Perturbative QCD and Jets

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Lecture 2

2013 CERN-Fermilab Hadron Collider Physics School

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
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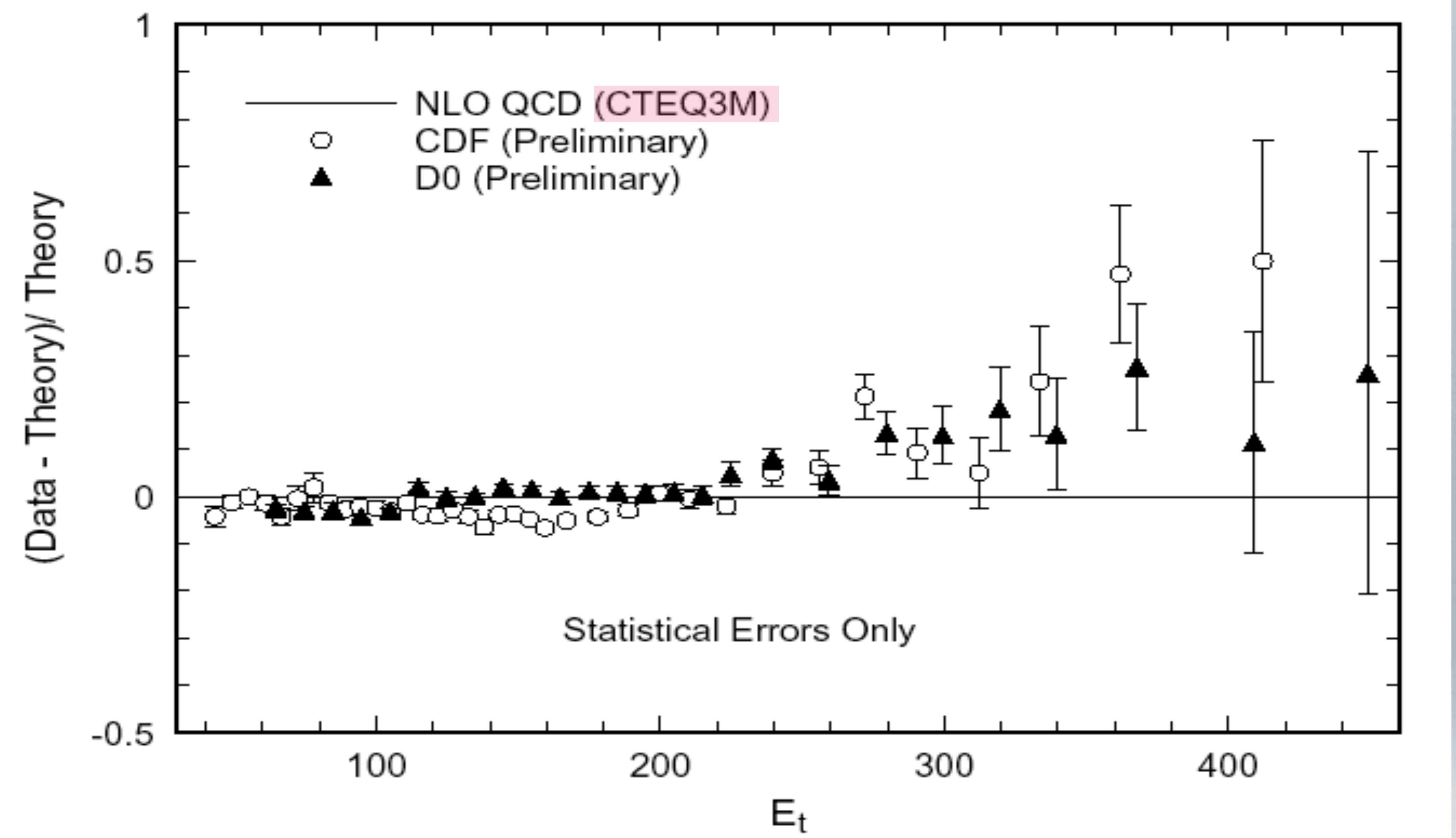
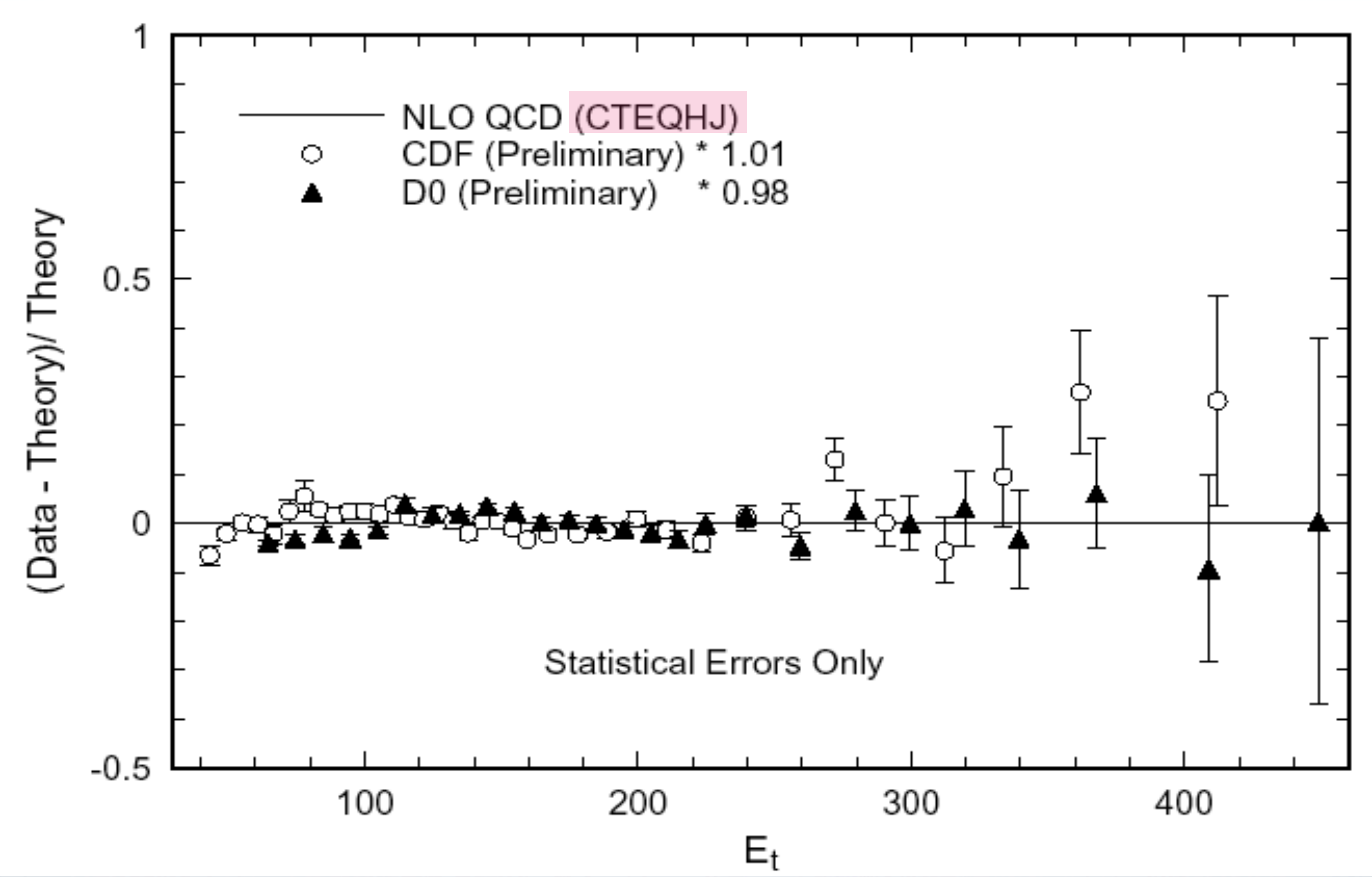
Historic example from Tevatron Run I



Excess in inclusive jet cross section at high Et

New Physics ! (?)

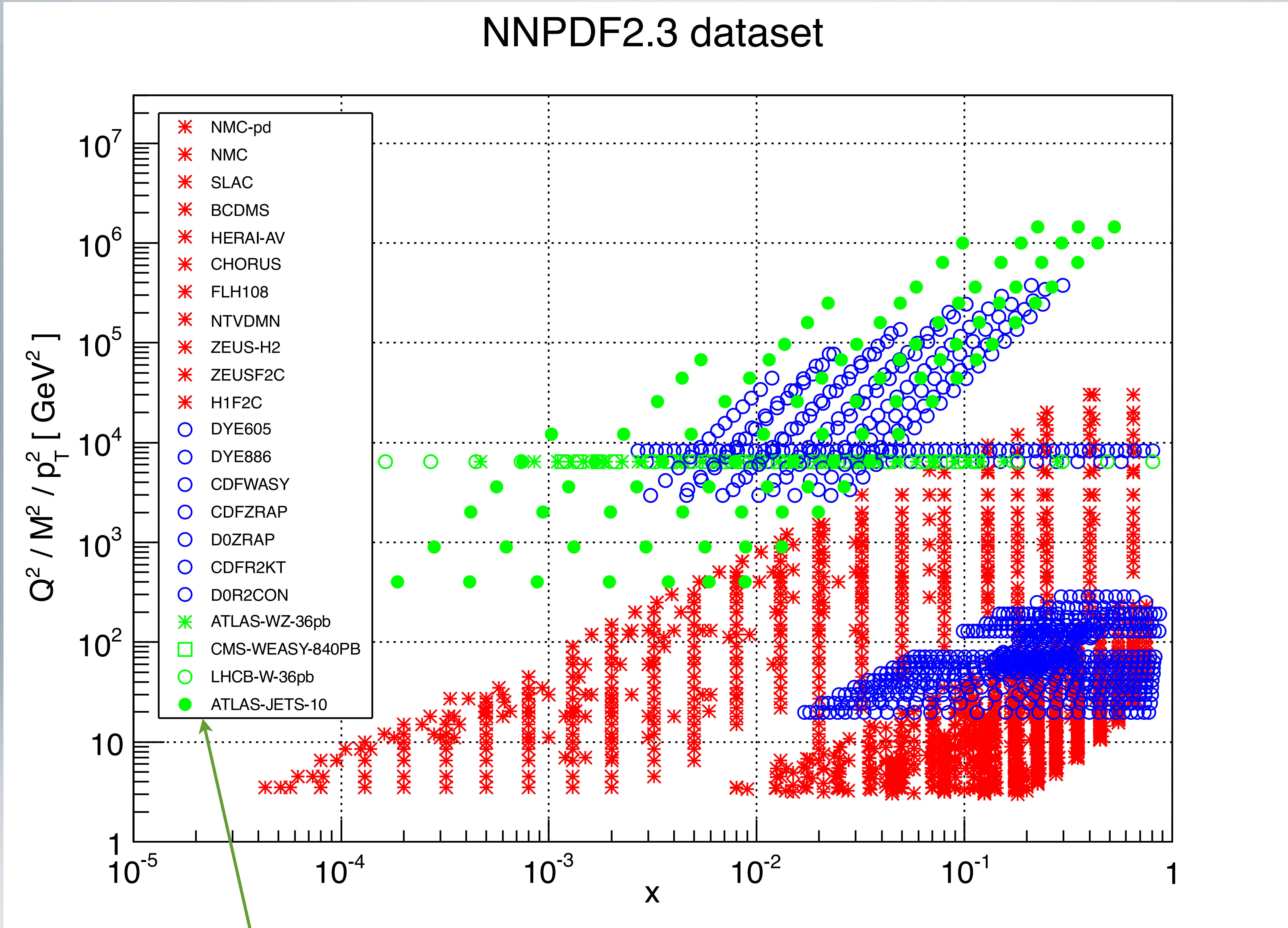
after update of PDFs including high-Et jet data:



