

# Neutrino-Nucleus QE Scattering

*GTG*

*Los Alamos Nat. Lab.*

# Aim of this Talk

**I hope to convince you that :**

- Present day neutrino event generators for  $0.2 > E_\nu > \sim 2$  GeV are inadequate
- These generators use 40 year old nuclear physics, produce wrong cross sections, assign incorrect neutrino energies, with a possible serious impact on the determination of neutrino oscillation parameters.
- Nucleon – Nucleon interactions are ignored. Mean field (eg. Fermi Gas) momentum distributions for nucleons in a nucleus are seriously wrong.
- For  $A \geq 12$  20% of the nucleons are involved in short range correlations (SRC). These SRC typically generate nucleon momenta much greater than the Fermi momentum.
- Meson exchange + current conservation,  $\nabla \cdot \vec{j} = \frac{\partial \rho}{\partial t}$  gives rise to two body nucleonic weak currents that enhance the transverse vector cross section. The evidence for this has been around for 20 years but for the most part ignored.
- The physics to improve the CCQE sector in event generators is in hand.

# Why is QES Important?

Experiments investigating neutrino oscillations employ QES(CCQE) neutrino-nucleus interactions.

For  $0.3 < E_\nu < 3.0$  GeV it is the dominant interaction.

CCQE is assumed to be readily calculable, experimentally identifiable, allowing assignment of the neutrino energy.

*Some 40 calculations published since 2005*

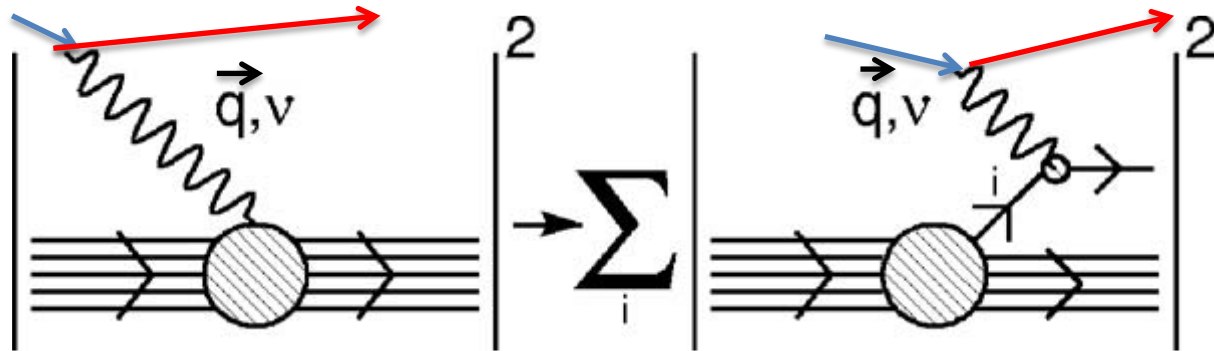
Relevant neutrino oscillation period:

$$1.27 \Delta m_{ij}^2 (eV^2) \times (L_\nu(km) / E_\nu(GeV))$$

$$\Delta m_{22}^2 = 10^{-3} L(10^3) / E(1) \quad \text{LBNE}$$

# Quasi-elastic Scattering on Nuclei

In the Impulse Approximation, CCQE is just the charge changing scattering off independent single nucleons incoherently summed over all nucleons in the nucleus.



If the nucleon is assumed to be at rest, the neutrino energy inferred from the muon energy and angle is:

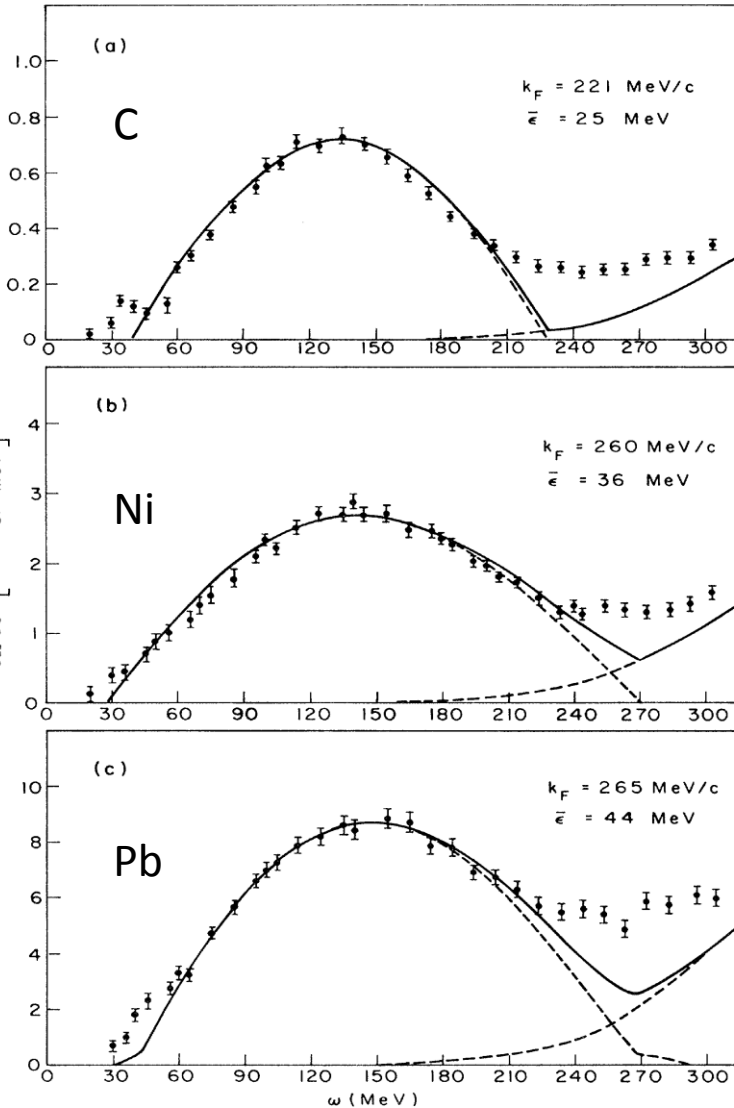
$$E_{n_m}^{QE} = \frac{1}{2} \frac{2(m + S)E_\mu - (2mS + S^2 + m_\mu^2 + Dm_{n,p}^2)}{m + B - E_\mu + \sqrt{E_\mu^2 - m_\mu^2} \cos \varphi}$$

$m$ =nucleon mass,  $E_\mu$ =detected muon energy,  $m_\mu$ =mass of the muon,  
 $S$ = average separation energy

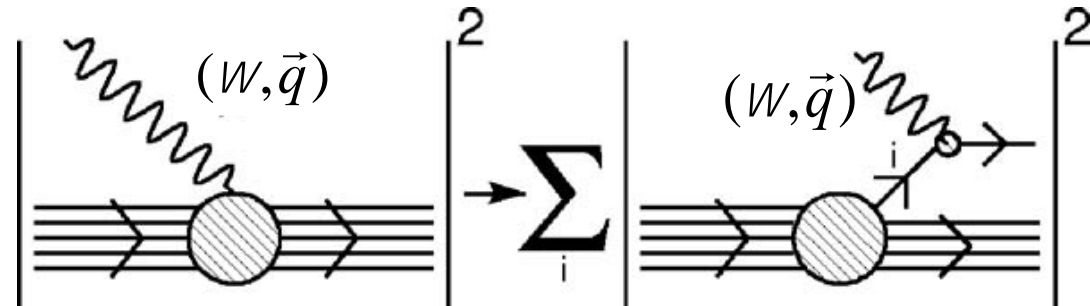
This inferred neutrino energy is uncertain by  $\sim \pm \frac{|\vec{p}_F| |\vec{q}|}{m}$

# Quasi-Elastic Scattering in Nuclear Physics originated with Electron-Nucleus Scattering

Moniz et al PRL 1971



Impulse Approximation



Simple Fermi Gas: 2 parameter,  $S_E$ ,  $p_F$

$$(E, \vec{p}) - (E', \vec{p}') = (W, \vec{q}), \quad Q^2 = \vec{q}^2 - W^2$$

$$W = \frac{Q^2}{2m} + S_E$$

$$DW = \frac{(\vec{q} + \vec{p}_F)^2}{2m}$$

$$V_N = 4\pi/3(1.2)^3 A 10^{-39} \text{cm}^3 \quad p_F = 250 \text{MeV}/c$$

# electron-Nucleus QES

Quasi-Elastic Electron Scattering:

$$\frac{dS^2}{dW_e dW} = \left( \frac{dS}{dW_e} \right)_{Mott} \left\{ \left( \frac{Q}{|\vec{q}|} \right)^4 R_L(|\vec{q}|, W) + \left( \frac{1}{2} \left( \frac{Q}{|\vec{q}|} \right)^2 + \tan^2 \frac{q}{2} \right) R_T(|\vec{q}|, W) \right\}$$

$$(dS/dW_e)_{Mott} = a^2 \cos^2(q/2) / E \sin^4(q/2)$$

$$t = \frac{Q^2}{4m^2}$$

$$\langle \vec{p}', N | J_m | \vec{p}, N \rangle = \frac{i}{W} \langle \bar{u}_N(\vec{p}' = \vec{p} + \vec{q}) | [F_1^N(q^2) g_m + F_2^N(q^2) S_{mn} q_n] | u_N(\vec{p}) \rangle$$

$$F_i^{t_3}(q^2) = \frac{1}{2} (F_i^S(q^2) + t_3 F_i^V(q^2)) \quad 2mF_2^S(0) = m'_p + m_n = -0.120$$

$$F_1^S(0) = 1 \quad F_1^V(0) = 1 \quad 2mF_2^V(0) = m'_p - m_n = +3.706$$

Free Nucleon Cross Section,  $W = \frac{Q^2}{2m}$

$$\frac{dS}{dW_e} = S_{Mott} \frac{E'}{E_0} \left[ \frac{G_E^{N,2}(q^2) + t G_M^{N,2}(q^2)}{1+t} + 2t G_M^2(q^2) \tan^2 \left( \frac{q}{2} \right) \right]$$

$$G_E(q^2) = F_1(q^2) + t F_2(q^2) \quad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Heart of the  
nuclear  
problem

# Scaling in Electron Quasi-elastic Scattering (1)

The energy given up by the electron, to a nucleon with initial momentum  $\vec{k}$

$$E_{e,i} - E_{e,f} = W = T_N + E_s + E_R$$

$T_N$  is the final kinetic energy of the struck nucleon,  $E_s$  the separation energy of the struck nucleon,  $E_R$  the recoil kinetic energy of the nucleus.  $\vec{q}$  is the 3 momentum transferred to the nucleon by the scattered electron.

$$W = [(\vec{k} + \vec{q})^2 + m^2]^{\frac{1}{2}} - m + E_s + E_{recoil} \quad E_{recoil} = \frac{k^2}{2(A-1)m}$$

$$= [k_{\parallel}^2 + 2k_{\parallel}q + q^2 + k_{\perp}^2 + m^2]^{\frac{1}{2}} - m + E_s + E_{recoil} \quad q \rightarrow \infty$$

*neglecting  $E_s$ ,  $E_{recoil}$ , and  $k_{\perp}$  relative to  $2k_{\parallel}q$*

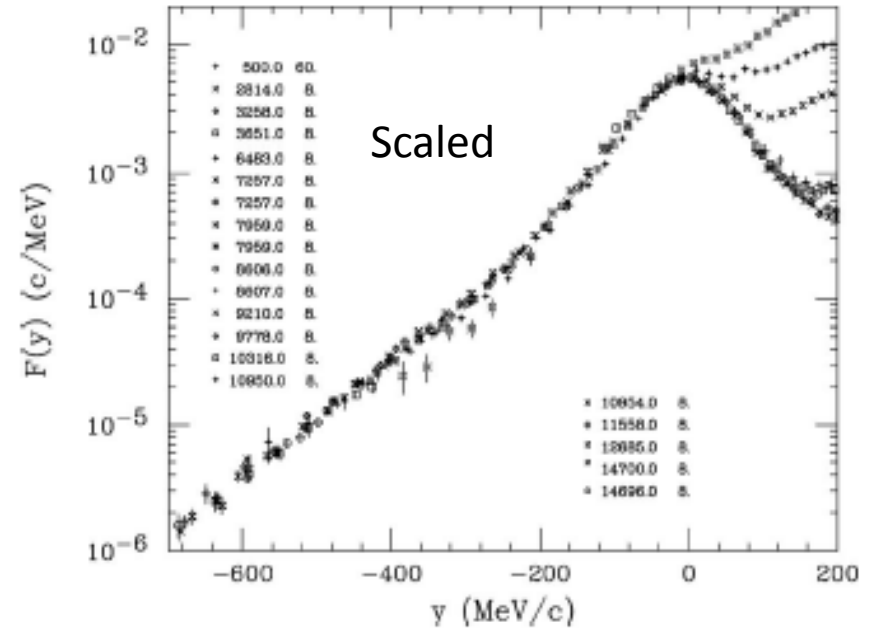
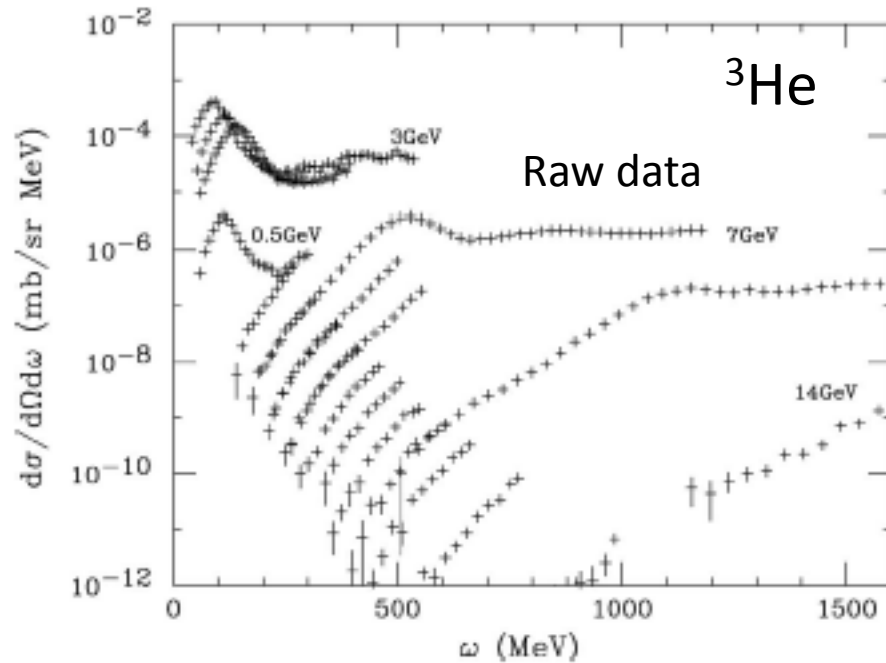
$$k_{\parallel} = \sqrt{W^2 + 2mW} - q \quad y$$

Instead of presenting the data as a function of  $q$  and  $\omega$ , it can be expressed in terms of the single variable  $y$

$$F(y, q) = \left( \frac{d^2 S}{d\Omega dW} \right)_{EXP} \left( \frac{1}{Z S_{ep}(q) + N S_{en}(q)} \right) \frac{dW}{dy}$$

The scaling function  $F(y, q)$  is formed from the measured cross section at 3-momentum transfer  $q$ , dividing out the incoherent single nucleon contributions at that three momentum transfer.

