Progress toward NNLO cross sections

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Energy frontier workshop on QCD physics

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NORTHWESTERN UNIVERSITY

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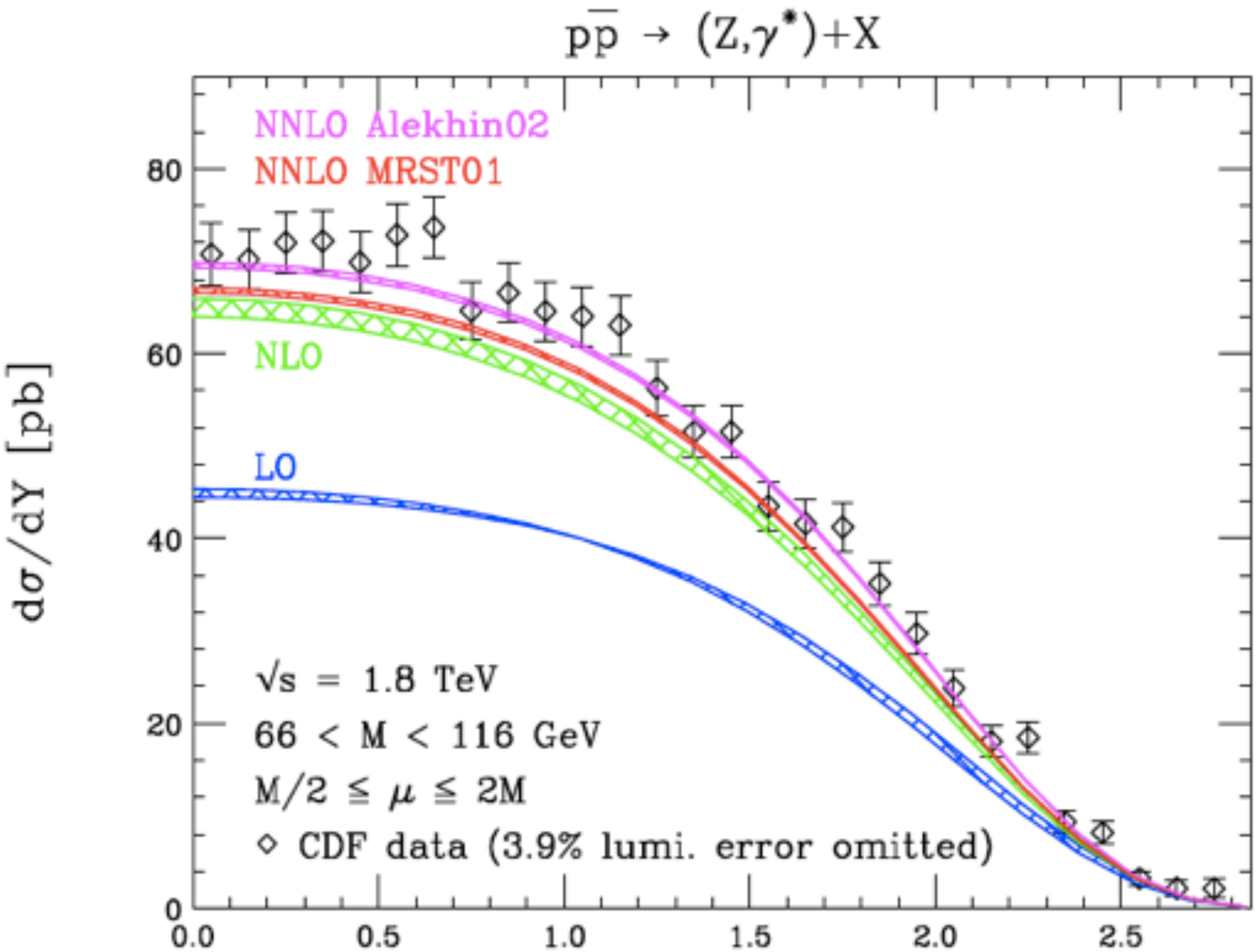
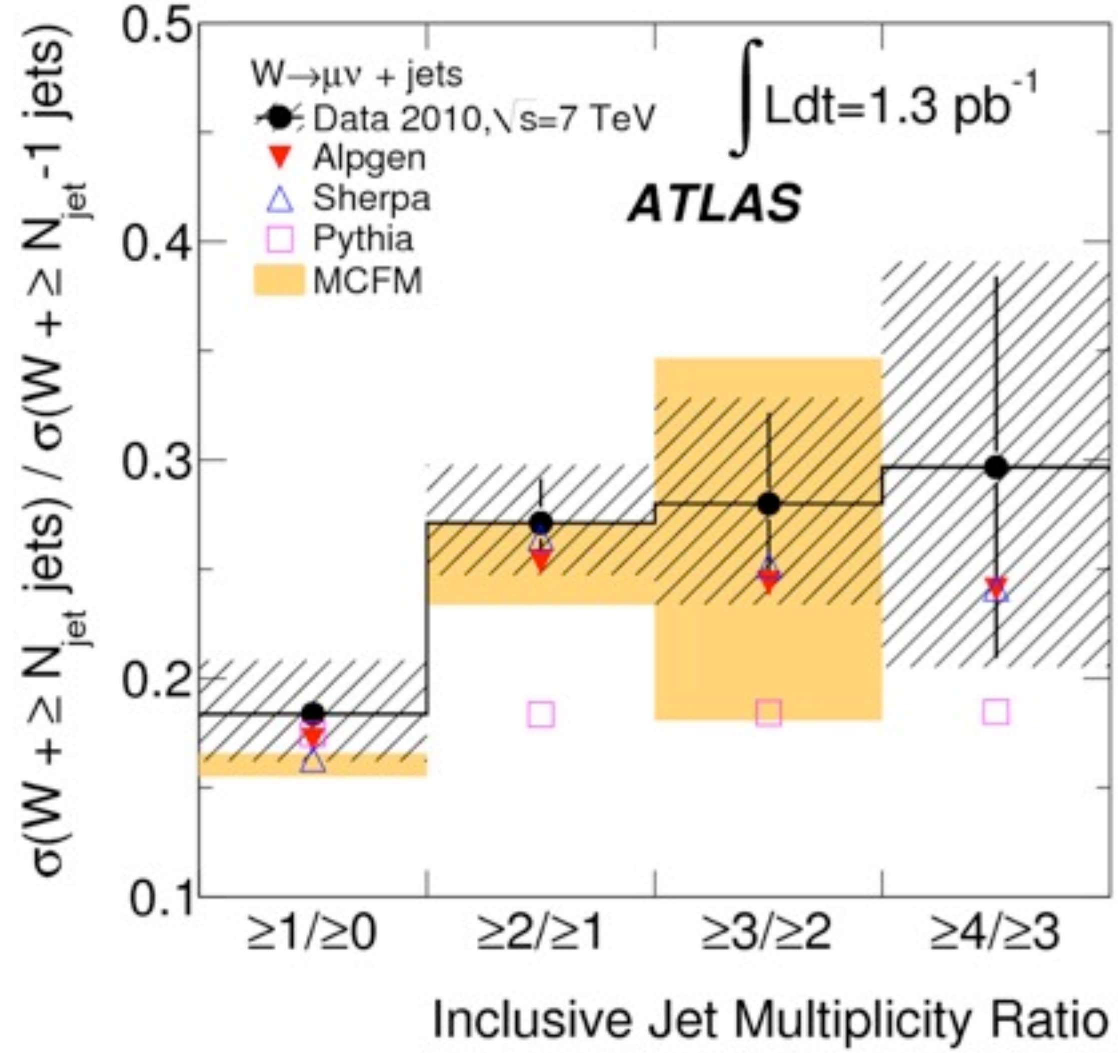
Outline