problem: constraining the gluon PDFs, especially at larger x values

DIS and fixed target experiments mostly cover low x

Gluon enters only at NLO in DIS structure functions

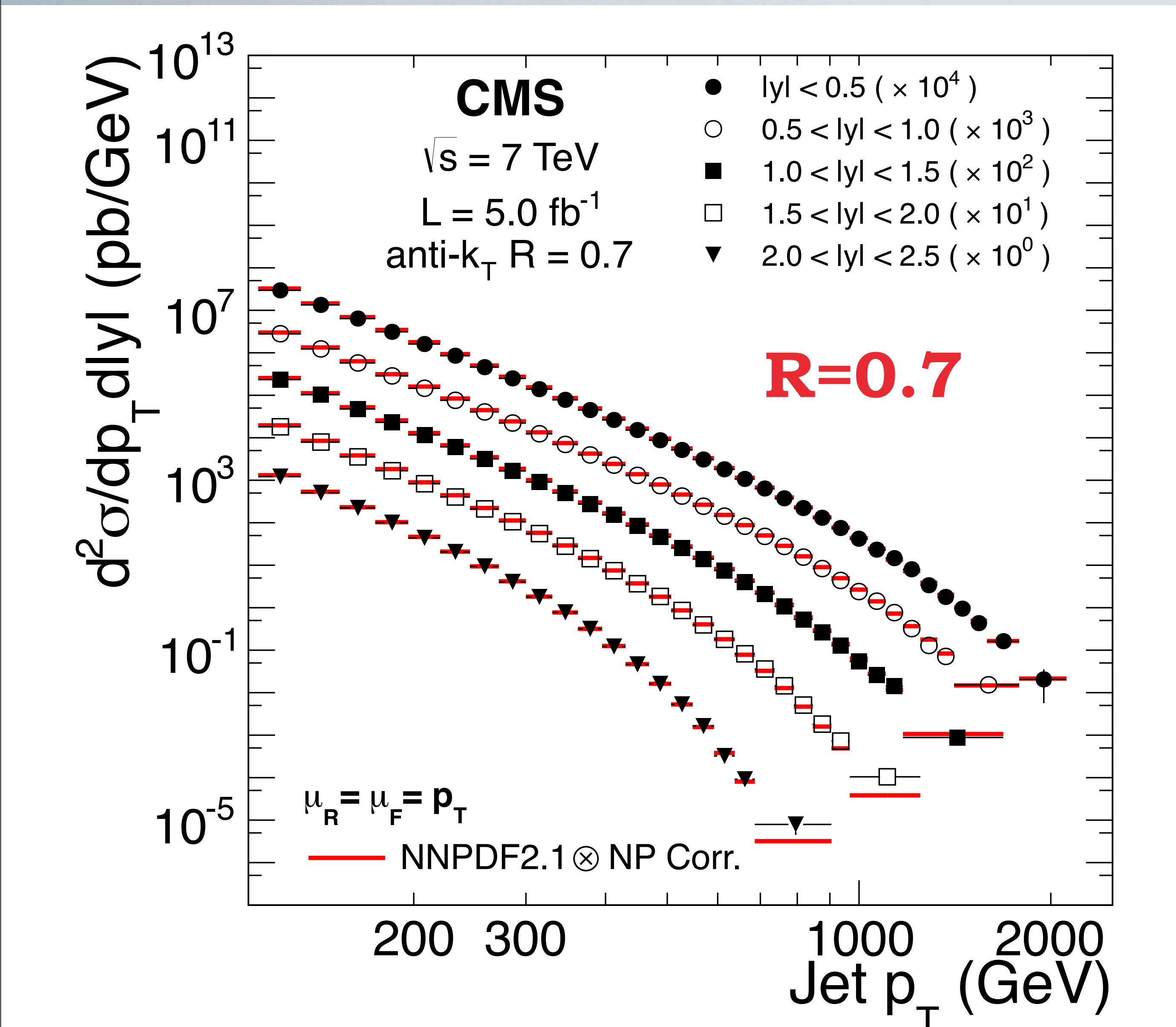


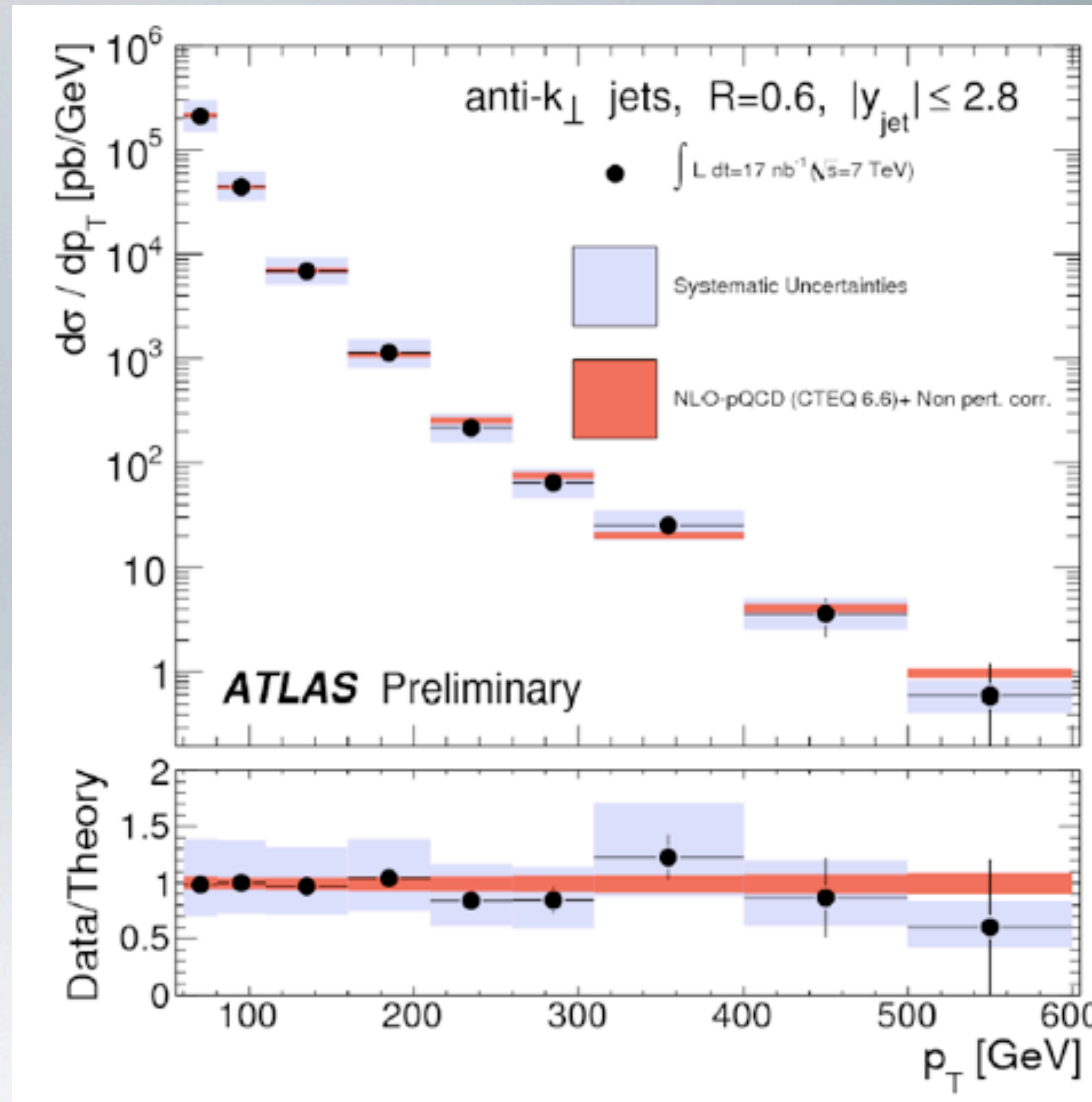
LHC data very important to cover larger range in (x,Q^2) plane

NNPDF collaboration, 1207.1303

Tevatron data as well !

at LHC:



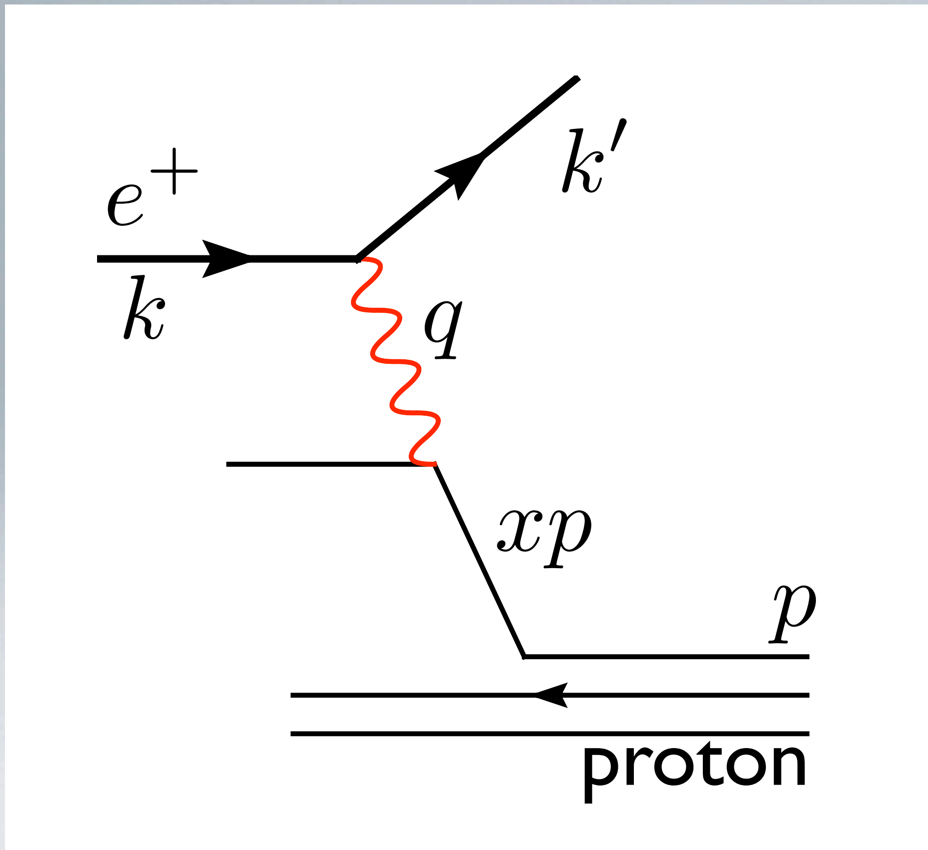


good description over
may orders of magnitude

Theory intermezzo

(What do I mean by DIS structure functions)

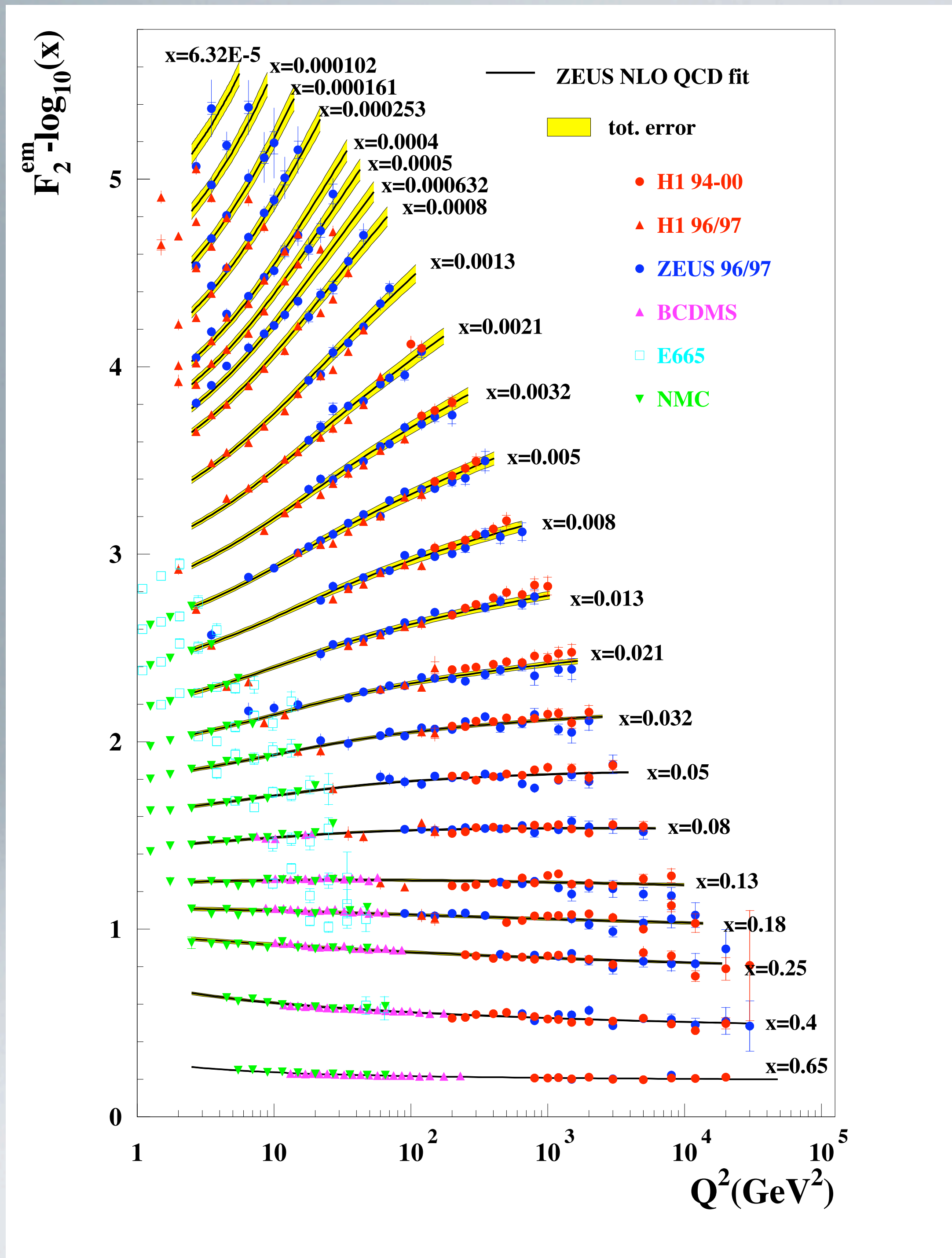
DIS: Deeply inelastic scattering



kinematics:

$$Q^2 = -q^2 \, , \; x = \frac{Q^2}{2p \cdot q} \, , \; y = \frac{p \cdot q}{p \cdot k} = \frac{Q^2}{xs}$$

DIS (e-p scattering) convenient to extract quark PDFs:



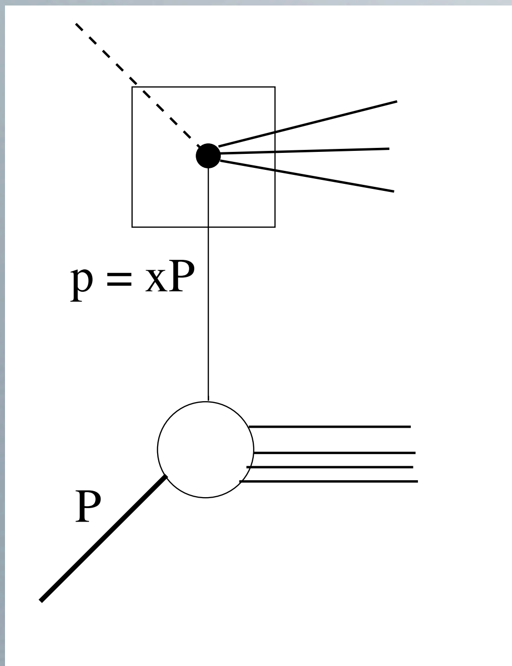
$$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha_{em}^2}{xQ^4}\left[xy^2\, F_1(x,Q^2) + (1-y)\, F_2(x,Q^2)\right]$$

$$F_2(x,Q^2) \sim \sum_i q_i^2\, x\, f_{i/P}(x,Q^2) \qquad \text{(LO)}$$

sum over quark flavours

important test of factorisation and "improved parton model" (Q^2 dependence)

PDFs and DGLAP evolution



Consider the scattering of a hadron H with a high momentum probe (e.g. energetic electron ⇒ DIS).

At high scattering energies, the partons inside a hadron H can be considered as point-like particles, each carrying a fraction x of the hadron's longitudinal momentum P.

hadronic cross section: $\sigma_H(P) = \sum_i \int_0^1 dx\, f_{i/H}(x)\, \hat{\sigma}_i(xP)$

The function $f_{i/H}(x)$

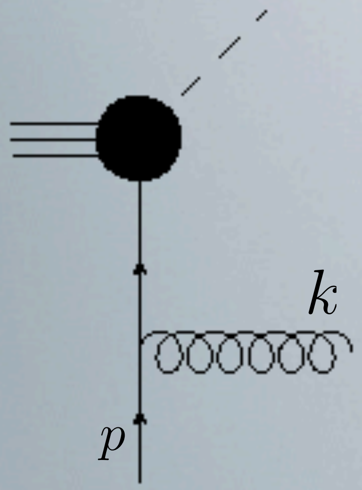
denotes the **probability** that parton i with momentum xP can be found in hadron H. It is a probability density in x-space.

The **partonic** cross section $\hat{\sigma}_i(xP)$

will receive radiative corrections from initial state gluon emission ⇒ need to extend the "naive parton model".

PDFs and DGLAP evolution

Now consider the emission of one gluon in the initial state.



Phase space factor for one gluon emission:

$$d\Phi \sim \frac{d^{D-1}k}{2k_0} \sim dz\,(1-z)^{-1-\epsilon} dk_\perp^2 (k_\perp^2)^{-\epsilon}$$

In the collinear limit $k_\perp^2 \to 0$

$$d\Phi \left|\bar{M}_1^{\mathrm{real}}(p,k)\right|^2 \sim \frac{\alpha_s}{2\pi} \frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}} dz\,(1-z)^{-\epsilon} P_{qq}(z,\epsilon) \left|\bar{M}_0(zp)\right|^2$$

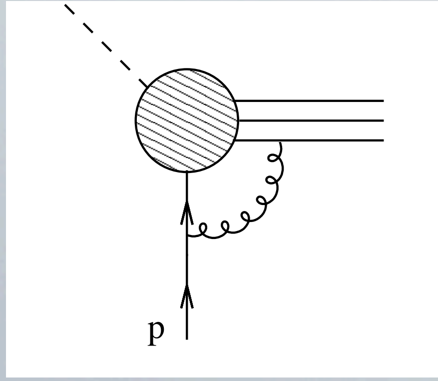
$$P_{qq}(z,\epsilon) = C_F \frac{1+z^2}{1-z} - \epsilon\,(1-z)$$

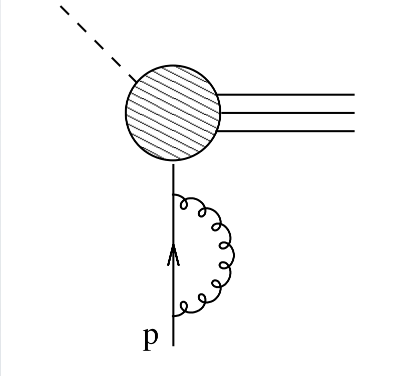
(note soft limit is z ⟶ 1)

Altarelli-Parisi splitting function

PDFs and DGLAP evolution

Are these singularities cancelled by the virtual corrections?





$$d\Phi\,\left|\bar{M}_1^{\mathrm{virt}}\right|^2 \;\sim\; \frac{\alpha_s}{2\pi}\,C_F\,\left|\bar{M}_0(zp)\right|^2\,\frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}}\,dz\,(1-z)^{-\epsilon}\left\{\frac{3}{2}-\frac{2}{1-z}\right\}$$

No ! Only the soft singularities cancel.

