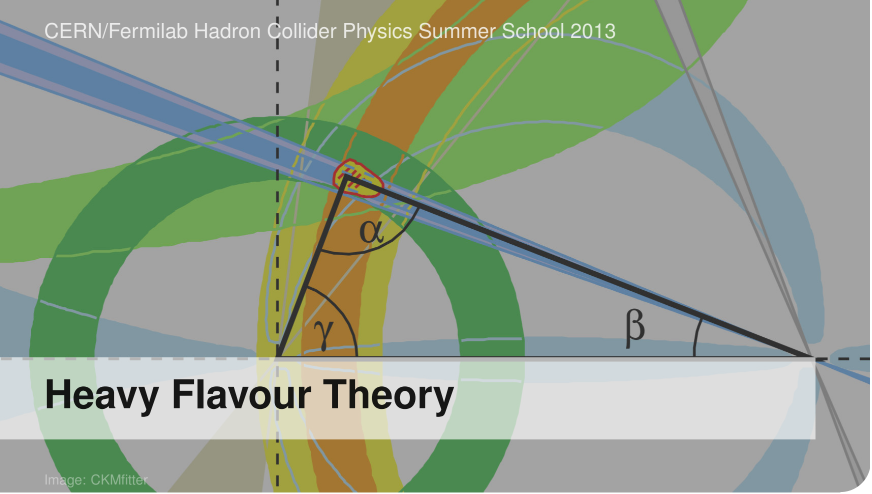
CERN/Fermilab Hadron Collider Physics Summer School 2013



Heavy Flavour Theory

Image: CKMfitter

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Plan of Part I (today)

1. CKM mechanism
2. Theoretical tools

Plan of Part II (tomorrow)

3. Meson-antimeson mixing
4. Three types of CP violation
5. Rare decays

Part I

Flavour in the Standard Model

Flavour = replication of fields

The Standard Model fermions come in 3 copies with the same gauge quantum numbers

$$Q_L^{1,2,3} \sim (3,2)_{\frac{1}{6}} \qquad U_R^{1,2,3} \sim (3,1)_{\frac{2}{3}} \qquad D_R^{1,2,3} \sim (3,1)_{-\frac{1}{3}}$$

$$L_L^{1,2,3} \sim (1,2)_{\frac{1}{2}} \qquad E_R^{1,2,3} \sim (1,1)_{-1}$$

under $SU(3)_c \times SU(2)_L \times U(1)_Y$

Flavour physics deals with the interactions that distinguish these copies

Why flavour physics is important

1. Most of the *free parameters* of the SM are related to flavour
   - what are their values?
   - why do they have these values?
2. In the SM, the flavour sector is the only source of *CP violation*
   - CPV is one of the Sakharov conditions for baryogenesis, but is too weak in the SM
   - are there other sources of CPV?
3. *Flavour-changing neutral currents* are sensitive probes of physics beyond the SM
   - Rare decays
   - Meson-antimeson mixing

Flavour in the SM

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} - \mathcal{V}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{\text{gauge}} = \sum_{\psi} \bar{\psi} i \not{D} \psi + |D_\mu H|^2 - \sum_a \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

The *gauge interactions* are completely flavour symmetric, i.e. they are invariant under individual rotations of the five fermion fields in complex 3-dimensional flavour space

$$Q_L^i \to U^{ij} Q_L^j \qquad\qquad \text{etc.}$$

Group theoretically: the gauge sector has a $U(3)^5 = [SU(3) \times U(1)]^5$ flavour symmetry!

Flavour in the SM

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} - \mathcal{V}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

The *Higgs potential* is of course also flavour invariant

$$\mathcal{V}_{\text{Higgs}} = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Flavour in the SM

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} - \mathcal{V}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

Yukawa couplings are the only interactions distinguishing between flavours. Focusing on quarks from now on:

$$\mathcal{L}_{\text{Yukawa}} = \tilde{H}\,\bar{Q}_L^i\,Y_u^{ij}\,U_R^j + H\,\bar{Q}_L^i\,Y_d^{ij}\,D_R^j + \text{h.c.}$$

The flavour symmetry is badly broken!

The only flavour symmetry left in the quark sector is baryon number, i.e. an overall phase rotation of all fields

$$U(3)_Q \times U(3)_U \times U(3)_D \to U(1)_B$$

Yukawa couplings

$$\mathcal{L}_{\text{Yukawa}} = \tilde{H}\,\bar{Q}_L^i\,Y_u^{ij}\,U_R^j + H\,\bar{Q}_L^i\,Y_d^{ij}\,D_R^j + \text{h.c.}$$

- after electroweak symmetry breaking $\langle H \rangle = (0\ v)^T$, $\mathcal{L}_{\text{Yukawa}}$ turns into a quark *mass term*
- by unitary field redefinitions, we can diagonalize *one of the two* mass matrices, e.g.

$$v\,V_Q^\dagger\,Y_u\,V_U = \text{diag}(m_u, m_c, m_t)$$
$$v\,V_Q^\dagger\,Y_d\,V_D = V^\dagger \text{diag}(m_d, m_s, m_b)$$

Flavour change in the mass basis

- we can get to the mass eigenstate basis, where both mass terms are diagonal, by rotating the up- and down components of $Q_L^i = (u_L^i \; d_L^i)^T$ independently. This *violates* the $SU(2)_L$ gauge symmetry, so the unitary matrix $V^\dagger$ shows up in the W boson vertex

- V is the Cabibbo-Kobayashi-Maskawa matrix V_{CKM}
- Crucially, *neutral* vertices are always *flavour diagonal* due to the unitarity of the mixing matrices
  This is called the *GIM* (Glashow-Iliopoulos-Maiani) *mechanism*

Physical parameters in the Yukawa couplings

Quark sector:

- $Y_{u,d}$ contain 18 real parameters and 18 phases
- Using the $U(3)$ field redefinitions of Q_L, U_R and D_R, we can absorb 9 angles and 18 phases, which are therefore unphysical = unobservable
- Actually, this is not quite true since an overall phase rotation of all fields does not change the Yukawa couplings, so we can only absorb 18 — 1 phases

We end up with 9 real angles and 1 phase that can be identified with

- the *masses* of 3 up-type and 3 down-type quarks
- 3 angles and 1 phase in the *CKM matrix*

Free parameters of the SM

Let's count the number of physical parameters in $\mathcal{L}_{\text{SM}}$.

