Perturbative QCD and Jets

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Lecture 1

2013 CERN-Fermilab Hadron Collider Physics School

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

Outline

• Basics of QCD

- Lagrangian and Feynman rules
- Colour
- QCD beta-function and asymptotic freedom
- Factorisation
- QCD concepts in Phenomenology
	- e+e- to hadrons and infrared singularities
	- Scale variations
	- Hadronic collisions and PDFs
	- Jets
	- Selected topics: NLO automation, prompt photons, ... (time permitting)

Literature

• R. K. Ellis, W. J. Stirling and B. R. Webber, *QCD and collider physics*, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 8 (1996).

Some literature for

- *•* G. Dissertori, I. Knowles, M. Schmelling, *Quantum Chromodynamics: High energy experiments and theory* International Series of Monographs on Physics No. 115, Oxford University Press, Feb. 2003, 2005.
- *•* J. M. Campbell, J. W. Huston and W. J. Stirling, *Hard Interactions of Quarks and Gluons: A Primer for LHC Physics*, Rept. Prog. Phys. 70 (2007) 89 [hep-ph/0611148].
- *•* J. Alcaraz Maestre et al., *The SM and NLO Multileg and SM MC Working Groups: Summary Report of the Les Houches 2011 workshop on Physics at TeV Colliders*, arXiv:1203.6803 [hep-ph].
- *•* G. P. Salam, *Towards Jetography*, Eur. Phys. J. C 67 (2010) 637, arXiv:0906.1833 [hep-ph].
- *•* M. Dasgupta, A. Fregoso, S. Marzani and G. P. Salam, *Towards an understanding of jet substructure*, arXiv:1307.0007 [hep-ph].
- *•* J. Shelton, *TASI Lectures on Jet Substructure*, arXiv:1302.0260 [hep-ph].

Motivation

Why do we care about QCD ?

- we have to : it dominates hadronic collisions
- can hide New Physics effects
- can fake New Physics effects
- is interesting by itself

the precision we can achieve on important measurements (e.g. Higgs properties) is directly linked to the control of QCD effects!

e.g. Higgs production in gluon fusion at NNLO:

scale $pdf + \alpha_S$

 $\sigma(m_H = 125 \,\text{GeV}) = 19.27^{+7.2\%}_{-7.8\%}~^{+7.5\%}_{-6.9\%}~\text{pb}$

D. de Florian, EPS '13

magnitudes of cross sections:

QCD dominates

Basics of QCD

strong interactions are described by SU(3) gauge theory

Evidence for 3 Colours

Data lies systematically higher that

Data lies systematically higher that

Evidence for 3 Colours

zeigt daher nur die Produktion von Hadronen über Quarks, ist also sozusa-

QCD Lagrangian $\overline{\mathbf{QCD}}$

ian gauge theory flavourse ortianal transmit content from QED ! important content

about the mortant content from a both important content in the mortant content of the main term and the main t different from QED ! important consequences $\frac{1}{2}$ non-Abelian gauge theory and different from QED ! important consequences

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#² (covariant gauges)

$$
\mathcal{L}_{\text{fermion}}\ =\ \sum_{\text{flavours}} \bar{q}_a\,(i\rlap{\,/}D^{ab}-m\,\delta^{ab})\,q_b
$$

^µ − g fABCA^B

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<u> 2 (covariant gauges) et anno 1994.</u>
1995 - Covariant gauges, politik gauges (covariant gauges) et anno 1994.
1996 - Covariant gauges, politik gauges (covariant gauges) et anno 1996.

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Lgauge fixing and the set

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$$
\rlap{\,/}\hspace{.1in}D_{ab} \; = \; \gamma_\mu D_{ab}^\mu \; ; \; D_{ab}^\mu = \partial^\mu \delta_{ab} + i \; g \; (t^A \mathcal{A}_A^\mu)_{ab}
$$

 $a, b \in \{1, 2, 3\}$ quarks in **rundamental** representation d $a, b \in \{1, 2, 3\}$ quarks in **fundamental** representation of SU(3)

$$
t^A = \lambda^A/2
$$
 λ^A : Gell-Mann matrices
generators of SU(3)

KAB µν

 $[t_A, t_B] = i \int_{ABC} t^C$ $[t_A, t_B] = i f_{ABC} t^C$ fabre: structure constants

 $\mathcal{L} = \mathcal{L} \mathcal$

indices are summed over NB conventions: doubly occurring indices are summed over er NB conventions: doubly occurring indices are summed over

$$
\mathcal{L}_{\text{gauge fixing}} = -\frac{1}{2\lambda} \left(\partial^{\mu} \mathcal{A}_{\mu}^{A} \right)^{2} \text{ (covariant gauges)}
$$

"² (axial gauges)

"2 (axial gauges: n · A = 0) (axial gaug

 $\lambda \rightarrow 0$: Landau gauge $\lambda = 1$: Feynman gauge $\lambda \to 0$: Landau gauge

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n^µA^A

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^Lgauge fixing ⁼ [−] ¹

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\mathcal{L}_{\text{gauge fixing}} = -\frac{1}{2\lambda} \left(n^{\mu} \mathcal{A}_{\mu}^{A} \right)^{2} \text{ (axial gauges: } n \cdot A = 0)
$$

$$
n^{2} = 0: \text{ light-cone gauge}
$$

reminder: classical equation of motion $K_{\mu\nu}^{AB}{\cal A}_{\nu}^{\nu}$ $K^{AB}_{\mu\nu}{\cal A}^{\nu}_{B}=\delta^{AB}(-\Box g_{\mu\nu}+\partial_{\mu}\partial_{\nu})\,{\cal A}^{\nu}_{B}=J^{A}_{\mu}$ $L_R^{\nu} = \delta^{A}$ $\frac{1}{2}$

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"² (axial gauges: n · A = 0)

 $-AB$: $\qquad \qquad \qquad \bullet$ cannot be solved because $K_{\mu\nu}^{AB}$ is not invertible \implies need gauge fixing τ \sim AB . The set of \sim \sim $\mu\nu$ eed ga $K_{\mu\nu}^{AB}$ not be solved because $K_{\mu\nu}^{AB}$ is not invertib

and General Ge
Short fields **Ghost fields** Ghost fields

^Lgauge fixing ⁼ [−] ¹

 $\mathcal{L}_{\text{ghost}} = \partial_{\mu}(\eta^A)^{\dagger} (D^{\mu}_{AB} \eta^B)$ (covariant gauges) $\mathcal{L} = \partial (\rho^A)^\dagger (\mathcal{D}^\mu, \rho^B)$ (covariant gauges)

 $\mathcal{L}_{\text{ghost}} = -(\eta \text{)} \eta_{\mu} (D_{AB} \eta) = -(\eta \text{)} \eta_{\mu} (\sigma \eta_A)$ (axial gauges) $\mathcal{L}_{\text{ghost}} = -(\eta^A)^{\dagger} n_{\mu} (D^{\mu}_{AB} \eta^B) = -(\eta^A)^{\dagger} n_{\mu} (\partial^{\mu} \eta_A)$ (axial gauges)

 η complex scalar field obeying Fermi statistics (related to Jacobian of gauge transformations in path integral formulation) auge transformations

• Covariant gauges introduce unphysical gluon polarisations at quantum level which are cancelled by ghost-gluon interactions.

t gluon polarisations propagate. nosis do not couple to
propagate. • In axial gauges ghosts do not couple to gluons, only *physical*

Therefore axial gauges are also called *physical* gauges.