But the collinear singularities factorise from the hard scattering cross section

Defining the “plus distribution” as

$$\int_0^1 dz\,\left[\frac{p(z)}{1-z}\right]_+ f(z) = \int_0^1 dz\,p(z)\,\left(\frac{f(z)-f(1)}{1-z}\right)$$

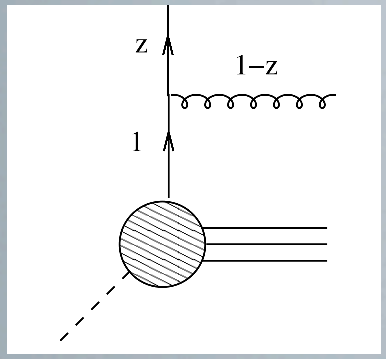
where f(z) is a smooth test function, we obtain

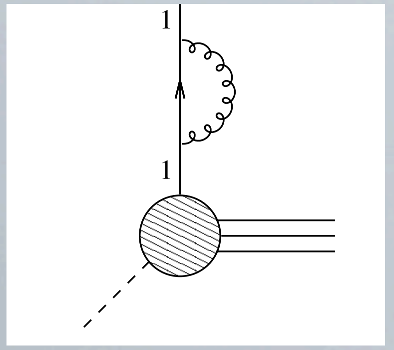
$$\hat{\sigma}_1(p) = \frac{\alpha_s}{2\pi}\int\frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}}\,dz\,P_{qq}(z)_+\,\hat{\sigma}_0(zp) \qquad P_{qq}(z)_+ = \left[\frac{1+z^2}{(1-z)}\right]_+ + \frac{3}{2}\,\delta(1-z)$$

PDFs and DGLAP evolution

Recap:

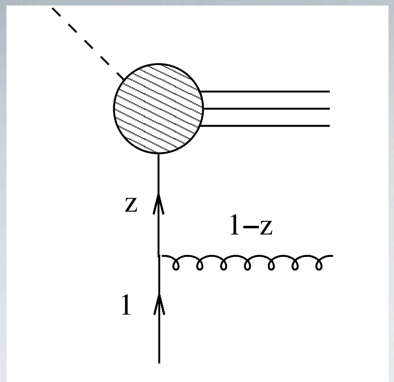
gluon emission in final state:

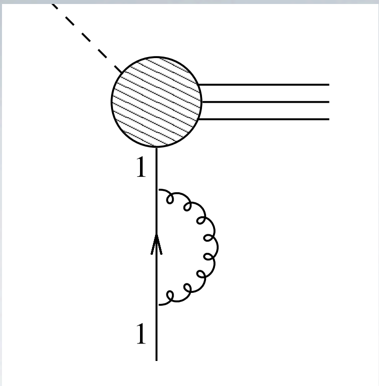




both soft and collinear singularities cancel between real and virtual corrections

gluon emission in initial state:





only soft singularities cancel between real and virtual corrections

PDFs and DGLAP evolution

Absorb initial state singularities by defining a "renormalised" parton distribution function (PDF)

$$f_{q/H}(x,\mu) = \int_x^1 \frac{dz}{z} \left\{ \delta(1-z) + \frac{\alpha_s}{2\pi} \int_0^{\mu^2} \frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}} \left[P_{qq}(z)\right]_+ \right\} f_{q/H}(x/z)$$

PDFs have to be determined from fits to data but evolution with μ^2 can be predicted by perturbative QCD.

$$\mu^2 \frac{\partial f_{i/H}(x,\mu)}{\partial \mu^2} = \frac{\alpha_s}{2\pi} \sum_j \int_x^1 \frac{dz}{z} \left[P_{ij}(z)\right]_+ f_{j/H}(\frac{x}{z},\mu)$$

DGLAP equation

(Dokshitzer, Gribov, Lipatov, Altarelli, Parisi)

Can be extended to all orders

$$\mu^2 \frac{\partial f_{i/H}(x,\mu)}{\partial \mu^2} = \sum_j \int_x^1 \frac{dz}{z} \left[\mathcal{P}_{ij}(\alpha_s(\mu),z)\right]_+ f_{j/H}(\frac{x}{z},\mu)$$

$$\mathcal{P}_{ij}(\alpha_s(\mu),z) = P_{ij}^{(0)}(z) + \frac{\alpha_s(\mu)}{2\pi} P_{ij}^{(1)}(z) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 P_{ij}^{(2)}(z) + \dots$$

LO (1974) NLO (1980) NNLO (2004)

(flavour) singlet evolution equations:

$$\Sigma(x,Q^2) \equiv \sum_{i=1}^{n_f} \left(q_i(x,Q^2) + \bar{q}_i(x,Q^2) \right)$$

$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} \Sigma(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \int_x^1 \frac{\mathrm{d}y}{y} \begin{pmatrix} P^S_{qq}\left(\frac{x}{y},\alpha_S(Q^2)\right) & 2n_f P^S_{qg}\left(\frac{x}{y},\alpha_S(Q^2)\right) \\ P^S_{gq}\left(\frac{x}{y},\alpha_S(Q^2)\right) & P^S_{gg}\left(\frac{x}{y},\alpha_S(Q^2)\right) \end{pmatrix} \begin{pmatrix} \Sigma(y,Q^2) \\ g(y,Q^2) \end{pmatrix}$$

non-singlet:

$$q^{\mathrm{NS}}_{ij}(x,Q^2) = q_i(x,Q^2) - q_j(x,Q^2)$$

$$\frac{\partial}{\partial \ln Q^2} q^{\mathrm{NS}}_{ij}(x,Q^2) = \int_x^1 \frac{\mathrm{d}y}{y} P^{\mathrm{NS}}_{ij}\left(\frac{x}{y},\alpha_S(Q^2)\right) q^{\mathrm{NS}}_{ij}(y,Q^2)$$

constraints:

$$\int_0^1 \mathrm{d}x\, x \left[\sum_{i=1}^{n_f} \left(q_i(x,Q^2) + \bar{q}_i(x,Q^2) \right) + g(x,Q^2) \right] = 1.$$

$$\int_0^1 \mathrm{d}x \left(q_i(x,Q^2) - \bar{q}_i(x,Q^2) \right) = n_i \qquad (n_u = 2, n_d = 1, n_{s,c,b,t} = 0)$$

(baryon number conservation)

PDFs and DGLAP evolution

Q^2 evolution can be predicted using the DGLAP equations,
x-dependence needs to be extracted from data

Process	Subprocess	Partons	x range	
$\ell^\pm \{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$	fixed target
$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$	
$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$	
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$	
$\nu(\bar{\nu}) N \to \mu^-(\mu^+) X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$	
$\nu N \to \mu^-\mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$	
$\bar{\nu} N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$	
$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$	HERA
$e^+ p \to \bar{\nu} X$	$W^+ \{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$	
$e^\pm p \to e^\pm c\bar{c} X$	$\gamma^* c \to c,\ \gamma^* g \to c\bar{c}$	$c,\ g$	$0.0001 \lesssim x \lesssim 0.01$	
$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$	
$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$	Tevatron
$p\bar{p} \to (W^\pm \to \ell^\pm \nu) X$	$ud \to W, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$	
$p\bar{p} \to (Z \to \ell^+\ell^-) X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$	

example: data set used for MSTW08

typical x values for Higgs production: $x \sim M_H/\sqrt{s} \sim 0.016$ (at 8 TeV)

today: LHAPDF interface provides PDF sets in standardized form

Parton distribution functions (PDFs)

- Several groups provide pdf fits + uncertainties
- Differ by: data input, TH/bias, HQ treatment, coupling, etc

up to 5% ! >15% in Higgs cross section

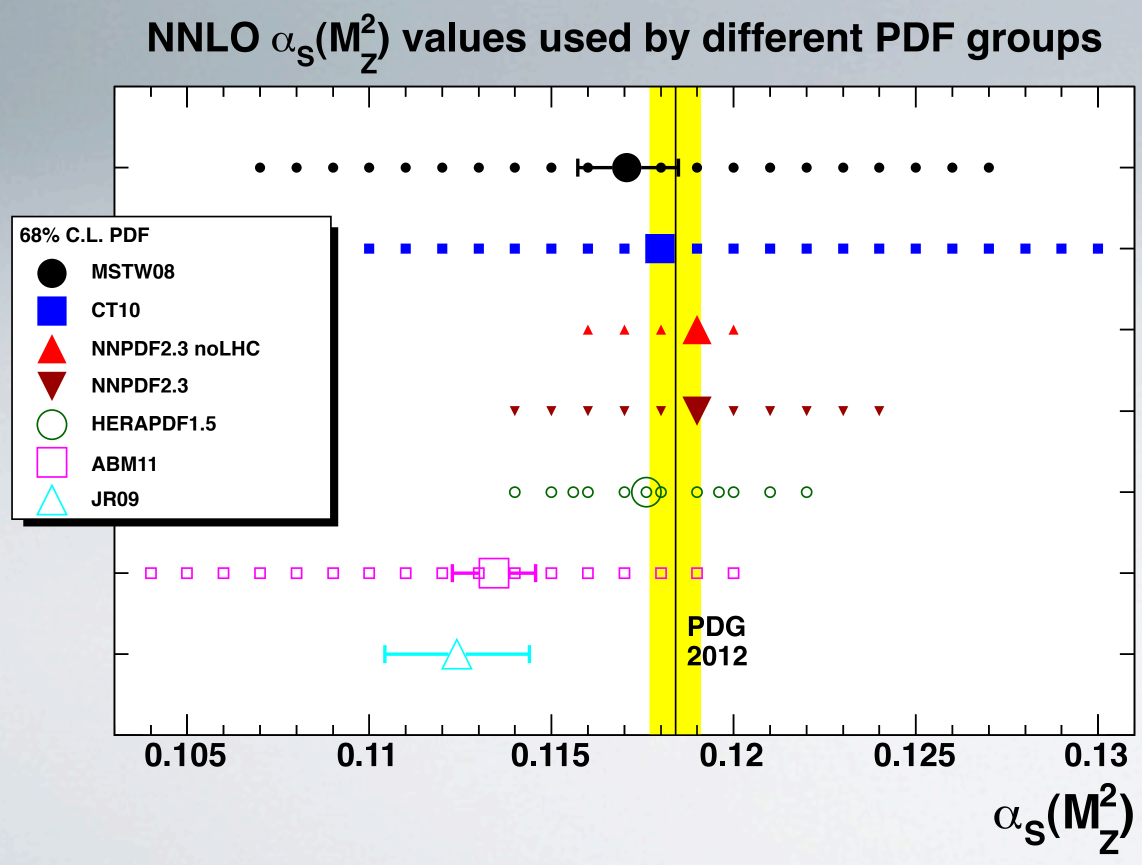
set	H.O.	data	$\alpha_s(M_Z)@NNLO$	uncertainty	HQ	Comments
MSTW 2008	NNLO	DIS+DY+Jets	0.1171	Hessian (dynamical tolerance)	GM-VFN (ACOT+TR')	old HERA DIS
CT10	NNLO	DIS+DY+Jets	0.118	Hessian (dynamical tolerance)	GM-VFN (SACOT-X)	New HERA DIS
NNPDF 2.3	NNLO	DIS+DY+Jets +LHC	0.1174	Monte Carlo	GM-VFN (FONLL)	New HERA DIS
ABKM	NNLO	DIS+DY(f.t.)	0.1135	Hessian	FFN BMSN	New HERA DIS
(G)JR	NNLO	DIS+DY(f.t.)+ some jet	0.1124	Hessian	FFN (VFN massless)	valence like input pdfs
HERA PDF	NNLO	only DIS HERA	0.1176	Hessian	GM-VFN (ACOT+TR')	Latest HERA DIS

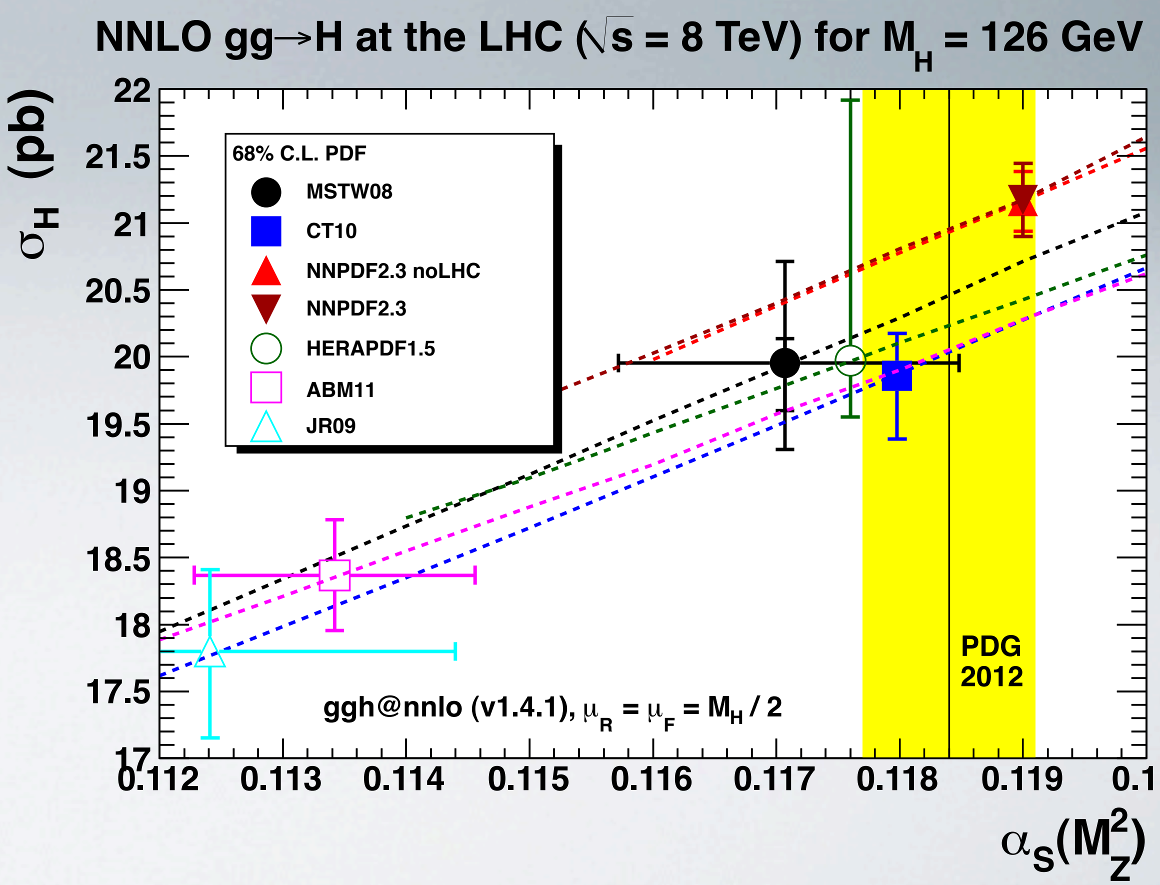
Jet data not used because no full NNLO calculation available

compiled by D.de Florian, EPS '13

	MSTW08	CT10	NNPDF2.3	HERAPDF1.5	ABM11	JR09
HERA DIS	✔	✔	✔	✔	✔	✔
Fixed-target DIS	✔	✔	✔	✗	✔	✔
Fixed-target DY	✔	✔	✔	✗	✔	✔
Tevatron $W+Z+$jets	✔	✔	✔	✗	✗	✗
LHC $W+Z+$jets	✗	✗	✔	✗	✗	✗