- gauge sector: $g_{1,2,3}$ — or α_{em}, α_s and $\sin\theta_w$
- QCD vacuum angle θ_{QCD}
- Higgs potential: μ and λ — or G_F and m_h
- Yukawa sector (massless neutrinos): 9 masses, 4 CKM parameters

$$3 + 1 + 2 + 13 = 19$$

Two thirds from the flavour sector!

CKM matrix: standard parametrization

The CKM matrix has 3 mixing angles and 1 phase.

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM matrix: standard parametrization

The CKM matrix has 3 mixing angles and 1 phase.

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$

CKM matrix: standard parametrization

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$c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$

Experimentally: $(s_{12}, s_{13}, s_{23}, \delta) \approx (0.225, 0.042, 0.0036, 70^\circ)$

CKM matrix: Wolfenstein parametrization

Since V_{CKM} turns out to be very hierarchical, it is often very useful to consider a different parametrization, expanding in $\lambda \equiv s_{12} = \sin\theta_C \approx 0.22$

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

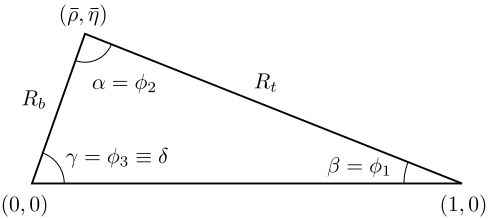
$$(\lambda, A, \bar{\rho}, \bar{\eta}) \approx (0.225, 0.82, 0.13, 0.35)$$

Unitarity triangle

V_{CKM} has to be a *unitary* matrix. This implies certain relations among its elements, in particular

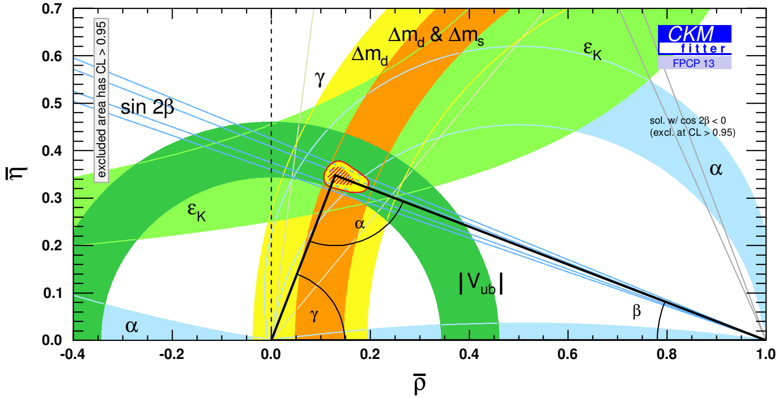
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

This can be represented as a *triangle* in a complex plane



$$(\alpha, \beta, \gamma) \approx (89^\circ, 22^\circ, 70^\circ)$$

Experimental status of the CKM mechanism



The CKM mechanism seems to be fundamentally at work

1. CKM mechanism

2. Theoretical tools

Multiple scales

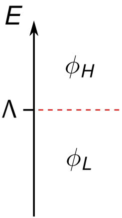
Weak decays always involve physics at vastly disparate energy scales, e.g.

- non-perturbative QCD interactions describing the hadrons, $\Lambda_{\text{QCD}} \sim 0.2$ GeV
- b quark mass ~ 4 GeV
- mass of the W mediating FCNCs ~ 80 GeV
- top quark mass ~ 170 GeV
- new heavy particles in loops $\sim$ TeV?

Such a multitude of scales can only be tackled with a powerful tool: *Effective field theory*

Effective field theory

We want to study physics at energies much lower than some scale Λ in a theory where particles lighter and heavier than Λ are present.



To this end, we can replace the complicated Lagrangian of the "full" theory by an *effective Lagrangian* containing only the light fields and a series of local operators built out of the light fields

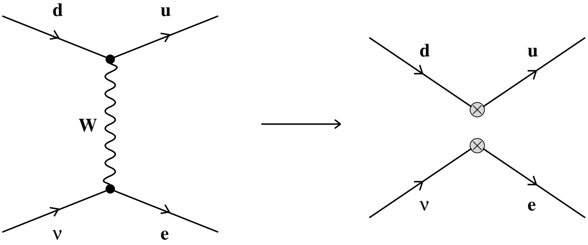
$$\mathcal{L}(\phi_L, \phi_H) \to \mathcal{L}(\phi_L) + \mathcal{L}_{\text{eff}} = \mathcal{L}(\phi_L) + \sum_i C_i\, Q_i(\phi_L)$$

This expansion is called the *operator product expansion*

Example: modern view of Fermi theory

In Fermi's model of β decay, the full weak Lagrangian (that he didn't know of course) is effectively replaced by the low-energy (QED) Lagrangian plus a single operator

$$\mathcal{L}_{\text{ew}} \to \mathcal{L}_{\text{QED}} + \frac{G_F}{\sqrt{2}}(\bar{u}d)(e\bar{\nu})$$



Local operator $\equiv$ effective vertex!

More about the OPE

$$\mathcal{L}_\text{eff} = \sum_i C_i \, Q_i(\phi_L)$$

- the local operators have mass dimension > 4, i.e. they are *non-renormalizable*
- operators with dimension $4 + n$ contribute with strength $(E/\Lambda)^n$ to a process with energy E; thus, the OPE can be *truncated* at some dimension d and typically a small number of operators is important
- C_i are called *Wilson coefficients* $\equiv$ effective coupling constants
- $\mathcal{H}_\text{eff} \equiv -\mathcal{L}_\text{eff}$

Weak effective Hamiltonian

$$\mathcal{H}_\text{eff} = \frac{4G_F}{\sqrt{2}} \sum_i \xi^i_\text{CKM}\, C_i\, Q_i$$

- we only have to consider operators up to *dimension 6*
- since flavour-change is always mediated by the W boson, one can factor out the Fermi constant $\frac{4G_F}{\sqrt{2}} = \frac{g^2}{2m_W^2} \Rightarrow$ the WC of dimension-6 operators are *dimensionless*
- factoring out the CKM elements, the WC are *real* in the SM
- the amplitude of a weak decay takes the generic form

$$A(i \to f) = \langle f|\mathcal{H}_\text{eff}|i\rangle = \frac{4G_F}{\sqrt{2}} \sum_i \xi^i_\text{CKM}\, C_i(\mu)\, \langle f|Q_i(\mu)|i\rangle$$