Edition Rules

September 1988 = 0 : light-cone gauge \mathcal{A}

 \boldsymbol{p} B, ν A, μ gluon propagator $\begin{picture}(150,10) \put(0,0){\dashbox{0.5}(10,0){ }} \put(150,0){\circle{10}} \put(150,$

 \mathcal{A}

$$
\Delta_{\mu\nu}^{AB}(p) = \frac{i \, \delta^{AB}}{p^2 + i \, \varepsilon} \, d_{\mu\nu}
$$

$$
d_{\mu\nu} = \sum_{\text{relativities}} \epsilon^*_{\mu}(p, \alpha) \epsilon_{\nu}(p, \alpha)
$$

 $\mathop{\rm polar}$ isations α

$$
= \begin{cases} -g_{\mu\nu} + (1-\lambda) \frac{p_{\mu}p_{\nu}}{p^2} & \text{covariant gauge} \\ -g_{\mu\nu} + \frac{p_{\mu}n_{\nu} + p_{\nu}n_{\mu}}{p \cdot n} & \text{light-cone gauge} \end{cases}
$$

 $\frac{p_{\nu}n_{\mu}}{2}$ light-cone gauge covariant gauge $\frac{\partial \rho + p_\nu n_\mu}{p\cdot n} \qquad \text{light-cone gauge}$

Feynman Rules

conventions from Ellis, Stirling, Webber QCD and Collider Physics

$$
\frac{a}{\cos \theta} \int_{-\infty}^{\infty} \frac{b}{\cos \theta} \cos \theta \, d\theta
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= \frac{a}{\cos \theta} \int_{-\infty}^{\infty} \frac{b}{\cos \theta} \cos \theta \, d\theta
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\int_{\text{max b}} = T_R \text{ and } Tr(t^A) = 0
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\n
$$
\begin{cases}\n\text{b} & \text{if } Tr(t^A) = 0 \\
\text{a} & \text{if } Tr(t^A) = 0 \\
\text{b} & \text{if } Tr(t^A) = 0\n\end{cases}
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Andrea Banfi Lecture 1

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Colour Algebra

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generators of SU(Nc):

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Trace identities: Tr(ta)=0 and Tr(tatb) = TRδab

a b = 1 march 2008 and 2008 a

generators of SU(Nc):

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^a = 0 ^R ^a ^T ^b ⁼ ^a ^b Adjoint representation 8 ^a ^b = !ab ^c = i fabc b a Andrea Banfi Lecture 1 ^a = 0 ^R ^a ^T ^b ⁼ ^a ^b Adjoint representation 8 ^a ^b = !ab ^c = i fabc b a Andrea Banfi Lecture 1 Pictorial representation of SU(Nc) identities Casimir factors Fundamental representation 3: kj = C^F δij C^F = ^c − 1 ²N^c = CF Adjoint representation 8: facdf bcd ⁼ ^CAδab ^C^A ⁼ ^N^c CA ⁼ Fierz identity: ^k (t 2 δi ^j ^δ^l ^k [−] ¹ 2Nc δ1 ^k ^δ^l 2 1 2Nc = ! Gluons as carriers of colour in the large-Nc limit Casimir factors Fundamental representation 3: kj = C^F δij C^F = N² ^c − 1 2N^c = CF facdf bcd ⁼ ^CAδab ^C^A ⁼ ^N^c CA ⁼ Fierz identity: ^j = ^j δ^l ^k [−] ¹ δ1 ^k ^δ^l 2 1 2Nc 1 = ! Gluons as carriers of colour in the large-Nc limit Pictorial representation of SU(Nc) identities Casimir factors Fundamental representation 3: ikt kj = C^F δij C^F = ^c − 1 ²N^c = CF Adjoint representation 8: X facdf bcd ⁼ ^CAδab ^C^A ⁼ ^N^c CA ⁼ ^a)ⁱ ^k (t a) l ^j = 1 2 δi ^j ^δ^l ^k [−] ¹ 2Nc δ1 ^k ^δ^l j 1 1 = ! Gluons as carriers of colour in the large-Nc limit + O(1/N)c 1 2 = Casimir factors Fundamental representation 3: a ikt a kj = C^F δij C^F = N² ^c − 1 2N^c = CF Adjoint representation 8: X facdf bcd ⁼ ^CAδab ^C^A ⁼ ^N^c CA ⁼ ^a)ⁱ ^k (t a) ^j = 2 δi ^j δ^l ^k [−] ¹ 2Nc δ1 ^k ^δ^l j 2 1 2Nc 1 = ! Gluons as carriers of colour in the large-Nc limit + O(1/N)c 1 2 = A, µ B, ν a b A B T^R C^A C^F = N² ^c − 1 2N^c Tr(t At ^B) = TRδAB 1 2 C,D f CDAf CDB = C^A δAB , C^A = N^c (t ^A)ab(t ^A)cd = ^δadδbc [−] ¹ δabδcd f ABEf CDE + f BCEf ADE + f CAEf BDE = 0 generators of SU(Nc): ^c − 1 hermitean traceless matrices (t^A)ab Tr(t At ^B) = TRδAB , T^R = 1 2 ! act = C^A δAB , C^A = N^c ^A)cd = 1 ^δadδbc [−] ¹ δabδcd f ABEf CDE + f BCEf ADE + f CAEf BDE generators of SU(Nc): ^c − 1 hermitean traceless matrices (t^A)ab C^F = N² ^c − 1 2N^c ^A) = 0 At ^B) = TRδAB , T^R = 1 2 ! A t A act A cb = C^F δab ^A)ab(t ^A)cd = 1 ^δadδbc [−] ¹ 2N^c δabδcd f ABEf CDE + f BCEf ADE + f CAEf BDE generators of SU(Nc): ^c − 1 hermitean traceless matrices (t^A)ab p a b A B T^R C^A 1 color = ^C^F ⁼ ^N² Tr(t ^A) = 0 ^B) = TRδAB , ^T^R ⁼ ¹ 2 A cb = C^F δab ! C,D f CDAf CDB = C^A δAB , C^A = N^c f ABEf CDE+ f BCEf ADE + f CAEf BDE = 0 generators of SU(Nc): ^c − 1 hermitean traceless matrices (t^A)ab a b A B T^R C^A 2N^c = N² ^c − 1 2N^c ^B) = TRδAB , T^R = 2 ! A t act A cb = C^F δab ! f CDAf CDB , C^A = N^c (t ^A)ab(t ^A)cd = 1 2 ^δadδbc [−] ¹ 2N^c δabδcd f ABEf CDE + f BCEf ADE + f CAEf BDE = 0 generators of SU(Nc): ^c − 1 hermitean traceless matrices (t^A)ab A, µ a b A B T^R C^A colour = ^A) = 0 Tr(t At ^B) = TRδAB cb = C^F δab ! C,D f CDAf CDB = C^A δAB ^A)ab(t ^A)cd = ^δadδbc [−] ¹ Tr(t At ^B) = TRδAB , T^R = 1 2 A cb = C^F δab C,D f CDAf CDB = C^A δAB , C^A = N^c ^A)cd = 1 2 ^δadδbc [−] ¹ 2N^c f ABEf CDE + f BCEf ADE + f CAEf BDE = 0 generators of SU(Nc): ^c − 1 hermitean traceless matrices (t^A)ab colour = C^F = N² Tr(t Tr(t At ^B) = TRδAB ! C,D f CDAf CDB = C^A δAB ^A)ab(t ^A)cd = ^δadδbc [−] ¹ f ABEf CDE + f BCEf ADE + f CAEf BDE generators of SU(Nc): N² ^c − 1 hermitean traceless matrices (t^A)ab B, ν a b A B T^R C^A 1 2N^c colour = C^F = N² ^c − 1 2N^c Tr(t ^A) = 0 Tr(t At ^B) = TRδAB , T^R = ! 2 A act A cb = C^F δab , C^A = N^c (t ^A)ab(t ^A)cd = ^δadδbc [−] ¹ 2N^c δabδcd f ABEf CDE + f BCEf ADE + f CAEf BDE generators of SU(Nc): N² ^c − 1 hermitean traceless matrices (t^A)ab a b A B T^R C^A colour = ^c − 1 2N^c ^A) = 0 Tr(t At ^B) = TRδAB ! A t cb = C^F δab ! f CDAf CDB = C^A δAB Fundamental representation 3 !ij t a ij ⁱ ^j i j = = Trace identities: Tr(ta)=0 and Tr(tatb) = TRδab ^a = 0 ^R ^a ^T ^b ⁼ ^a ^b Adjoint representation 8 ^a ^b = !ab ^c = i fabc b a A, µ B, ν p a b A B T^R C^A 1 2N^c colour C^F = Tr(t ^A) = 0 Tr(t At ^B) = TRδAB , T^R = 2 ! A t A act cb = C^F δab (Fierz identity) ,