S.Forte, G.Watt,
1301.6754



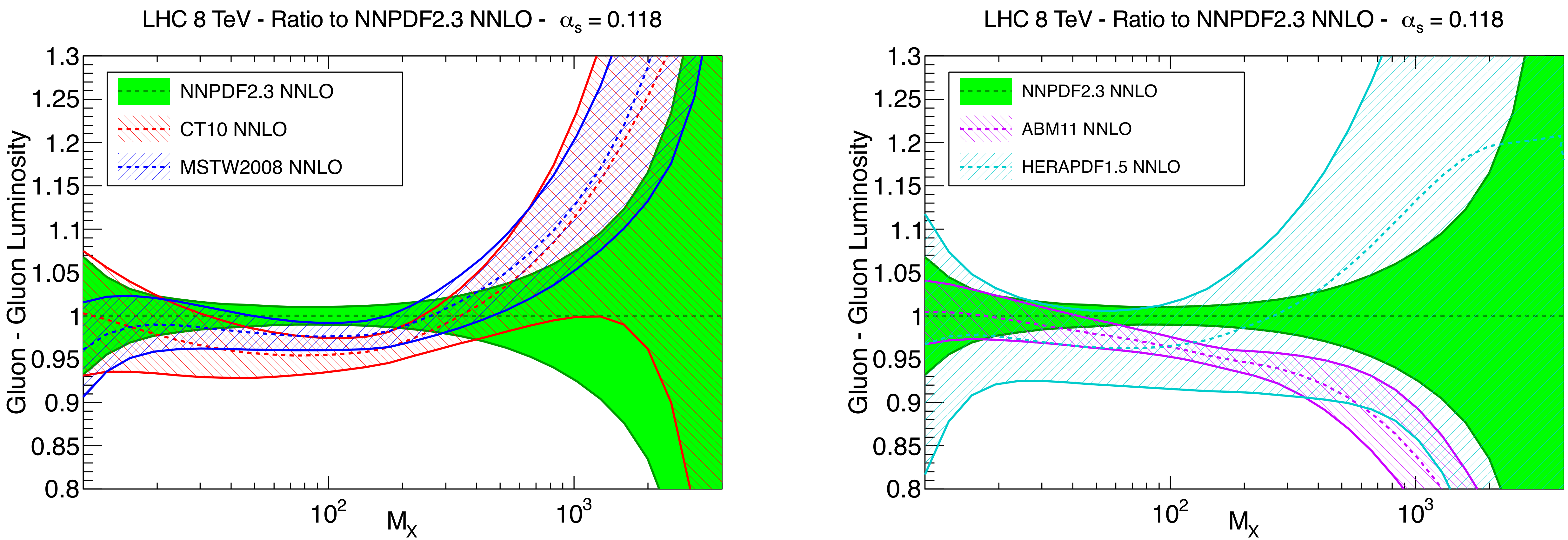


Luminosities with common $\alpha_s = 0.118$

PDF4LHC, Ball et al

$$\mathcal{L}_{ij}(\tau \equiv M_X^2/S) = \frac{1}{S}\int_\tau^1 \frac{dx}{x} f_i(x, M_X^2) f_j(\tau/x, M_X^2)$$

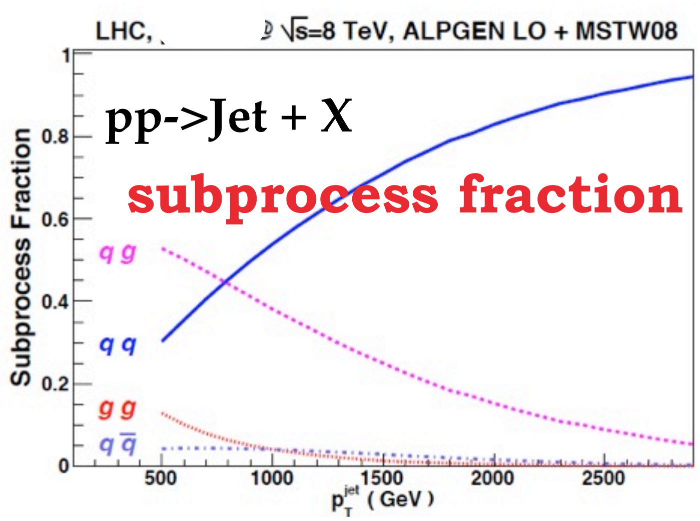
gluon-gluon

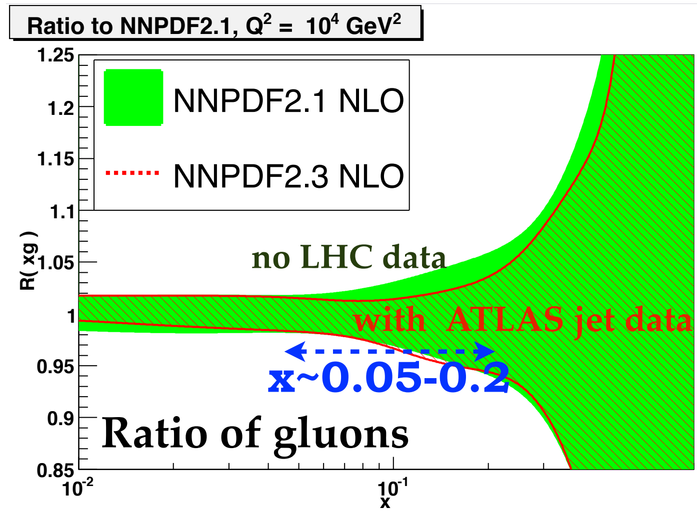


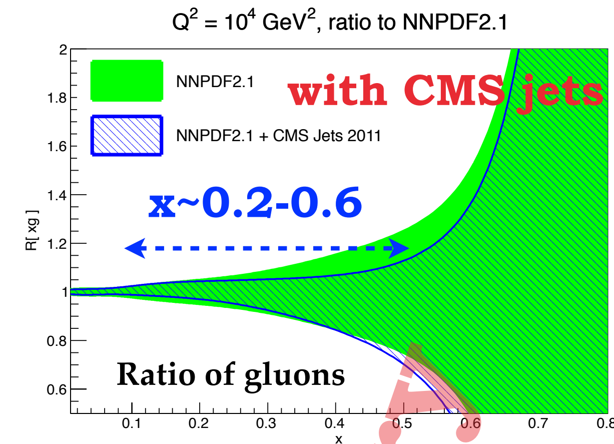
- Good agreement for global fits but deviations as large as uncertainties
- Larger differences with “non-global” results

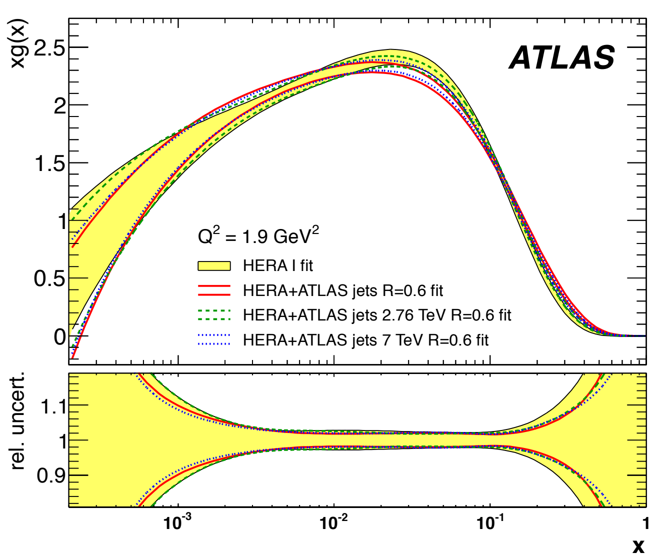
The story of further constraining the gluon pdfs continues...

Impact of jet measurements on PDFs

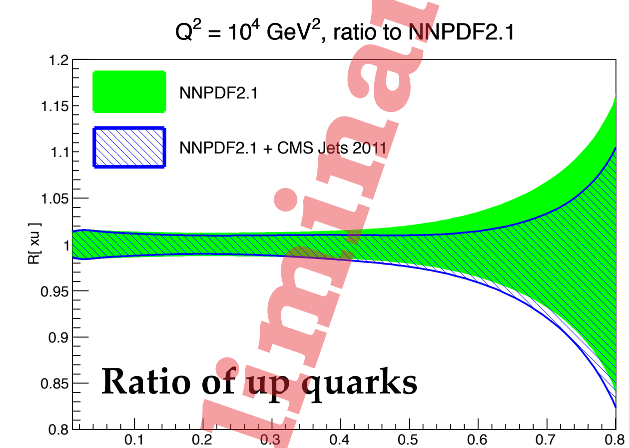


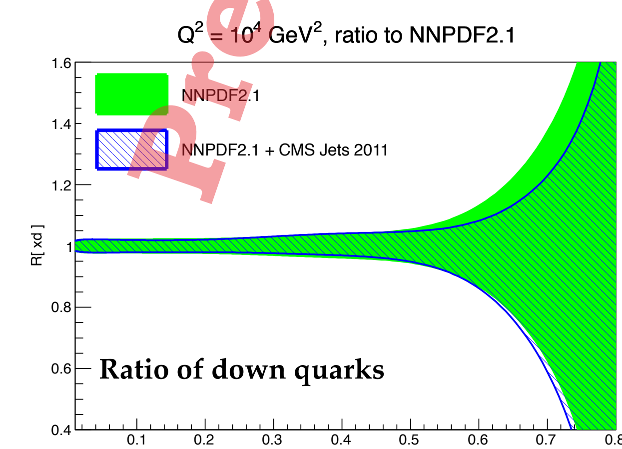






- ATLAS and CMS data public
- first attempts from PDF fitters to include the LHC jet data
  - preliminary studies: jet data constrain the gluon PDF up to x~0.6 but also the u,d PDFs at higher x
- ratios between c.m. energies can constrain the PDFs further

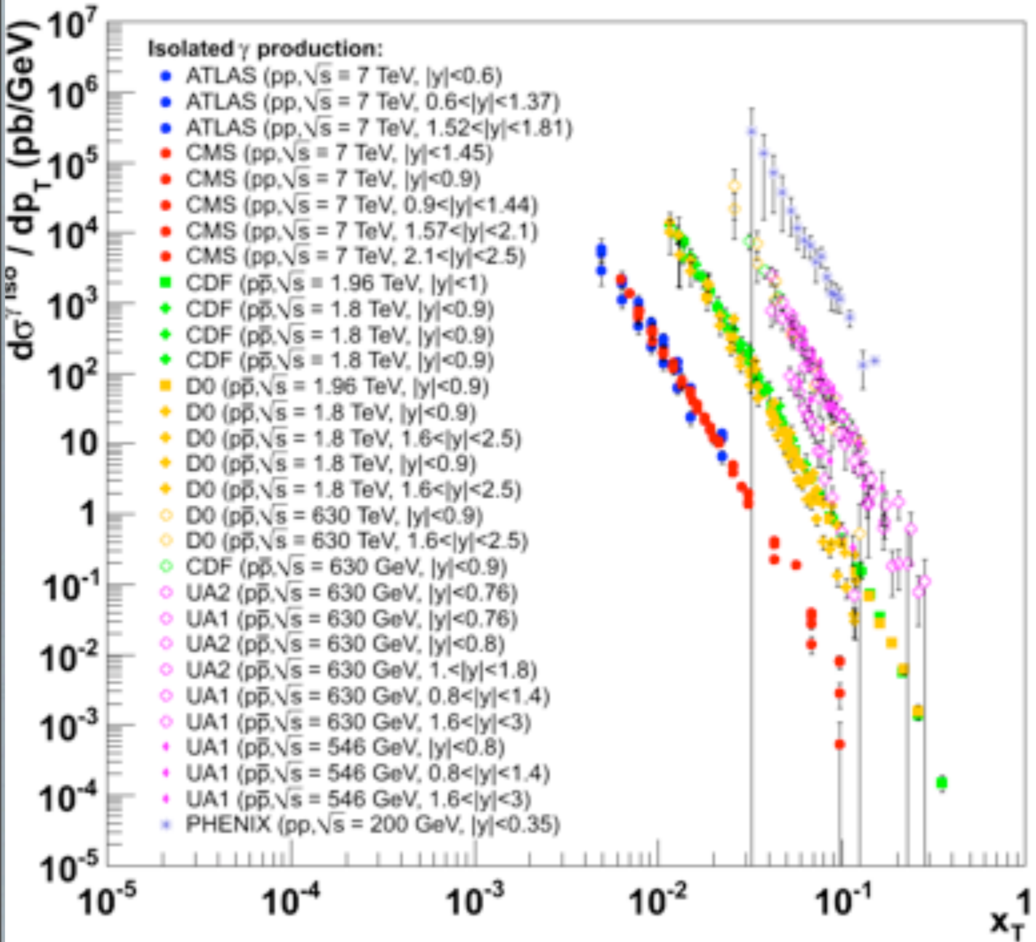


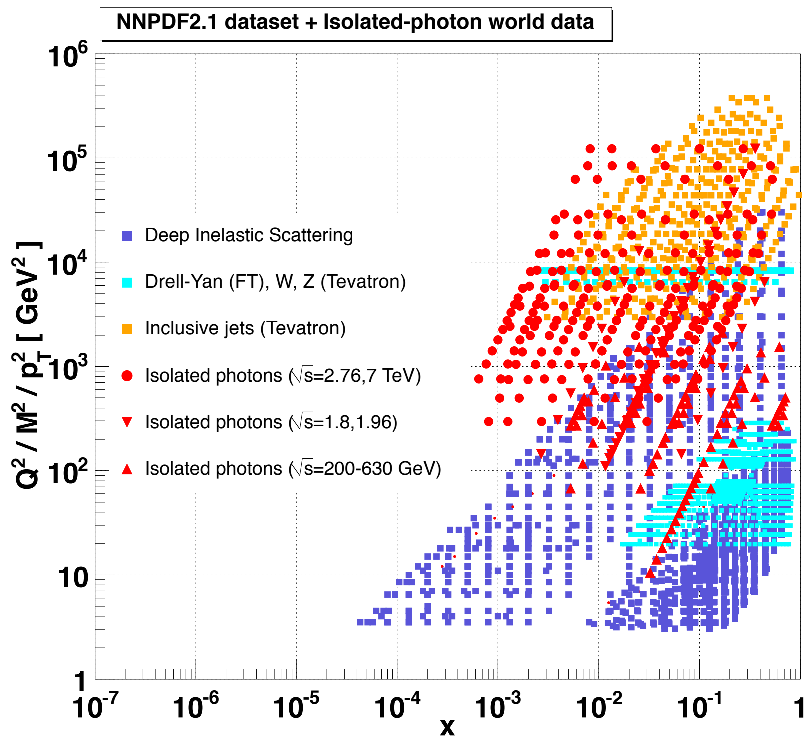


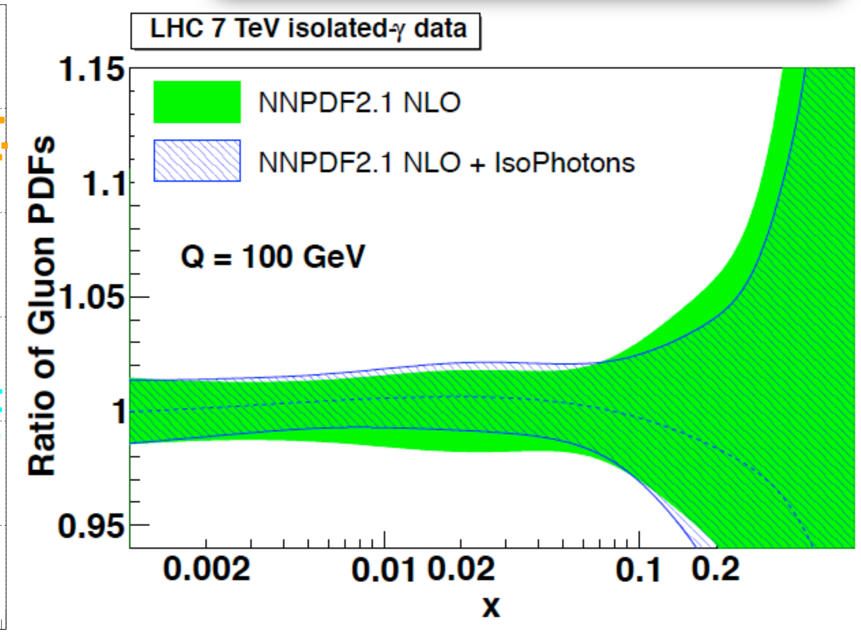
K.Kousouris, EPS ’13

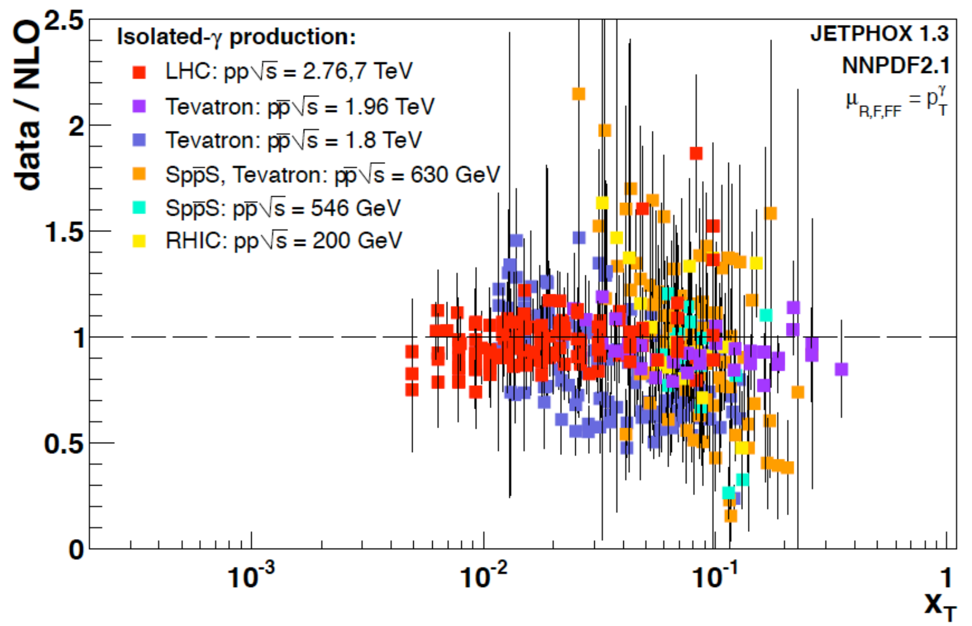
Direct photon production

Nucl. Phys. B860 (2012) 311









- large amount of photon measurements accumulated over the years
- not used so far in the PDF fits
  - probably missing the correlation of uncertainties?
- first attempt to include the photon measurements
  - in the NNPDF framework
  - moderate impact on the gluon PDF
- more photon data needed

EPS 2013

Recommendations for PDF determinations

(from S.Forte, G.Watt, 1301.6754)

• The range of data sets must be as wide as possible.