# Scaling in Electron Quasi-elastic Scattering (2)



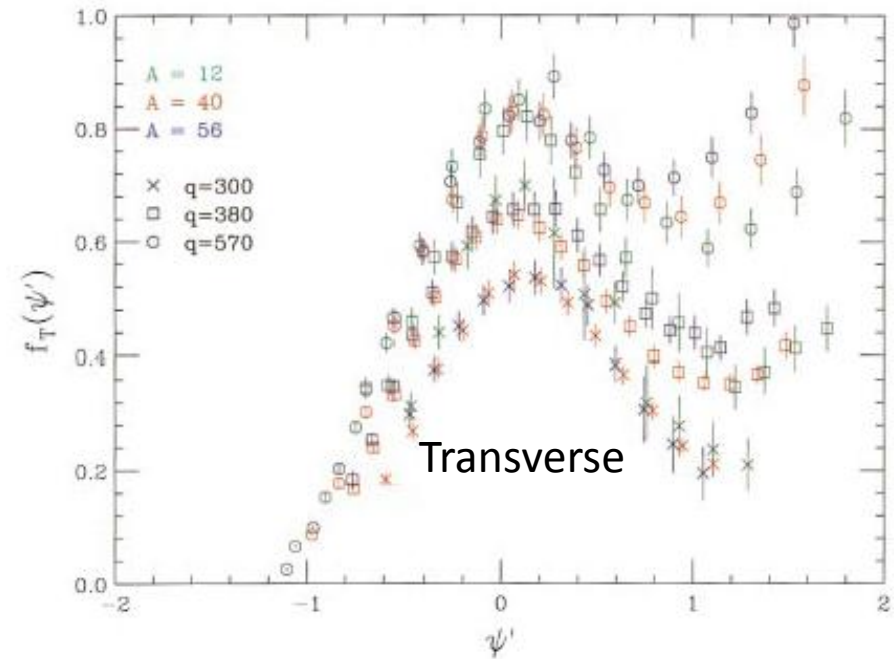
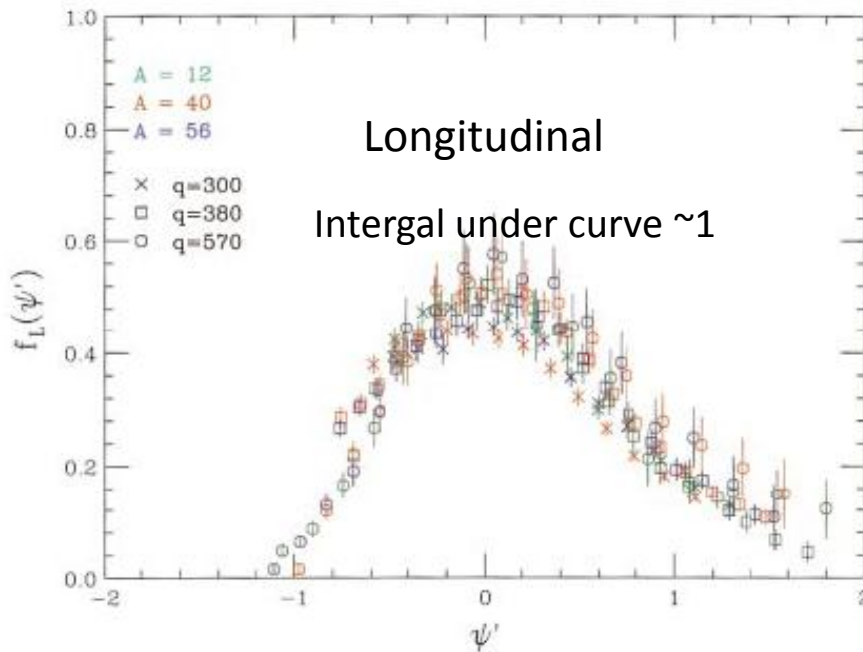
*At  $y = (\omega^2 + 2m\omega)^{1/2} - q = 0$   $\omega = Q^2/2m$  scattering off nucleon at rest  
 $y < 0$  smaller energy loss  
 $y > 0$  greater energy loss*

*Excuses (reasons) for failure  $y > 0$ : meson exchange, pion production, tail of the delta.*



# Separating Scaling into its Longitudinal and Transverse Responses

Phys. Rev. **C60**, 065502 (1999)



Dimensionless scaling variable:  $y' \gg \frac{y}{k_F}$  allows comparing different nuclei: superscaling

The responses are normalized so that in a Relativistic Fermi Gas:  $f_L(y') = f_T(y')$

$f_L(y')$  satisfies the expected Coulomb sum rule, but its asymmetry in  $y'$  indicates an energy loss greater than impulse approximation scattering off a single nucleon.

$f_T(y')$  shows clear enhancement for  $q > 300$  MeV/c

# Neutrino –Nucleon Cross Section

While inclusive electron scattering and CCQE neutrino experiments are very different, *the lepton-nucleon hardly changes.*

Neutrino (+), Anti-Neutrino(-) Nucleon CCQE Cross Section

**Charged lepton mass=0**

$$\frac{dS}{dQ^2} = \frac{G_F^2 \cos^2 q_C}{8\rho E_n^2} \left\{ A(Q^2) \pm B(Q^2) \left[ \frac{s-u}{M^2} \right] + C(Q^2) \left[ \frac{s-u}{M^2} \right]^2 \right\}$$

$$A(Q^2) = \frac{Q^2}{4} \left[ f_1^2 \left( \frac{Q^2}{M^2} - 4 \right) + f_1 f_2 \left( \frac{4Q^2}{M^2} \right) + f_2^2 \left( \frac{Q^2}{M^2} - \frac{Q^4}{4M^4} \right) + g_1^2 \left( 4 + \frac{Q^2}{M^2} \right) \right]$$

$$B(Q^2) = Q^2 (f_1 + f_2) g_1$$

$$C(Q^2) = \frac{M^2}{4} \left( f_1^2 + f_2^2 \frac{Q^2}{4M^2} + g_1^2 \right)$$

$$s - u = 4ME_n + Q^2$$

The  $f_1$  and  $f_2$  are isovector **vector** form factors that come from electron scattering.  $g_1$  is the isovector **axial form factor** fixed by neutron beta decay at  $Q^2=0$ , with a dipole form,  $1.27/(1+Q^2/M_A^2)^2$ ;  $M_A=1.02 \pm 0.02$

# More Familiar Representation

$$j_V^\mu(\mathbf{P}', \mathbf{P}) = \bar{u}(\mathbf{P}') \left[ 2F_1^V \gamma^\mu + i \frac{F_2^V}{m_N} \sigma^{\mu\nu} Q_\nu \right] u(\mathbf{P})$$

$$j_A^\mu(\mathbf{P}', \mathbf{P}) = \bar{u}(\mathbf{P}') \left[ G_A \gamma^\mu + G_P \frac{Q^\mu}{2m_N} \right] \gamma^5 u(\mathbf{P}) ,$$

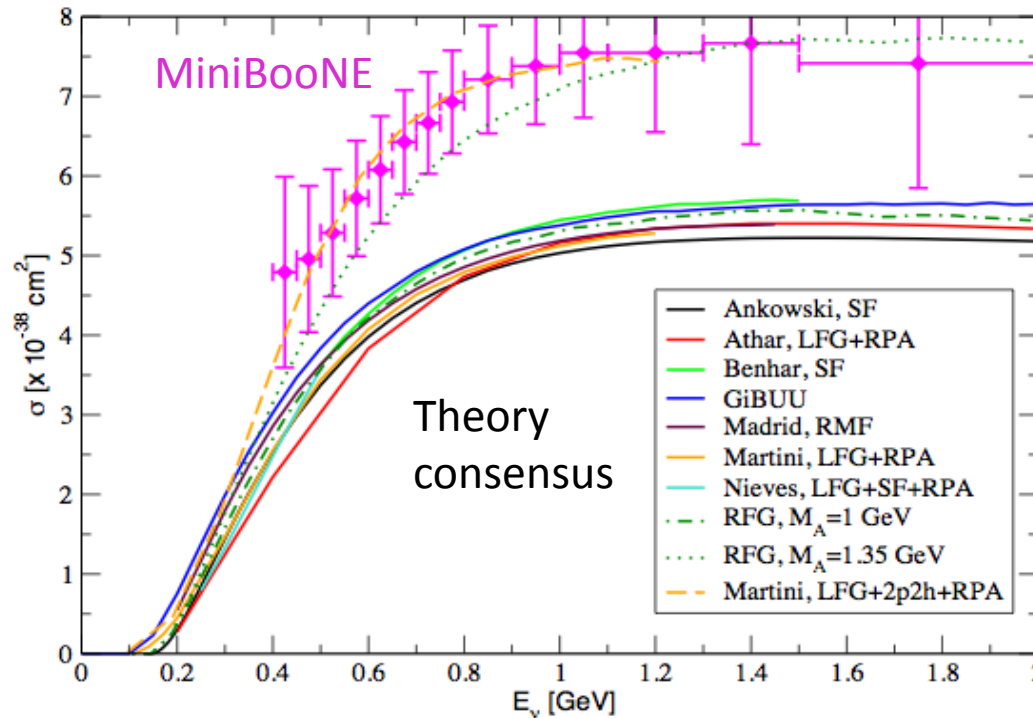
$$G_A = \frac{g_A}{1 - Q^2/M_A^2}$$

$$G_P = \frac{4m_N^2}{m_\pi^2 - Q^2} G_A$$

$$G_E = F_1 - \tau F_2 \text{ and } G_M = F_1 + F_2,$$

**Nucleon one body current!!**

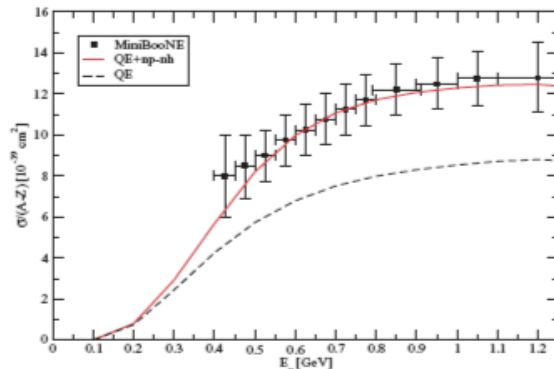
# What did MB Observe? CCQE $n_m + {}^{12}\text{C} \rightarrow m + 7p, 5n(\otimes)$



MB fits the observed  $Q^2$  distribution and crosssection by increasing  $M_A$  to 1.35 GeV

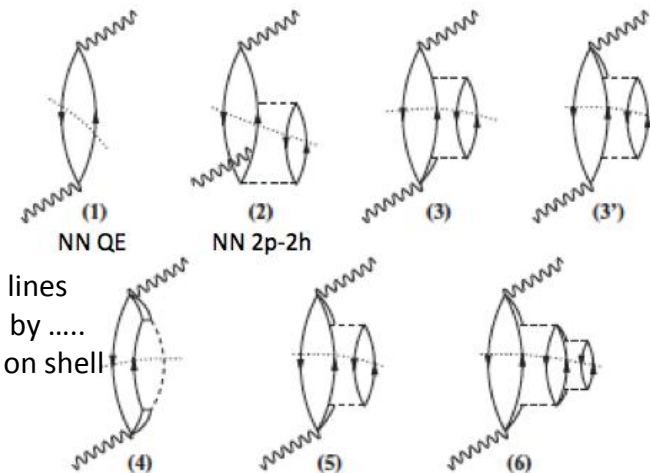
M. Martini, M. Ericson, G. Chanfray, and J. Marteau, PHYS. REV. C 80, 065501 (2009)

They use  $M_A=1.03$  GeV, in an RPA formalism



*"We suggest that the proposed increase of the axial mass from the standard value to a larger one to account for the quasielastic data, reflects the presence of a polarization cloud, mostly due to tensor interaction, which surrounds a nucleon in the nuclear medium. It translates into a final state with ejection of two nucleons, which in the present stage of the experiments is indistinguishable from the quasielastic final state."*

Some RPA p-h diagrams from Martini et al



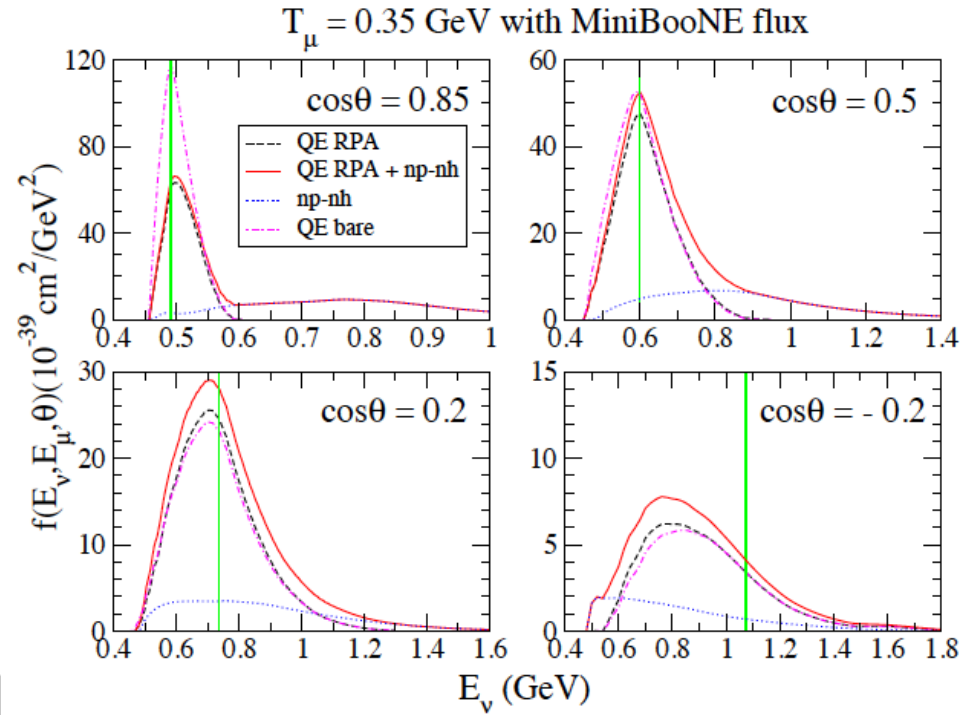
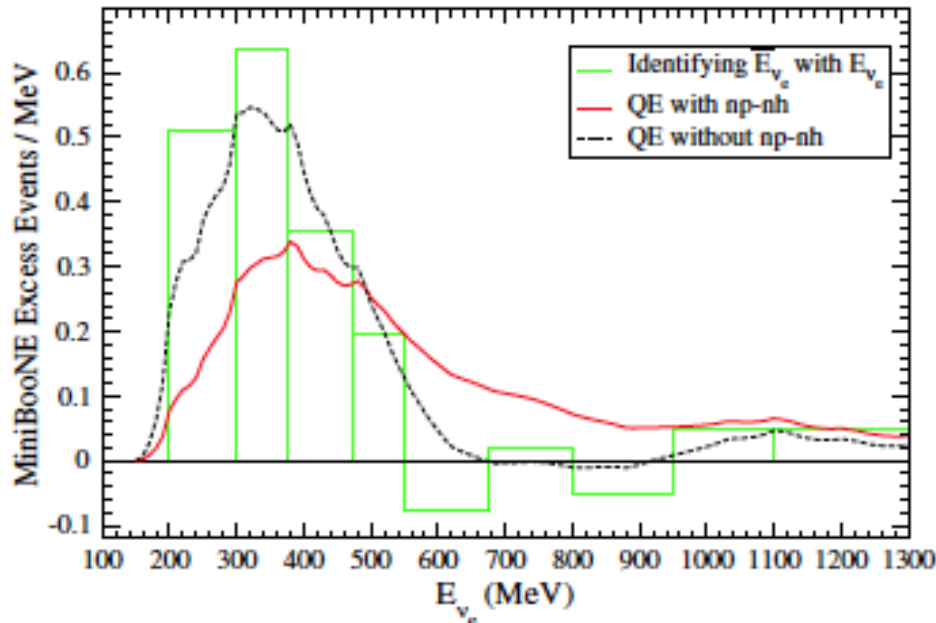
Particle lines crossed by ..... are put on shell