- Lengthy introduction and motivation on why NNLO QCD calculations are needed, illustrated by numerous examples at the LHC
- Short status of what is the bottleneck for providing needed calculations: double-real emission corrections
- Will attempt to describe a new technique that satisfies the following conditions: can provide 2→2 scattering processes at NNLO in finite (~1 year) time, and is extendable to higher-multiplicities in a simple way
- Initial results for ttbar cross section at the Tevatron (Baernreuther, Czakon, Mitov); results expected soon for Higgs+jet (Boughezal, Caola, et al.)

The need for higher-order QCD



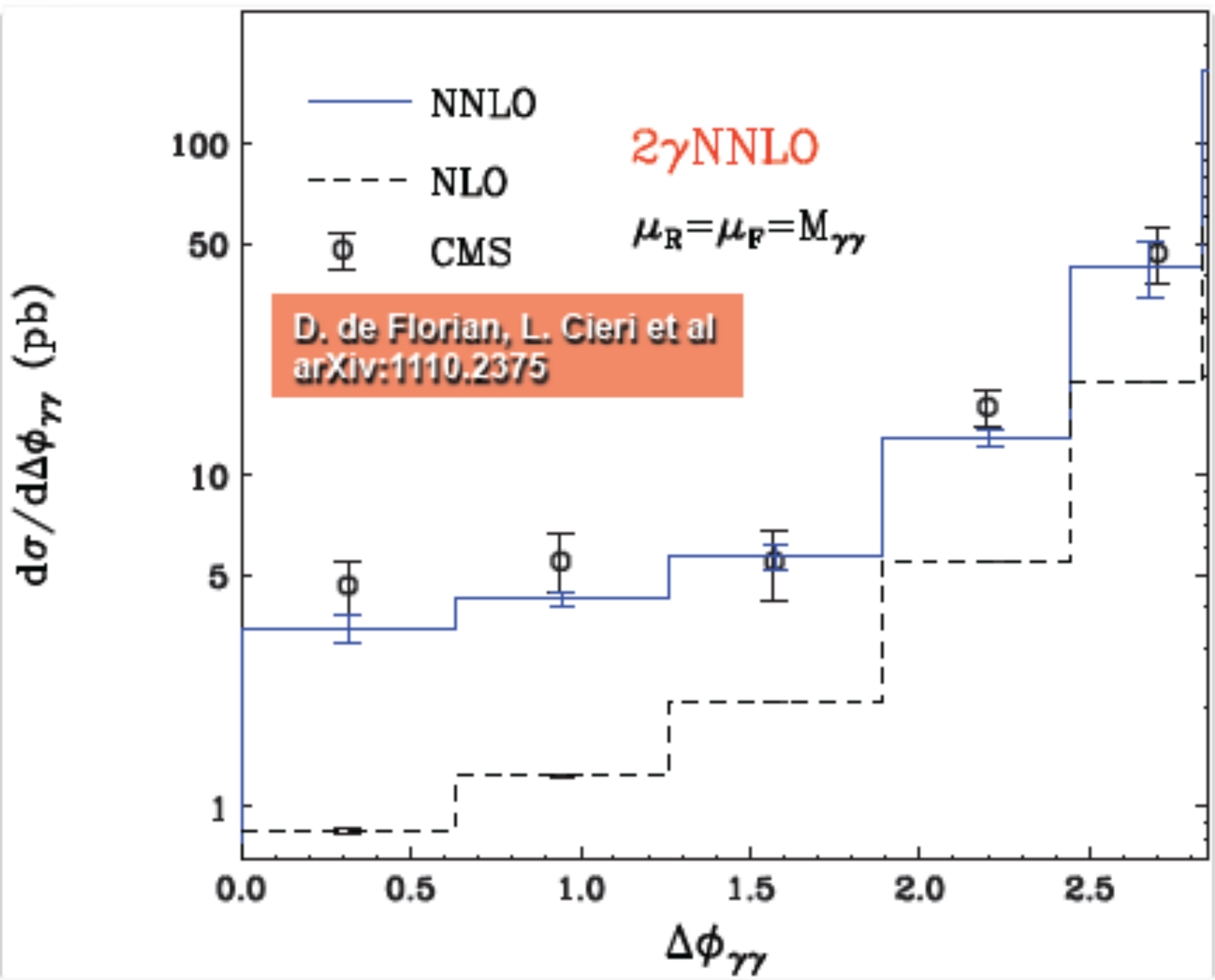
Anastasiou, Dixon, Melnikov, FP 2004



- The need to go beyond leading order QCD, or the parton-shower approximation, to understand hadron-collider data is by now unquestioned. NLO and matched parton-shower+NLO now standard tools used.