• The parametrization should be sufficiently general and demonstrably unbiased, either by using a sufficiently large number of parameters, or by careful a posteriori checks of parametrization independence.

• The experimental uncertainties should be understood and carefully propagated.

• Computations should be performed at the highest available perturbative order, and in particular, at the order which is subsequently to be used in the computation of partonic cross sections.

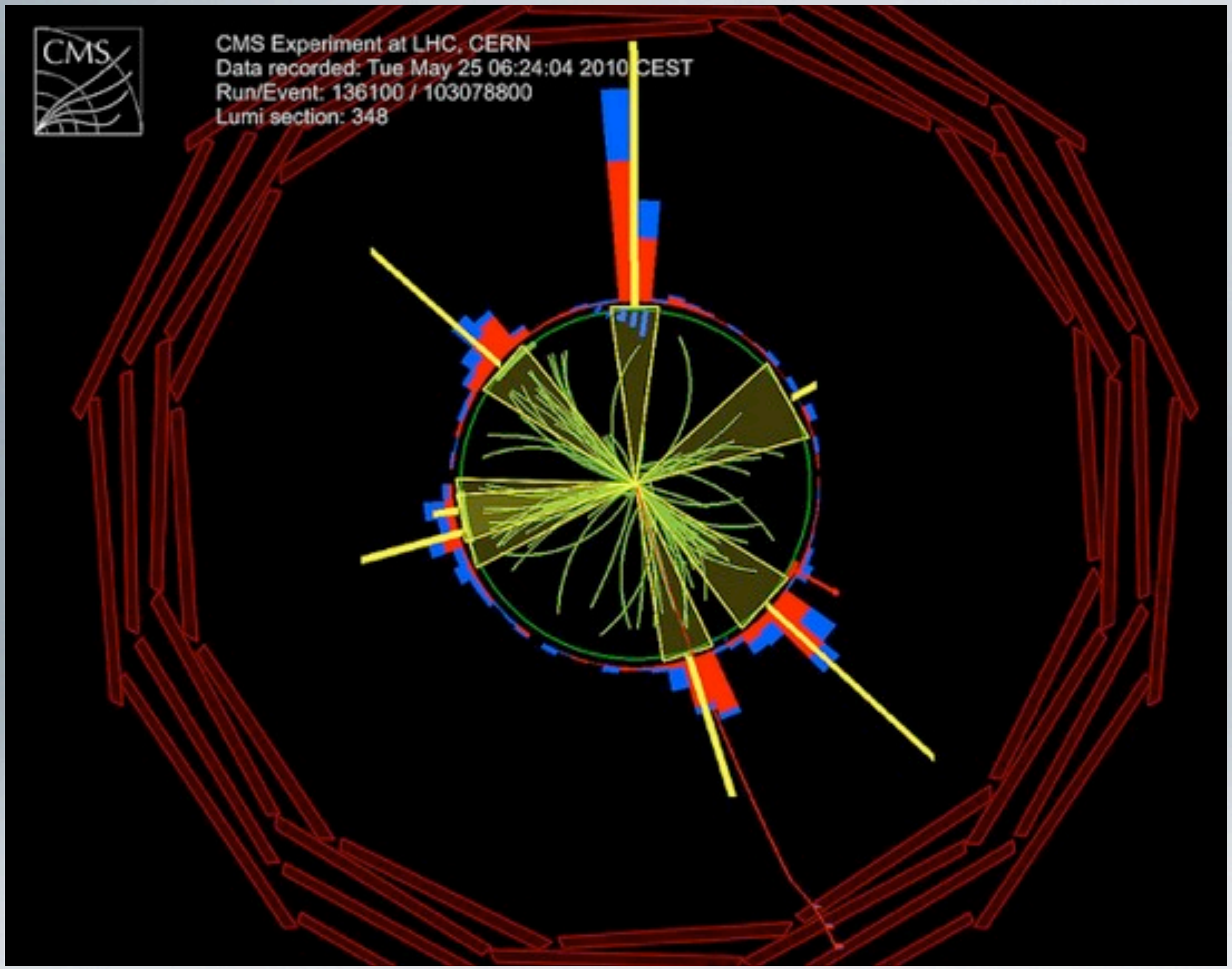
• PDFs including electroweak corrections will have to be constructed.

• The treatment of heavy quarks will have to include mass-suppressed terms.

• The strong coupling, in addition to being determined simultaneously with PDFs, should also be decoupled from the PDF determination, with PDF sets available for a range of fixed α_s values, and full PDF uncertainty determination for each value of α_s

• An estimate of theoretical uncertainties should be performed together with PDF sets, and such uncertainties should be provided each time they become comparable with other sources of PDF uncertainty. This is presently an almost unexplored territory.

Jets



- jets are bundles of hadrons observed experimentally
- clustering based on a **jet algorithm**
- a good jet algorithm should:
  - **reflect the parton dynamics**
  - **be infared safe (well defined at any order in perturbation theory)**
  - **be efficient and fast to implement**

Jet algorithm development

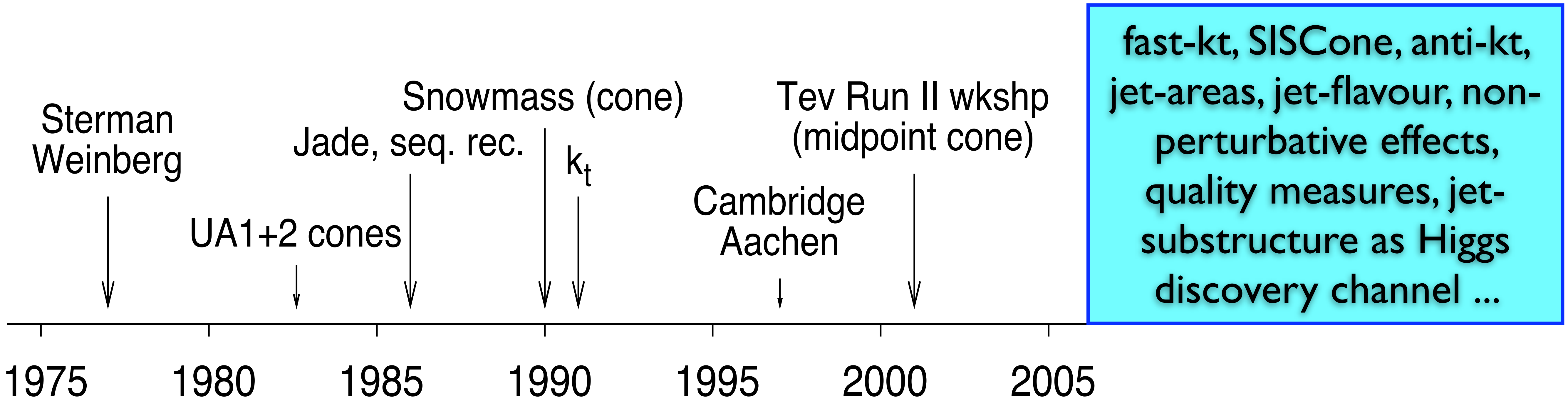


figure from G. Zanderighi

"Snowmass accord on jets" (1995)

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross sections at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronisation.

• two main categories of jet algorithms:

cone based

e.g. midpoint, SISCone, ...

cluster particles within a cone of radius R in rapidity and azimuthal angle space:

- take particle i (e.g. with largest pT) as a seed
- sum momenta of all particles j within a cone of radius R

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 < R^2$$

- take the resulting sum as a new seed and iterate
- iterate until stability is reached

sequential

e.g. Jade, Durham kt, Cambridge/Aachen, anti-kt, ...

cluster particles according to distance in momentum space

example Jade algorithm:

- For each pair of particles i, j work out the distance

$$y_{ij} = \frac{2E_i E_j(1 - \cos\theta_{ij})}{Q^2}$$

- Find the minimum $y_{\rm min}$ of all the y_{ij}
- If $y_{\rm min}$ is below some jet resolution threshold $y_{\rm cut}$ then recombine i and j into a new pseudo-particle and iterate the procedure
- Otherwise declare all remaining particles as jets and stop the iteration

Things to note:

- The distance measure $y_{ij} = \frac{2E_iE_j(1-\cos\theta_{ij})}{Q^2}$ contains the total energy Q in the event.

  Therefore it is not applicable in this form to hadronic collisions.

- Different sequential algorithms mainly differ by their distance measure.

- Durham kt-algorithm in e+e- : $y_{ij} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{Q^2}$

- Durham kt-algorithm in hadronic collisions : (variables invariant under longitudinal boosts)

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2)\Delta R_{ij}^2\,, \qquad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$d_{iB} = p_{ti}^2\,,$ (particle-beam distance)

- Anti-kt-algorithm :

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p})\frac{\Delta R_{ij}^2}{R^2}\,, \qquad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$d_{iB} = p_{ti}^{2p}\,,$

p = -1 (p=1: kt)

(p=0: Cambridge-Aachen)

Recombination schemes

need to define how to merge two particles

basically two approaches

- “E-scheme” : combine 4-vectors
- “Snowmass-scheme” :

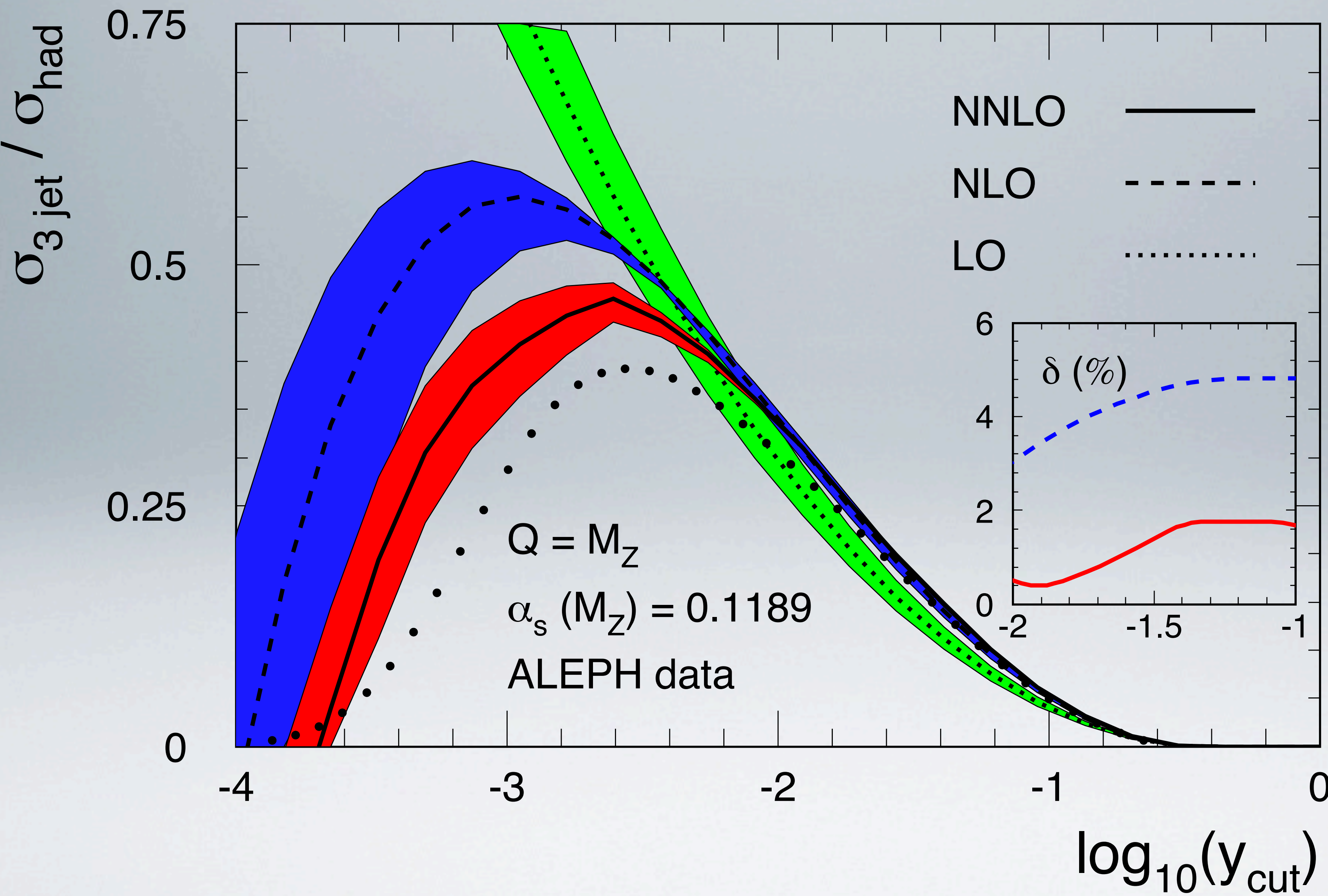
(not invariant under longitudinal boosts for massive particles)

$$E_{t,\text{jet}} = \sum_i E_{ti} \; ,$$

$$\eta_{\text{jet}} = \frac{1}{E_{t,\text{jet}}} \sum_i E_{ti}\eta_i \; ,$$

$$\phi_{\text{jet}} = \frac{1}{E_{t,\text{jet}}} \sum_i E_{ti}\phi_i \; ,$$

Example: 3-jet rate in e+e- (kt-algorithm)



ycut:
"jet resolution parameter"