Calculating Wilson coefficients: matching

- The values of the effective coupling constant should be such that amplitudes in the effective theory reproduce the ones in the full theory.

$$\mathcal{L}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{ud}\, C\, Q$$
$$Q = (\bar{u}_L \gamma^\mu d_L)(\bar{e}_L \gamma_\mu \nu_L)$$

- Requiring the amplitudes to coincide, one finds

$$\mathcal{A}_{\text{full}} = \frac{g^2}{2m_W^2} V_{ud} \langle Q \rangle \stackrel{!}{=} \frac{4G_F}{\sqrt{2}} V_{ud} C \langle Q \rangle = \mathcal{A}_{\text{eff}}$$
$$\Rightarrow C = 1$$

- This process is called *matching*

Detour: renormalization

In a QFT, infinities in calculations have to be removed by *renormalizing* bare parameters in the Lagrangian, e.g. in QCD

$$G^a_{0\mu} = \sqrt{Z_3} G^a_\mu \qquad q_0 = \sqrt{Z_q} q \qquad g_{0s} = Z_g g_s \mu^\epsilon \qquad m_0 = Z_m m$$

- 0: unrenornormalized = bare fields/parameters
- Z_i: renormalization constants
- μ: renormalization scale

Dimensional regularization + minimal subtraction: only poles in $\epsilon = 2 - d/2$ subtracted

$$Z_i = \frac{\alpha_s}{4\pi} \frac{a_{1i}}{\epsilon} + O(\alpha_s^2)$$

Renormalization scale

- g_s and m (in fact, all couplings in a QFT) become μ-dependent
- μ dependent terms are renormalization scheme dependent
- physical *observables* have to be *μ-independent*
- values of the parameters at different scales are connected by *renormalization group equations (RGE)*, e.g.

$$\frac{dg_s(\mu)}{d\ln\mu} = \beta(g_s(\mu))\,, \qquad \frac{dm(\mu)}{d\ln\mu} = -\gamma_m(g_s(\mu))\,m(\mu)$$

Operator renormalization

- Also Q_i have to be renormalized:

$$Q_i^0 = Z_{ij} Q_j$$

- C_i become scale-dependent
- The scale dependence is cancelled by the scale dependence of the matrix element

$$A(i \to f) = \frac{G_F}{\sqrt{2}} \sum_i \xi^i_{\text{CKM}} \, C_i(\mu) \, \langle f | Q_i(\mu) | i \rangle$$

Renormalizing non-renormalizable operators?

- In a *renormalizable* theory, all infinities can be removed to all orders in perturbation theory by a finite number of counterterms
- In a *non-renormalizable* theory, infinities have to be removed at any order and the number of subtraction terms is infinite
- In the OPE, once we renormalize the (finite number of) operators of a given dimension, the higher-dimensional ones are still divergent, but we don’t care since they are *not relevant for low-energy physics*

Modern view of renormalization: the low-energy limit of every EFT is a renormalizable theory

RGE for the Wilson coefficients

- Recall our multi-scale problem: which renormalization scale μ to choose for $C_i(\mu)$?
- C_i obey a RGE

$$\frac{d}{d\ln\mu} C_j(\mu) = \sum_i C_i(\mu)\, \gamma_{ij}(\mu)$$
$$\Rightarrow C_j(\mu_1) = U_{ji}(\mu_1, \mu_2)\, C_i(\mu_2)$$

- we calculate (match) C_i at a high scale where QCD is perturbative and use the RGE to evolve it down to the appropriate scale
- by “running” the RGEs, we are in effect running through a series of EFTs where μ playing the role of the scale Λ

Generic weak decay amplitude

$$A(i \to f) = \frac{4G_F}{\sqrt{2}}$$

$\sum_i \xi^i_{\text{CKM}}$	$C_i(m_W)$	$U(\mu_l, m_W)$	$\langle f \vert Q_i(\mu_l) \vert i \rangle$
CKM factors	short-distance	QCD corrections	hadronic matrix element
	— perturbative —		non-perturbative
	— indep. of external states —		specific for ext. state
	sensitive to NP	— independent of NP —	

The OPE has achieved a *separation of scales*

Part II

Flavour-changing neutral currents

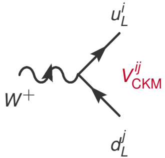
3. Meson-antimeson mixing

4. Three types of CP violation

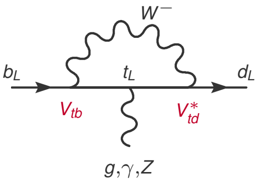
5. Rare decays

Flavour-changing neutral currents

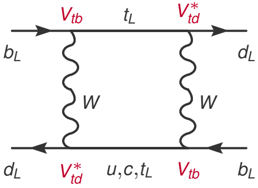
In the SM, the only flavour-changing coupling is the W vertex that changes also the electric charge:



However, flavour-changing neutral currents can be generated at loop level!



“Penguin diagram”

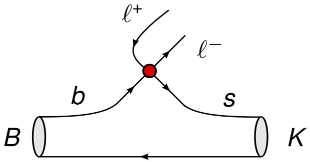


“Box diagram”

Two classes of FCNC processes

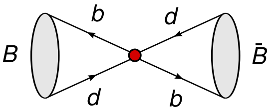
$\Delta F = 1$ = rare decays

e.g.



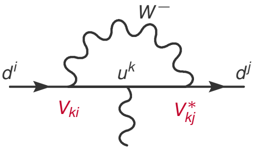
$\Delta F = 2$ = meson-antimeson mixing (K, B, B_s, D)

e.g.