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some pictorial identities:

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we can write every n-gluon tree graph colour factor as a sum of graph colour factor as a sum of
traces of matrices: $Tr(t^{A_1}t^{A_2}\cdots t^{A_n}) +$ all non-cyclic permutations • Always single traces (at tree level) \mathbf{e} a cum of ictor as a st te eve $\frac{1}{2}$ and traceless of final fields

 $\begin{aligned} \textsf{similarity} \quad & q\bar{q}gggg \, . \end{aligned}$ similarly $q\bar{q}ggggg \ldots \Rightarrow Tr(t^{A_1}t^{A_2}\cdots t^{A_n})_{ab}$ + permutations $q\,q\,g\,g\,g\,g\ldots \Rightarrow$

^A)ab(t

f ABC

Tr(t

 $\mathcal{M}_n^{\text{tree}}(\{p_i, a_i, h_i\}) = g^{n-2}Tr(t^{A_1}t^{A_2}\cdots t^{A_n}) M_n^{\text{tree}}(1^{h_1}, 2^{h_2}\cdots n^{h_n}) + \text{all non-cyclic permutations}$ momenta colour helicities ncs

colour ordered subamplitude, colour factors stripped off ^A¹t ^A² · · ·t ^Aⁿ) + all non-cyclic permutations $\frac{1}{2}$ UUI TICHUI

important: as $M_n^{\text{tree}}(1^{h_1}, 2^{h_2} \ldots n^{h_n})$ comes from diagrams with cyclic ordering of external legs, it only has singularities in adjacent invariants $|s_{i,i+1} - (p_i + p_{i+1})^2|$ (see later)

ⁿ ({pi, ^ai, ^hi}) ⁼ ^gⁿ−² Colour expansion generators of SU(Nc):

(as well as helicities):

(as well as helicities):

Inserting:

Æ Up to 1*/Nc*

$$
d\sigma^{\rm tree}(\{p_i, a_i, h_i\}) \sim \sum_{a_i} \sum_{h_i} |M_n^{\rm tree}(\{p_i, a_i, h_i\})|^2
$$

insert colour ord ered amplitude and perform the colour sum " solour ordered amplitude and perform the colou \mathbf{a}_i and doing the color sums diagrammatically: \mathbf{a}_i and doing the color sums diagrammatically: $\frac{1}{2}$ insert colour ordered amplitude and perform the colour sum:

$$
\begin{pmatrix}\n\begin{b
$$

$$
d\sigma^{\text{tree}}(\{p_i, a_i, h_i\}) \sim N_c^n \sum_{\sigma \in S_n/Z_n} \sum_{h_i} \left| M_n^{\text{tree}}(\sigma(1^{h_1}), \sigma(2^{h_2}) \dots \sigma(n^{h_n})) \right|^2 + \mathcal{O}(N_c^{n-2})
$$

Non-planar topologies are subleading in colour definite color flow definite color flow definite color flow definite to part of the part o

L. Dixon, 7/20/06 Higher Order QCD: Lect. 1 24

 $\frac{1}{2}$ Note: parton showers usually do not take $\mathbf{L} = \mathbf{L} \mathbf$ subleading colour into account

QCD beta-function

... contains one of the most important minus signs in physics !

QED:

Roughly speaking, the gluon self couplings reverse the sign of the beta-function. In more detail ...

QCD beta-function

 g^2

 4π

- consider a dimensionless observable R which can be expanded in $\alpha_s=$ and which depends on a single large energy scale Q **CONSIDER A** dimensionle
	- dimensional analysis R should be independent of Q Consider dimensionless physical observable R which depends on a single large
	- however, R needs UV renormalisation !
	- this introduces another mass scale μ : the point at which the subtractions of the UV divergences are performed
	- therefore R will depend on the ratio Q/μ subtractions which remove divergences are performed. Then R depends on the
	- the renormalized coupling α_s will also depend on μ \cos and the construction of $\cos \beta$ and the renormalized on β
	- as μ is arbitrary, R can not depend on it \implies

$$
\mu^2 \frac{d}{d\mu^2} R\left(\frac{Q^2}{\mu^2}, \alpha_S\right) \equiv \left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_S}{\partial \mu^2} \frac{\partial}{\partial \alpha_S}\right] R = 0
$$

QCD beta function Introducing unction

 define $\tau = \ln \left(\frac{Q^2}{2} \right)$ μ^2 \int , $\beta(\alpha_S) = \mu^2 \frac{\partial \alpha_S}{\partial \alpha_S^2}$ $\frac{\partial \mu}{\partial \mu^2}$, $\tau = \ln |\mathbf{r}|$ \mathbf{n} $\left(\frac{Q^2}{\beta(\alpha s)}\right)_{\alpha=0}$ $=\mu^2$ $\left(\mu^2\right)^{-1}$ $\beta(\alpha_S) = \mu^2 \frac{\partial \alpha_S}{\partial \alpha_S}$

then

Introducing

we have

Introducing

Then

Then

and hence R(Q2/µ²

from running of αS(Q).

order allows us to predict variation of R with Q.

then
$$
\left[-\frac{\partial}{\partial \tau} + \beta(\alpha_S) \frac{\partial}{\partial \alpha_S}\right] R = 0
$$
 renormalisation group equation

renormalisation group equation ∂τ ⁺ ^β(αS) [∂] ∂α^S

QCD and Monte Carlo MethodsLecture I: QCD, asymptotic freedom and infrared safety – p.25/38

c − 5NcMf − 3CF

QCD and Monte Carlo MethodsLecture I: QCD, asymptotic freedom and infrared safety – p.25/38

solved by running coupling
$$
\alpha_s(Q)
$$
: $\tau = \int_{\alpha_S}^{\alpha_S(Q)} \frac{dx}{\beta(x)}, \quad \alpha_S(\mu) \equiv \alpha_S$

$$
\frac{\partial \alpha_S(Q)}{\partial \tau} = \beta(\alpha_S(Q)), \quad \frac{\partial \alpha_S(Q)}{\partial \alpha_S} = \frac{\beta(\alpha_S(Q))}{\beta(\alpha_S)}
$$

beta fur $\frac{1}{2}$ beta function has the expansion α = β (β) , β , β the beta function has the expansion
■ **and scale of the expansion**

order allows us to predict variation of R with Q.