# Enhancement $\rightarrow$ Uncertainty in Assigned $E_\nu$

Martini et al: arXiv 1211.1523, Phys.Rev. D85, 093012

*Multiparticle final states, RPA, formalism somewhat opaque*

$$\begin{aligned} \frac{\partial^2 \sigma}{\partial \Omega \partial k'} = & \frac{G_F^2 \cos^2 \theta_c (k')^2}{2 \pi^2} \cos^2 \frac{\theta}{2} \left[ G_E^2 \left( \frac{q_\mu^2}{q^2} \right)^2 R_\tau^{NN} \right. \\ & + G_A^2 \frac{(M_\Delta - M_N)^2}{2 q^2} R_{\sigma\tau(L)}^{N\Delta} + G_A^2 \frac{(M_\Delta - M_N)^2}{q^2} R_{\sigma\tau(L)}^{\Delta\Delta} \\ & + \left( G_M^2 \frac{\omega^2}{q^2} + G_A^2 \right) \left( -\frac{q_\mu^2}{q^2} + 2 \tan^2 \frac{\theta}{2} \right) (R_{\sigma\tau(T)}^{NN} + 2 R_{\sigma\tau(T)}^{N\Delta} + R_{\sigma\tau(T)}^{\Delta\Delta}) \\ & \left. \pm 2 G_A G_M \frac{k+k'}{M_N} \tan^2 \frac{\theta}{2} (R_{\sigma\tau(T)}^{NN} + 2 R_{\sigma\tau(T)}^{N\Delta} + R_{\sigma\tau(T)}^{\Delta\Delta}) \right] \end{aligned}$$



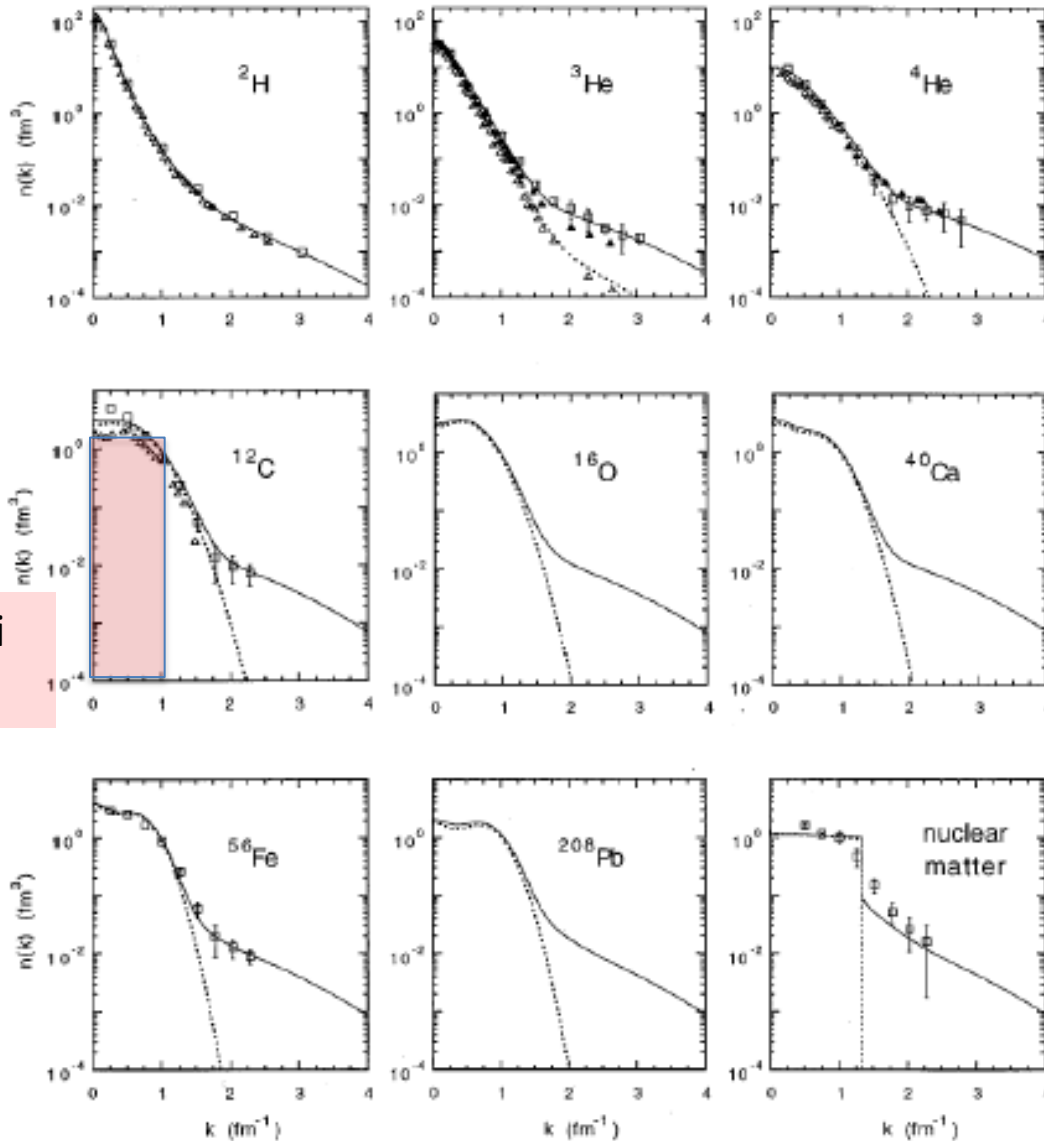
Impact on neutrino energy assignment

- Looks like there are problems!
- Can we do better? Yes.
- Much of the physics that is needed is already out there.

# Momentum Distribution in Nuclei

arXiv:1104.1196v3 [nucl-ex] 26 Mar 2012

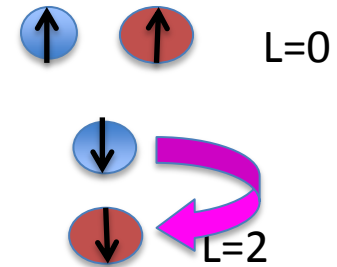
The momentum distributions are similar for  $k > 1.5 \text{ fm}^{-1}$



**Actual distribution requires multiplication by  $4\pi k^2 dk$ .**

High momentum tails look like deuteron!!

Mostly due to tensor force,  $\Delta L=2, T=0, S=1$



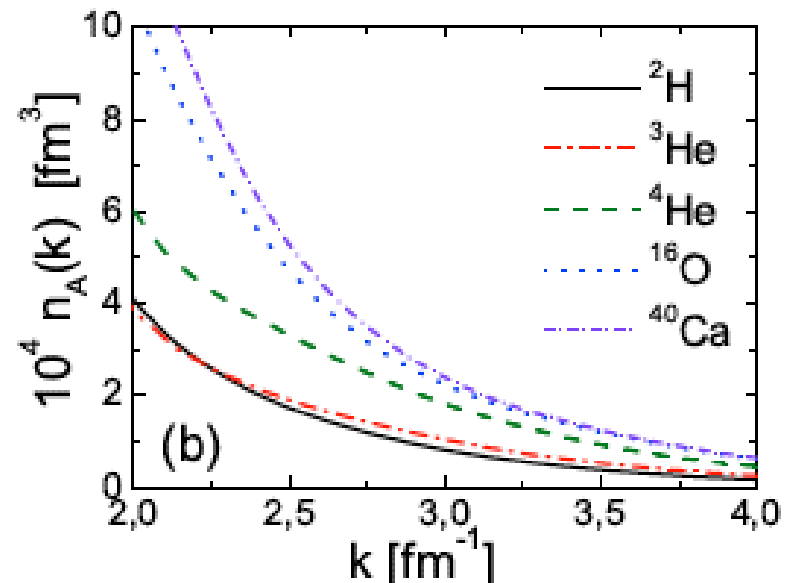
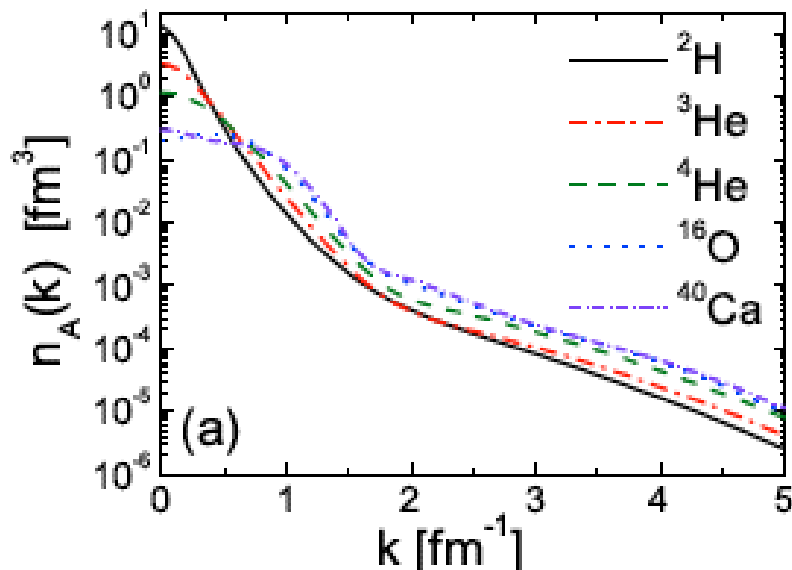
This correlation is neglected when treating the nucleus as an ensemble of free nucleons in a mean field.

# Recent Calculation of Nucleon Momentum Distributions using Realistic Interactions

arXiv 1211.0134, Alvioli, degli Atti, et al.

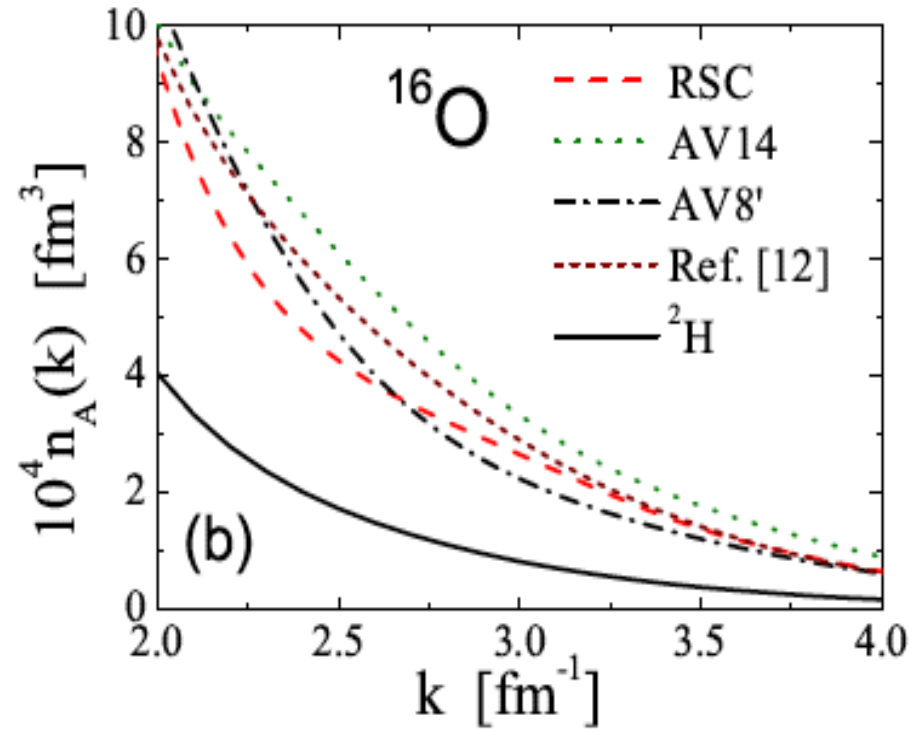
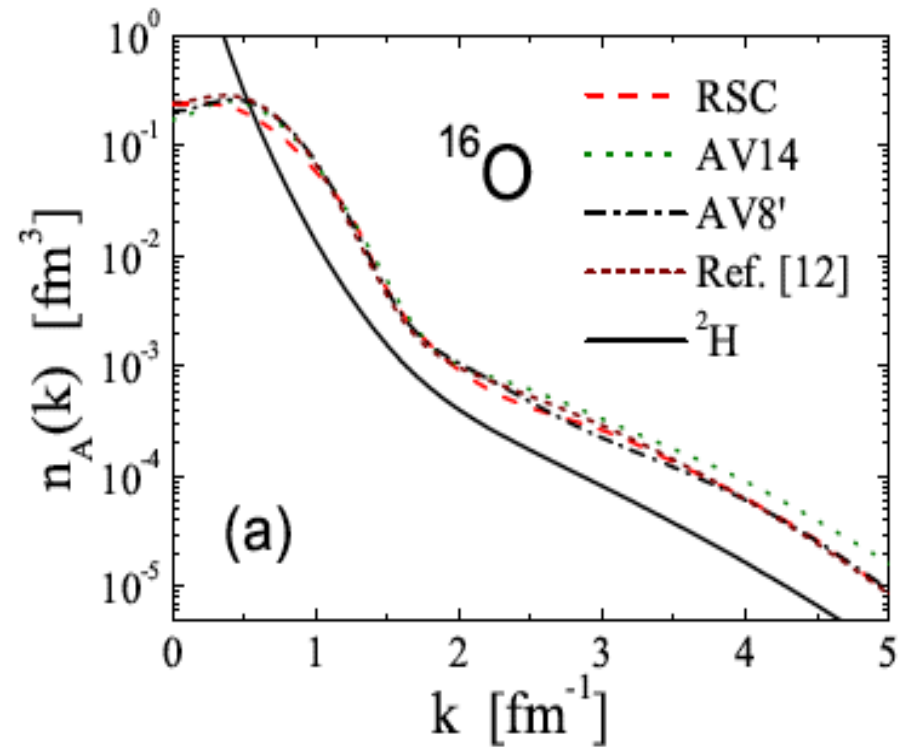
$$\mathcal{P}_{0(1)}^{N_1}(k_1^\pm) = 4\pi \int_{k^-}^{k_1^+} n_{0(1)}^{N_1}(\mathbf{k}_1) k_1^2 dk_1$$

	${}^2\text{H}$	${}^3\text{He}(\text{n})$	${}^3\text{He}(\text{p})$		${}^4\text{He}$		${}^{16}\text{O}$		${}^{40}\text{Ca}$	
$k_1^- [\text{fm}^{-1}]$	$\mathcal{P}$	$\mathcal{P}_1$	$\mathcal{P}_0$	$\mathcal{P}_1$	$\mathcal{P}_0$	$\mathcal{P}_1$	$\mathcal{P}_0$	$\mathcal{P}_1$	$\mathcal{P}_0$	$\mathcal{P}_1$
0.00	1.000	0.999	0.677	0.323	0.84621	0.15285	0.79999	0.20016	0.80	0.19321
0.50	0.3078	0.568	0.277	0.201	0.53643	0.14032	0.66972	0.19635	0.69997	0.18301
1.00	0.081	0.163	0.038	0.0723	0.10479	0.1045	0.17588	0.14794	0.24706	0.13771
1.50	0.0366	0.067	0.0049	0.036	0.0079	0.0791	0.00792	0.09417	0.01022	0.10143
2.00	0.0221	0.041	0.0015	0.024	$6.9512 \cdot 10^{-4}$	0.06156	$5.9 \cdot 10^{-5}$	0.06344	$3.28 \cdot 10^{-4}$	0.07124





# Differences Produced by Different Interactions



*Don't forget  $k^2 dk$*

- For 40 years theorists maintained there were high momentum components in the nuclear wave function due to short range nucleon-nucleon correlations.
- Some manifestations are the deuteron quadruple moment (SR tensor force), depletion of shell model orbits, saturation of nuclear matter (short range repulsion).
- “Direct evidence” has been hard to come by until middle of last decade.  
PRL **90** 042301  $^{12}\text{C}(p,2p+n)$ , PRL **99**,072501 (e,e’p)