GIM mechanism at 1-loop

Generic form of a $\Delta F = 1$ amplitude:



$$\sum V_{ki} V_{kj}^* F(m_{u^k}) = V_{ui} V_{uj}^* F(m_u) + V_{ci} V_{cj}^* F(m_c) + V_{ti} V_{tj}^* F(m_t)$$
$$\approx \left(V_{ui} V_{uj}^* + V_{ci} V_{cj}^*\right) F(0) + V_{ti} V_{tj}^* F(m_t)$$
$$= V_{ti} V_{tj}^* \left[F(m_t) - F(0)\right]$$

- FCNC amplitude would be zero if all masses were degenerate!
- FCNC amplitudes in B physics dominated by internal top quark exchange

3 Meson-antimeson mixing

4 Three types of CP violation

5 Rare decays

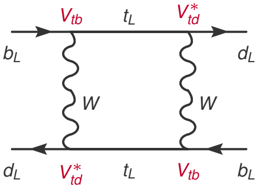
Computing the B^0 mixing amplitude

$$A(B^0 \to \bar{B}^0) \propto \sum_i \xi_{\text{CKM}}\, C_i(m_W)\, U(\mu_l, m_W)\, \langle \bar{B} | Q_i(\mu_l) | B \rangle$$

In the SM, the $\Delta B = 2$ effective Hamiltonian only contains a single operator:

$$Q_{LL} = (\bar{b}_L \gamma_\mu d_L)(\bar{b}_L \gamma^\mu d_L)$$

Evaluation of the box diagram



Taking into account the GIM mechanism, one finds

$$A(B^0 \to \bar{B}^0) = \frac{G_F^2}{4\pi^2} m_W^2 \, \xi_{\text{CKM}} \; C_{LL}(m_W) \; U(\mu_l, m_W) \; \langle \bar{B} | Q_{LL}(\mu_l) | B \rangle$$

$$= \frac{G_F^2}{4\pi^2} m_W^2 (V_{tb}^* V_{td})^2 \; S_0(x_t) \; \eta_B(\mu_l) \; \langle \bar{B} | Q_{LL}(\mu_l) | B \rangle$$

where $x_t = m_t^2 / m_W^2$ and $\eta_B(\mu_l)$ summarizes the (perturbative) QCD corrections.

Missing piece: matrix element

The quantity $\langle \bar{B}|Q_{LL}(\mu_l)|B\rangle$ cannot be calculated by perturbative means and one has to rely on *lattice QCD*. One can write

$$\eta_B(\mu_l)\langle \bar{B}|Q_{LL}(\mu_l)|B\rangle = \frac{2}{3} f_B^2 m_B^2 \eta_B(\mu_l) B(\mu_l) = \frac{2}{3} f_B^2 m_B^2 \hat{\eta}_B \hat{B}$$

- f_B: B meson decay constant
- $\hat{B}$: bag parameter for B mixing
- NB: the μ_l dependence in η_B cancels with the one of B
- an average of recent computations yields $f_B\sqrt{\hat{B}} = (227 \pm 19)$ MeV
- This is the main limiting factor in the theoretical precision

Summary

Our final result reads

$$A(B^0 \to \bar{B}^0) = \frac{G_F^2}{6\pi^2} m_W^2 m_B \, (V_{tb}^* V_{td})^2 \; S_0(x_t) \; \hat{\eta}_B \; f_B^2 \hat{B}$$

and we needed

- CKM elements,
- short-distance contributions (box diagram!),
- QCD corrections, and
- input from the lattice.

Time evolution of an unstable meson

Consider the time evolution of a meson state $|M\rangle$

$$i\frac{d}{dt}|M(t)\rangle = \left(M_M - i\frac{\Gamma}{2}\right)|M(t)\rangle$$

where M_M is the meson mass and $\Gamma = 1/\tau$ the decay width

$$|M(t)\rangle = e^{-iMt}\,e^{-\Gamma t/2}\,|M(0)\rangle$$

Quantum mechanics of B-$\bar{B}$ mixing

Now: consider a coupled meson-antimeson system

$$i\frac{d}{dt}\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix} = \begin{pmatrix}M - i\frac{\Gamma}{2} & M_{12} - i\frac{\Gamma_{12}}{2}\\ M_{12}^* - i\frac{\Gamma_{12}^*}{2} & M - i\frac{\Gamma}{2}\end{pmatrix}\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix}$$

- *mass* M is of the order of m_b
- *lifetime* Γ is determined by the weak interaction, roughly $\Gamma \propto G_F^2 m_b^5 V_{cb}^2$
- M_{12} is the mixing amplitude we calculated ($\to$ box diagram)

$$M_{12} = \frac{1}{2M}A(B^0 \to \bar{B}^0) = \frac{1}{2M}\langle\bar{B}^0|\mathcal{H}_{\text{eff}}^{\Delta B=2}|B^0\rangle$$

- Γ_{12} is the absorptive part of the box diagram, i.e. with real (not virtual) intermediate states. Dominated by light states (= long distance), hard to estimate

Diagonalizing the system

Let us start in the limit of CP symmetry: $\delta_{\text{CKM}} \to 0 \Rightarrow M_{12}, \Gamma_{12} \in \mathbb{R}$.

We obtain two mass eigenstates after diagonalization,

$$B_{L,H} = \frac{1}{\sqrt{2}} \left(|B\rangle \pm |\bar{B}\rangle \right)$$

$$i \frac{d}{dt} \begin{pmatrix} |B_L(t)\rangle \\ |\bar{B}_H(t)\rangle \end{pmatrix} = \begin{pmatrix} M_L - i\frac{\Gamma_L}{2} & 0 \\ 0 & M_H - i\frac{\Gamma_H}{2} \end{pmatrix} \begin{pmatrix} |B_L(t)\rangle \\ |\bar{B}_H(t)\rangle \end{pmatrix}$$

The mass and width differences are

$$\Delta M = M_H - M_L = 2|M_{12}| \qquad |\Delta\Gamma| = |\Gamma_H - \Gamma_L| = 2|\Gamma_{12}|$$