E^W

^T =

$$
\beta(\alpha_s) = -b_0 \alpha_s^2 (1 + b_1 \alpha_s) + \mathcal{O}(\alpha_s^4)
$$

$$
b_0 = \frac{1}{12\pi} (11N_c - 2N_f), b_1 = \frac{17N_c^2 - 5N_cN_f - 3C_FN_f}{2\pi (11N_c - 2N_f)}
$$

M²

b<mark>o</mark>on ah boo

where N_f is the number of active flavours. Terms up to $\mathcal{O}(\alpha^4)$ where N_f is the number of active flavours. Terms up to $\mathcal{O}(\alpha_s^4)$ are known.

^W + p²

^T (W)

F) KG − 2Nf) , b1 = 2Nf (200) , b1 = 2Nf (200) , b1 = 2Nf)

asymptotic freedom

QCD:
$$
\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2) b_0 \ln\left(\frac{Q^2}{2}\right)}
$$
 $b_0 = \frac{1}{12\pi}(11N_c - 2N_f)$

$$
\text{QCD:} \quad \alpha_s(Q^2) = \frac{\alpha_s(\mu)}{1 + \alpha_s(\mu^2) b_0 \ln\left(\frac{Q^2}{\mu^2}\right)} \qquad b_0 = \frac{1}{12\pi}
$$

From earlier slides, coupling decreases with energy ⇒

> \overline{a} = \overline{b} **asymptotic freedom**

 $\mathcal{L}=\mathcal{L}^{\mathcal{L}}$ and $\mathcal{L}^{\mathcal{L}}=\mathcal{L}^{\mathcal{L}}$ and $\mathcal{L}^{\mathcal{L}}$ and $\mathcal{L}^{\mathcal{L}}$ and $\mathcal{L}^{\mathcal{L}}$

QED:
$$
\alpha(Q^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{Q^2}{m_e^2}\right)}
$$

coupling grows with energy

 $\overline{}$

at small scales. Turning coupling diverges, so per turbation theory cannot be applied

⇒ domain of **lattice QCD**

 $T_{\rm eff}$ is a scale that is introduced dynamically introduced dynamically in the theory, it is the price of \sim confinement: partons (quarks and gluons) are only found in The running of the coupling tells us that we cannot be coupling theory for scales us that we can not scale sca colour singlet bound states (hadrons)

The group of the main scales in groups at small scales in the processes is consistent with the fact that the f quarks critical be obtained as formal be objected as free objects, but are always confined to form α **hadronisation:** partons produced in hard scattering processes reorganize themselves to form hadrons

Lambda Parameter β(αs) = −b0 α2)
β(αs) = −b0 α2)

b⁰ =

 \mathbf{h}_1 , but \mathbf{h}_2 , \mathbf{h}_3 and \mathbf{h}_4 , \mathbf{h}_5

^s (1 ⁺ ^b¹ ^αs) ⁺ ^O(α⁴

E^W

 $\mathcal{F}_{\mathcal{A}}$

It is useful to define a dimensionful parameter Λ (integration constant) setting the scale at which the coupling becomes large. \ddot{e} at which the coupling becomes large. aimensioniui paramet r Λ (integration constant $\overline{11}$ $\overline{10}$ $\overline{0}$ $\overline{0}$ $\overline{0}$ $\overline{10}$ $\overline{10}$ define a dimensionful parar ln "Q22"
Eln "Q22" am \sim $\sqrt{ }$ (integration) $\mathsf{constant}$)

$$
\ln\left(\frac{Q^2}{\Lambda^2}\right) = -\int_{\alpha_s(Q)}^{\infty} \frac{dx}{\beta(x)} = \int_{\alpha_s(Q)}^{\infty} \frac{dx}{b_0 x^2 (1 + b_1 x + \ldots)}
$$

Keeping only
$$
b_0(\text{LO}), b_1(\text{NLO})
$$

\n
$$
\alpha_s(Q) = \frac{1}{b_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)} \quad \text{(LO)} \qquad \alpha_s(Q) = \frac{1}{b_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)} \left[1 - \frac{b_1 \ln \ln\left(\frac{Q^2}{\Lambda^2}\right)}{b_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}\right] \quad \text{(NLO)}
$$

αs (Q) = αs
(Q) = αs (Q) = αs (Q)
= αs (Q) = αs (Q) = αs (Q) \mathbf{S} $\overline{}$ ber of active $\frac{1}{2}$ $\overline{}$ bte that Λ deper $\overline{\mathbf{M}}$ Note that Λ depends on the number of active flavours N_f .

 \mathbb{H} . The \mathbb{H}

S! ^b⁰ ln % ^Q² Λ2 Not as famous as the Higgs though . . . $\frac{1}{\sqrt{2}}$ T + Elepton El T + Emission Comment: as it sets the scale of hadron masses, it is quite an important parameter in particle physics!

Ejet

 $\mathcal{L}(\mathbf{Q}^{\mathsf{T}}\mid \mathbf{W}^{\mathsf{T}}\mathbf{E})$, $\mathcal{L}(\mathbf{Z}^{\mathsf{T}}\mid \mathbf{W}^{\mathsf{T}}\mathbf{E})$

The Emission of the Emission

F. Krauss IPPPP in der Regission in der Re
Der Regission in der Regis

how can we describe this?

Factorisation: separate hard and soft scales **autonoation.** Superature

$$
d\sigma = \sum_{ab} \int_{a}^{b} dx_a \int dx_b f_{a}(\mathbf{x}_a, \mu_F^2) f_b(\mathbf{x}_2, \mu_F^2) \otimes \hat{\sigma}_{ab}(p_1, p_2, \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_F^2}, \alpha_s(\mu_F^2))
$$

$$
\otimes D_{c \to X}(z, \mu_F^2) + \mathcal{O}(\Lambda/Q)
$$

 f_a, f_b : parton distribution(μ f μ) etidns (fdôm fit \mathcal{B} dio $^{(0)}$ data) $^{n+1}_s d\hat{\sigma}^{(1)} + ...$

 $\hat{\sigma}_{ab}$: partonic hard scattering cross section

calculable order by order in perturbation theory Standard Model Theory for Collider Physics Daniel de Florian

 $D_{c \to X}(z,\mu_f^2)$: describing the final state e.g. fragmentation function, jet observable, etc.

Without factorisation we would be quite lost, but there are still a number of open (QCD) questions, e.g.

• hard scattering cross section:

 $\frac{1}{200}$ which order in the perturbative expansion is precise enough? (LO, NLO, NNLO ...)

 $\frac{1}{2}$ is fixed order adequate, or do we need to resum large logarithms?

• how to combine the partonic hard scattering result with a parton shower?