repeat clustering of particles i, j as long as $y_{ij,D} < \mathrm{ycut}$

Gehrmann-De Ridder, Gehrmann, Glover, GH 2009

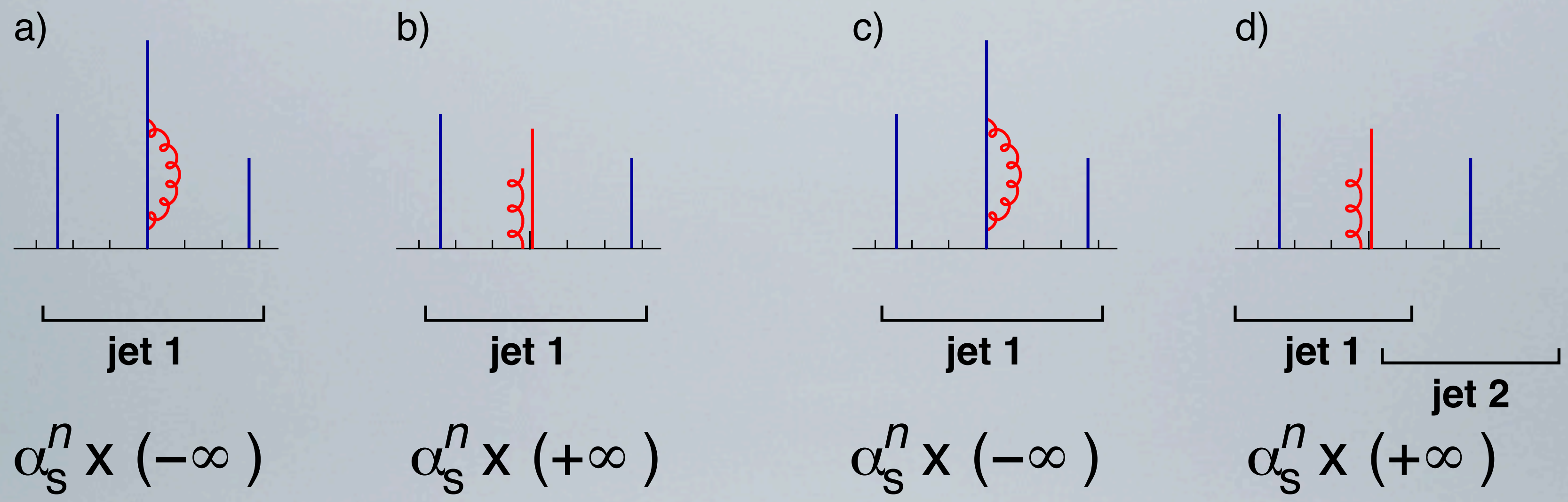
large values of ycut: cluster all particles in very few jets

small values of ycut: higher jet multiplicities more frequent

Infrared Safety

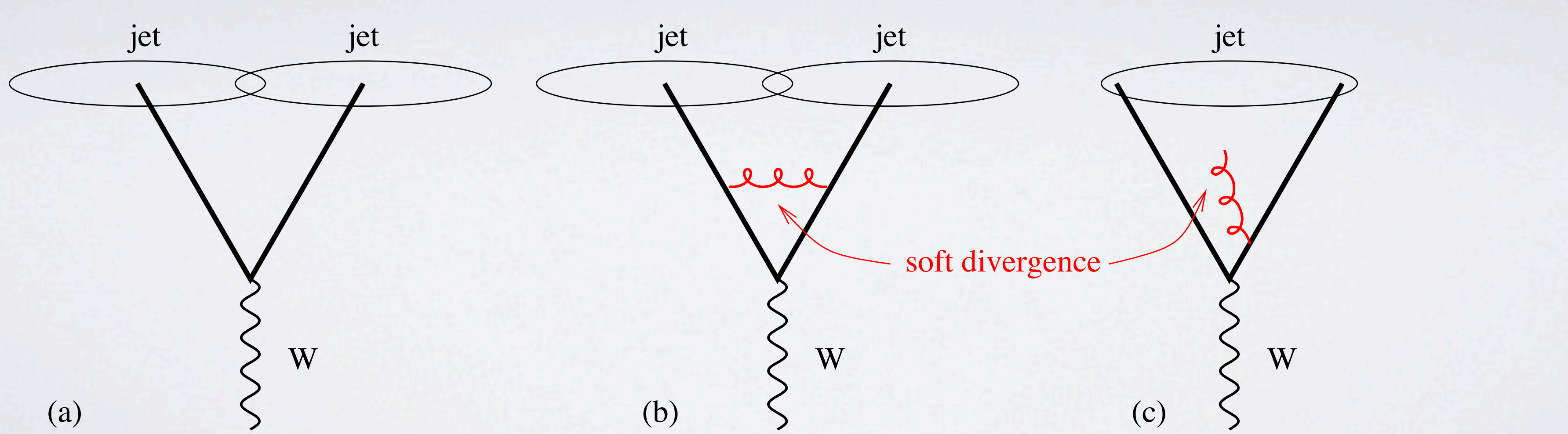
A jet algorithm is called infrared unsafe when the addition of a soft particle changes the configuration of jets found by the algorithm.

Collinear safe jet alg. | **Collinear unsafe jet alg**



Infinities cancel | **Infinities do not cancel**

literature recommendation:
G. P. Salam, 0906.1833



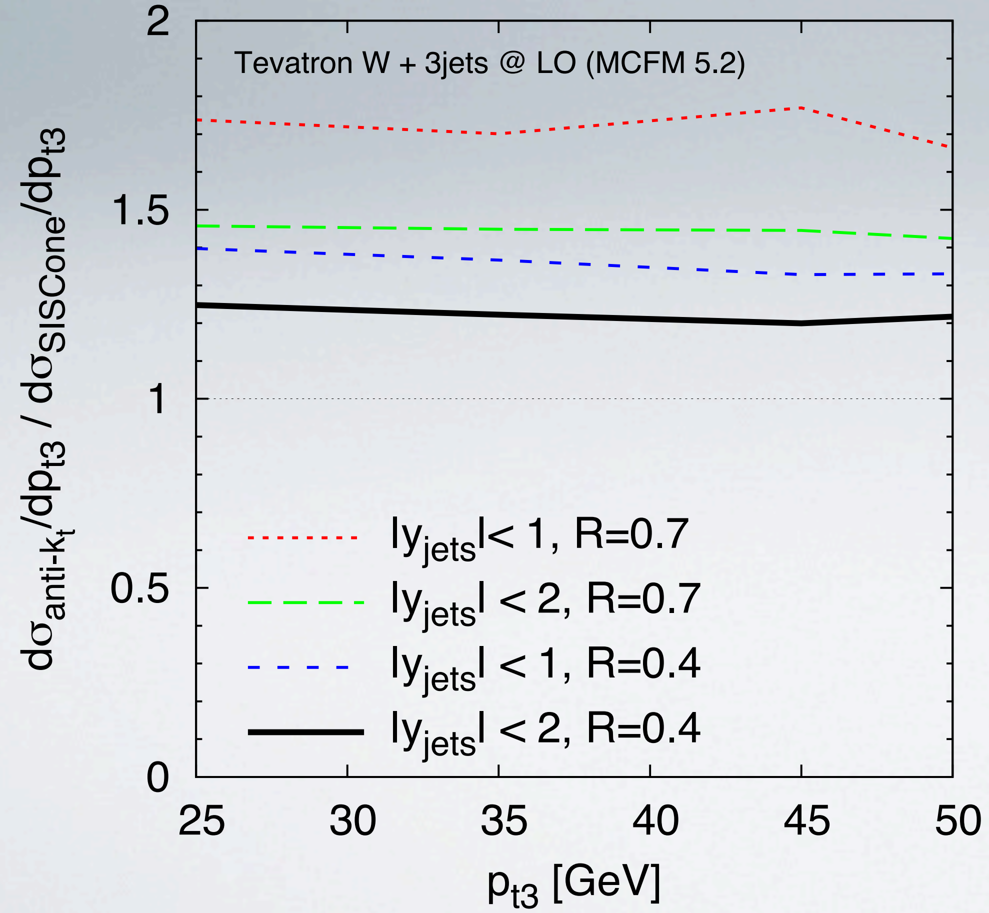
examples of infrared unsafe jet algorithms which were used at the Tevatron:

Midpoint, JetClu

algorithms used at the LHC are infrared safe

Two algorithms of most importance:

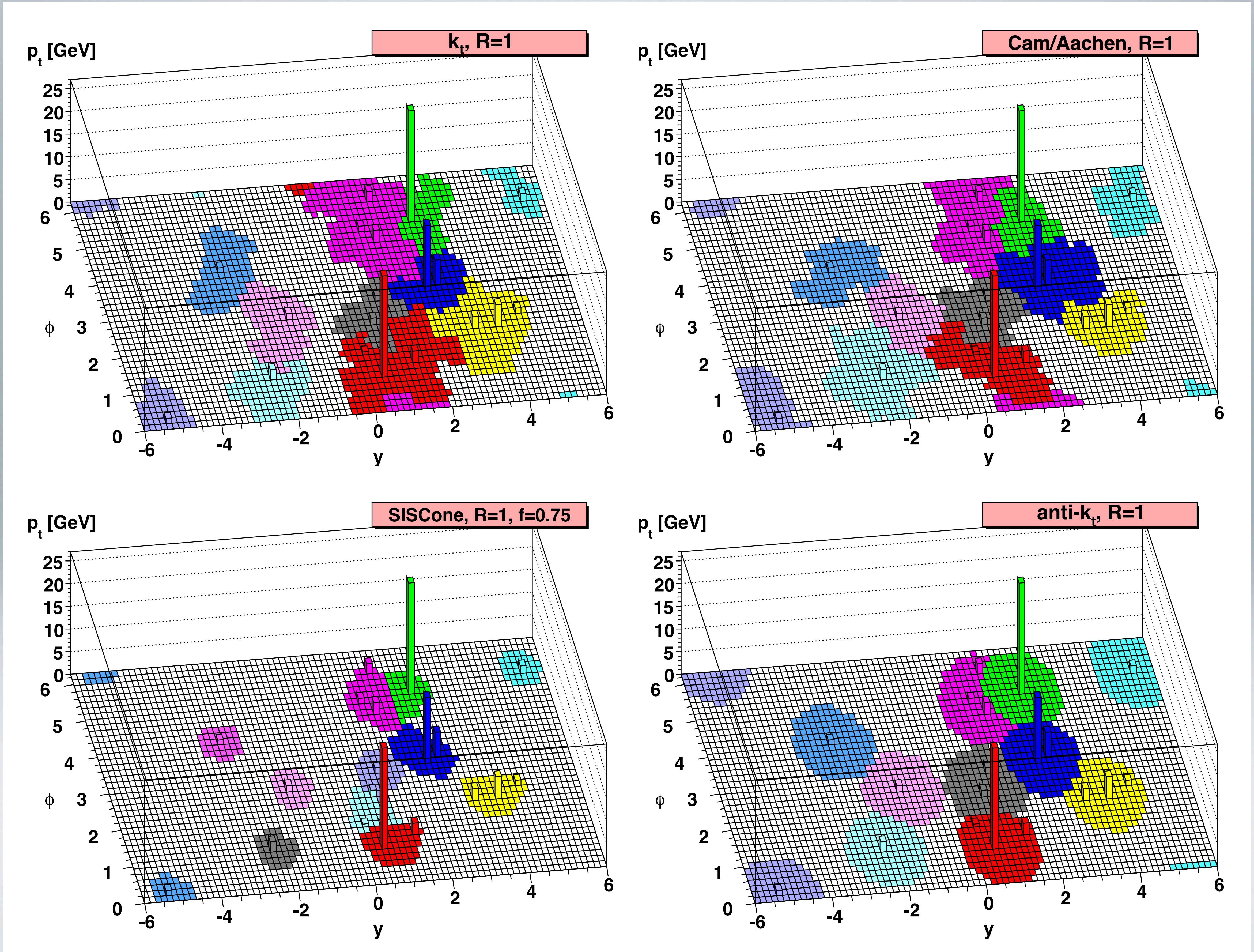
SISCone	Salam and Soyez (2007)
anti-k_T	Cacciari, Salam and Soyez (2008); Delsart



Still jet algorithm can lead to sizable differences in some observables.

Example transverse momentum of third hardest jet

Jet areas

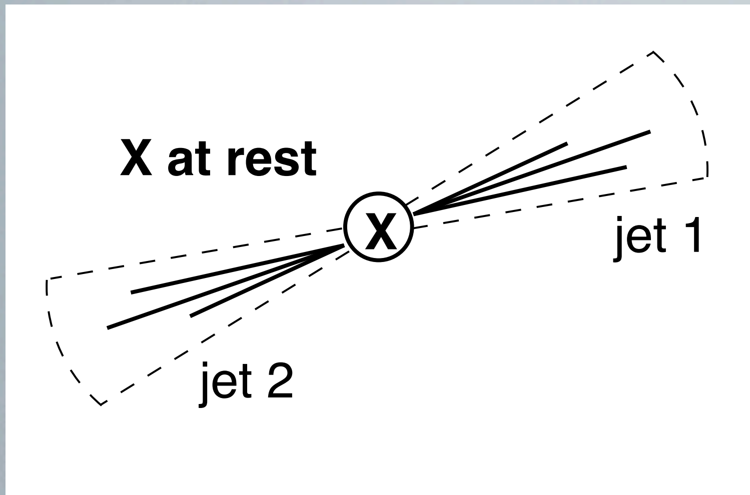


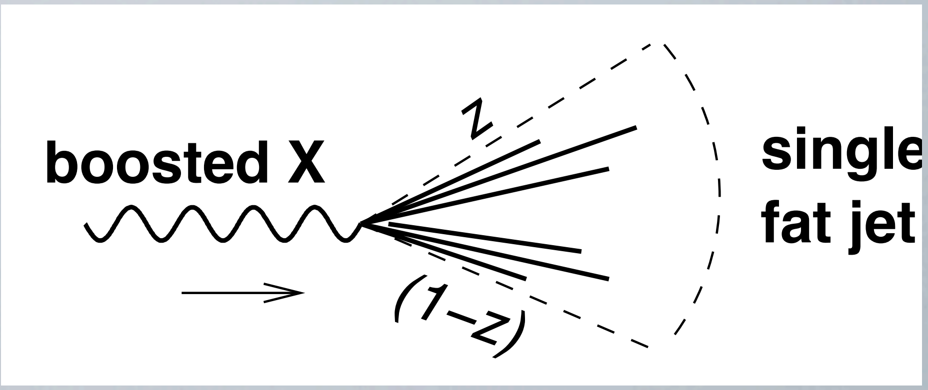
G. P. Salam, 0906.1833

Jet substructure

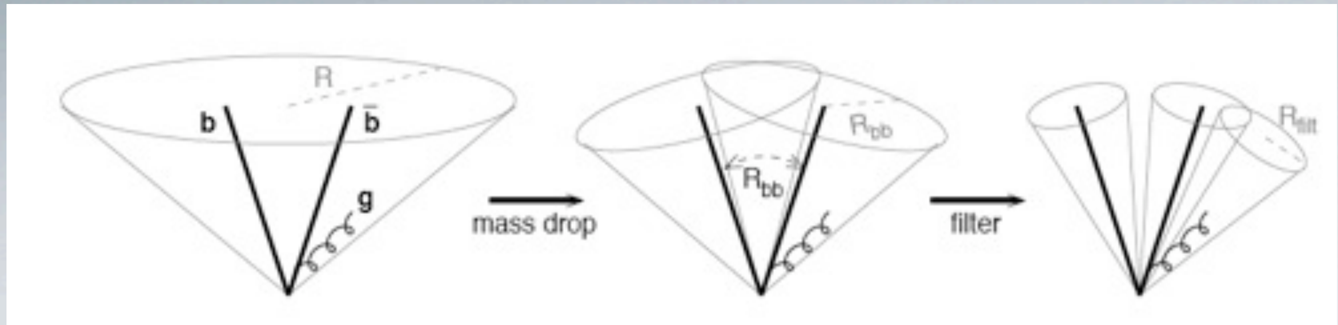
interesting to control backgrounds in Higgs to bb: **"fat jets"**