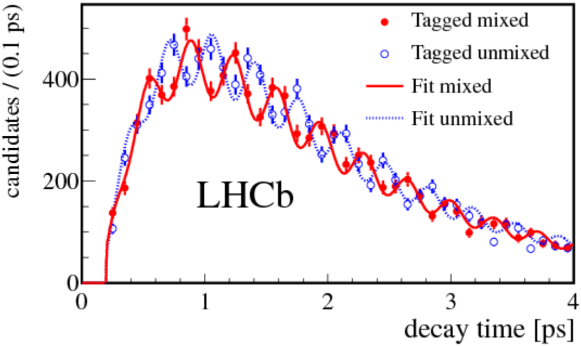
Solving the Schrödinger equation

In the meson rest frame, an initially pure flavour eigenstate evolves according to

$$|B(t)\rangle = e^{-iMt}\, e^{-\Gamma t/2}\, [a(t)|B\rangle + b(t)|\bar{B}\rangle]$$

$$\begin{aligned} a(t) &= \quad \cosh\left(\frac{\Delta\Gamma t}{4}\right)\cos\left(\frac{\Delta M t}{2}\right) - i\sinh\left(\frac{\Delta\Gamma t}{4}\right)\sin\left(\frac{\Delta M t}{2}\right) \\ b(t) &= -\sinh\left(\frac{\Delta\Gamma t}{4}\right)\cos\left(\frac{\Delta M t}{2}\right) + i\cosh\left(\frac{\Delta\Gamma t}{4}\right)\sin\left(\frac{\Delta M t}{2}\right) \end{aligned}$$

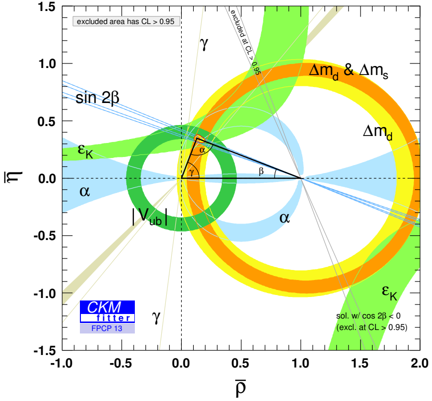
Measurement



LHCb measurement of B_s-$\bar{B}_s$ mixing, April 2013

ΔM_d and the unitarity triangle

$$\Delta M_d = 2|M_{12}| \propto |(V_{tb}V_{td}^*)^2| \approx (A\lambda^3)^2 \left[(1-\rho)^2 + \eta^2\right]^2$$



Enter CP violation

We know that the weak interactions don't respect CP, so we expect $M_{12} \neq M_{12}^*$ and $\Gamma_{12} \neq \Gamma_{12}^*$

$$B_{L,H} = p|B\rangle \pm q|\bar{B}\rangle$$

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}$$

- By rephasing $|B\rangle$ or $|\bar{B}\rangle$, we can remove all phases in M_{12}, Γ_{12} and q/p except one
- We end up with 3 physical *meson mixing parameters*

$$|M_{12}| \qquad |\Gamma_{12}| \qquad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

Three types of CP violation

- CP violation in *mixing* (= indirect CPV)
- CP violation in *decay* (= direct CPV)
- CP violation in the *interference of mixing and decay* (= mixing-induced CPV)

1. CP violation in mixing

CP violation in mixing caused by $M_{12} \neq M_{12}^*$ and/or $\Gamma_{12} \neq \Gamma_{12}^*$

$$B_{L,H} = p|B\rangle \pm q|\bar{B}\rangle$$

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}$$

Basis of physical observables:

$$|M_{12}| \qquad |\Gamma_{12}| \qquad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

CPV in mixing $\Leftrightarrow \phi \neq 0 \Leftrightarrow |q/p| \neq 1$

Simplification: assume $\Delta\Gamma \ll \Delta M$

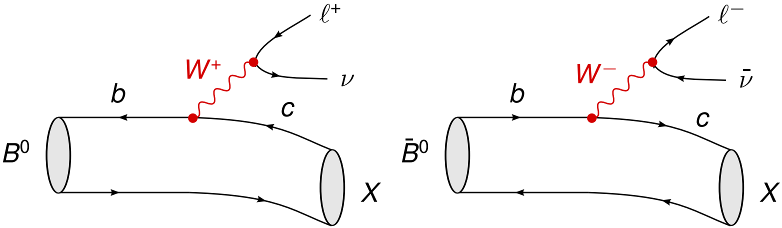
This holds in the case of B and B_s mixing. Then,

$$\Delta M \approx 2|M_{12}| \qquad \Delta\Gamma \approx 2|\Gamma_{12}|\cos\phi \qquad \phi = \arg\left(-M_{12}/\Gamma_{12}\right)$$

$$\frac{q}{p} \approx -\frac{M_{12}^*}{|M_{12}|}$$

CP violation in mixing

- To isolate CPV in mixing, consider decays that are only allowed in the presence of mixing
- Semi-leptonic asymmetry: “wrong-charge” decays



$$A_{sl}(B) = \frac{\Gamma(\bar{B}^0(t) \to l^+ \nu X) - \Gamma(B^0(t) \to l^- \bar{\nu} X)}{\Gamma(\bar{B}^0(t) \to l^+ \nu X) + \Gamma(B^0(t) \to l^- \bar{\nu} X)}$$

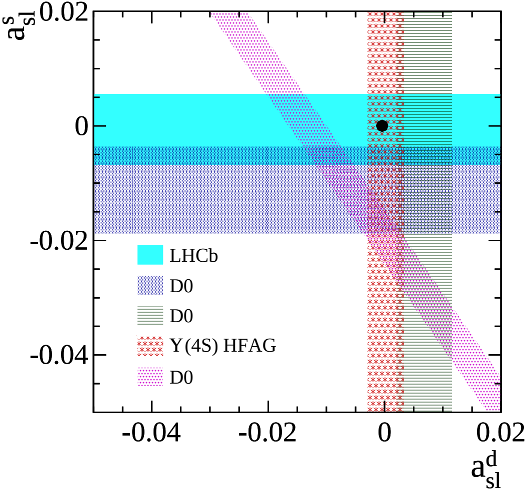
Semi-leptonic CP asymmetry

$$A_{\rm sl}(B) = \frac{\Gamma(\bar{B}^0(t) \to l^+ \nu X) - \Gamma(B^0(t) \to l^- \bar{\nu} X)}{\Gamma(\bar{B}^0(t) \to l^+ \nu X) + \Gamma(B^0(t) \to l^- \bar{\nu} X)} = \frac{1 - |q/p|^4}{1 + |q/p|^4}$$

- This is very small for B and B_s mixing, where $|q/p| \approx 1$

$$A_{\rm sl}(B) \approx \text{Im}\frac{\Gamma_{12}}{M_{12}} = \frac{\Delta\Gamma}{\Delta M} \tan\phi$$

Measurements



[arXiv:1308.1048]

2. CP violation in decay

- CPV in decay (= direct CPV) is best isolated in charged *B* decays

$$A_{\text{CP}}^{\text{dir}}(B \to f) = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)}$$

- Decay rate and amplitude are related as $\Gamma \propto |\mathcal{A}|^2$. How to get $A_{\text{CP}}^{\text{dir}} \neq 0$?

