to DGLAP evolution of quarks

Special features:

- Gluon recombination/ rescattering effects
- Impact parameter dependence in the local approximation



Unintegrated gluon — x -dependence

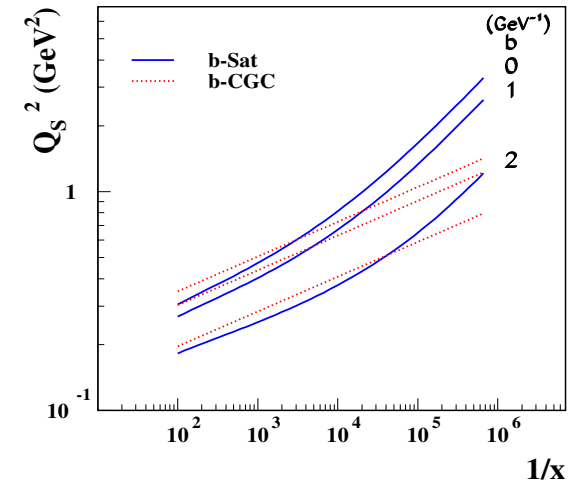
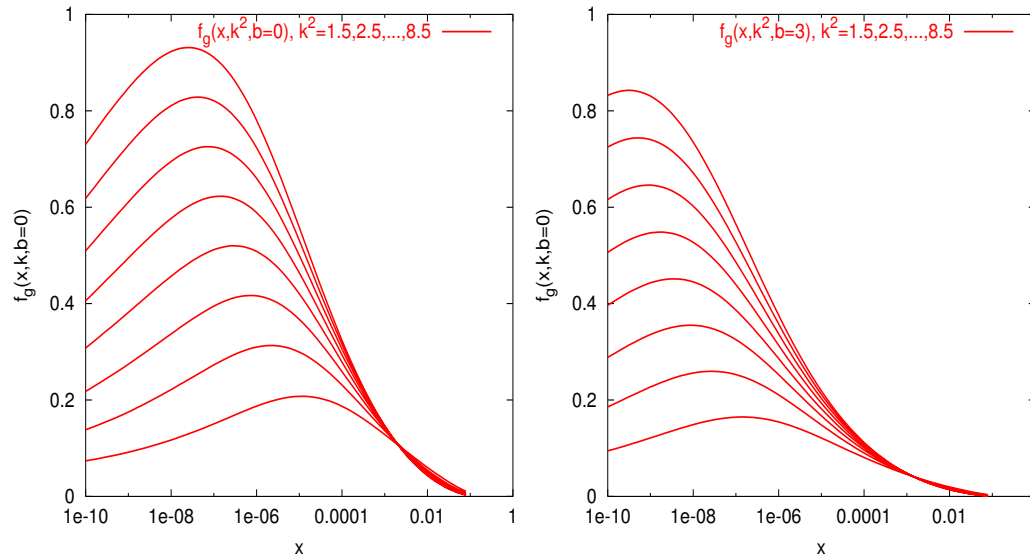
$\tilde{f}_g(x, k^2, b)$ plotted as a function of x for $k^2 = 1.5, 2.5, \dots, .5 \text{ GeV}^2$ and for

$b = 0$

and

$b = 0.6 \text{ fm}$

consistent with fits to HERA data



- The maxima signal that the saturation scale $Q_s(x)$ reached the momentum k
- Decreasing distribution for $k^2 < Q_s^2(x)$
- Saturation scale grows with decreasing x
- Saturation scale decreases with growing b

Infra-red behaviour of the correction

$$Q_R \sim \int_{k_0^2} \frac{dk_1^2}{k_1^2} \int_{k_1^2} \frac{dk_2^2}{k_2^4} \int_{k_1^2} \frac{dk_4^2}{k_4^4} f_g^{(a)}(x_2, k_1^2) f_g^{(b)}(x_3, k_2^2) f_g^{(c)}(x_4, k_4^2) T_s(k_2, M_H/2)$$

k_0 is a perturbative scale that cuts off the integrals

Assume for simplicity:

$$f_g^{(a)}(x_2, k_1^2) \sim \text{const}(k_1), \quad f_g^{(b)}(x_3, k_2^2) \sim \text{const}(k_2), \quad f_g^{(c)}(x_4, k_4^2) \sim \text{const}(k_4)$$