• do we know the parton distribution functions (PDFs) well enough?

• how to model hadronisation?

• how to combine the partonic hard

we will concentrate mostly on the hard scattering cross section in the following

e+e- annihilation

start with simple example, with simple example, with simple example example example: with simple example: with

In case of Rˆ there is complete cancellation of soft and collinear radiation, Rˆ

۰

à.

start with simple example, with simple example, with simple example example example: Δ

or if quark and gluon become collinear

Renormalisation group

Note: \mathbb{C} are UV divergent. These UV singularities cancel with vertex diagram due to Ward Identity

(ω → 0)

(θ → 0)

 p

emission and virtual corrections of the corrections of the corrections of

k

Rˆ(αs(Q), 1), obtained with partons instead than hadrons, is in agreement with experi-

Besides UV divergences, any gauge theory with massless particles can have

Infrared (IR) and collinear (together IRC) singularities are present both in real present both in real present

For soft and collinear radiation the characteristic emission time τ ∼ 1/(ωθ2) is

Any observable that is sensitive to soft and collinear gluons will acquire a

 \mathbb{R}^2

much larger than 1/Q, the characteristic time of the the the time of the hard collision of the hard collision

 $m = \frac{1}{2}$, which $\frac{1}{2}$ and $\frac{1}{2}$ $\frac{1}{2$

divergences when the propagator goes on shell

Soft singularities

Consider real emission diagrams in more detail:

2 IR divergences

$$
\mathcal{M}^{\mu}_{q\bar{q}g} = \bar{u}(p_1) \left(-ig t^A \phi \right) \frac{i(p_1 + k)}{(p_1 + k)^2} \left(-ie \gamma^{\mu} \right) v(p_2) \n+ \bar{u}(p_1) \left(-ie \gamma^{\mu} \right) \frac{-i(p_2 + k)}{(p_2 + k)^2} \left(-ig t^A \phi \right) v(p_2)
$$

If gluon becomes soft: neglect *k* except if it is in denominator: becomes soπ: heglect *κ* $\frac{1}{2}$ except if it is in der $\frac{1}{2}$

$$
\mathcal{M}^{\mu}_{q\bar{q}g} \stackrel{soft}{=} -iegt^A \bar{u}(p_1) \gamma^{\mu} \left(\frac{\rlap{\hspace{0.02cm}/}{p_1}}{2p_1k} - \frac{\rlap{\hspace{0.02cm}/}{p_2}\rlap{\hspace{0.02cm}/}{p_2}}\right) v(p_2)
$$

$$
|\mathcal{M}_{q\bar{q}g}|^2 \stackrel{soft}{\rightarrow} |\mathcal{M}_{q\bar{q}}|^2 g^2 C_F \frac{p_1 p_2}{(p_1 k)(p_2 k)} \times
$$

Note: colour will in general **not** factorize in the soft limit

=

Factorisation into Born matrix element and Eikonal factor

Soft singularities

Consider real emission diagrams in more detail:

2 IR divergences

$$
\mathcal{M}^{\mu}_{q\bar{q}g} = \bar{u}(p_1) \left(-ig t^A \phi \right) \frac{i(p_1 + k)}{(p_1 + k)^2} \left(-ie \gamma^{\mu} \right) v(p_2) \n+ \bar{u}(p_1) \left(-ie \gamma^{\mu} \right) \frac{-i(p_2 + k)}{(p_2 + k)^2} \left(-ig t^A \phi \right) v(p_2)
$$

If gluon becomes soft: neglect *k* except if it is in denominator: becomes soπ: heglect *κ* $\frac{1}{2}$ except if it is in der $\frac{1}{2}$

$$
\mathcal{M}_{q\bar{q}g}^{\mu} \stackrel{soft}{=} -iegt^A \bar{u}(p_1) \gamma^{\mu} \left(\frac{\rlap{\hspace{0.1cm}/}{p_1}}{2p_1k} - \frac{\rlap{\hspace{0.1cm}/}{p_2}\rlap{\hspace{0.1cm}/}{p_2}}\right) v(p_2)
$$

$$
|\mathcal{M}_{q\bar{q}g}|^2 \stackrel{soft}{\rightarrow} |\mathcal{M}_{q\bar{q}}|^2 g^2 C_F \frac{p_1 p_2}{(p_1 k)(p_2 k)} \times
$$

Factorisation into Born matrix element and Eikonal factor

Note: colour will in general **not** factorize in the soft limit

=

Collinear singularities \sim 2ar sind v(p2) ar singularities 2 soft

"

! #/p/¹

p

written as

 $(p_1$ $\sqrt{2}$ $(+ 6)$ - $\overline{1}$ note: if n1 is a m po singular denominat $\frac{10}{2}$ no singular denominator for $\theta \to 0$ note: if p1 is a massive particle: $p_1 = E(1,0,0,v)$, $v =$ $p(1 - \cos \theta) \to 0$ for $\theta \to 0$ $\sqrt{1 - \frac{m_1^2}{E^2}}$ 1 $E(1,0,0,v)$, $v = \sqrt{1 - \frac{m_1}{E^2}}$ $(1)^2$ $2E(1)$ $\frac{1}{2}$ p = 0.000 k $\frac{1}{2}$ n = 0.000 k = 2E1ω (1 − cos θ) → 0 for θ → 0 $(p_1 + k)^2 = 2E\omega (1 - v \cos \theta)$ \sim \sim \sim $(p_1 + k)^2 = 2E \omega (1 - \cos \theta) \rightarrow 0$ for $\theta \rightarrow 0$ \sim \sim soft ! #/p/¹ $\sqrt{m_1^2}$ ω (1) \overline{v} cos \overline{O} $v = \sqrt{1 - E^2}$ $= 2E\omega (1 - v \cos \theta)$

 \mathbb{R}^n and \mathbb{R}^n and \mathbb{R}^n

M^µ

es can lead to a c ⇒ only massless particles can lead to a collinear singularity \mathcal{I} p^µ n^µ k⊥p = k⊥n = 0

convenient parametrisation of momenta z = r amatı

$$
p_1 = z p^{\mu} + k^{\mu}_{\perp} - \frac{k^2_{\perp}}{z} \frac{n^{\mu}}{2p_1 n}
$$

\n
$$
k = (1 - z) p^{\mu} - k^{\mu}_{\perp} - \frac{k^2_{\perp}}{1 - z} \frac{n^{\mu}}{2p_1 n}
$$

\n
$$
\Rightarrow 2p_1 k = -\frac{k^2_{\perp}}{z(1 - z)}
$$

\n
$$
p^{\mu}
$$

\n
$$
k^2 p =
$$

("Sudakov parametrisation") [⊥] [−] ^k² \equiv 2Ew \equiv

 γ^{μ} = γ p^μ collinear direction k is not an independent k if $p = k_1 n_2 = 0$ $\overline{ }$ z(1 − z) n^{μ} light-lik \int_{-a}^{a} auxiliary vector $\lbrack p^{\mu }]$ = 2

collinear direction

⊔ight-like auxiliary vector \overline{a} $k_{\perp}p = k_{\perp}n = 0$ $z = \frac{E_1}{E_1 + E_2}$ p $k_{\perp} p = k_{\perp} n = 0$ $z = \frac{E_1}{E_1 + E_g}$ ur directio $z =$