Strong & weak phases

- Imaginary part of the amplitude can originate from
  - *CP violating phases*, i.e. CKM elements or new physics contributions, also called *"weak" phases*
  - Rescattering effects from on-shell intermediate states: *"strong" phases*
- If amplitude receives several contributions from different types of diagram

$$\Gamma(B^+ \to f^+) = A_1 e^{i(\delta_1+\phi_1)} + A_2 e^{i(\delta_2+\phi_2)}$$
$$\Gamma(B^- \to f^-) = A_1 e^{i(\delta_1-\phi_1)} + A_2 e^{i(\delta_2-\phi_2)}$$
$$\Rightarrow A_{\text{CP}}^{\text{dir}}(B \to f) \propto -2A_1 A_2 \sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2)$$

- $A_{\text{CP}}^{\text{dir}}$ only non-zero in the presence of ≥ 2 contributions with *different weak and strong phases*

Triple product asymmetries

- In some decays, one can form asymmetries in kinematical quantitites that are a triple product of 3 independent 3-vectors (spins or momenta)

$$A_T = \frac{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) - \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0)}{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) + \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0)}$$

$$\mathcal{A}_T = \frac{1}{2}(A_T + \bar{A}_T)$$

- This quantity is odd under a T tansformation and, by means of CPT, CP-odd
- It can be shown that $\mathcal{A}_T$ is not suppressed by small strong phases:

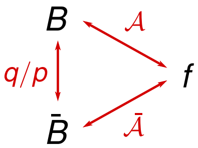
$$\mathcal{A}_T \propto \cos(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)$$

3. CP violation in the interference of mixing and decay

Time-dependent CP asymmetry in neutral B decays to CP eigenstates ($f = \bar{f}$)

$$A_{\text{CP}}(t, f) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)}$$

$$\approx A_{\text{CP}}^{\text{dir}}(f) \cos(\Delta M t) + A_{\text{CP}}^{\text{mix}}(f) \sin(\Delta M t)$$



Mixing-induced CP asymmetry

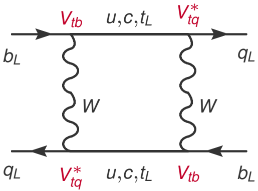
$$\xi_f = \frac{q}{p}\frac{\bar{\mathcal{A}}}{\mathcal{A}} \qquad \Rightarrow A_{\mathrm{CP}}^{\mathrm{dir}}(f) = \frac{1-|\xi_f|^2}{1+|\xi_f|^2}\,, \qquad A_{\mathrm{CP}}^{\mathrm{mix}}(f) = \frac{2\,\mathrm{Im}\xi_f}{1+|\xi_f|^2}$$

- In the $B_{d,s}$ system, $\Delta\Gamma \ll \Gamma \Rightarrow q/p \approx e^{-i\phi}$
- Particularly interesting: *"Golden modes"* where all contributions to the decay amplitude carry the same weak phase

$$\Rightarrow \frac{\bar{\mathcal{A}}}{\mathcal{A}} = -\eta_f e^{-2i\phi_D}\,, \quad A_{\mathrm{CP}}^{\mathrm{dir}}(f) = 0\,, \quad A_{\mathrm{CP}}^{\mathrm{mix}}(f) = -\sin(2\phi_D - \phi)$$

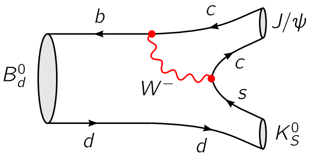
where $\eta_f = \pm 1$ is the CP parity of the final state

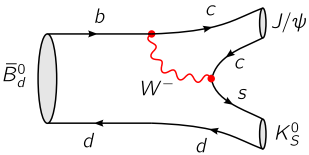
B_q ($q = d, s$) mixing phases



$$\phi(B_q) = \arg(V_{tb}^* V_{tq})$$
$$= 2\beta \qquad \text{for } B_d$$
$$= 2\beta_s \qquad \text{for } B_d$$

Golden mode $B^0 \to J/\psi K_S$

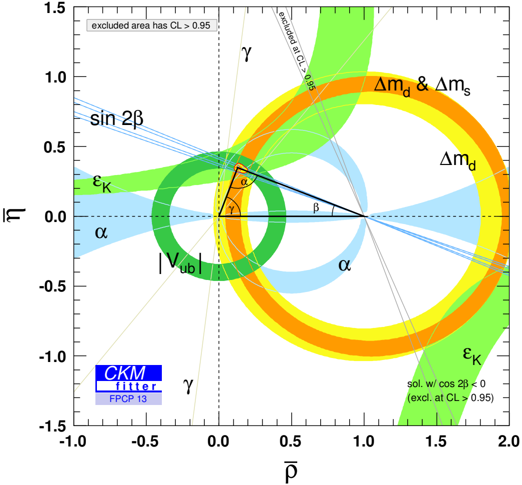




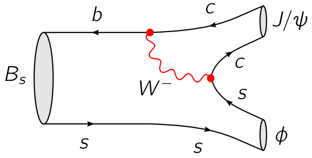
$$\phi_D = \arg(V_{cb}^* V_{cs}) = 0$$

$$A_{CP}^{\text{mix}}(J/\psi K_S) = -\sin(2\beta)$$

sin 2β and the unitarity triangle



Another golden mode: $B_s \to J/\psi\phi$



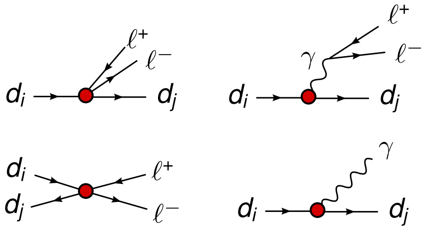
$$A_{CP}^{\text{mix}}(J/\psi\phi) = \sin(2\beta_s) \approx 0.04$$

LHCb 2013:

$$\sin(2\beta_s) = -0.01 \pm 0.07$$

Rare B (and K) decays

- Rare decays = FCNC decays with $\Delta F = 1$
- particularly interesting: leptonic, semi-leptonic and radiative decays