Integral over k_2 is stabilised by the Sudakov form-factor but

$$Q_R \sim \int_{k_0^2} \frac{dk_1^2}{k_1^2} \int_{k_1^2} \frac{dk_4^2}{k_4^4} \sim \frac{1}{k_0^2}$$

Sensitivity to the details of the proton structure at low momenta

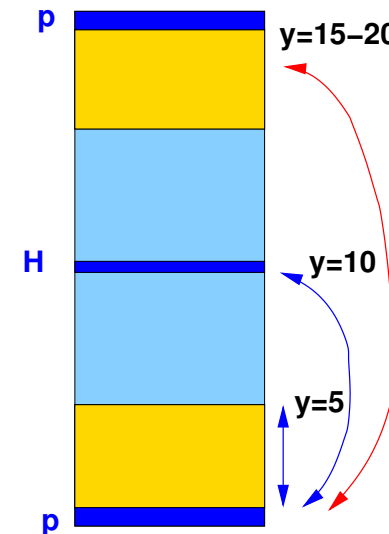
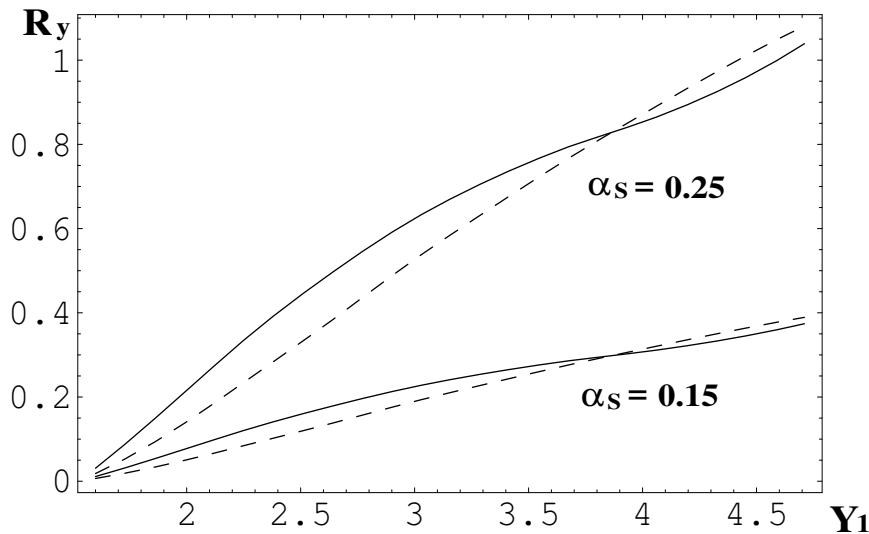
Nonlinear low- x evolution generates perturbative saturation scale $Q_s(x) > k_0$

$$f_g^{(c)}(x_4, k_4^2) \sim [k_4^2/Q_s^2(x_4)]^{1+\delta} \quad \int_{Q_s^2(x_4)} \frac{dk_4^2}{k_4^4} \int_{k_0^2}^{k_4^2} \frac{dk_1^2}{k_1^2} \sim \frac{\log(Q_s^2(x_4)/k_0^2)}{Q_s^2(x_4)}$$

Rescattering correction – results

Y_1 \longrightarrow rapidity distance of the Triple Pomeron Vertex from the *projectile proton* at $b = 0$ and $b_1 = 0$. $M_H = 120$ GeV, the LHC energy

$$R_y(y_1) = \left[\frac{|d\tilde{M}_{\text{corr}}(y, \mathbf{b}, \mathbf{b}_1)/dy_1|}{|\tilde{M}_0(y, \mathbf{b}, \mathbf{b}_1)|} \right]_{b=b_1=0, y=0}$$