$A(e,e')$  PRL **108**, 092502 (2012)

$$r(A,D) = \frac{2 S_A}{A S_D} \quad 1.5 < x < 1.9$$

A	$\theta_e = 18^\circ$	$\theta_e = 22^\circ$	$\theta_e = 26^\circ$	Inel. sub.
$^3\text{He}$	$2.14 \pm 0.04$	$2.28 \pm 0.06$	$2.33 \pm 0.10$	$2.13 \pm 0.04$
$^4\text{He}$	$3.66 \pm 0.07$	$3.94 \pm 0.09$	$3.89 \pm 0.13$	$3.60 \pm 0.10$
Be	$4.00 \pm 0.08$	$4.21 \pm 0.09$	$4.28 \pm 0.14$	$3.91 \pm 0.12$
C	$4.88 \pm 0.10$	$5.28 \pm 0.12$	$5.14 \pm 0.17$	$4.75 \pm 0.16$
Cu	$5.37 \pm 0.11$	$5.79 \pm 0.13$	$5.71 \pm 0.19$	$5.21 \pm 0.20$
Au	$5.34 \pm 0.11$	$5.70 \pm 0.14$	$5.76 \pm 0.20$	$5.16 \pm 0.22$
$\langle Q^2 \rangle$	$2.7 \text{ GeV}^2$	$3.8 \text{ GeV}^2$	$4.8 \text{ GeV}^2$	
$x_{\min}$	1.5	1.45	1.4	

A	$R_{2N}$ (E02-019)	SLAC	CLAS	$F_{\text{CM}}$
$^3\text{He}$	$1.93 \pm 0.10$	$1.8 \pm 0.3$	...	$1.10 \pm 0.05$
$^4\text{He}$	$3.02 \pm 0.17$	$2.8 \pm 0.4$	$2.80 \pm 0.28$	$1.19 \pm 0.06$
Be	$3.37 \pm 0.17$	...	...	$1.16 \pm 0.05$
C	$4.00 \pm 0.24$	$4.2 \pm 0.5$	$3.50 \pm 0.35$	$1.19 \pm 0.06$
Cu(Fe)	$4.33 \pm 0.28$	$(4.3 \pm 0.8)$	$(3.90 \pm 0.37)$	$1.20 \pm 0.06$
Au	$4.26 \pm 0.29$	$4.0 \pm 0.6$	...	$1.21 \pm 0.06$
$\langle Q^2 \rangle$	$\sim 2.7 \text{ GeV}^2$	$\sim 1.2 \text{ GeV}^2$	$\sim 2 \text{ GeV}^2$	
$x_{\min}$	1.5	...	1.5	
$\alpha_{\min}$	1.275	1.25	1.22–1.26	

# Energy Transfer ( $\omega$ )

*In Mean Field:*

$$W_1 = \left( \sqrt{(\vec{q} + \vec{p})^2 + m^2} - m \right) + \frac{p^2}{2(A-1)m} + S_1$$

*In 2 body Correlation assuming  $p_{CM}=0$ :*

$$W_2 = \left( \sqrt{(\vec{q} + \vec{p})^2 + m^2} + \sqrt{p^2 + m^2} - 2m \right) + S_2$$

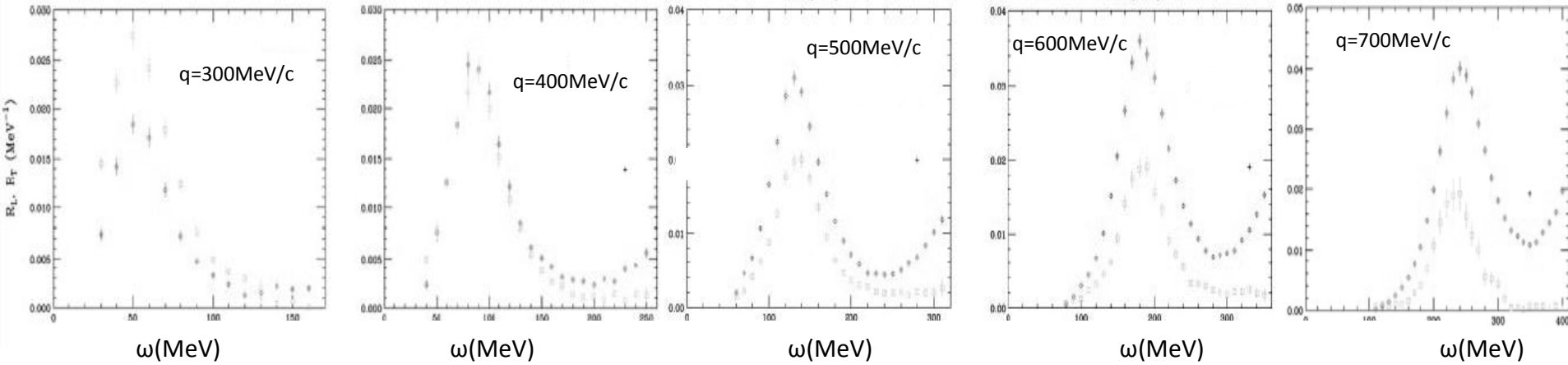


Correlated partner

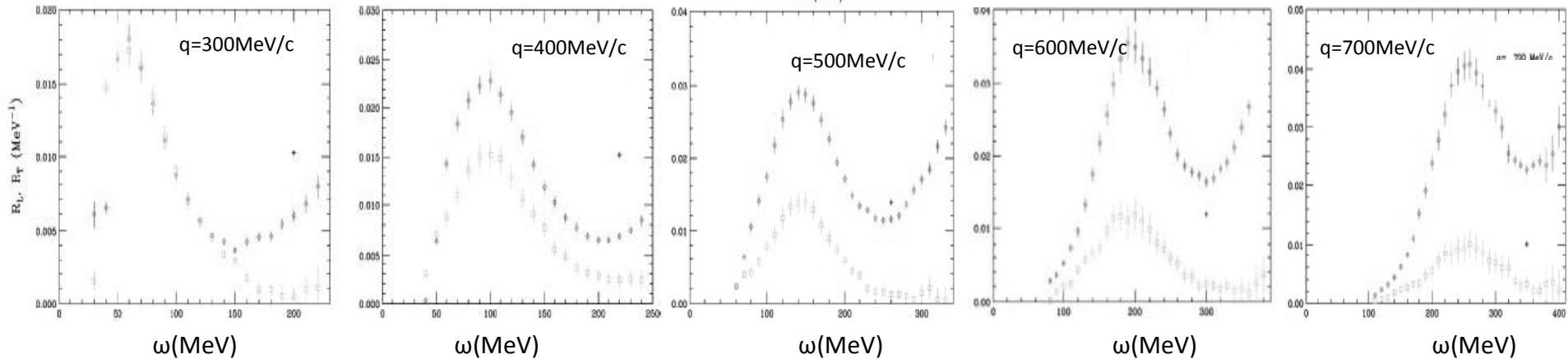
# Carlson et al Phys. Rev. C65 024002 (2002)

## Longitudinal and Transverse Response Functions from $^3\text{He}$ and $^4\text{He}$ from $(e,e')$ Quasi-elastic Scattering

### $^3\text{He}$



### $^4\text{He}$

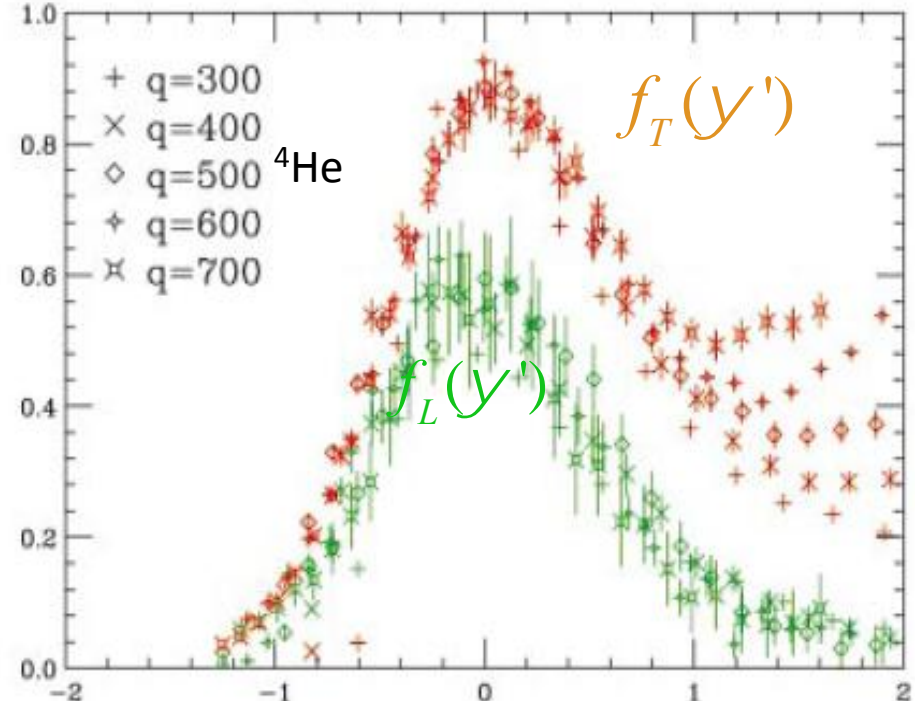
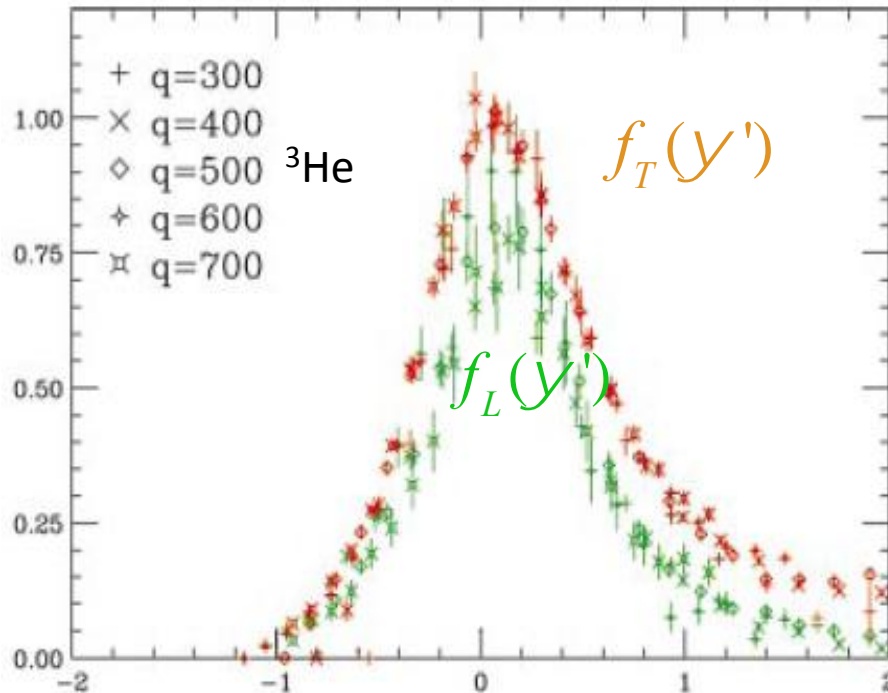


# $^3\text{He}$ and $^4\text{He}$ Longitudinal and Transverse Scaled Response Functions

*Phys. Rev. C65 024002 (2002)*

$$f_{L,T}(\psi', q) = k_F \frac{R_{L,T}(q, W)}{G_{L,T}(q)}$$

$$f_T(\psi', q) = \frac{k_F 2mq R_T(q, \omega)}{Q^2 (ZG_{M_n}^2(q) + NG_{M_n}^2(q))}$$



$$y' = \frac{\sqrt{W^2 + 2mW} - q}{k_F}$$

Note : Change in  $f_T/f_L$  and shift to higher values of  $y'$  between  $^3\text{He}$  and  $^4\text{He}$ ,

# $^4\text{He}$ Longitudinal and Transverse $e, e'$ QE Response

$$\tilde{E}_{T,L}(q, t) = \int_{W_{th}}^{\infty} e^{-(W-E_0)t} R_{T,L}(q, W) dW \quad (\text{definition of Euclidian response function, } \tau)$$

$$\tilde{E}_T(q, t) = \langle 0 | j_T^*(\vec{q}) e^{-(H-E_0)t} j_T(\vec{q}) | 0 \rangle - e^{\frac{q^2 t}{2AM}} \left| \langle 0(\vec{q}) | j_T(\vec{q}) | 0 \rangle \right|^2 \quad (\text{mode of calculation})$$

$$E_{T,L}(q, t) = \frac{e^{\frac{q^2 t}{2m}}}{G_{E,N}(\tilde{Q}^2)^2} \tilde{E}_{T,L}(q, t) \quad (\text{scaled response presented below})$$

**Results of calculation; Uses 2 & 3 body NN force, includes 2 body current operators.**

TABLE I. Longitudinal sum rule obtained with one body only and with both one- and two-body charge operators.

$q$ (MeV/c)	$^3\text{He}$		$^4\text{He}$		$^6\text{Li}$	
	1	1+2	1	1+2	1	1+2
300	0.787	0.763	0.670	0.649	0.977	0.933
400	0.921	0.875	0.859	0.815	0.995	0.932
500	0.964	0.901	0.941	0.881	0.990	0.921
600	0.982	0.908	0.973	0.910	0.990	0.924
700	0.994	0.914	0.994	0.942	0.994	0.938

TABLE II. Transverse sum rule obtained with one body only and with both one- and two-body current operators.