[⇒] ²p1^k ⁼ [−] ^k²

→ 1999年 → 1999年

participation of the control of the
Participation of the control of the

= −iegt

soft

g2

C^F

 $\mathbf{p}^{\text{max}}_{\text{max}}$

soft

= −iegt

(p¹ ⁺ ^k)² (−ieγ^µ) ^v(p2)

 \mathcal{A} , provided by

(p² ⁺ ^k)² (−igt

2p1n

 \mathbb{R}^n

⊥

[⇒] ²p1^k ⁼ [−] ^k²

 $\frac{1}{2}$

particular particular and particular and particular and particular and particular and particular and particular
Particular and particular and particular and particular and particular and particular and particular and parti

 $|\mathcal{M}_1(p_1, k, p_2)|$ 2 coll $\stackrel{coll}{\rightarrow} \quad g^2 \; \frac{1}{n_1} \, .$ $\frac{1}{p_1 \cdot k} P_{qq}(z) | \mathcal{M}_0(p_1 + k, p_2)|^2$ $p_1 + k, p_2$

Cross sections for a scattering process q^a + q^b → p¹ + . . . + p^N can be

: splitting functions n functions $\frac{1}{2}$ <u>Inctions</u> $\overline{ }$ $\overline{$ $P_{\bm{q}\bm{q}}(z)$

[⇒] ²p1^k ⁼ [−] ^k²

k = (1 − z) p
(1 − z) pµ − kµ pµ − kµ
(2 − z) pµ − kµ pµ − kµ p

DGLAP splitting functions Including all the color factors we find the unregulated branching for th

(Dokshitzer, Gribov, Lipatov, Altarelli, Parisi)

$$
\hat{P}_{qg}(z) = T_R \left[z^2 + (1-z)^2 \right], \quad T_R = \frac{1}{2},
$$
\n
$$
\hat{P}_{qg}(z) = C_F \left[\frac{1+z^2}{(1-z)} \right],
$$
\n
$$
\hat{P}_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right],
$$
\n
$$
\hat{P}_{gg}(z) = C_A \left[\frac{1+(1-z)^2}{z} \right],
$$
\n
$$
\hat{P}_{gg}(z) = C_A \left[\frac{z}{(1-z)} + \frac{1-z}{z} + z(1-z) \right]
$$

t

2π

(details see later in PDF discussion)

QCD and Monte Carlo methodsLecture II: Proton structure and Parton Showers – p.16/37

Real radiation networks and the set $\frac{1}{2}$ the initial state and sum over colonized in-the final state and in-the final state particles are not measured, coming particles and if the spins of the spins of the spins of the final state particles are not measured, the **the same is done for the same is done for the polarism**

produits and average over initial state polynomials and average over initial state polynomials. in general we sum over final state polarizations and the polarizations and the polarisations. urs and average over initial e over initial state |
— p over initial state $|\mathsf{p}|$

$$
|\overline{\mathcal{M}}|^2 \, \, \rightarrow \, \, \overline{\sum}_{\lambda,c} |\mathcal{M}_{\lambda,c}|^2 = \frac{1}{\prod}
$$

at LO, we obtain

3 Phase space integrals

3 Phase space integrals

Define

Define

Define

Phase space: In the space of the space of the

Phase space:

Phase space:

gluon energy:

Define

$$
|\overline{\mathcal{M}}_0|^2 = \frac{1}{3} 4e^2 Q_q^2 N_c s
$$

 $\mathbf{1}$

 $\frac{1}{2}$

 $\begin{array}{c}\n 3 \cdot \cdot \cdot \\
 2 \cdot \cdot \cdot\n \end{array}$

The modulus of the modulus of the matrix element involves the average over colours in the average over colours

the initial state and sum over colours in the final state and sum over colours in the final state. For unpolarized in-

gluon energy: **iation:** $p_1 = E_1 (1, 0, 0, 1)$
 $p_2 = E_2 (1, 0, \sin \theta, \cos \theta)$ $p^{\gamma} = \sqrt{s} (1, 0, 0, 0)$ with extra gluon radiation: $p_1 = E_1(1,0,0,1)$ **adiation.** $p_1 = E_1 (1, 0, 0, 1)$
 $p_2 = E_2 (1, 0, \sin \theta, \cos \theta)$ $k \equiv p_3 = p^{\gamma} - p_1 - p_2$ $s_{ij} = (p_i + p_j)^2$ \overline{a} $\left(\begin{array}{cc} 1 & 0 & 1 \end{array} \right)$ $\begin{array}{ccc} \n\frac{1}{2} & - \\
\frac{1}{2} & - \n\end{array}$ $, 3 = 5$ $\frac{1}{2}$, $\frac{0}{2}$ $-p_1 - p_2$

$$
|\overline{\mathcal{M}}_1|^2 = |\overline{\mathcal{M}}_0|^2 \frac{2g^2 C_F}{s} \left(\frac{s_{13}}{s_{23}} + \frac{s_{23}}{s_{13}} + 2s \frac{s_{12}}{s_{13}s_{23}} \right)
$$

 $\sqrt{M_0^2 + 2 \frac{2g^2 C_F}{r^2}} \left(\frac{x_1^2 + x_2^2}{r^2} \right)$ becomes completely insensitive to soft and collinear emissions up to the hard $\overline{1}$ In the IRC region the matrix elements for real and virtual corrections are equal but $\overline{1}$ C_F / s s²³ x_{\cdot} $\overline{}$ $\overline{\ }$ x_2^2 $\frac{1}{\sqrt{2}}$ delli ning x_1 $\overline{2}$ 2 \overline{F} \overline{a} $\sqrt{ }$ $\frac{3}{2}$ \hat{x} $2E_{1/}$ \sqrt{s} $=$ $|V_0|$ \overline{s} $\sqrt{1}$ defining $x_1 = 2E_1/\sqrt{s}$, $x_2 = 2E_1/\sqrt{s}$ $s \sqrt{(1-x_1)(1-x_2)}$ $|\mathcal{M}_1|$ 2 = $|\overline{\mathcal{M}}_0|$ $_2$ $2g^2$ C_F s $\begin{pmatrix} x_1^2 + x_2^2 \end{pmatrix}$ $(1-x_1)(1-x_2)$ \setminus gluon energy:

 $(1 - x)^2 + x^2$

Define

gluon energy:

$$
E_g = \sqrt{s} (1 - x_1 - x_2)
$$

² The observable assigns the same weight to real emissions and virtual corrections

s¹²

s13s²³

Real radiation matrix element s
Santa Santa S
Santa Santa S ^s13s²³ " **ION** leal ra $\frac{1}{2}$ $R6$ \overline{a} l li adiation

s²³

^E^g ⁼ [√]^s (1 [−] ^x¹ [−] ^x2)

 \mathbb{R}^n

s¹³

Real radiation matrix element
\n
$$
|\overline{\mathcal{M}}_1|^2 = |\overline{\mathcal{M}}_0|^2 \frac{2g^2 C_F}{s} \left(\frac{s_{13}}{s_{23}} + \frac{s_{23}}{s_{13}} + 2s \frac{s_{12}}{s_{13}s_{23}} \right)
$$
\n
$$
= |\overline{\mathcal{M}}_0|^2 \frac{2g^2 C_F}{s} \left(\frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} \right)_{x_1 = 2E_1/\sqrt{s}, x_2 = 2E_1/\sqrt{s}}
$$