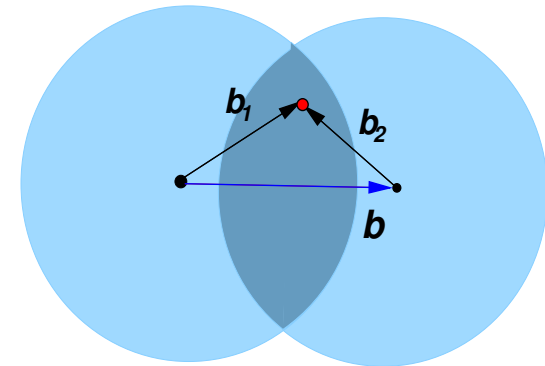
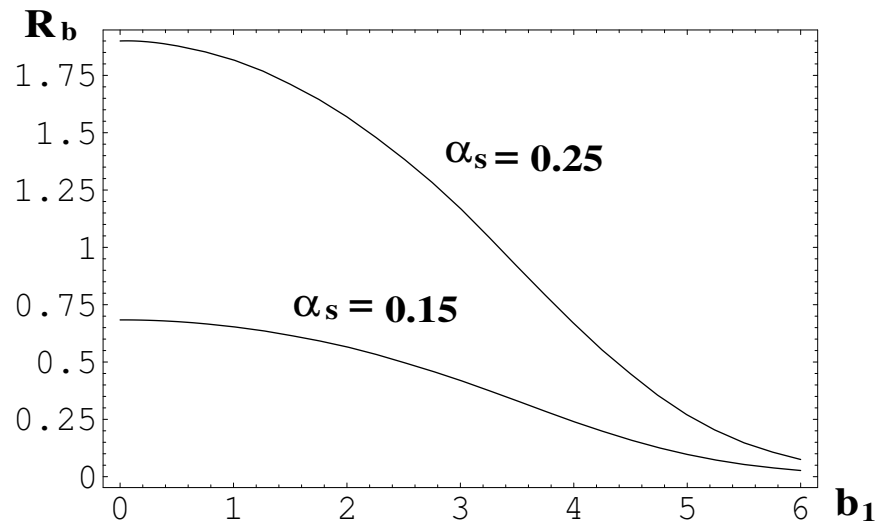
BFKL Green's function (continuous) and the two gluon exchange (dashed)

- \longrightarrow Screening is strongest for rapidities close to the production vertex
- \longrightarrow Multiple scattering of the large mass intermediate state
- \longrightarrow The correction is large

Rescattering correction – results

Relative correction as a function of the distance of the production vertex from the centre of the target proton – at zero impact parameter of the collision

$$R_b(b_1) = \left[\frac{|\tilde{M}_{\text{corr}}(y, b, b_1)|}{|\tilde{M}_0(y, b, b_1)|} \right]_{y=0, b=0}$$



→ The rescattering correction amplitude is more concentrated close to the centre of proton than the production amplitude

→ The correction should experience stronger suppression by the soft rescattering than the production amplitude

Effect of hard rescattering on the cross section

$\left(\frac{d\sigma_{pp \rightarrow pHp}}{dy}\right)_{y=0}$ (in fb) with hard and soft rescattering corrections included
 — for $M_H = 120$ GeV at the LHC

TPF	$\alpha_s = 0.15$	$\alpha_s = 0.2$	$\alpha_s = 0.25$
0.4	0.12	0.042	0.14

Hard rescattering correction \longrightarrow soft gap survival probability, \hat{S}^2 , is modified

TPF)	$\alpha_s = 0.15$	$\hat{S}^2 (0.2)$	$\hat{S}^2 (0.25)$
0.024	0.037	0.051	0.016

\longrightarrow Correction amplitude equals to leading term at $\alpha_s = 0.2$

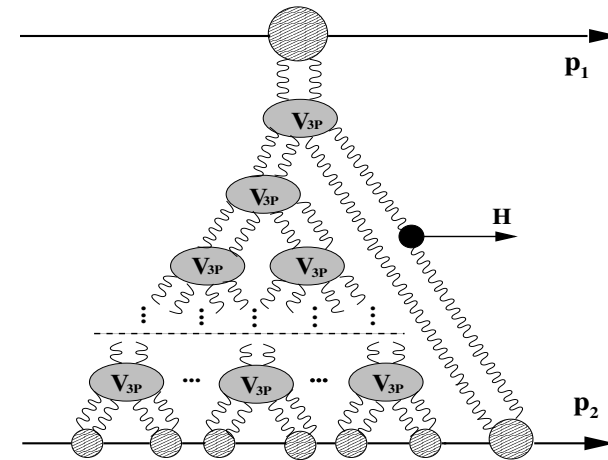
\longrightarrow Important effect, need for higher order contributions