Generic form of $\Delta F = 1$ amplitude

$$A(B \to f) = V_{tb}^* V_{tq} \sum_i C_i(m_W)\, U(\mu_l, m_W)\, \langle f | Q_i(\mu_l) | B \rangle$$

- C_i and U can be calculated perturbatively
- Matrix element:
  1. *Inclusive decays* (sum over all possible hadronic final states): Related to (calculable) quark level decay by *heavy quark effective theory*. E.g.

$$\text{BR}(B \to X_s\gamma) = \text{BR}(b \to s\gamma) + O(\Lambda_{\text{QCD}}^2 / m_b^2)$$

     Theoretically clean, but experimentally challenging, in particular at hadron machines

Generic form of $\Delta F = 1$ amplitude

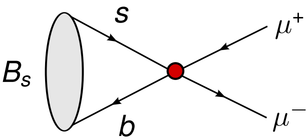
$$A(B \to f) = V_{tb}^* V_{tq} \sum_i C_i(m_W)\, U(\mu_l, m_W)\, \langle f | Q_i(\mu_l) | B \rangle$$

- C_i and U can be calculated perturbatively
- Matrix element:
  2. *Exclusive decays* (definite hadronic final state): need for hadronic *form factors*, e.g.

$$\langle \bar{K}^* | \bar{s}\gamma_5 b | \bar{B} \rangle \propto A_0(q^2)$$

  Non-perturbative quantities $\Rightarrow$ lattice QCD or models

$B_s \to \mu^+\mu^-$



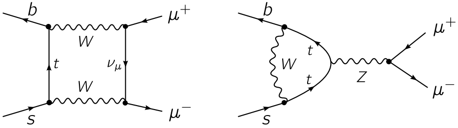
- helicity-suppressed since it vanishes for massless leptons (in addition to the loop- and CKM-suppression) $\Rightarrow$ one of the rarest B decays
- non-hadronic final state $\Rightarrow$ relatively clean theoretically (for an exclusive decay)
- clean experimental signature

$B_s \to \mu^+\mu^-$ amplitude

- Operators: in the SM only 1

$$Q_{10} = (\bar{s}_L \gamma_\mu b_L)(\bar{\mu}\gamma^\mu \gamma_5 \mu)$$

- Wilson coefficient:



- Matrix element:

$$\langle 0|\bar{s}\gamma^\mu \gamma_5 b|\bar{B}_s\rangle = i f_{B_s} p^\mu$$

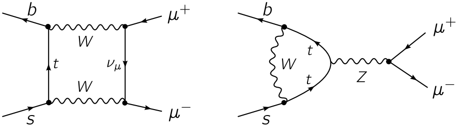
only 1 number, the *decay constant* $\Leftarrow$ lattice

$B_s \to \mu^+\mu^-$ amplitude

- Operators: in the SM only 1

$$Q_{10} = (\bar{s}_L \gamma_\mu b_L)(\bar{\mu}\gamma^\mu \gamma_5 \mu)$$

- Wilson coefficient:



- Matrix element:

$$\langle 0|\bar{s}\gamma^\mu \gamma_5 b|\bar{B}_s\rangle = i f_{B_s} p^\mu$$

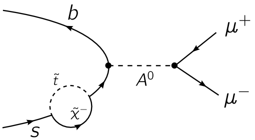
only 1 number, the *decay constant* $\Leftarrow$ lattice

$$\text{BR}_{\text{SM}} = (3.2 \pm 0.2) \times 10^{-9} \qquad \text{BR}_{\text{LCHb+CMS 2013}} = (2.9 \pm 0.7) \times 10^{-9}$$

$B_s \to \mu^+\mu^-$ beyond the SM

Example: in supersymmetry, new operators are generated by heavy Higgs exchange

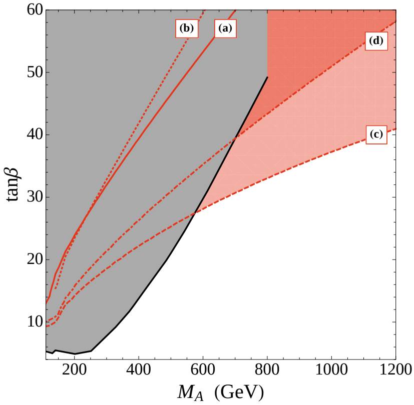
$$Q_S = (\bar{s}_L b_R)(\bar{\mu}\mu)\,, \qquad Q_P = (\bar{s}_L b_R)(\bar{\mu}\gamma_5\mu)$$



- This contribution can greatly enhance the branching ratio for large $\tan\beta = v_u/v_d$

$$C_S \simeq -C_P \propto \frac{\mu A_t}{m_{\tilde{t}}^2}\frac{m_{B_s} m_\mu}{m_A^2}\tan^3\beta$$

$B_s \to \mu^+\mu^-$ constraint on the MSSM



gray: $A, H \to \tau^+\tau^-$

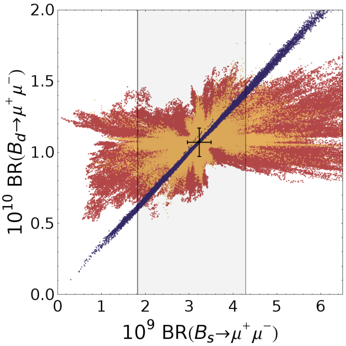
[Altmannshofer, Carena, Shah (2012)]

- measurement constrains the parameters tan β and M_A
- large tan β + light M_A disfavoured
- constraint is complementary to direct searches for heavy Higgs

$B_{s,d} \to \mu^+\mu^-$ beyond the SM

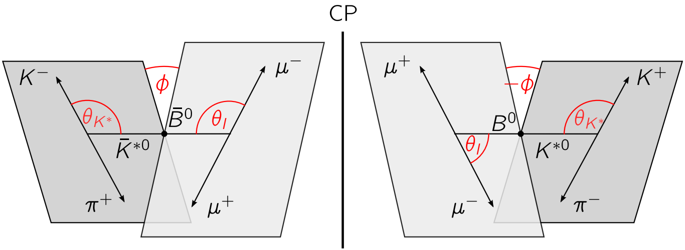
- Even in the absence of scalar/pseudoscalar operators, visible new physics effects can be generated in $B_s \to \mu^+\mu^-$ and $B_d \to \mu^+\mu^-$
- Example: models with partial compositeness. Contributions to

$$Q_{10} = (\bar{s}_L \gamma_\mu b_L)(\bar{\mu}\gamma^\mu \gamma_5 \mu) \qquad Q'_{10} = (\bar{s}_R \gamma_\mu b_R)(\bar{\mu}\gamma^\mu \gamma_5 \mu)$$