 \mathcal{L} . The \mathcal{L}

^s13s²³ "

singularity γ collinear singularity $\begin{array}{c|c} p_1 \parallel p_3 \end{array}$, $\begin{array}{c} x_2 \rightarrow 1 \end{array}$ collinear singularity $\begin{array}{c} p_2 \parallel \end{array}$ $\sqrt{1}$ $x_1 \rightarrow 1 - x_2$ soft gluon $E_q = \sqrt{s(1-x_1-x_2)}$ **qularit** $p_2 \parallel q$ $\frac{1}{2}$ $\frac{2}{3}$ $\left\|\begin{array}{ccc} x_1\rightarrow 1 \end{array}\right.$ collinear singularity $\left\|p_1\right\|p_3$, $\left\|x_2\rightarrow 1 \right.$ collinear singularity $\left\|p_2\right\|p_3$:
: $x_1 \rightarrow 1 - x_2$; soft gluon $\frac{1}{2}$: sij = (pⁱ + p^j) $E_g = \sqrt{s} (1 - x_1 - x_2)$

are matrix element is singular:
element is singular: **in these limits the matrix element is singular !**

dΦ^D \cdot how can w $n₀$ $n₁$ $n₁$ $n₂$ • how can we interpret this ?

|M1|

² 2g² C^F

Q → p¹ + . . . + p^N

 \mathbf{S}_{max}

s

Q → p¹ + . . . + p^N In the following consider mass consider mass consider mass consider mass case parties of the constant of the c • how can we remedy this ?

2Eine and 2Eine and 2Eine

d^D−¹

p¹ \$ p³ x¹ → 1 x² → 1 x¹ → 1 − x²

p \$i (

 \mathbb{Z}_2

Cancellation of IR divergences

• interpretation: A quark-antiquark pair with a soft an collinear gluon cannot be distinguished experimentally from just a q qbar pair, so this is not an observable final state. Physical final states are hadrons or jets.

Kinoshita, Lee, Nauenberg, 60's

Soft and collinear singularities cancel in the sum over degenerate states

• what are degenerate states?

For example, a quark emitting a soft gluon cannot be distinguished from simply a quark. Exchange of virtual gluons also leads to IR singularities (same oder in alpha_s).

Singularities cancel between real and virtual corrections.

Dimensional Regularization ensional F 2 $\overline{}$ $\frac{1}{2}$ x22 $\frac{1}{2}$ $\frac{1}{2}$ Dimensio nal Reg s $\overline{}$ $\overline{\ }$ Z_0^{\prime} $\overline{1}$ $\overline{}$ $\overline{1}$

, x2 = 2E1/2 = 2E1/2 = 2E1/2 = 2E1/2 = 2E2/2 = 2E2/2 = 2E2

1 + x2

s¹²

^s13s²³ "

√s

"

A convenient way to isolate singularities is dimensional regularisation: p¹ \$ p³ x¹ → 1 x² → 1 x¹ → 1 − x² ² 2g² C^F ! x² ¹ + x² Define "

we work in $|D = 4 - 2\epsilon|$ dimensions. $\sin \left(D - 4 - 2\epsilon \right)$ din -2ϵ dimensions.

x1 = 2E1/2E1/2E1/2E1/2E1/2E1/2E1

- regulates both UV and IR divergences p¹ \$ p³ x¹ → 1 x² → 1 x¹ → 1 − x²
- does not violate gauge invariance lolate gauge litvarian
.
	- \bullet poles can be isolated in terms of $\quad 1/e^b$ s
	- need phase space integrals in D dimensions **•** need phase space integrals in D dir $\frac{1}{2}$ $\frac{1}{2}$
		- need integration over virtual loop momenta in D dimensions ial lo

$$
g^2 \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \longrightarrow g^2 \mu^{2\epsilon} \int_{-\infty}^{\infty} \frac{d^Dk}{(2\pi)^D}
$$

Virtual corrections Virtu $\frac{R_{\rm{max}}}{\sqrt{2}}$ \mathbf{I} experience

^RPT

defined the control of the control of

α,

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"

√s

c
c₂ + π_β

 α will not as through the calculation but only quote the regult: we will not go through the calculation but only quote the result: alculation but only −∞ e the r

$$
R^{\text{virt}} = R^{LO} \times \frac{\alpha_s}{2\pi} C_F \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{-s}{4\pi\mu^2}\right)^{-\epsilon} \left\{-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon)\right\}
$$

Phase space in D dimensions Dhase spa es in D dimensions real axis. **the integral over the contour integral over the contour is an use the identity**

that the interest on \mathbb{R} prescription is exactly such that the contour does not enclose any poles. Therefore

1 to N particle phase space: $\frac{1}{3}$ portiole space space: is N particle phase space: integrals we make the transformation in the transformation in the transformation in \blacksquare integration contour in the integration contour in the complex location complex location \blacksquare # ∂ ace:

!

de Constantinople

 $\mathcal{A}=\mathcal{A}+\mathcal{$

Phase space:

$$
Q \to p_1 + \dots + p_N
$$

$$
\int d\Phi_N^D = (2\pi)^{N-D(N-1)} \int \prod_{j=1}^N d^D p_j \, \delta^+(p_j^2 - m_j^2) \delta^{(D)} \Big(Q - \sum_{i=1}^N p_i\Big)
$$

ponents are not one not on the poles of the poles of

In the following consider massless case $p_j^2 = 0$. Use for $i = 1, ..., N - 1$

$$
\int d^D p_i \delta^+(p_i^2) = \int d^D p_i \delta(p_i^2) \theta(E_i) = \int d^{D-1} \vec{p}_i dE_i \delta(E_i^2 - \vec{p}_i^2) \theta(E_i)
$$

=
$$
\frac{1}{2E_i} \int d^{D-1} \vec{p}_i \Big|_{E_i = |\vec{p}_i|}
$$

and eliminate p_N by momentum conservation $n_{\rm M}$ by momentu $\overline{1}$ $\frac{1}{1}$ $\frac{1}{2}$ $P_N \sim_J$ montent and conservation

$$
\Rightarrow \int d\Phi_N^D = (2\pi)^{N-D(N-1)} 2^{1-N} \int \prod_{j=1}^{N-1} d^{D-1} \vec{p}_j \frac{\Theta(E_j)}{E_j} \delta^+ ([Q - \sum_{i=1}^{N-1} p_i]^2) \Big|_{E_j = |\vec{p}_j|}
$$

phase space volume of unit sphere in D dimensions

ddan adda

p \$1

|p

<u> 2012 - 11</u>

#'Q² [−] ²|^p

adele de Co

1|

 15111

 $\mathbb{S}_{\mathbb{Z}}$

phase space volume of unit sphere in D dimensions 0

 \mathbb{R} .