Discussion of hard rescattering

The relative magnitude of the correction is large and the sign is negative

Factorisation between hard production amplitude and rescattering is strongly broken

- The magnitude of the higher order unitarity corrections is expected to be large as well
- Theoretical uncertainty of $\sigma_{excl}(pp \rightarrow pHp)$ is higher than expected
- Suppression or enhancement?
- Tests of the framework needed



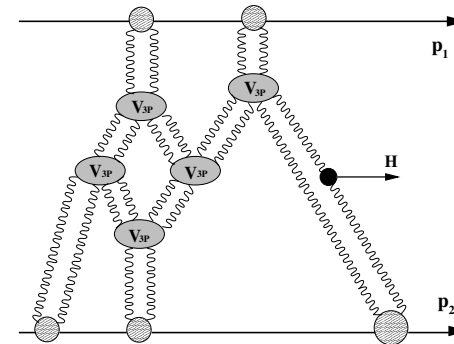
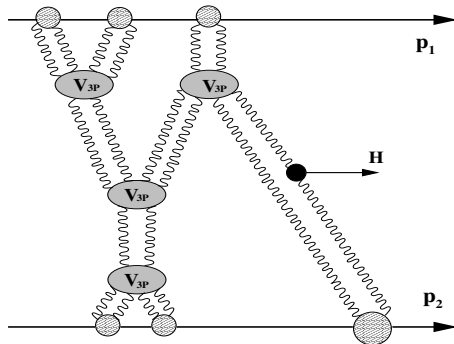
Key ingredients:

- Large rapidity available for the screening pomeron $Y \sim 15 - 20$
- Perturbative momenta and large mass of the rescattering state
- Partial resummation of unitarity corrections

Outlook

→ Symmetric and unified QCD treatment of rescattering is necessary to understand high density QCD at the LHC. Initially this task seemed to be *prohibitively difficult*. Fortunately, it was not end of the story

→ A candidate formalism was proposed based on effective QCD Pomeron interacting by triple pomeron vertices [M.A. Braun; S. Bondarenko, LM]. All order resummation of QCD pomeron trees, also with Higgs emission (work in progress)



→ Large rescattering correction to CED Higgs boson production in dipole-dipole collision was found in a QCD computation [J.S. Miller]

→ Analysis of CED within Pomeron Field Theory in zero transverse dimensions is a solvable problem. Important step was made recently to construct a phenomenological implementation of a model belonging to this class [KMR, A. Martin's talk]. Large bare intercepts $\Delta \sim 0.5$ and *sizeable bare Tripe Pomeron Vertex*

→ Still, we need QCD description of multiple scattering — and Central Exclusive Diffraction (esp. dijets) may be used to probe unitarity corrections in pp at small x