LHC examples of NLO versus data

- Sometimes even NLO is not enough... now there's data to illustrate the point

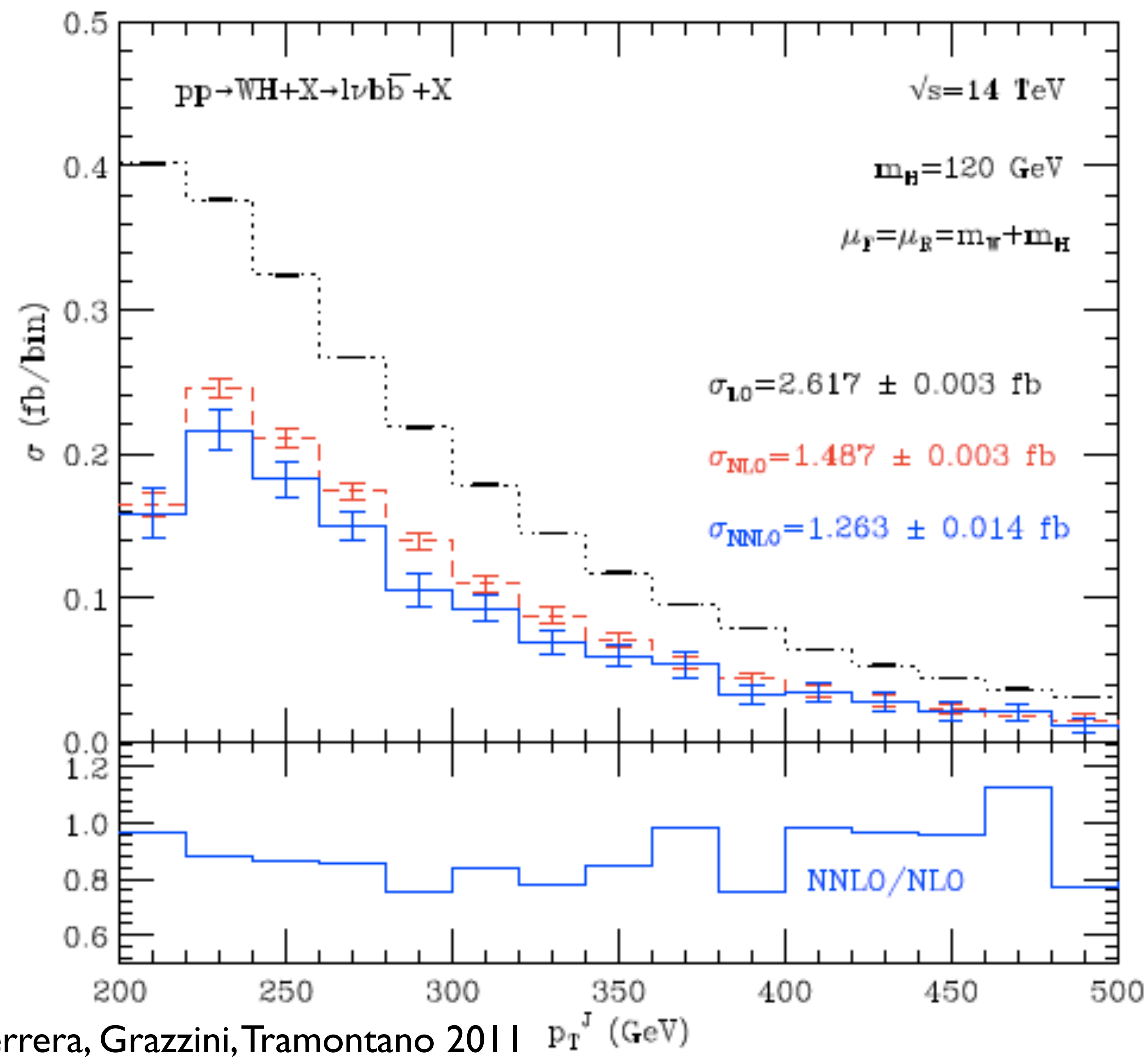


- At LO, opening angle in the transverse plane is π
- Distribution begins only at NLO
- NLO→NNLO shift large for two reasons: large first correction to the large qg channel which first opens at NLO, and new gg channel

The boosted Higgs

- Let's look at the Higgs in WH, following the original boosted study

Butterworth et al., 2008



Ferrera, Grazzini, Tramontano 2011

- Original analysis suggests vetoing extra jets with p_T>20 GeV
- Introduces large logarithms into the expansion; -50% from LO to NLO, additional -20% from NLO to NNLO
- But shouldn't tools like POWHEG, which have some resummation of these logs, save the experimentalists from making too big a mistake?