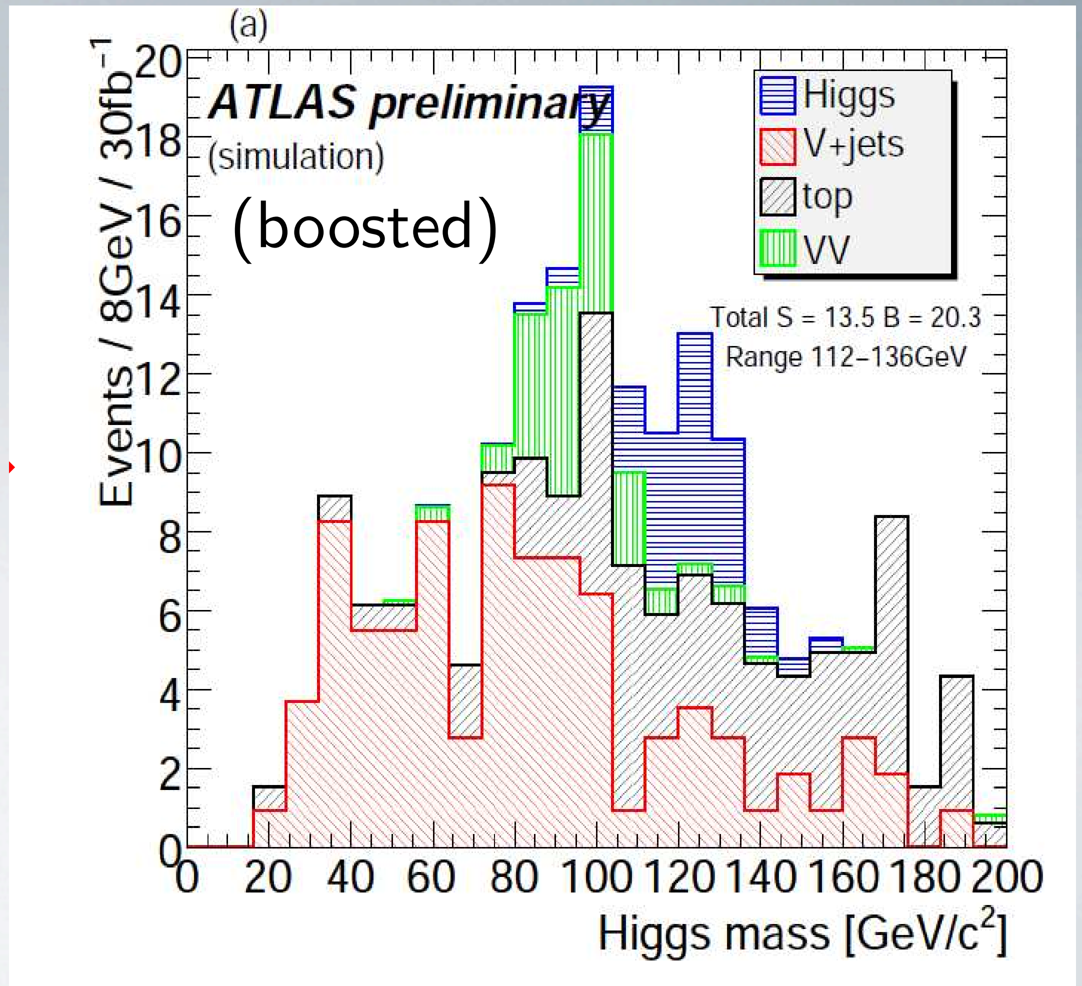
highly boosted Higgs will produce a fat jet with two b-jets inside





W/Z+H, H to bbar





idea: Undo last stage of clustering and look for significant mass drop, consistent with heavy particle decaying to jets

method pioneered by Butterworth, Davison, Rubin, Salam '08

Event shape observables

- characterise global properties of hadronic events
- extensively studied at LEP, Petra (Jade coll.)
- can also be defined for hadronic collisions
  (have been measured at LHC already)
- are infrared safe
- free from uncertainties related to jet energy measurements
- have been used extensively for measurements of the strong coupling constant

S. Bethke '06

Process	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	Theory	refs.
DIS [pol. SF]	0.7 - 8		$0.113^{+0.010}_{-0.008}$	±0.004	$^{+0.009}_{-0.006}$	NLO	[76]
DIS [Bj-SR]	1.58	$0.375^{+0.062}_{-0.081}$	$0.121^{+0.005}_{-0.009}$	–	–	NNLO	[77]
DIS [GLS-SR]	1.73	$0.280^{+0.070}_{-0.068}$	$0.112^{+0.009}_{-0.012}$	$^{+0.008}_{-0.010}$	0.005	NNLO	[78]
τ-decays	1.78	0.345 ± 0.010	0.1215 ± 0.0012	0.0004	0.0011	NNLO	[70]
DIS [ν; xF_3]	2.8 - 11		$0.119^{+0.007}_{-0.006}$	0.005	$^{+0.005}_{-0.003}$	NNLO	[79]
DIS [e/μ; F_2]	2 - 15		0.1166 ± 0.0022	0.0009	0.0020	NNLO	[80, 81]
DIS [e-p $\rightarrow$ jets]	6 - 100		0.1186 ± 0.0051	0.0011	0.0050	NLO	[67]
Υ decays	4.75	0.217 ± 0.021	0.118 ± 0.006	–	–	NNLO	[82]
$Q\overline{Q}$ states	7.5	0.1886 ± 0.0032	0.1170 ± 0.0012	0.0000	0.0012	LGT	[73]
e^+e^- [F_2^γ]	1.4 - 28		$0.1198^{+0.0044}_{-0.0054}$	0.0028	$^{+0.0034}_{-0.0046}$	NLO	[83]
e^+e^- [σ_{had}]	10.52	0.20 ± 0.06	$0.130^{+0.021}_{-0.029}$	$^{+0.021}_{-0.029}$	0.002	NNLO	[84]
e^+e^- [jets & shps]	14.0	$0.170^{+0.021}_{-0.017}$	$0.120^{+0.010}_{-0.008}$	0.002	$^{+0.009}_{-0.008}$	resum	[85]
e^+e^- [jets & shps]	22.0	$0.151^{+0.015}_{-0.013}$	$0.118^{+0.009}_{-0.008}$	0.003	$^{+0.009}_{-0.007}$	resum	[85]
e^+e^- [jets & shps]	35.0	$0.145^{+0.012}_{-0.007}$	$0.123^{+0.008}_{-0.006}$	0.002	$^{+0.008}_{-0.005}$	resum	[85]
e^+e^- [σ_{had}]	42.4	0.144 ± 0.029	0.126 ± 0.022	0.022	0.002	NNLO	[86, 32]
e^+e^- [jets & shps]	44.0	$0.139^{+0.011}_{-0.008}$	$0.123^{+0.008}_{-0.006}$	0.003	$^{+0.007}_{-0.005}$	resum	[85]
e^+e^- [jets & shps]	58.0	0.132 ± 0.008	0.123 ± 0.007	0.003	0.007	resum	[87]
$p\bar{p} \rightarrow b\bar{b}X$	20.0	$0.145^{+0.018}_{-0.019}$	0.113 ± 0.011	$^{+0.007}_{-0.006}$	$^{+0.008}_{-0.009}$	NLO	[88]
$p\bar{p}$, $pp \rightarrow \gamma X$	24.3	$0.135^{+0.012}_{-0.008}$	$0.110^{+0.008}_{-0.005}$	0.004	$^{+0.007}_{-0.003}$	NLO	[89]
$\sigma(p\bar{p} \rightarrow$ jets)	40 - 250		0.118 ± 0.012	$^{+0.008}_{-0.010}$	$^{+0.009}_{-0.008}$	NLO	[90]
e^+e^- $\Gamma(Z \rightarrow had)$	91.2	$0.1226^{+0.0058}_{-0.0038}$	$0.1226^{+0.0058}_{-0.0038}$	±0.0038	$^{+0.0043}_{-0.0005}$	NNLO	[91]
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022	0.1176 ± 0.0022	0.0010	0.0020	NLO	[92]
e^+e^- [jets & shps]	91.2	0.121 ± 0.006	0.121 ± 0.006	0.001	0.006	resum	[32]
e^+e^- [jets & shps]	133	0.113 ± 0.008	0.120 ± 0.007	0.003	0.006	resum	[32]
e^+e^- [jets & shps]	161	0.109 ± 0.007	0.118 ± 0.008	0.005	0.006	resum	[32]
e^+e^- [jets & shps]	172	0.104 ± 0.007	0.114 ± 0.008	0.005	0.006	resum	[32]
e^+e^- [jets & shps]	183	0.109 ± 0.005	0.121 ± 0.006	0.002	0.005	resum	[32]
$\mathbf{e^+e^-}$ [jets & shps]	189	0.109 ± 0.004	0.121 ± 0.005	0.001	0.005	resum	[32]
e^+e^- [jets & shps]	195	0.109 ± 0.005	0.122 ± 0.006	0.001	0.006	resum	[81]
e^+e^- [jets & shps]	201	0.110 ± 0.005	0.124 ± 0.006	0.002	0.006	resum	[81]
e^+e^- [jets & shps]	206	0.110 ± 0.005	0.124 ± 0.006	0.001	0.006	resum	[81]

Event shapes

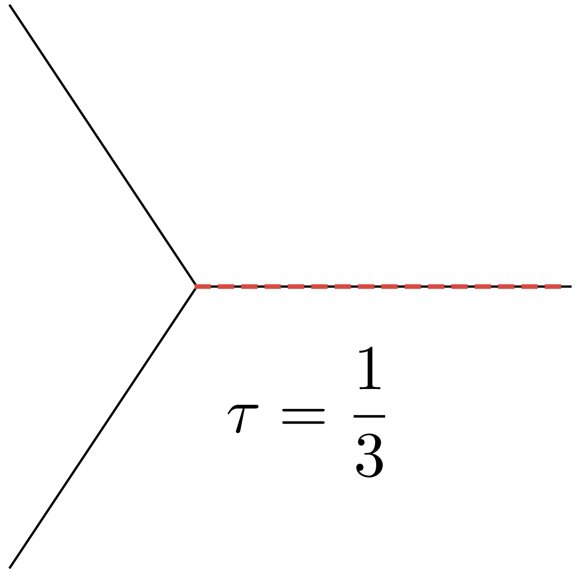
a classical event shape observable is thrust T

$$T = \frac{1}{Q} \max_{\vec{n}} \sum_i |\vec{n} \cdot \vec{p_i}|$$

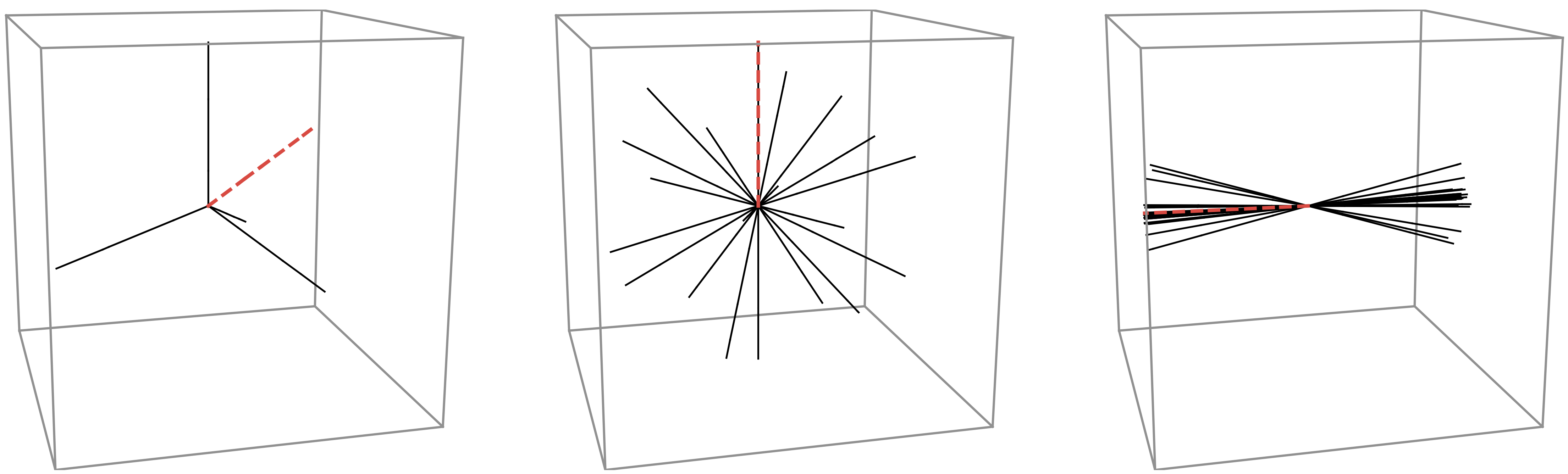
$$\tau = 1 - T$$

$\tau = 0$

(two-jet limit)



$\tau = \frac{1}{3}$



$\tau = 1 - \frac{1}{\sqrt{3}} = 0.42$

$\tau = 0.48$

$\tau = \frac{M_1^2 + M_2^2}{Q^2}$

Other "classical" event shape observables

- heavy hemisphere mass

$$\rho \equiv M_H^2/s = \max(M_1^2/s, M_2^2/s) \qquad M_i^2/s = \frac{1}{E_{\text{vis}}^2}\left(\sum_{k\in H_i} p_k\right)^2$$

- jet broadenings

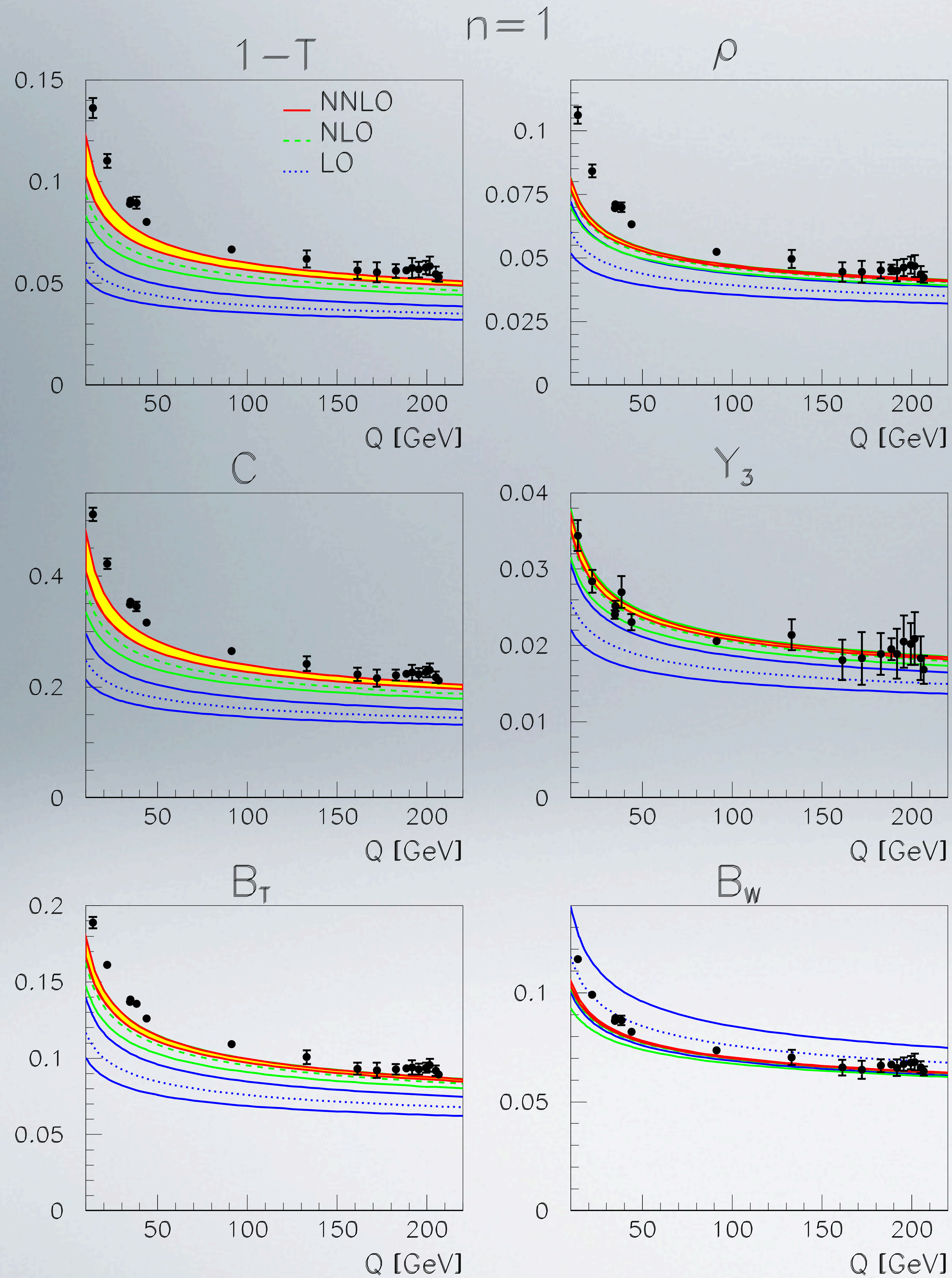
$$B_W = \max(B_1, B_2)$$
$$B_T = B_1 + B_2 \,.$$
$$B_i = \frac{\sum_{k\in H_i} |\vec{p_k} \times \vec{n}_T|}{2\sum_k |\vec{p_k}|}$$