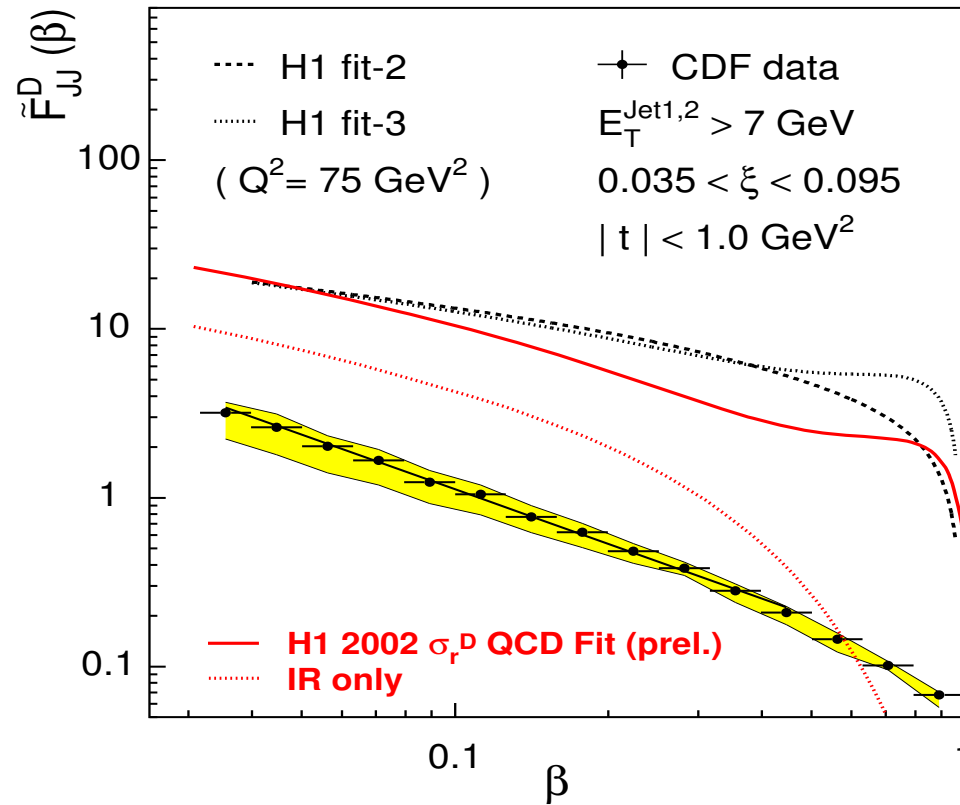
Conclusions

- The exclusive Higgs boson production may be possible at the LHC
- The standard QCD picture: hard matrix element \times gap survival factor coming from soft rescattering
- The hard rescattering correction to the exclusive Higgs boson production was evaluated and found to be large and clearly separated from soft rescattering
- Factorisation of the hard production process from the soft rescattering was found to be broken
- Theoretical uncertainty of the cross section for exclusive Higgs production was broadened
- Resummation of higher order unitarity corrections is necessary

BACKUP

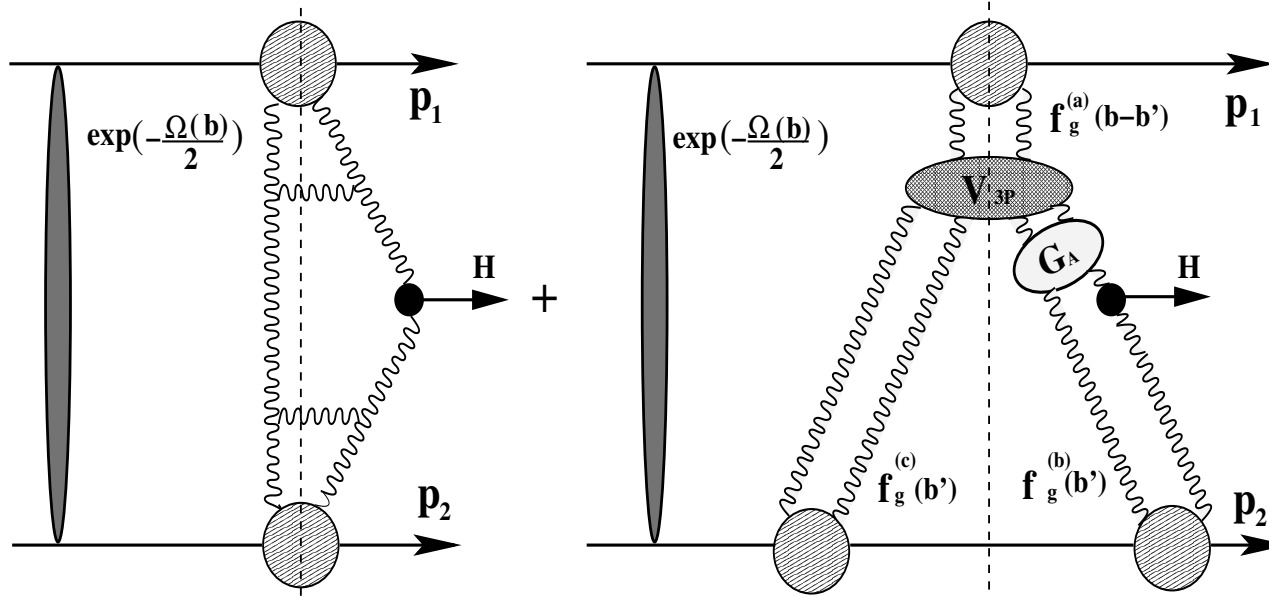
Example – single diffraction at the Tevatron