[arXiv:1302.4651]

$B \to K^* \mu^+ \mu^-$



- exclusive semi-leptonic decay probing the $b \to s$ transition
- 4-body decay: angular distribution with many observables sensitive to NP
- “self-tagging”: sensitive to CP violation

$B \to K^*(\to K\pi)\mu^+\mu^-$ angular decay distribution

$$\frac{1}{(\Gamma+\bar\Gamma)}\frac{d^4(\Gamma+\bar\Gamma)}{dq^2\,d\cos\theta_l\,d\cos\theta_{K^*}\,d\phi}=\frac{9}{32\pi}\times$$

$$\Big\{-\tfrac{3}{4}(F_L-1)\sin^2\theta_{K^*}+F_L\cos^2\theta_{K^*}$$

$$-\left(\tfrac{1}{4}(F_L-1)\sin^2\theta_{K^*}+F_L\cos^2\theta_{K^*}\right)\cos 2\theta_l$$

$$+S_3\sin^2\theta_{K^*}\sin^2\theta_l\cos 2\phi+S_4\sin 2\theta_{K^*}\sin 2\theta_l\cos\phi$$

$$+S_5\sin 2\theta_{K^*}\sin\theta_l\cos\phi$$

$$+\frac{4}{3}A_{FB}\sin^2\theta_{K^*}\cos\theta_l+S_7\sin 2\theta_{K^*}\sin\theta_l\sin\phi$$

$$+S_8\sin 2\theta_{K^*}\sin 2\theta_l\sin\phi+S_9\sin^2\theta_{K^*}\sin^2\theta_l\sin 2\phi\Big\}$$

Observables: differential branching ratio and 8 angular observables $S_i(q^2)$

$B \to K^* \mu^+ \mu^-$ amplitude

$$A(B \to K^* \mu^+ \mu^-) = V_{tb}^* V_{ts} \sum_i C_i(m_W)\, U(\mu_l, m_W)\, \langle K^* | Q_i(\mu_l) | B \rangle + \text{n.f.}$$

- Many operators contribute, some of them sensitive to NP, e.g.

$$Q_7 = (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu} \quad Q_9 = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell) \quad Q_{10} = (\bar{s}_L \gamma_\mu b)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

- Matrix elements $\langle K^* | Q_i(\mu_l) | B \rangle$ expressed in terms of 7 *form factors* $f_i(q^2)$, calculated e.g. on the lattice, with light-cone sum rules, . . .
- n.f. = non-factorizable corrections. Can be partially calculated in the heavy quark limit

$B \to K^* \mu^+ \mu^-$ amplitude

$$A(B \to K^* \mu^+ \mu^-) = V_{tb}^* V_{ts} \sum_i C_i(m_W)\, U(\mu_l, m_W)\, \langle K^* | Q_i(\mu_l) | B \rangle + \text{n.f.}$$

- Many operators contribute, some of them sensitive to NP, e.g.

$$Q_7 = (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu} \qquad Q_9 = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell) \qquad Q_{10} = (\bar{s}_L \gamma_\mu b)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

- Matrix elements $\langle K^* | Q_i(\mu_l) | B \rangle$ expressed in terms of 7 *form factors* $f_i(q^2)$, calculated e.g. on the lattice, with light-cone sum rules, ...
- n.f. = non-factorizable corrections. Can be partially calculated in the heavy quark limit
- Some tensions in exp. vs. SM predictions, but too early to draw firm conclusions [arXiv:1308.1701, 1307.5683, 1308.1501, 1308.1959]

$B \to K^*(\to K\pi)\mu^+\mu^-$ CP asymmetries

$$\frac{1}{(\Gamma+\bar\Gamma)}\frac{d^4(\Gamma-\bar\Gamma)}{dq^2\, d\cos\theta_l\, d\cos\theta_{K^*}\, d\phi} = \frac{9}{32\pi}\times$$
$$\Big\{ -\tfrac{3}{4}\big(F_L^{CP} - A_{CP}\big)\sin^2\theta_{K^*} + F_L^{CP}\cos^2\theta_{K^*}$$
$$-\Big(\tfrac{1}{4}\big(F_L^{CP} - A_{CP}\big)\sin^2\theta_{K^*} + F_L^{CP}\cos^2\theta_{K^*}\Big)\cos 2\theta_l$$
$$+ A_3\sin^2\theta_{K^*}\sin^2\theta_l\cos 2\phi + A_4\sin 2\theta_{K^*}\sin 2\theta_l\cos\phi$$
$$+ A_5\sin 2\theta_{K^*}\sin\theta_l\cos\phi$$
$$+\frac{4}{3}A_{FB}^{CP}\sin^2\theta_{K^*}\cos\theta_l + A_7\sin 2\theta_{K^*}\sin\theta_l\sin\phi$$
$$+ A_8\sin 2\theta_{K^*}\sin 2\theta_l\sin\phi + A_9\sin^2\theta_{K^*}\sin^2\theta_l\sin 2\phi\Big\}$$

- All ≈ 0 in the SM. Beyond the SM: $A_{7,8,9}$ are *triple product asymmetries* and not suppressed by small strong phases!

Reading list

A number of excellent lecture notes that cover most topics of this lecture in greater detail:

- A. J. Buras, "Weak Hamiltonian, CP violation and rare decays," hep-ph/9806471.
- A. J. Buras, "Flavor physics and CP violation," hep-ph/0505175.
- G. Isidori, "Flavor physics and CP violation," arXiv:1302.0661 [hep-ph].
- Y. Grossman, "Introduction to flavor physics," arXiv:1006.3534 [hep-ph].
- Y. Nir, "Flavour physics and CP violation," arXiv:1010.2666 [hep-ph].
- M. Neubert, "Effective field theory and heavy quark physics," hep-ph/0512222.