4

$$
\int d\Omega_{D-1} = V(D) = \frac{2\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})}
$$

$$
V(D) = \int_0^{2\pi} d\theta_1 \int_0^{\pi} d\theta_2 \sin \theta_2 \dots \int_0^{\pi} d\theta_{D-1} (\sin \theta_{D-1})^{D-2}
$$

dθ¹

Real radiation in D dimensions $200r$ **Preadise and ability and the transformation Engineeries** and the transformation Engineeries and the transformation E ³−² ^D 2⁴−^D $\frac{1}{2}$

 \sim

 \mathbb{R}

As in the following a parametrization in terms of the Mandelstam vari-

1 to 3 particle phase space: \mathbf{b} which leads to the leads ³−² ^D 2⁴−^D

de estas en la construcción de la
Desde estas en la construcción de la construcción de la construcción de la construcción de la construcción de

which leads to

 $\mathcal{D} = \{x_1, x_2, \ldots, x_n\}$

For N = 3 one can choose a coordinate frame such that

1 to 3 particle phase space:
$$
p_1 = E_1(1, \vec{0}^{(D-1)})
$$

\n $p_2 = E_2(1, \vec{0}^{(D-3)}, \sin \theta, \cos \theta)$ $x_i = \frac{2p_i \cdot p^{\gamma}}{s}$
\n $p_3 = p^{\gamma} - p_2 - p_1$

sD−3 dQ−3 dQ−2 dQ−3 dQ−3 dQ−2 dQ−3 in and an analysis of a substitute of a substitute of a substitute of a subs

32

$$
d\Phi_{1\to 3} = \frac{1}{4} (2\pi)^{3-2D} dE_1 dE_2 d\theta \left[E_1 E_2 \sin \theta \right]^{D-3} d\Omega_{D-2} d\Omega_{D-3}
$$

=
$$
(2\pi)^{3-2D} \frac{2^{4-D}}{32} s^{D-3} d\Omega_{D-2} d\Omega_{D-3} \left[(1-x_1)(1-x_2)(1-x_3) \right]^{D/2-2}
$$

$$
dx_1 dx_2 dx_2 \Theta(1-x_1) \Theta(1-x_2) \Theta(1-x_3) \delta(2-x_1-x_2-x_3)
$$

i=1

s

$$
|\overline{\mathcal{M}}_1|^2 = |\overline{\mathcal{M}}_0^{(D)}|^2 \frac{2g^2 C_F}{s} \left(\frac{(x_1^2 + x_2^2)(1 - \epsilon) + 2\epsilon(1 - x_3)}{(1 - x_1)(1 - x_2)} - 2\epsilon \right)
$$

Combine to final result ⁼ ^RLO [×] αs $\overline{}$ \overline{m} $\frac{1}{2}$ to final $\frac{1}{2}$ \overline{z} to \overline{m} .
2011 **pull** $\overline{}$ Combine \overline{c} \mathbf{r} ! x² 1 + x2 (1 − x1)(1 − x2)

x¹ = 2E1/

$$
R^{\text{real}} = R^{LO} \times \frac{\alpha_s}{2\pi} C_F \frac{\Gamma^2 (1 - \epsilon)}{\Gamma (1 - 3\epsilon)} \left(\frac{s}{4\pi \mu^2}\right)^{-\epsilon} \left\{ \frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right\}
$$
gluon both soft and collinear

 \ldots , \ldots , \ldots , \ldots , \ldots , \ldots

√s

"

 $\frac{1}{2}$ $\frac{1}{2}$

remember virtual corrections: −∞

$$
R^{\text{virt}} = R^{LO} \times \frac{\alpha_s}{2\pi} C_F \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{-s}{4\pi\mu^2}\right)^{-\epsilon} \left\{-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon)\right\}
$$

KLN theorem at work! $\overline{}$ αs Γ orem at work! **KLIV INCORTIL AL WORK!**

$$
R = R^{LO} \times \left\{ 1 + \frac{3}{4} C_F \frac{\alpha_s(\mu)}{\pi} + \mathcal{O}(\alpha_s^2) \right\}
$$
scale dependence

$$ ⊘Cale de nend ence

$$
R = R_0 \times \Delta_{QCD} = 3 \sum_{q} Q_q^2 \times \Delta_{QCD} \qquad \Delta_{QCD} = 1 + \frac{\alpha_s(\mu^2)}{\pi} + \sum_{n=2}^{\infty} C_n \left(\frac{s}{\mu^2}\right) \left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n
$$

$$
\frac{dR}{d\mu} = 0 \Rightarrow \mu^2 \frac{\partial R}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial R}{\partial \alpha_s} = 0
$$

∞

1995

Q2

#n

The more higher orders are included in R and **CIU** led in R and α_s the more the scale dependence is reduced

More on scale dependence 150 200 250 300 350 400 450 500 5

0

mann-De Ridder, Gehrmann, Glover, Pires 20 FIG. 2: Scale dependence of the inclusive jet cross section for Gehrmann-De Ridder, Gehrmann, Glover, Pires 2013 **Standard Boughezal, Caola, Melnikov, Petriello**, Schulze 2013 for details.

Standard Model Theory for Collider Physics Daniel de Florian

A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, J.Pires (2013)

Similar results expected for other partonic channels

Fire 2013 Roughezal Caola Melnikov Petriello Schulze 2013 $\mathcal{S}_{\mathcal{S}}$ 50 $\mathcal{S}_{\mathcal{S}}$ 70 $\mathcal{$ Boughezal, Caola, Melnikov, Petriello, Schulze 2013

section as a function as a function of the renormalization and factorization scale \mathcal{H}^*

of the calculation of the calculation of the corrections to the Higgs boson in the Higgs boson of production cross section c in gluon fusion. The reduced scale dependence is also apparent from Fig. 10, where we plot to talk to talk to Fig. 10: Scale dependence of the hadronic cross section in consecutive orders in perturbative Orders and texture text of the hadronic cross section in consecutive orders in perturbationfor details.

observed in the calculation of higher-order QCD corrections to the Higgs boson pro in gluon fusion. The reduced scale dependence is also apparent from Fig. 10, whe section as a function of the renormalization and factorization scale μ in the region

Finally, we comment on the phenomenological relevance of the "gluons-or sections and K-factors that we report. We note that at leading and next-to-leading smaller than the gluon ones. The gluon ones in the gluon-only results can be used for \mathbf{C} results can be used for reliable sections and K -factors that we report. We note that at leading and next-to-leading

More on scale dependence

e+e- to 3 jets up to NNLO:

bands from scale variations $M_Z/2 \leq \mu \leq 2 M_Z$

Gehrmann-De Ridder, Gehrmann, Glover, GH, 2008

- reduction of scale uncertainty
- better description of the data
- NNLO, NLO **not** within LO uncertainty band!

⇒

- scale variations of LO result do not necessarily give a realistic error estimate
- choice of a convenient central scale is important (and not straightforward)
- NLO does a reasonable job, LO does not

More on scale dependence

K-factors


```
example :
            pp \rightarrow W^+W^-b\bar{b}
```


00000

g

LO

t

W −

g

W+*W*−*b*¯*b* final state: one takes into account finite width effects and nonresonant contributions of the top quarks fully at NLO, while the other uses the narrow width approximation

 W^+

b

What is a convenient scale choice ? ⁼ ⁰ [⇒] ^µ² [∂]^R ↑
Mhat is a $=$ $M_Z \sim 2 \frac{M_Z}{\sqrt{2}}$

example from W+3 jets: possible scale choices: m n W+3 iets

R80'⊖…

de la P

dµ

possible scale choices:

! :

8)+)3\$%&#-&?%8#-/(, 8)+)3\$%&#-&?%8#-/(,

C.Berger et al (Blackhat) '09

;(/%&%11%"-&#-&<=:>&34-& ;(/%&%11%"-&#-&<=:>&34-& H_T much better reflects the scale of the hard interaction