$$p\bar{p} \rightarrow p + \text{gap} + jj$$



$$S^2 \simeq 0.1$$

Exclusive production in position space

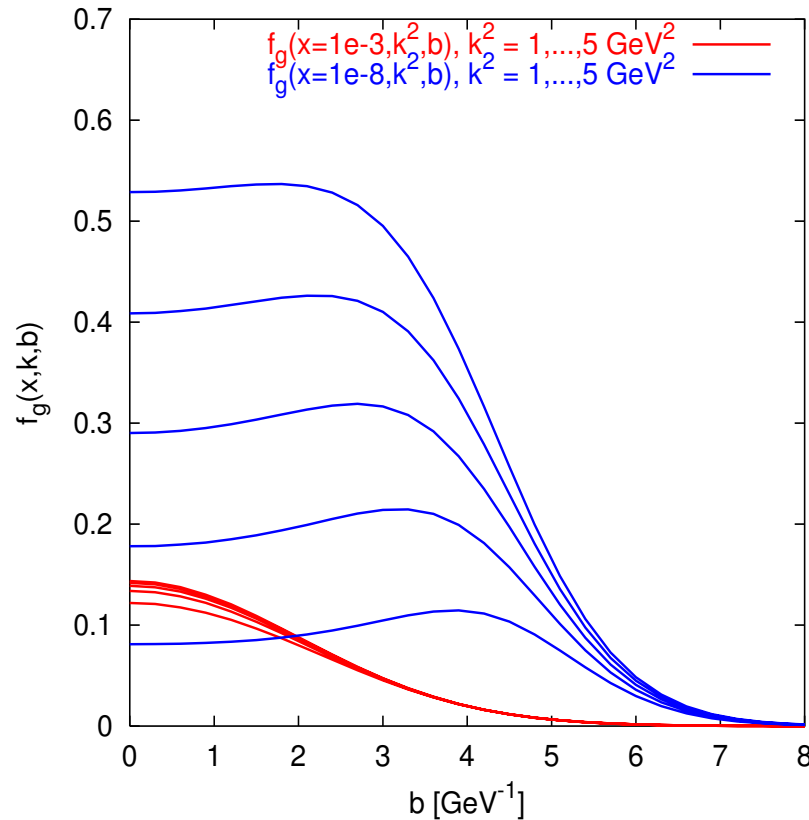


$$\begin{aligned} \text{Im } \tilde{M}_{\text{corr}}(y, \mathbf{b}, \mathbf{b}_1) = & - \left\{ \frac{9}{8} 16\pi^2 (2\pi^3 A) \int_{x_a}^{x_b} \frac{dx_4}{x_4} \int \frac{d^2 k_2}{2\pi k_2^4} \int \frac{d^2 k_3}{2\pi k_3^4} \int \frac{d^2 k_4}{2\pi k_4^4} \right. \\ & \times [V_{3P} \otimes \tilde{f}_g^{(a)}(x_1, \mathbf{b} - \mathbf{b}_1)](\mathbf{k}_2, \mathbf{k}_4) G_A(\mathbf{k}_2, \mathbf{k}_3; x_1, x_2; M_H/2) \\ & \left. \times \tilde{f}_g^{(b)}(x_3, k_3^2, \mathbf{b}_1; M_H/2) \tilde{f}_g^{(c)}(x_4, k_4^2, \mathbf{b}_1) \right\} - \{y \rightarrow -y\} \end{aligned}$$

$$\frac{d\sigma_{pp \rightarrow pHp}^{(0+1),\Omega}(y)}{dy} = \frac{1}{16\pi} \int d^2 b \int d^2 b_1 |S(\mathbf{b}_1)S(\mathbf{b} - \mathbf{b}_1)M_0(y) + M_{\text{corr}}(y, \mathbf{b}, \mathbf{b}_1)|^2 \exp(-\Omega(s, \mathbf{b}))$$

Unintegrated gluon — b -dependence

$\tilde{f}_g(x, k^2, b)$ plotted as a function of b for $k^2 = 1, 2, \dots, 5 \text{ GeV}^2$
and for $x = 10^{-3}$ and $x = 10^{-8}$



$$f_g(x, k^2) = \int d^2b f_g(x, k^2, b)$$

$$xg(x, Q^2) = \int^{Q^2} \frac{dk^2}{k^2} f_g(x, k^2)$$

- Broadening and box-like shape of the proton at lower momenta and very low x
- At very low x – gluon is a non-monotonical function of the distance from the centre