$q$ (MeV/c)	$^3\text{He}$		$^4\text{He}$		$^6\text{Li}$	
	1	1+2	1	1+2	1	1+2
300	0.929	1.31	0.893	1.67	0.912	1.57
400	0.987	1.30	0.970	1.62	0.974	1.52
500	1.01	1.28	1.00	1.55	0.999	1.46
600	1.01	1.25	1.01	1.49	1.01	1.41
700	1.01	1.23	1.01	1.44	1.011	1.37

# One and Two Body EM Currents and Charges

One-body current and charge:

$$\rho_{i,NR}^{(1)}(\mathbf{q}) = \epsilon_i e^{i\mathbf{q}\cdot\mathbf{r}_i}, \quad \mathbf{j}_i^{(1)}(\mathbf{q}) = \frac{1}{2m} \epsilon_i [\mathbf{p}_i, e^{i\mathbf{q}\cdot\mathbf{r}_i}]_+ - \frac{i}{2m} \mu_i \mathbf{q} \times \boldsymbol{\sigma}_i e^{i\mathbf{q}\cdot\mathbf{r}_i},$$

$$\epsilon_i = G_{E,p}(Q^2) \frac{1}{2} (1 + \tau_{z,i}) + G_{E,n}(Q^2) \frac{1}{2} (1 - \tau_{z,i}),$$

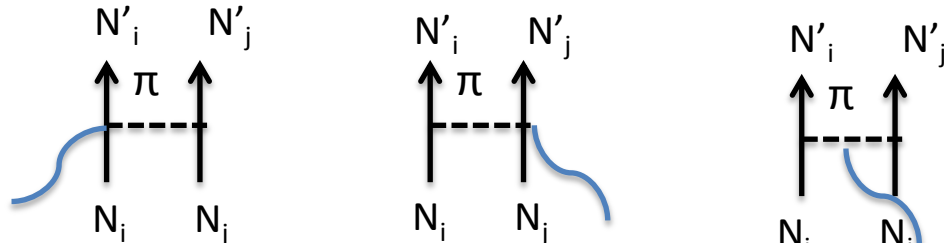
$$\mu_i \equiv G_{M,p}(Q^2) \frac{1}{2} (1 + \tau_{z,i}) + G_{M,n}(Q^2) \frac{1}{2} (1 - \tau_{z,i}),$$

Continuity eq.:

$$\nabla \cdot \vec{j} + i[H, r] = 0 \quad H = \sum_i T_i + \sum_{i>j} V_{i,j}$$

$$j = \underset{i}{\hat{a}} j_i^{(1)} + \underset{i>j}{\hat{a}} j_{i,j}^{(2)} \quad \nabla \cdot j_i^{(1)} + i[T_i, r_i^{(1)}] = 0 \quad \nabla \cdot j_{i,j}^{(2)} + i[V_{i,j}, r_i^{(1)} + r_j^{(1)}] = 0$$

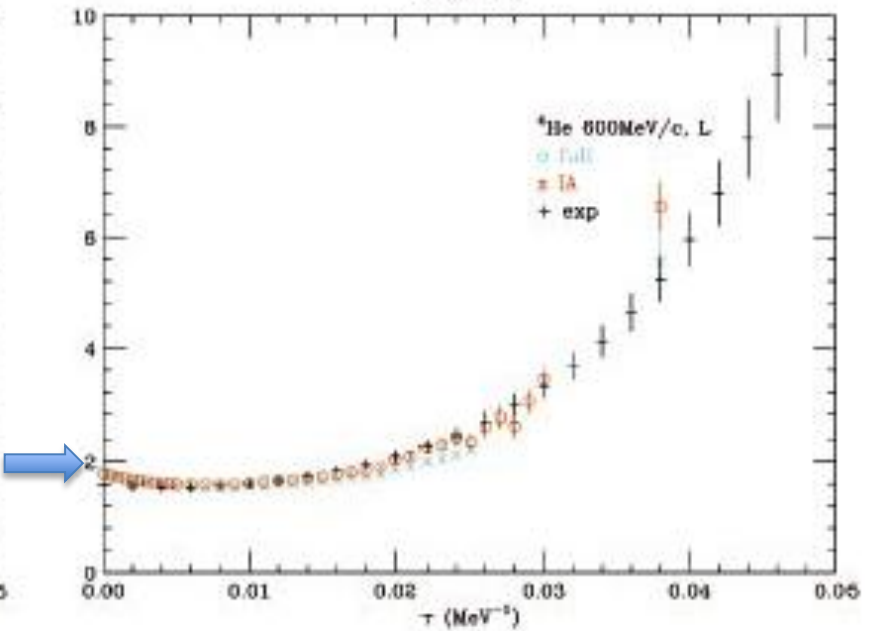
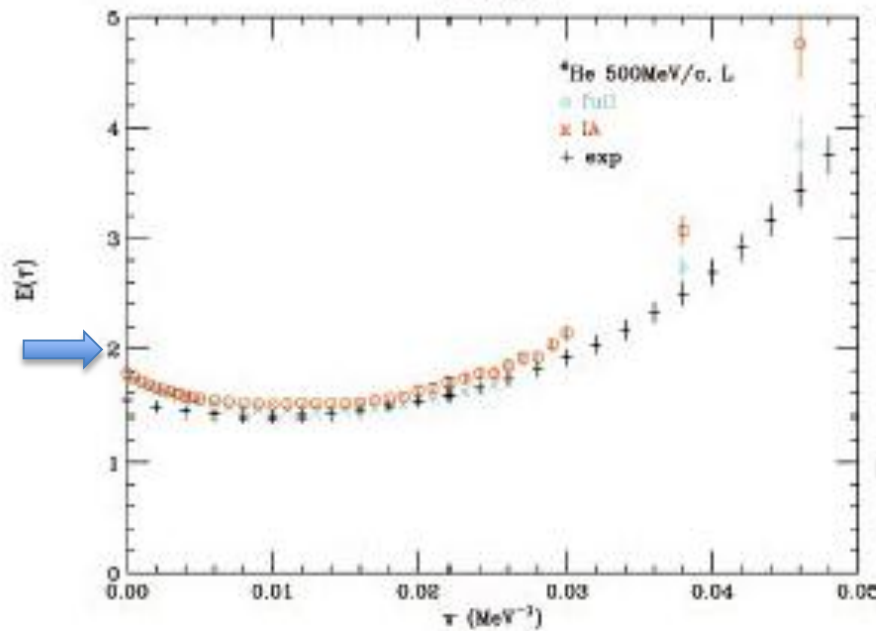
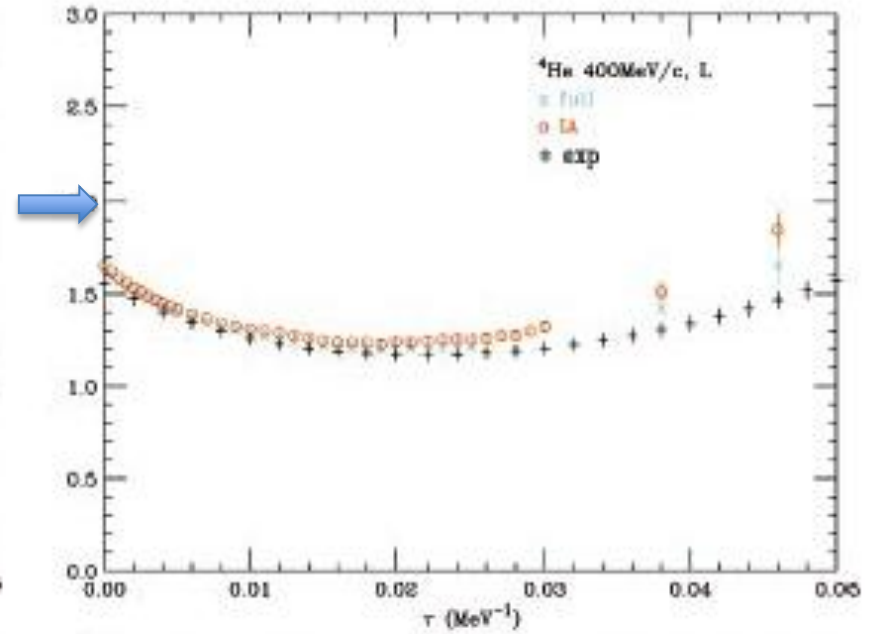
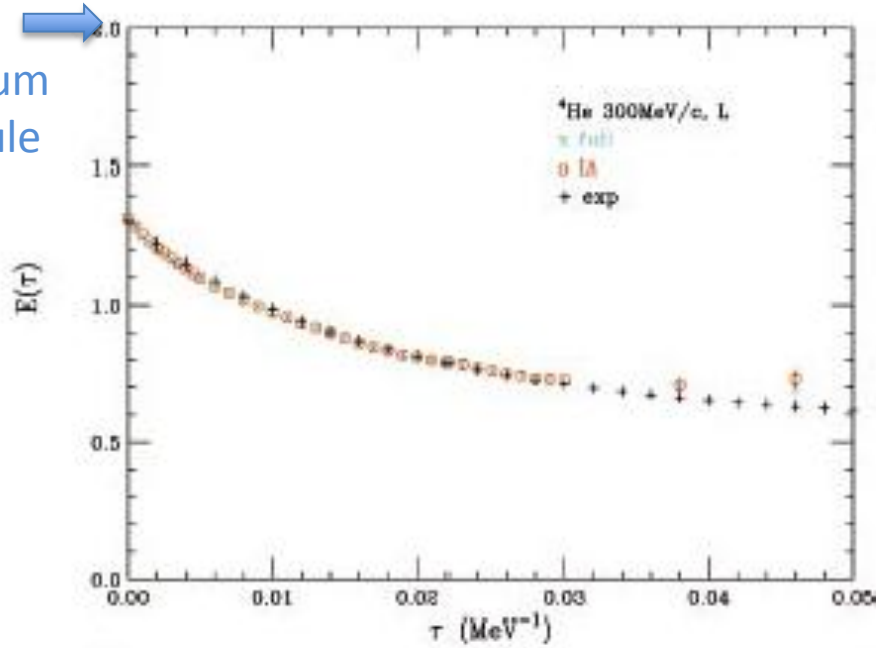
Two-body current:



$$\begin{aligned} \mathbf{j}_{ij}^{(2)}(\mathbf{q}; \pi) = & G_E^V(Q^2) (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z [e^{i\mathbf{q}\cdot\mathbf{r}_i} f_{PS}(\tau) \boldsymbol{\sigma}_i (\boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}) \\ & + e^{i\mathbf{q}\cdot\mathbf{r}_j} f_{PS}(r) \boldsymbol{\sigma}_j (\boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}) \\ & - (\boldsymbol{\sigma}_i \cdot \nabla_i) (\boldsymbol{\sigma}_j \cdot \nabla_j) (\nabla_i - \nabla_j) g_{PS}(\mathbf{q}; \mathbf{R}, \mathbf{r})], \end{aligned}$$

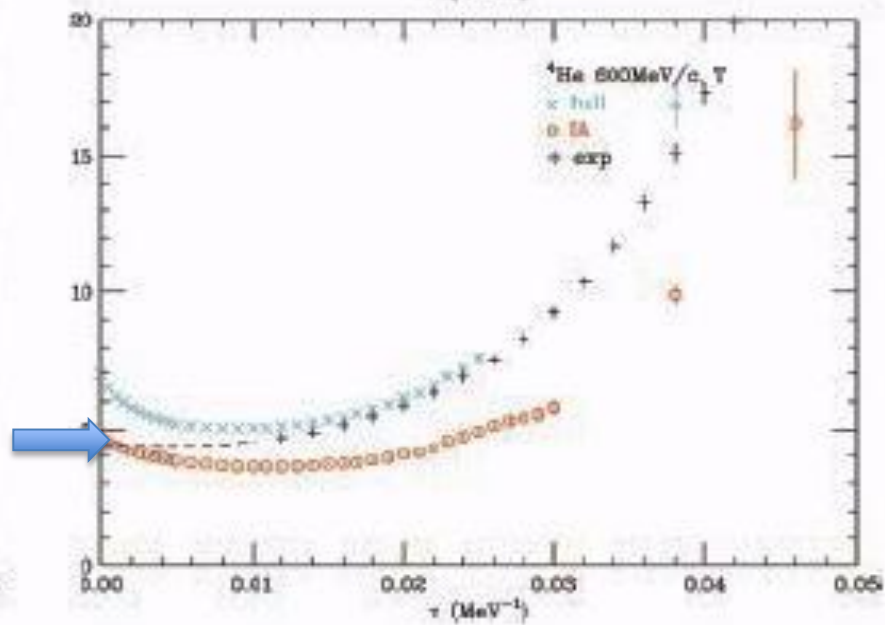
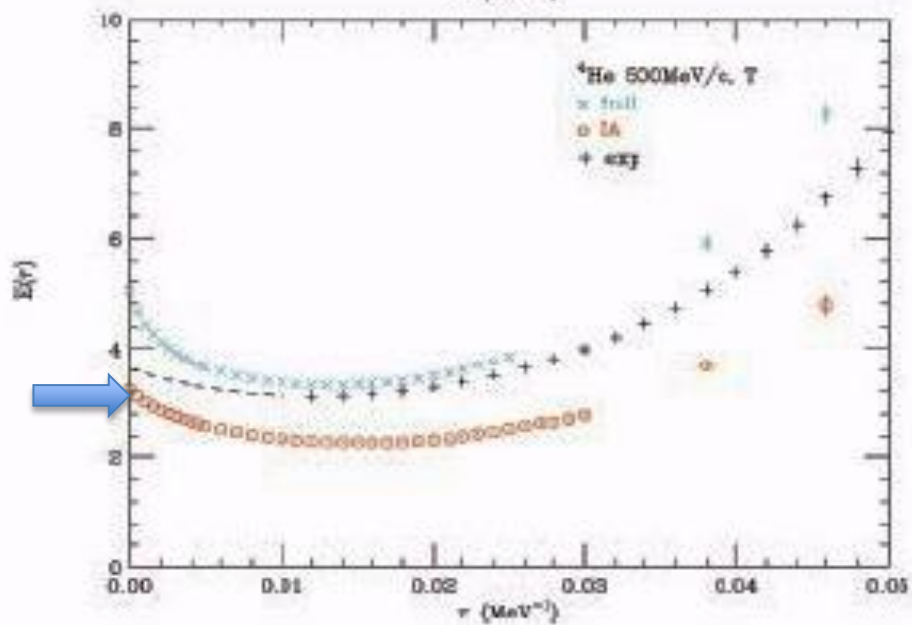
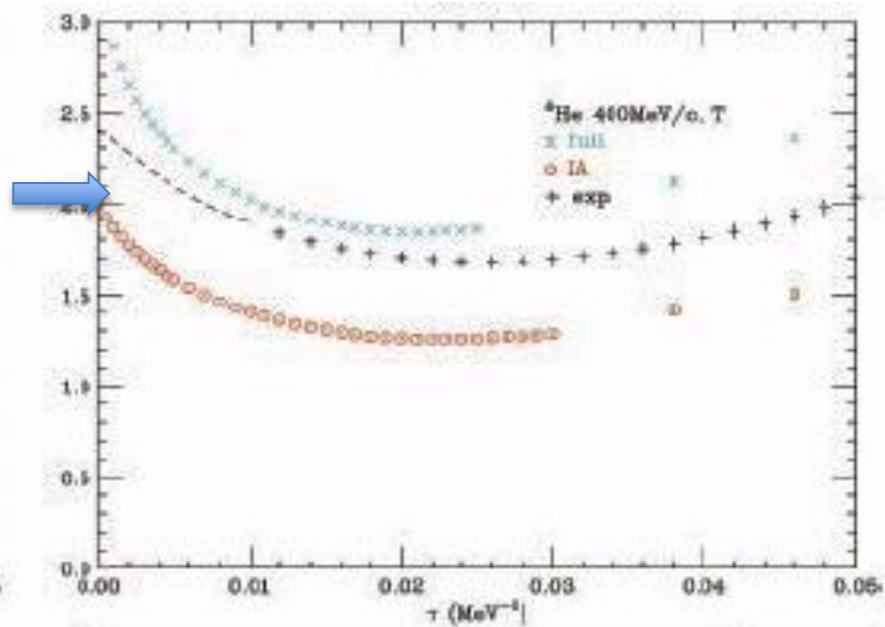
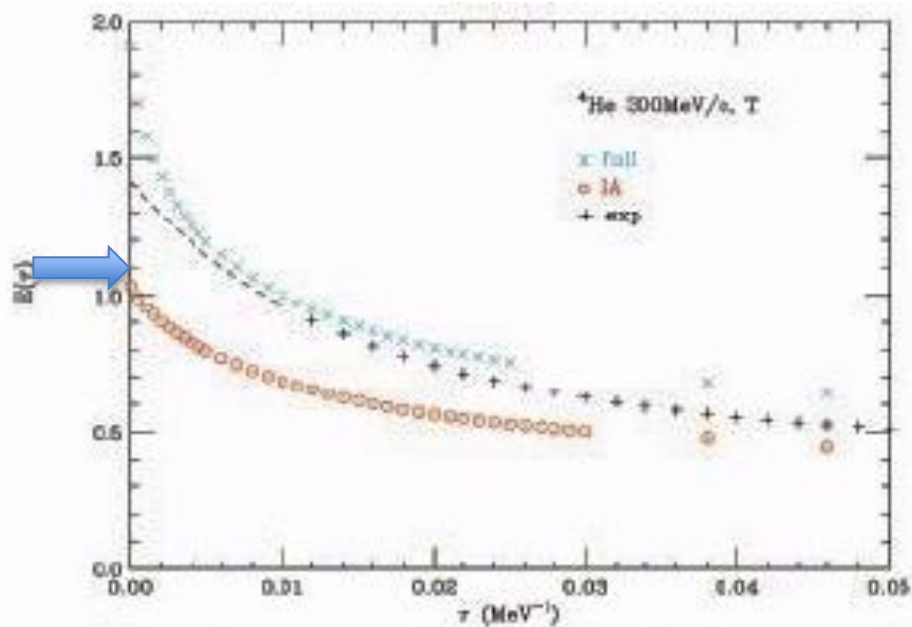
# $^4\text{He}$ Euclidian Longitudinal Response: Calculated versus Data

Sum rule





# $^4\text{He}$ Transverse Response Calculated Versus Data



# More from **PHYSICAL REVIEW C, VOLUME 65, 024002**

Fermi Gas= plane wave initial and final states

TABLE VII. Excess-strength contributions  $\Delta S_L$  and  $\Delta S_T$  to the Fermi-gas sum rules from terms involving two-nucleon currents.