Low-mass Drell-Yan

From: [redacted]
Subject: implementation of NLO calculation
Date: March 1, 2011 5:38:19 PM CST
To: Frank Petriello <f-petriello@northwestern.edu>

Dear Frank,

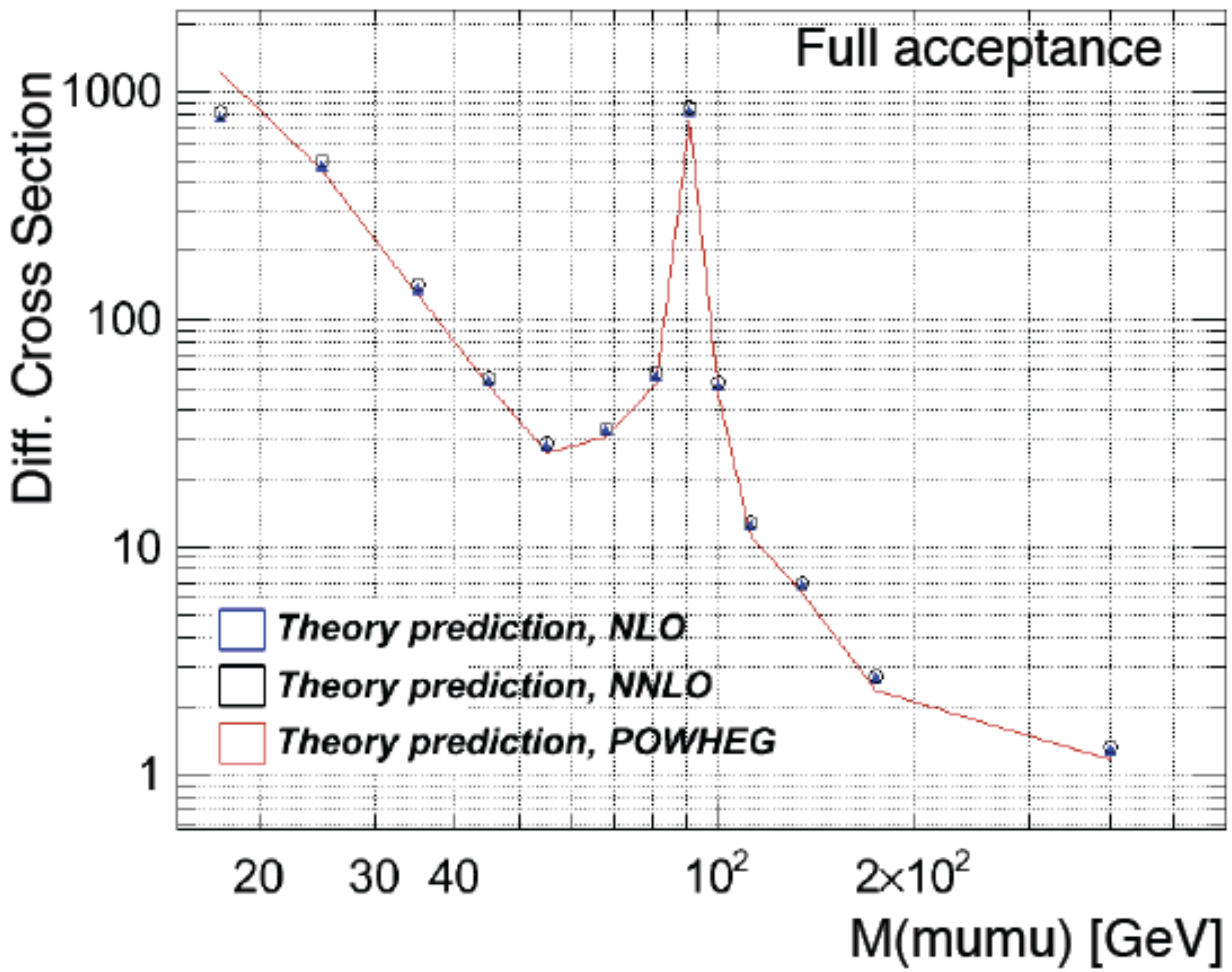
I'm comparing the implemenation (practically) of NLO calculation at FEWZ
and our POWHEG MC sample (NLO) because we observed about 25% difference
between FEWZ acceptance and POWHEG acceptance. Our POWHEG sample is also
based on NLO calculation, so it is very strange result. We have almost
no difference between FEWZ and POWHEG on full acceptance, but 25% is
arisen on CMS acceptance which we are using in the analysis.