- C-parameter

momentum tensor $\Theta^{\alpha\beta} = \frac{1}{\sum_k |\vec{p_k}|} \sum_k \frac{p_k^\alpha p_k^\beta}{|\vec{p_k}|}$ has 3 eigenvalues lambda

$$C = 3\,(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$$

- jet transition variable Y3: the value of the jet resolution parameter y_{cut} where an event changes from a 2-jet to a 3-jet configuration



n=1 means first moment of the observable y :

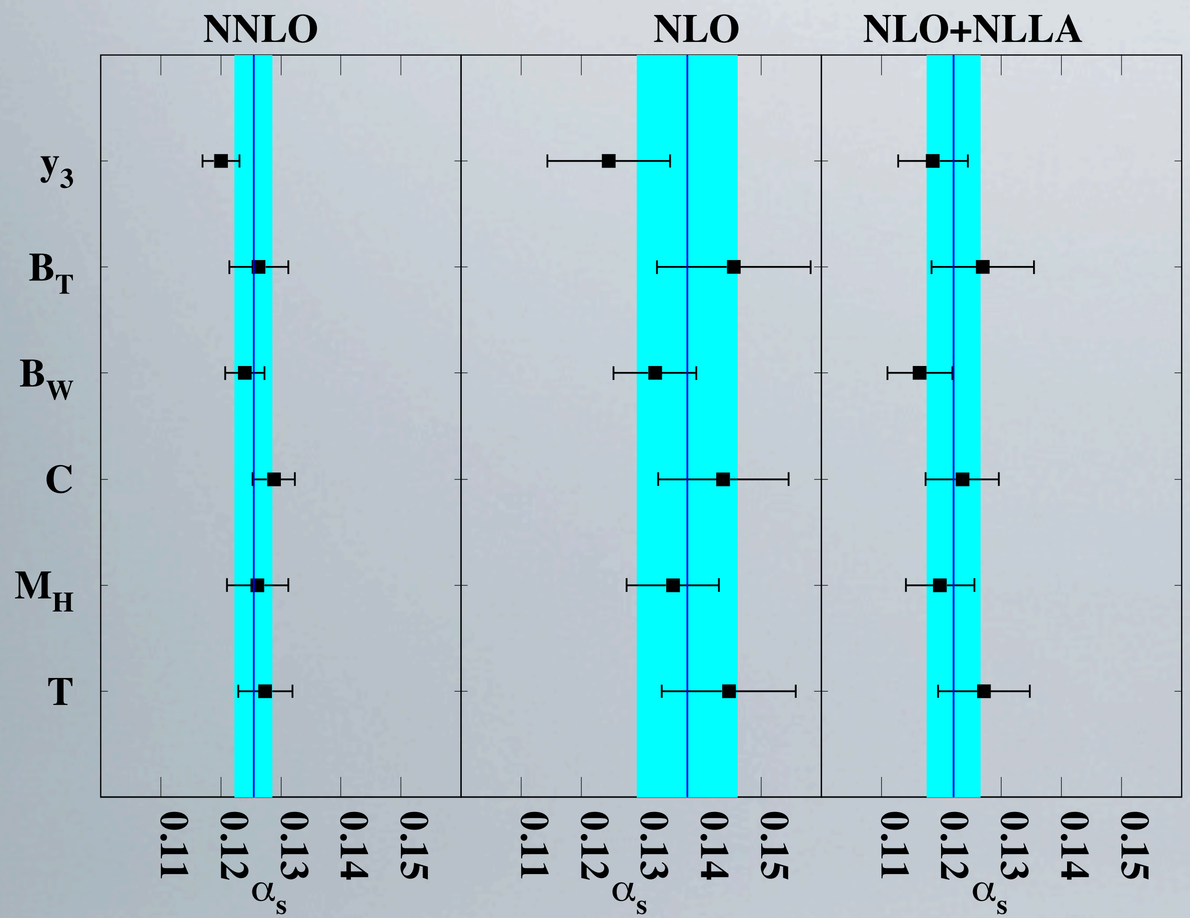
$$\frac{1}{\sigma_0}\int_0^{y_{\max}} \mathrm{d}y\, y^n \frac{\mathrm{d}\sigma}{dy}$$

comparison to JADE and OPAL data

resummation and hadronisation corrections are still important

Gehrmann-De Ridder, Gehrmann, Glover, GH 2009

Determinations of α_s from event shapes

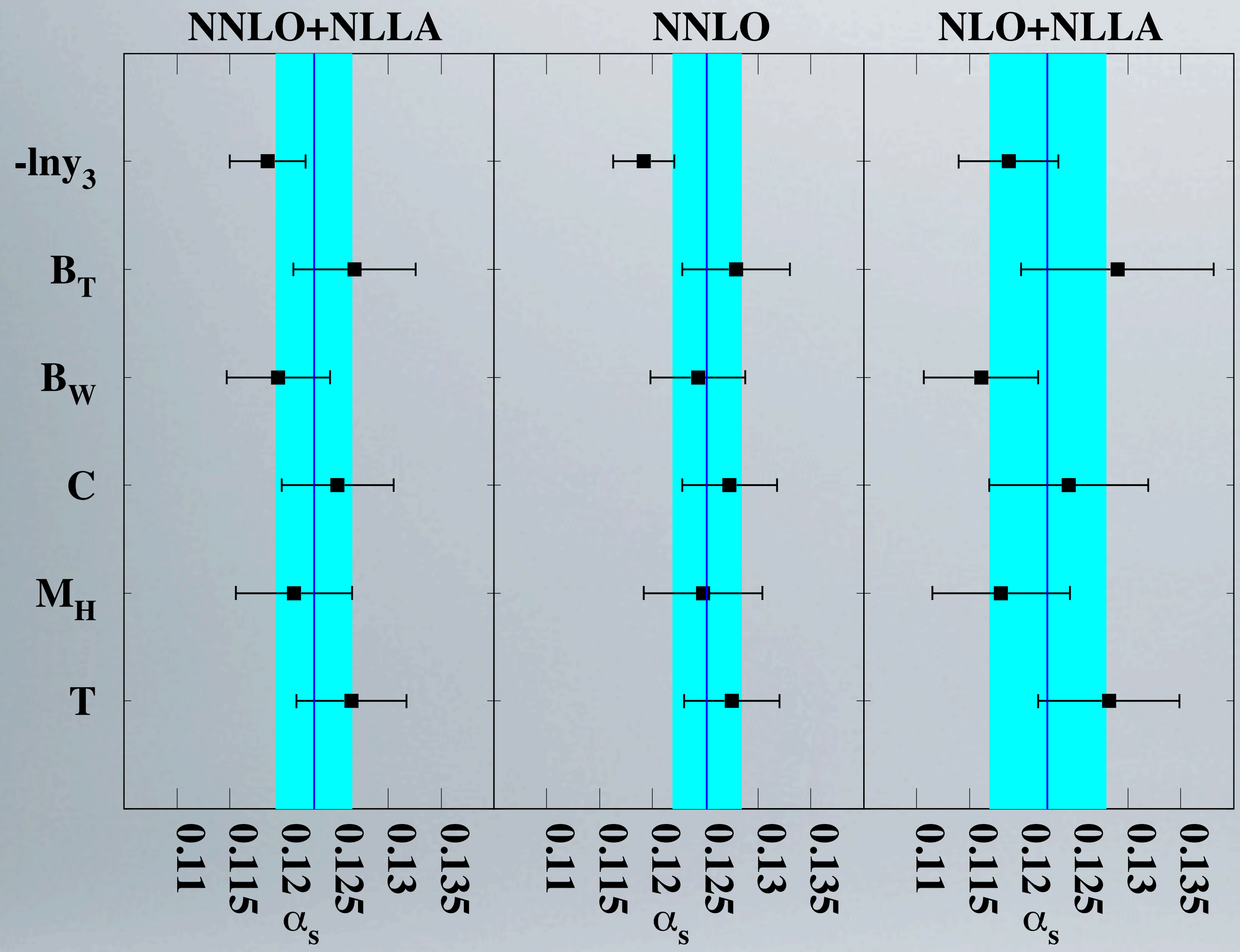


[G. Dissertori, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH, H. Stenzel, Dec '07]

$$\alpha_s(M_Z) = 0.1240 \quad \pm 0.0008\,(\text{stat}) \pm 0.0010\,(\text{exp}) \pm 0.0011\,(\text{had}) \pm 0.0029\,(\text{theo})$$

- reduction of theory error on α_s from new fit to LEP data by a factor of 2 (1.3) compared to NLO (NLO+resum.)
- scatter in different shapes greatly reduced

α_s fit based on NNLO+NLLA resummed event shapes



[G. Dissertori, A. Gehrmann-De Ridder,
T. Gehrmann, N. Glover,
GH, G. Luisoni, H. Stenzel '09]

$$\alpha_s(M_Z) = 0.1224 \quad \pm 0.0009\,(\text{stat}) \pm 0.0009\,(\text{exp})$$
$$\pm 0.0012\,(\text{had}) \pm 0.0035\,(\text{theo})$$

next-to-leading logarithmic approximation (NLLA) re-introduces larger scale dependence than pure NNLO -- why ?

In the two-jet region the NLLA+NLO and NLLA+NNLO predictions agree by construction, because the matching suppresses any fixed order terms.

→ Renormalisation scale uncertainty dominated by NLLA in this region .

Another interesting observation:

C-parameter and jet broadenings give a parton level prediction with PYTHIA, which is about 10% higher than the NNLO+NLLA prediction.

The PYTHIA result is obtained with tuned parameters, where the tuning to data had been performed at the hadron level.

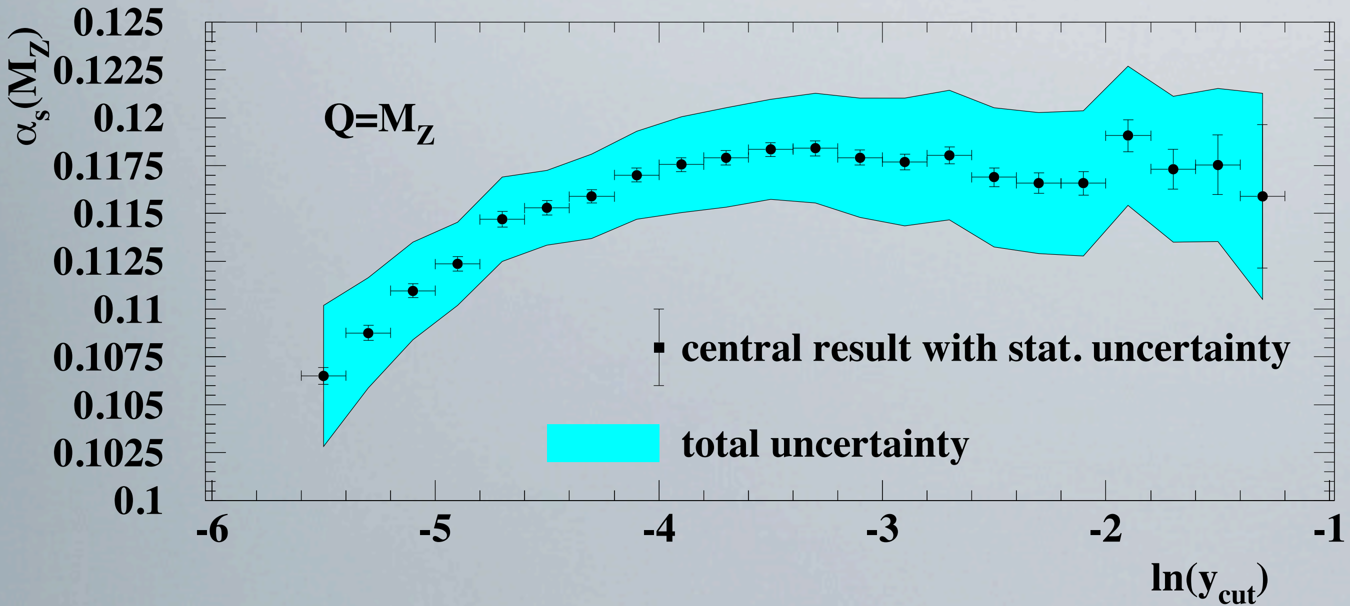
This tuning results in a rather large effective coupling in the parton shower.

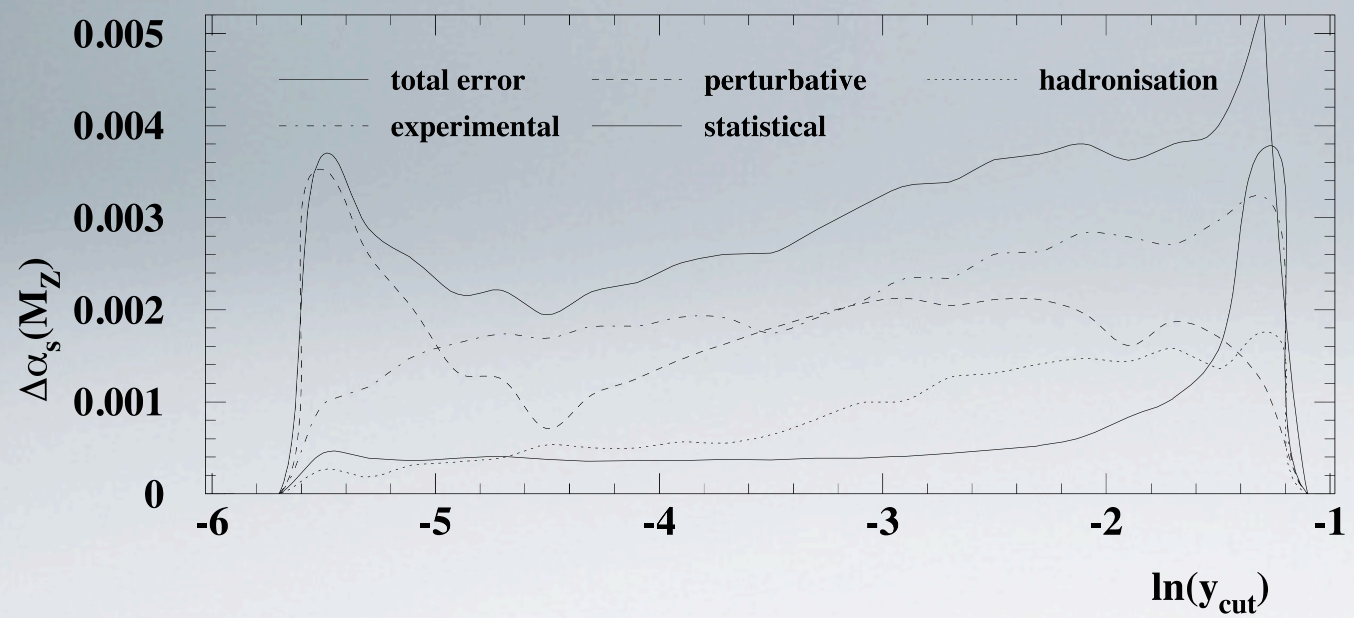
However, since the tuning has been performed at hadron level: hadronisation corrections come out to be smaller than what would have been found by tuning a hypothetical Monte Carlo prediction with a parton level corresponding to the NNLO+NLLA prediction.

→

The PYTHIA hadronisation corrections, applied in the alpha_s fit, might be too small, resulting in a larger alpha_s value.

α_s fit based on 3-jet rates





[G. Dissertori, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH, G. Luisoni, H. Stenzel, Oct ’09]

$$\alpha_s(M_Z) = 0.1175 \pm 0.0020\,(\mathrm{ex}) \pm 0.0015\,(\mathrm{theo})$$