$q$ (MeV/c)	$\Delta S_L$	$\Delta S_T$
300	0.004	0.114
400	0.007	0.081
500	0.011	0.066
600	0.017	0.060
700	0.024	0.056

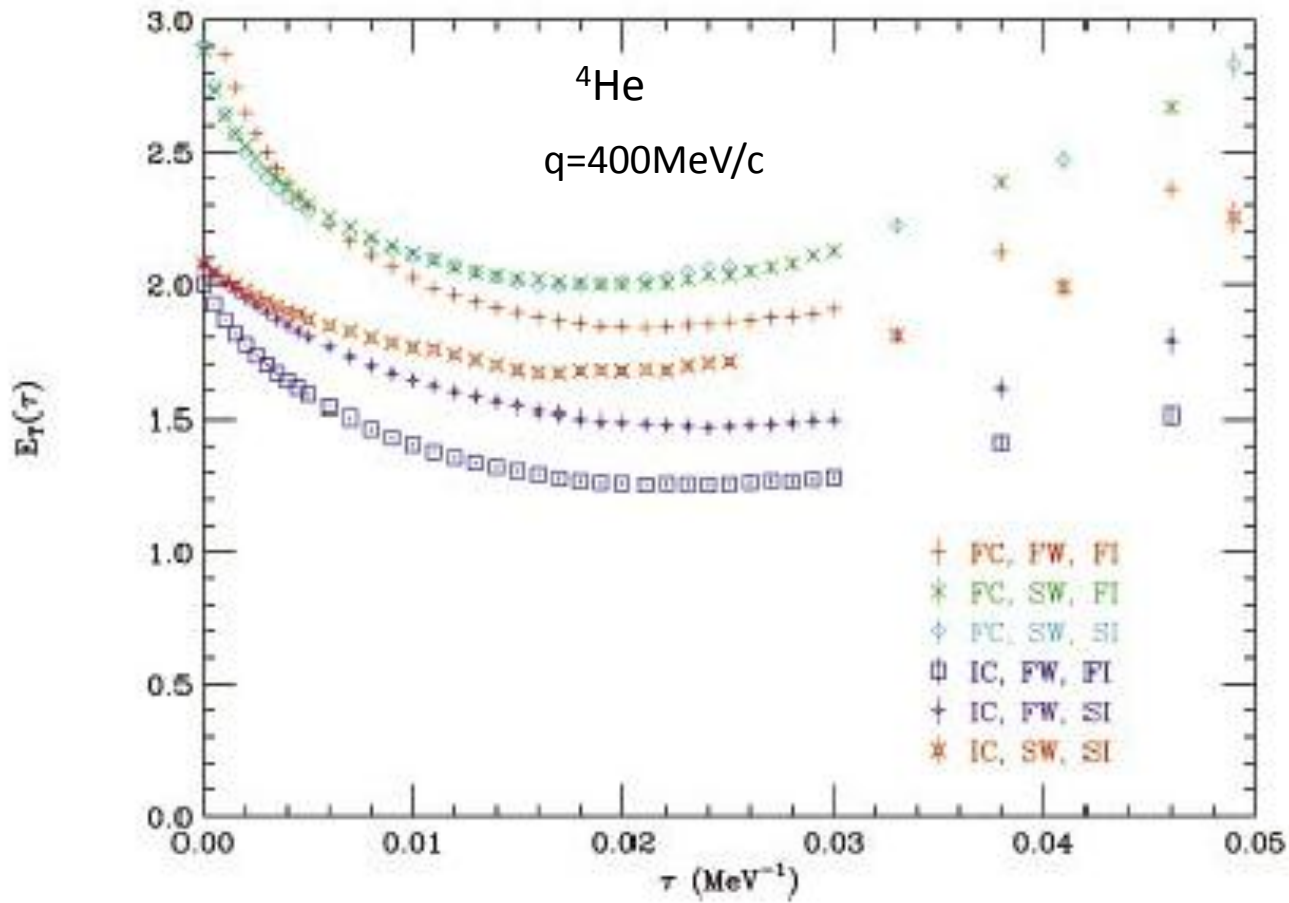
**Plane wave initial and final  
states don't work!!**

# Potentially Bad News!!

Conclusion from *Phys. Rev. C65 024002 (2002)*

“it is now clear that this enhancement arises from the concerted interplay of tensor interactions and correlations in both ground and scattering states. A successful prediction of the longitudinal and transverse response functions in the quasielastic region demands an accurate description of nuclear dynamics, based on realistic interactions and currents.”

*If true, how could all this be put into event generators??*



FC=Full Current  
SC=Simple Current

# What Can be Done?

- *Use better momentum distributions for nuclei* } yields  $\vec{q}$  and  $W$
- *Have a good model for energy loss in collision* }

*In Mean Field:*

$$W_1 = \left( \sqrt{(\vec{q} + \vec{p})^2 + m^2} - m \right) + \frac{p^2}{2(A-1)m} + S_1$$

*In 2 body Correlation assuming  $p_{CM}=0$ :*

$$W_2 = \left( \sqrt{(\vec{q} + \vec{p})^2 + m^2} + \sqrt{p^2 + m^2} - 2m \right) + S_2$$

- *With  $\vec{q}$  and  $W$  established, use the measured response functions,  $f_L(\psi')$  and  $f_T(\psi')$  to account for all the neglected nuclear physics.*

$$y' = \frac{1}{k_F} (\sqrt{W^2 + 2mW} - q)$$

- *Assume only the transverse vector response is enhanced*
- *The new momentum distribution, the new recipe for the energy loss, and enhanced transverse vector response will produce a higher apparent  $Q^2$ , more yield and higher incident neutrino energy.*

*With a known flux (??) of neutrinos one can then calculate the probability of a charged lepton with energy  $E_L$  and angle  $\theta$  created by a neutrino with energy  $E_\nu$ . Thus achieving a better representation of data and a more reliable estimate of neutrino energy and its uncertainty.*

*Note: Carlson, Schiavilla et al. say they will have computed the  $\nu_\mu + {}^{12}\text{C}$  CCQE cross-section by summer 2013 for  $\nu$  energies up to 2GeV with the full approach used in PR C65 024002. This can be compared both to MiniBooNE data and serve to test the simpler approaches suggested here.*

# Concluding Remarks

- *Better nucleon momentum distributions and a set of consistent 2-body currents should yield a better description of CCQE and NCE.*
- *It also provides a foundation to incorporate improvements in theory and new data particularly from electron scattering.*
- *Note all the theory addressed has been inclusive-lepton only*
- *Better cross sections will put greater emphasis on better neutrino flux determinations. Role for  $^2\text{H}$ ? Phys. Rev. C 86, 035503 (2012)*
- *These improvements are probably needed for reliable extensions of generators into the resonance region.*
- *Realization of the full capability of LAr detectors will require dealing with FSI-a difficult and messy task.*

# Supplemental Slides

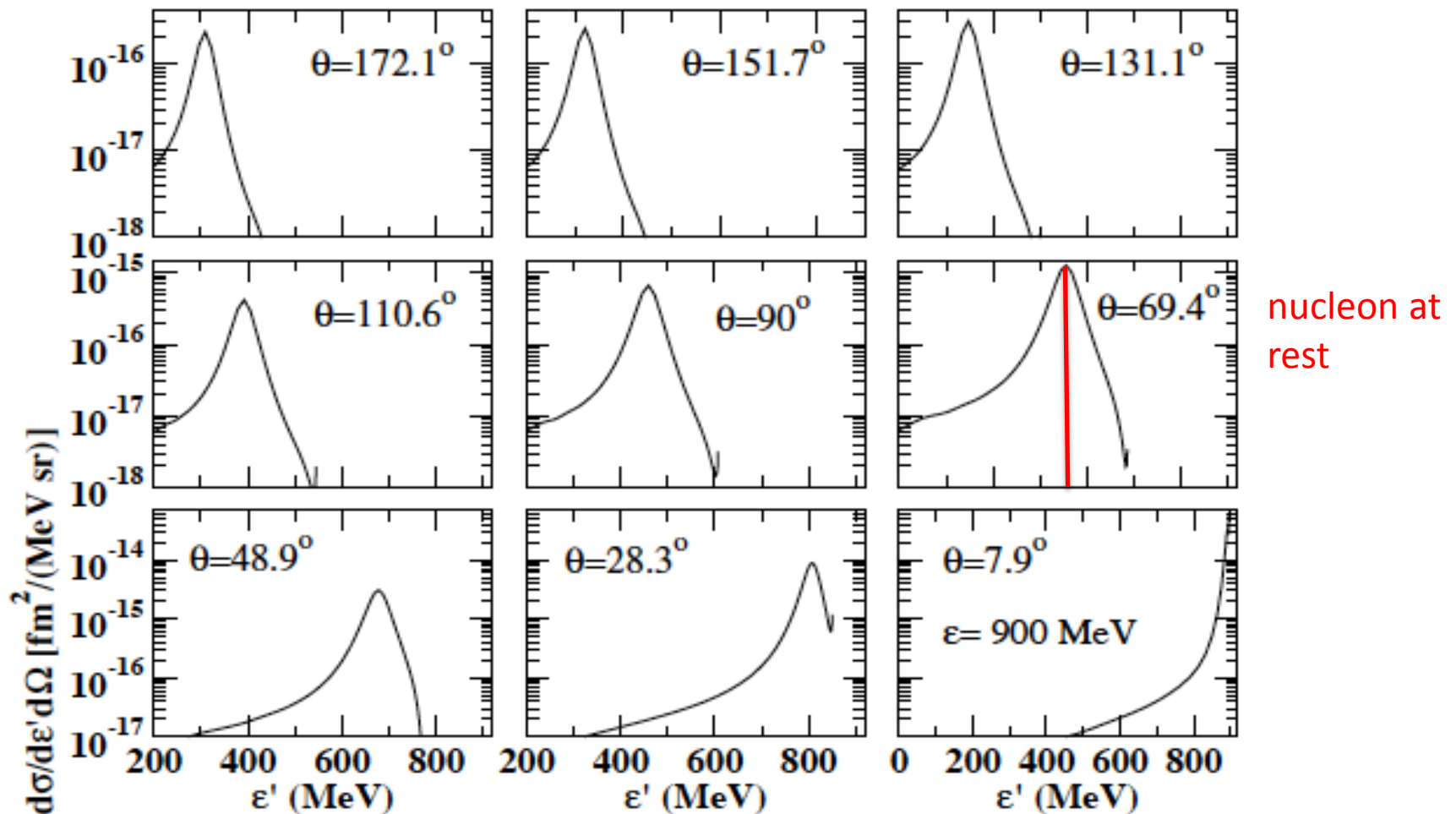


# $\nu$ - ${}^2\text{H}$ Scattering (Theory)

Phys. Rev. C 86, 035503 (2012)

$$\left(\frac{d\sigma}{d\varepsilon'd\Omega}\right)_{\nu/\bar{\nu}} = \frac{G^2}{2\pi^2} k' \varepsilon' F(Z, k') \cos^2\frac{\theta}{2} \left[ R_{00} + \frac{\omega^2}{q^2} R_{zz} - \frac{\omega}{q} R_{0z} + \left(\tan^2\frac{\theta}{2} + \frac{Q^2}{2q^2}\right) R_{xx+\nu y} \mp \tan\frac{\theta}{2} \sqrt{\tan^2\frac{\theta}{2} + \frac{Q^2}{q^2}} R_{xy} \right]$$

Calculated Lepton Energies for 900 MeV incident Neutrinos



Nucleus		(ST)			
		(10)	(01)	(00)	(11)
$^2\text{H}$		1	-	-	-
$^3\text{He}$	IPM	1.50	1.50	-	-
	SRC (Present work)	1.488	1.360	0.013	0.139
	SRC [44]	1.50	1.350	0.01	0.14
	SRC [23]	1.489	1.361	0.011	0.139
$^4\text{He}$	IPM	3	3	-	-
	IPM(0s states) [46]	3	3	-	-
	SRC (Present work)	2.99	2.57	0.01	0.43
	SRC [44]	3.02	2.5	0.01	0.47
	SRC [23]	2.992	2.572	0.08	0.428
$^{16}\text{O}$	IPM	30	30	6	54
	IPM(0s states) [46]	20	18	-	-
	SRC(Present work)	29.8	27.5	6.075	56.7
	SRC [44]	30.05	28.4	6.05	55.5
$^{40}\text{Ca}$	IPM	165	165	45	405
	IPM(0s states) [46]	90	20	-	-
	SRC(Present work)	165.18	159.39	45.10	410.34

## In Somewhat More Detail

*Take the nucleon momentum distributions as in arXiv 1211.0134*

*A neutrino of energy  $E_\nu$  imparts momentum  $q$  to one of the nucleons using one-body current.*

*The energy loss ( $\omega$ ) in mean field sector is standard:*

$$W_M = \sqrt{(\vec{p} + \vec{q})^2 + m^2} - m - B_M$$

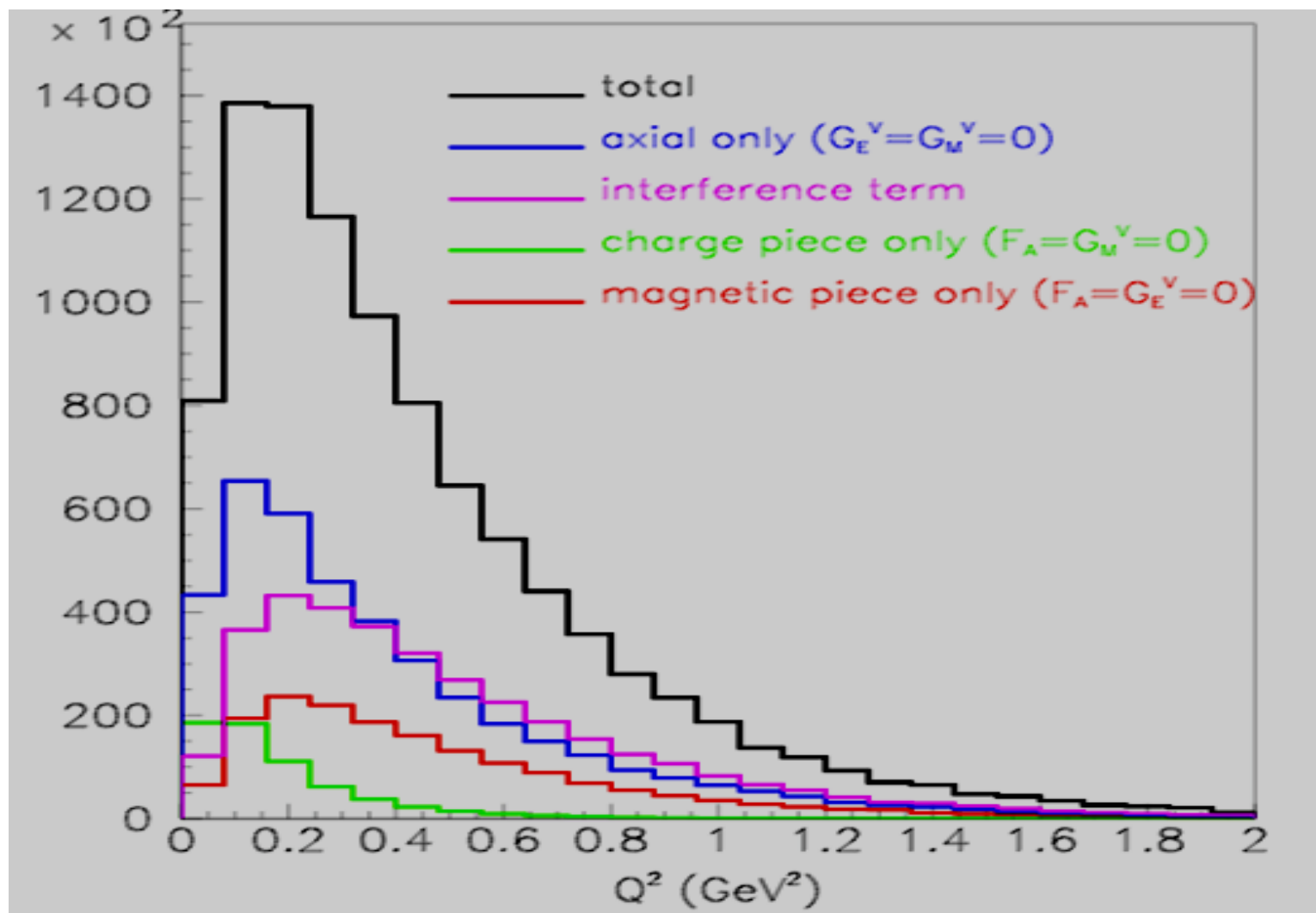
*The energy loss in the correlated sector is:*

$$W_C = (\sqrt{(\vec{p} + \vec{q})^2 + m^2} + \sqrt{\vec{p}^2 + m^2}) - 2m - B_C$$

*With  $q$  and  $\omega$ ,  $\psi'$  is obtained. The resulting  $R_{VL}(\psi')$  should be asymmetric in  $\psi'$  due to the increased energy loss when scattering off correlated nucleons.*

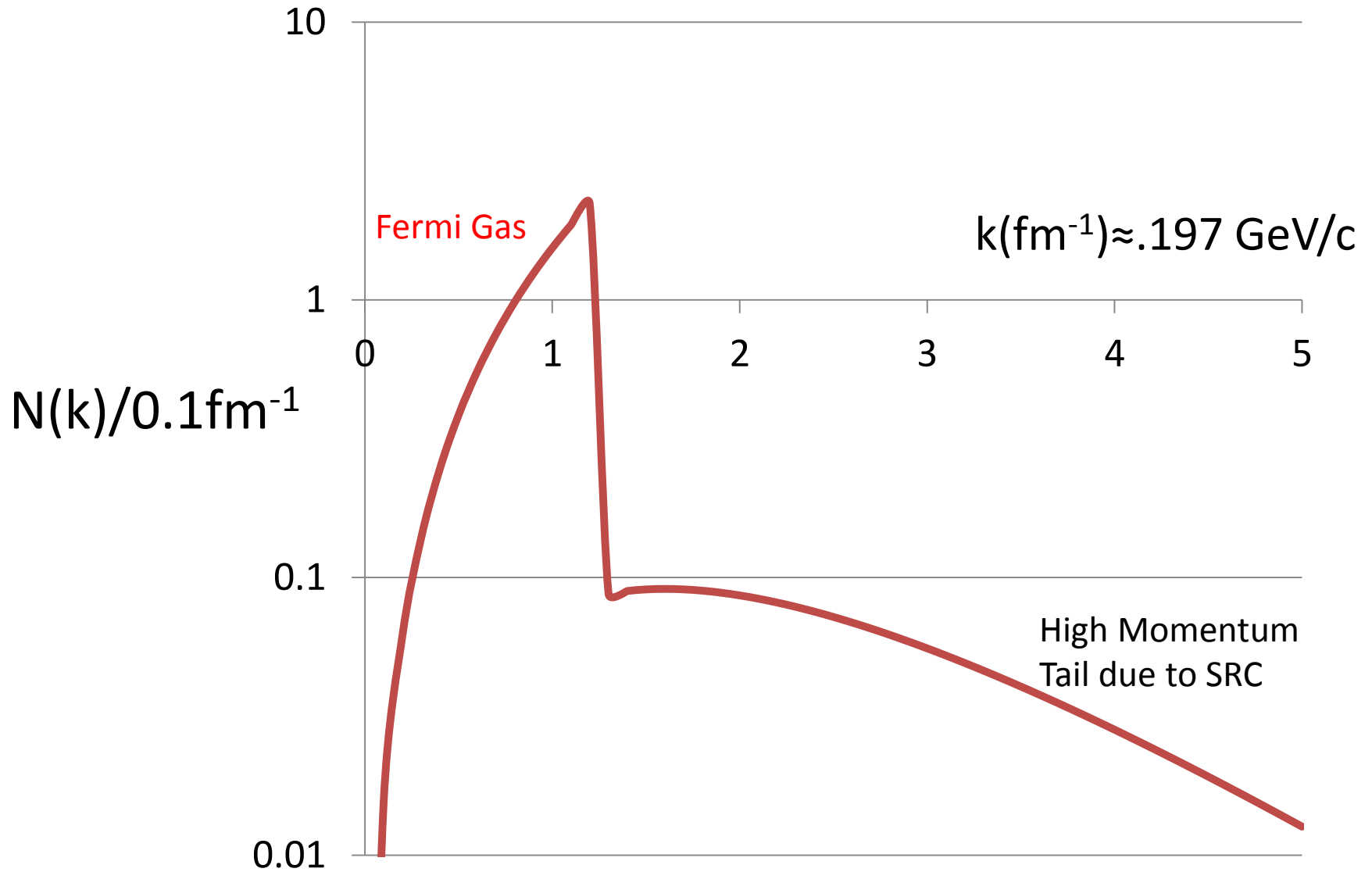
*The calculated value for  $R_{VT}(\psi')$  must be modified to account for neglected physics. The calculated one-body response must be enhanced by a factor  $R_{VT}(\psi') \times R_{VL}(\psi')$  ( $R_{T,V}(\psi')/R_L(\psi')$ ) where the latter ratio is say the one shown in PR C 65 024002.*

# NUANCE Breakdown of the QE Contributions to the MB Yields

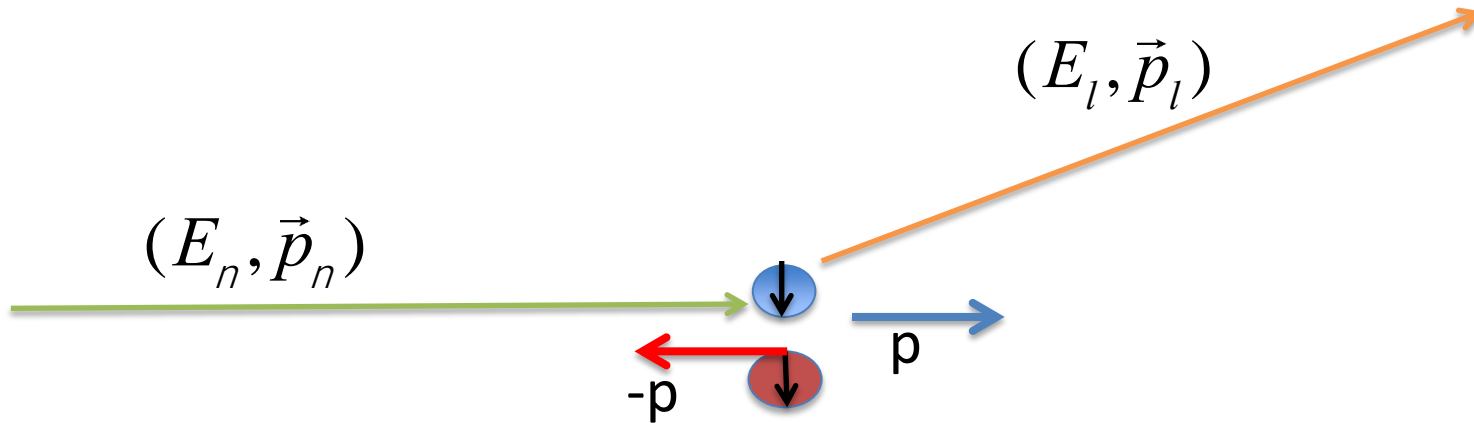


I will assume that only the Transverse Vector Response is effected by 2-n currents!!

# Simple Model for Momentum Distribution in $^{12}\text{C}$



# What's the Energy Loss in Collisions With High Momentum Tail?

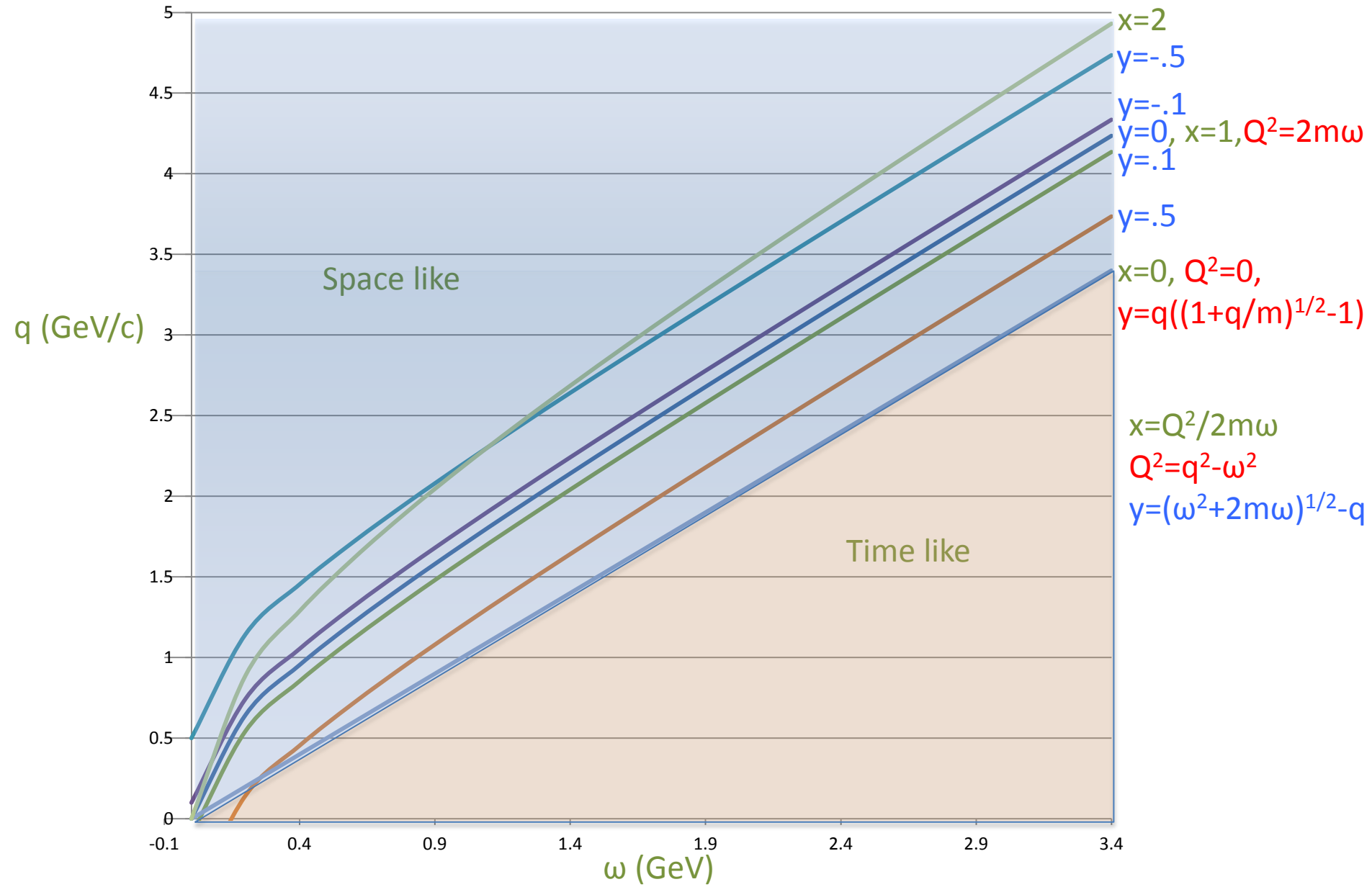


$$\vec{q} = \vec{p}_n - \vec{p}_l$$

$$W = \frac{(\vec{q} + \vec{p})^2}{2m} + \frac{p^2}{2m}$$

Different  
from FG

- It is impossible to capture all effects of the strong, short range N-N force with a mean field.
- For 40 years theorists maintained there were high momentum components in the nuclear wave function due to short range nucleon-nucleon correlations.
- Some manifestations are the deuteron quadruple moment (SR tensor force), depletion of shell model orbits, saturation of nuclear matter (short range repulsion).
- “Direct evidence” has been hard to come by until middle of last decade. PRL **90** 042301  $^{12}\text{C}(p,2p+n)$ , PRL **99**,072501 (e,e'p), PRL **108** 092502



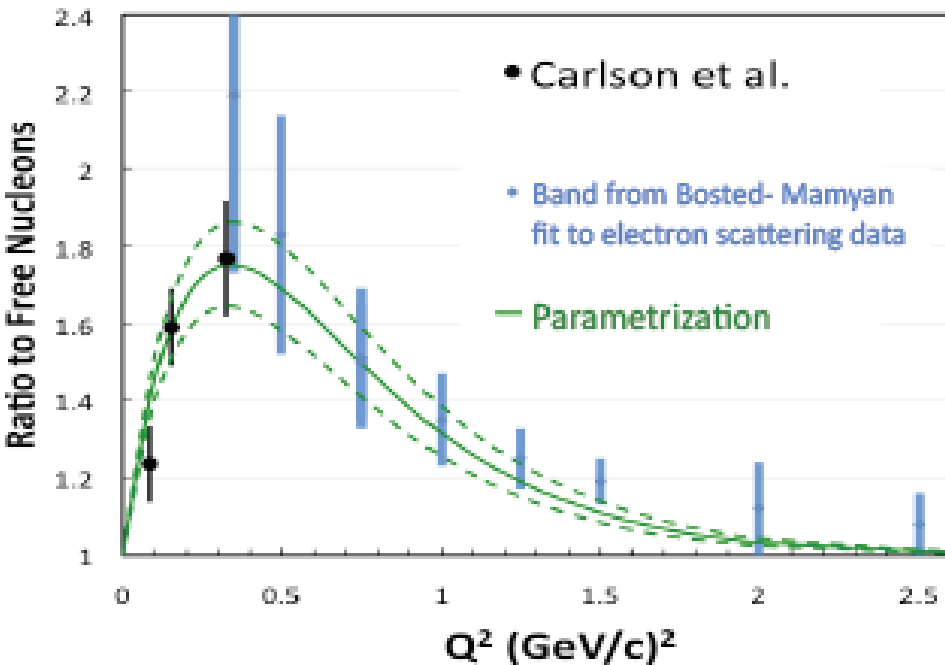


Why is the effect of correlations so evident in MB?

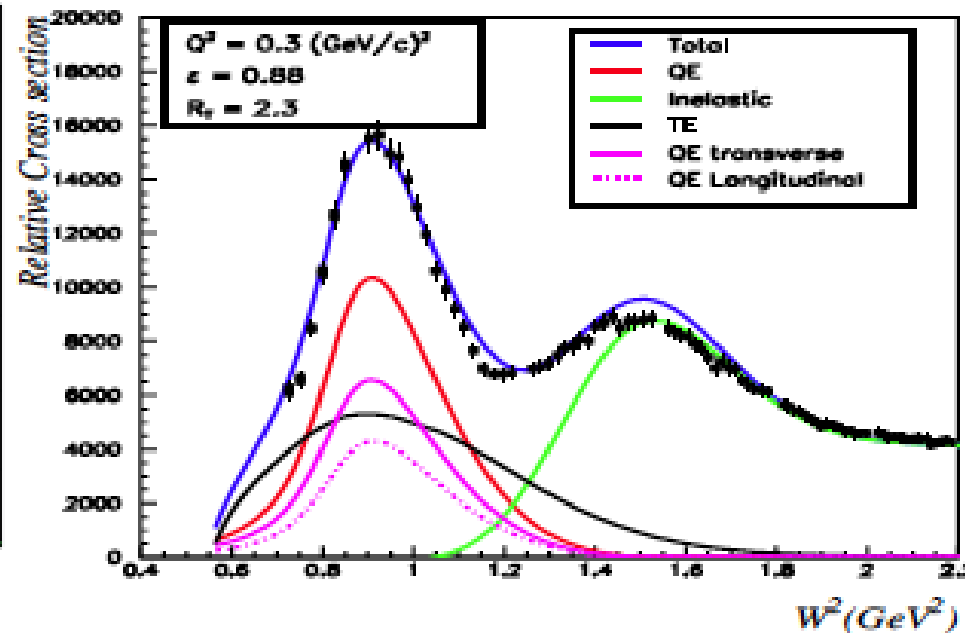
$$\frac{dS}{dQ^2} \propto V_L^2 + V_T^2 + A_T^2 \pm 2A_T V_T$$

Bodek et al Eur,Phys.J C71 1726 (2011); preliminary data from JUPITER coll. At JLAB (unpub.)

### Transverse Enhancement Carbon 12



### Preliminary E04-001, E = 1.204, $\theta = 28.011$

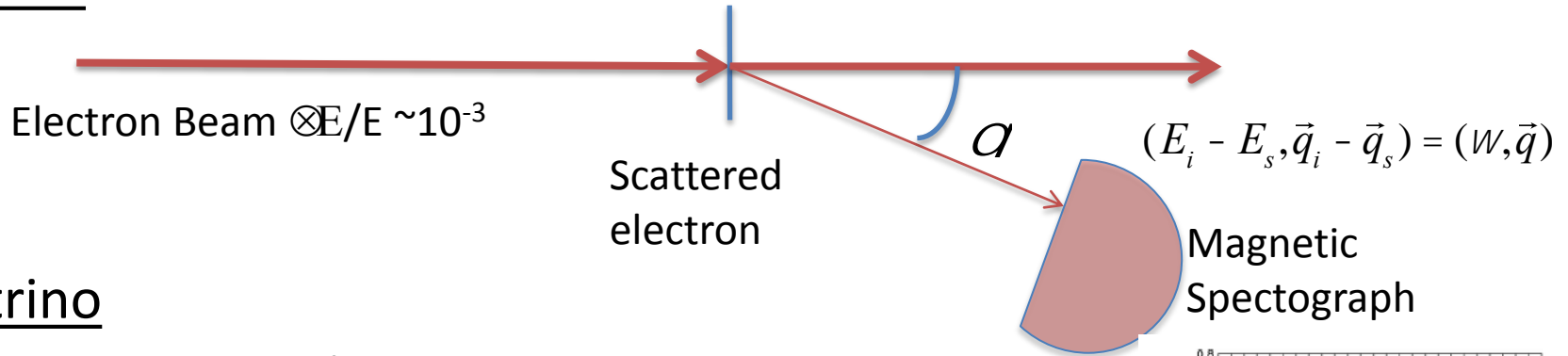


# What physics is required to Calculate “CCQE” scattering from Nuclei?

- “CCQE” events are those in which the weak interaction vertex creates only nucleons. Such events may have lepton energy transfer well beyond that inferred from the charged lepton momentum as the incident neutrino energy is unknown\*.  $(E_n, \vec{p}_n) - (E_l, \vec{p}_l) = (W, \vec{q})$
- Need an initial state momentum distribution of nucleons in the nucleus.
- Need an effective model for the energy transfer  $W$  for momentum transfer  $\vec{q}$ .
- Need the vector and axial vector form factors for nucleons at momentum transfer  $\vec{q}$ . ✓
- Need to know that nucleon structure not altered in nucleus. (y scaling) ✓
- Need the nuclear response for transfer  $(W, \vec{q})$ , likely using y scaling.
- With the above one can calculate  $\vec{p}_l$  for a flux of neutrinos  $N(E_n)dE_n$ , where each  $\vec{p}_l$  is associated with an  $E_n$ .

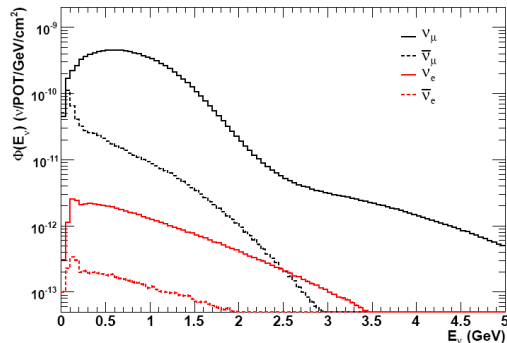
# Contrast of e-N with $\nu$ -N Experiments

## Electron



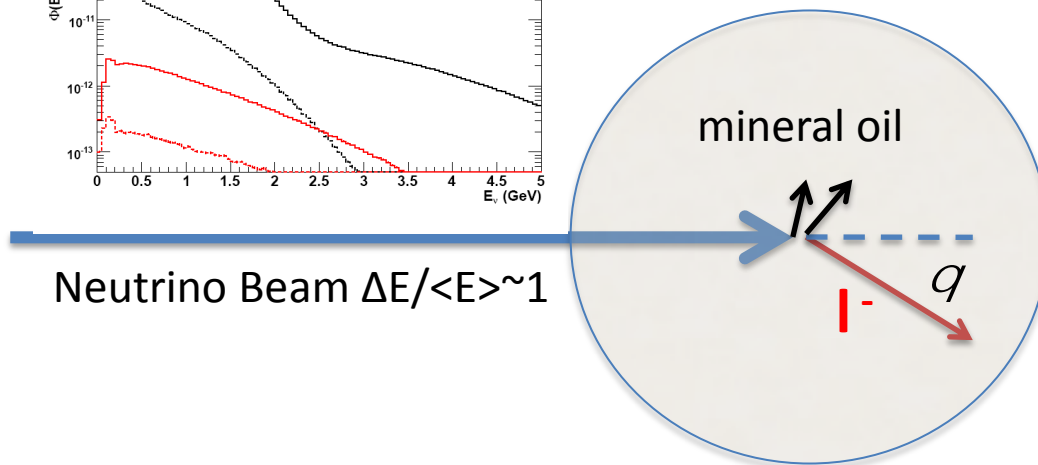
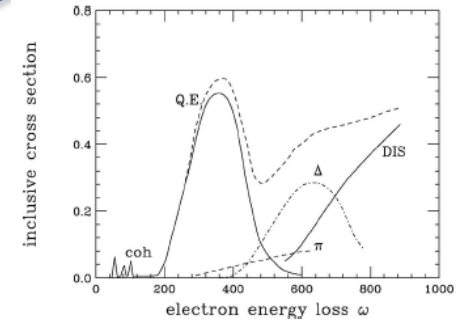
## Neutrino

### MiniBooNE Flux



$(E_i - E_s, \vec{q}_i - \vec{q}_s) = (W, \vec{q})$

MiniBooNE Detector



**Don't know  $E_\nu$  !!!**

**What's  $\omega$  ???**

**What's  $q$  ????**

**Very Different Situation from inclusive electron scattering!!**

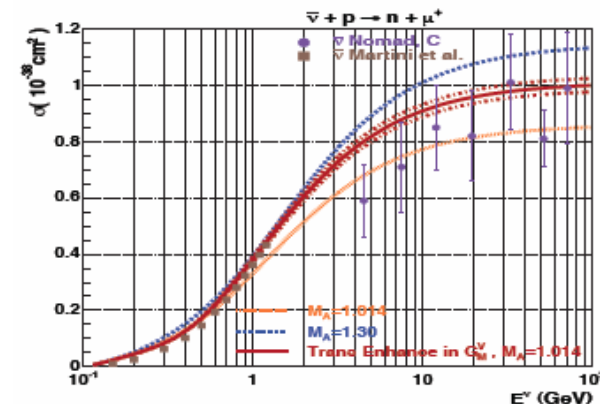
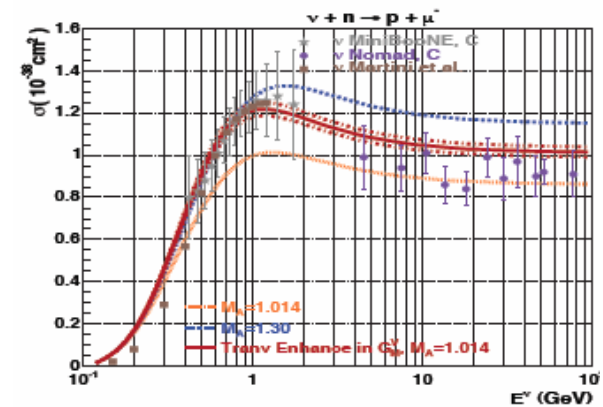
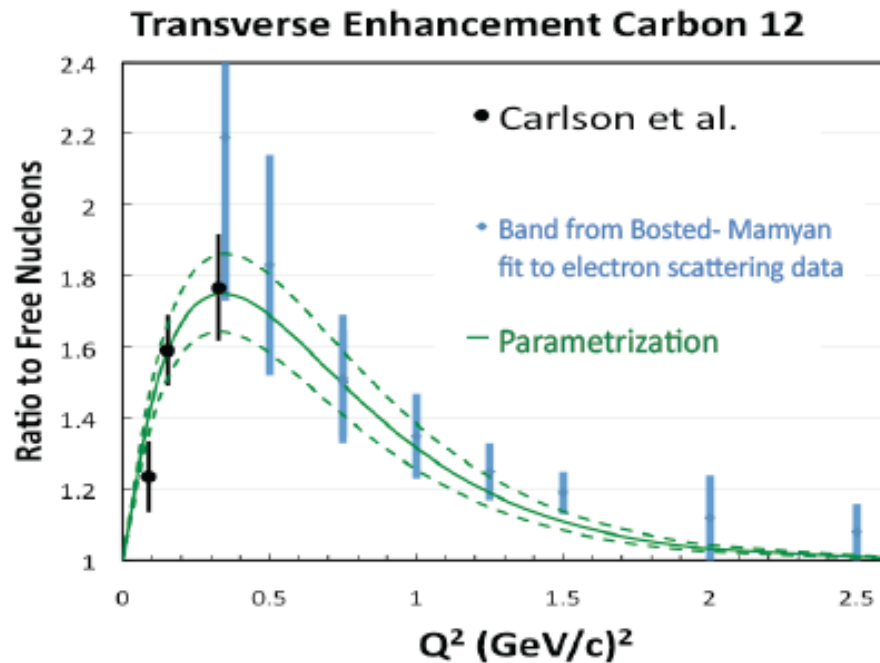
**Bodek et al:** Eur. Phys. J. **C71** (2011) 1726, much influenced by Carlson et al: PR **C65** 024002

Motivated by Carlson et al, Bodek et al. more correctly assigned the enhancement to the transverse vector response. In impulse approximation,

$$\sigma_{\nu,(\bar{\nu})} \sim V_L^2 + V_T^2 + A_T^2 + (-)2A_T V_T$$

Without addressing any dynamics Bodek et al. create the enhancement via increasing  $V_T$  as a function of  $Q^2$ , using  $Q^2 = 4E_\nu E_p \sin^2\theta/2$

$$R_\tau^V = 1 + 6.0Q^2 e^{-\frac{Q^2}{.34}}$$

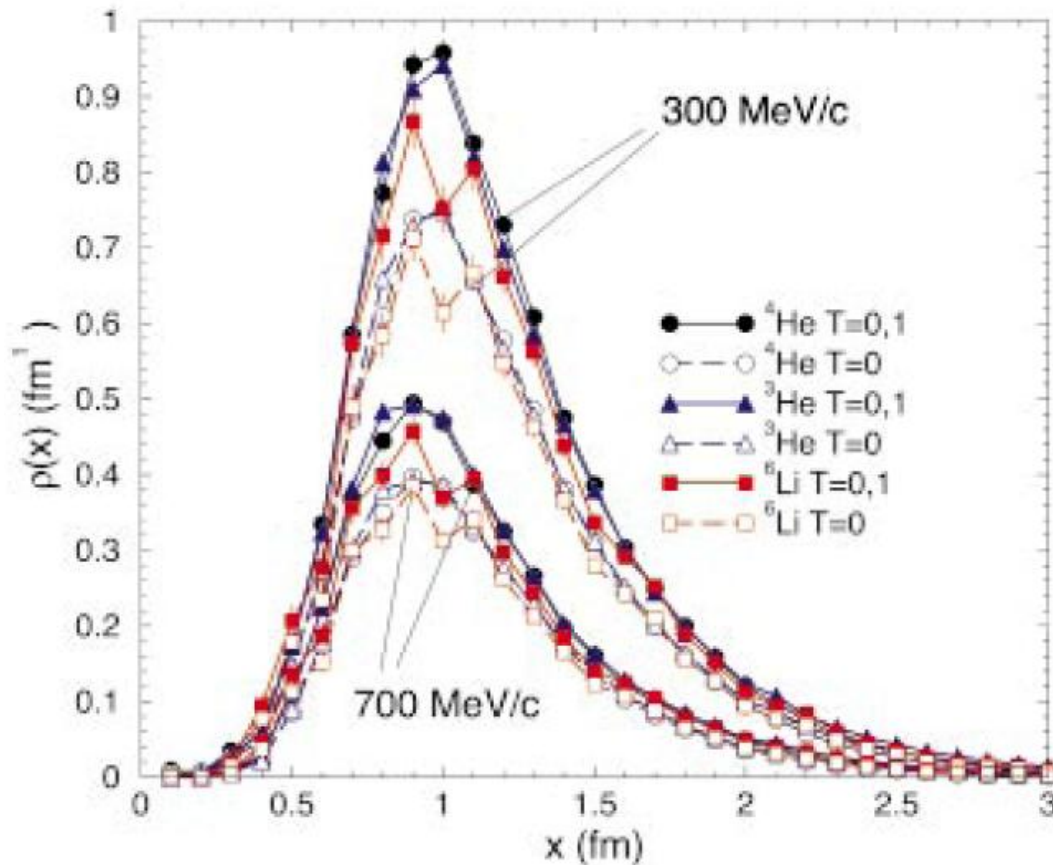


$$\Delta S_T(q) \equiv S_T(q) - S_T^{1B}(q) \quad \text{2-body contribution}$$

$$\Delta S_T(q) = \int_0^\infty dx \text{tr}[F(x; q) \rho(x; p_n)]$$

*2-body density*

*2-body current*



$x = \text{pair separation}$

# Some Observations

- *In mean field models  $f_L(\psi')$  would be symmetric about  $\psi' = 0$ , The asymmetric shift to more positive values of  $\psi'$  is due to the larger energy loss associated with the SRC pairs.*
- *The enhancement of  $f_T(\psi')$  becomes large for  $q > 300 \text{ MeV}/c$ .*
- *The large 2-body enhancement of  $f_T(\psi')$  requires adequate treatment of the initial and final nucleon states as well as 2-body currents.*
- *This is the likely source of the of the larger than expected MB cross section and the fact that the enhancement is associated with large energy loss indicate that its effects should be included when assigning incident neutrino energies.*