(FEWZ: Fully Exclusive W and Z production

Melnikov, FP 2006; Gavin, Li, Quackenbush, FP 2011-12)

Low-mass Drell-Yan

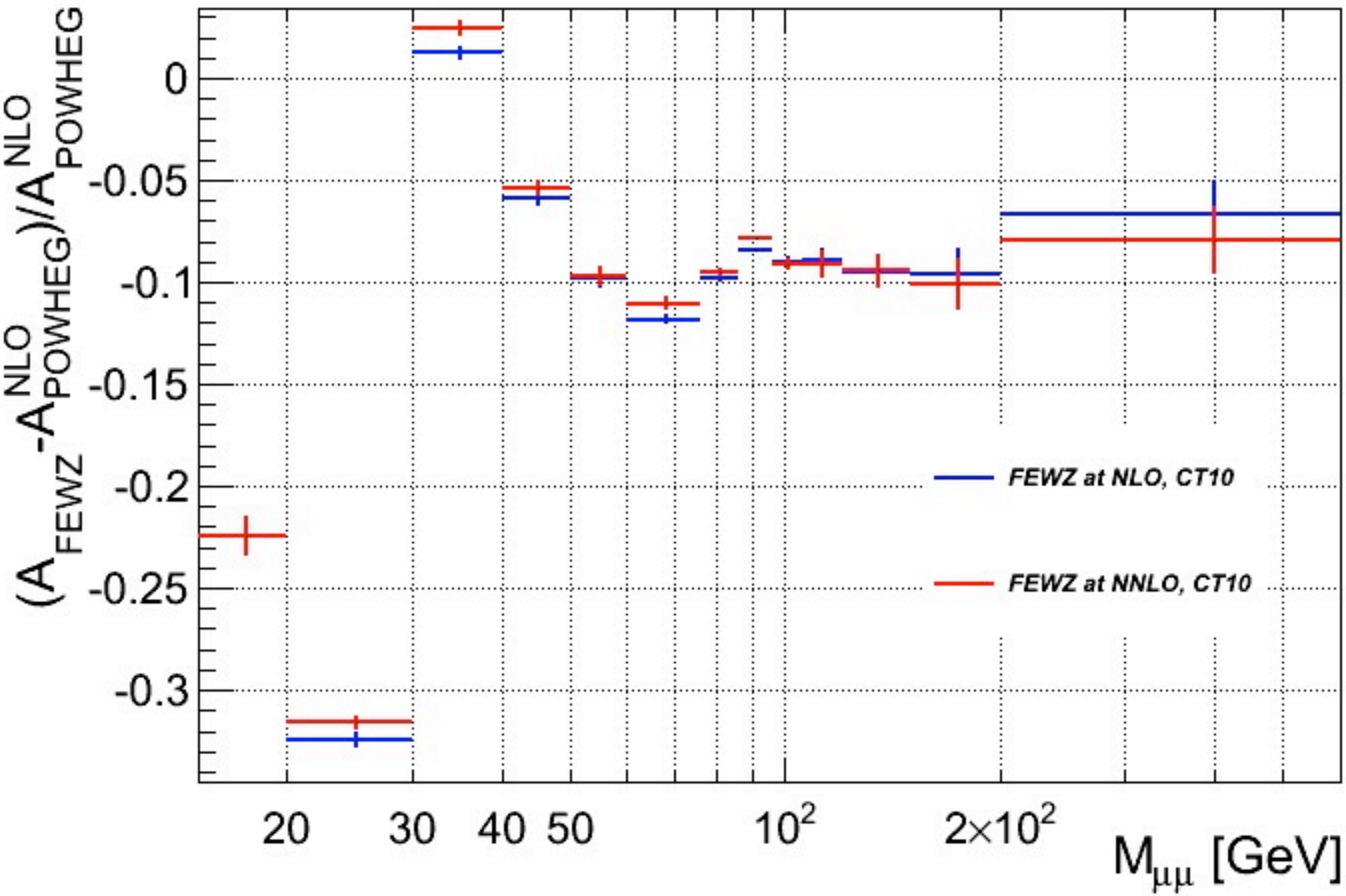
- From internal CMS talk:



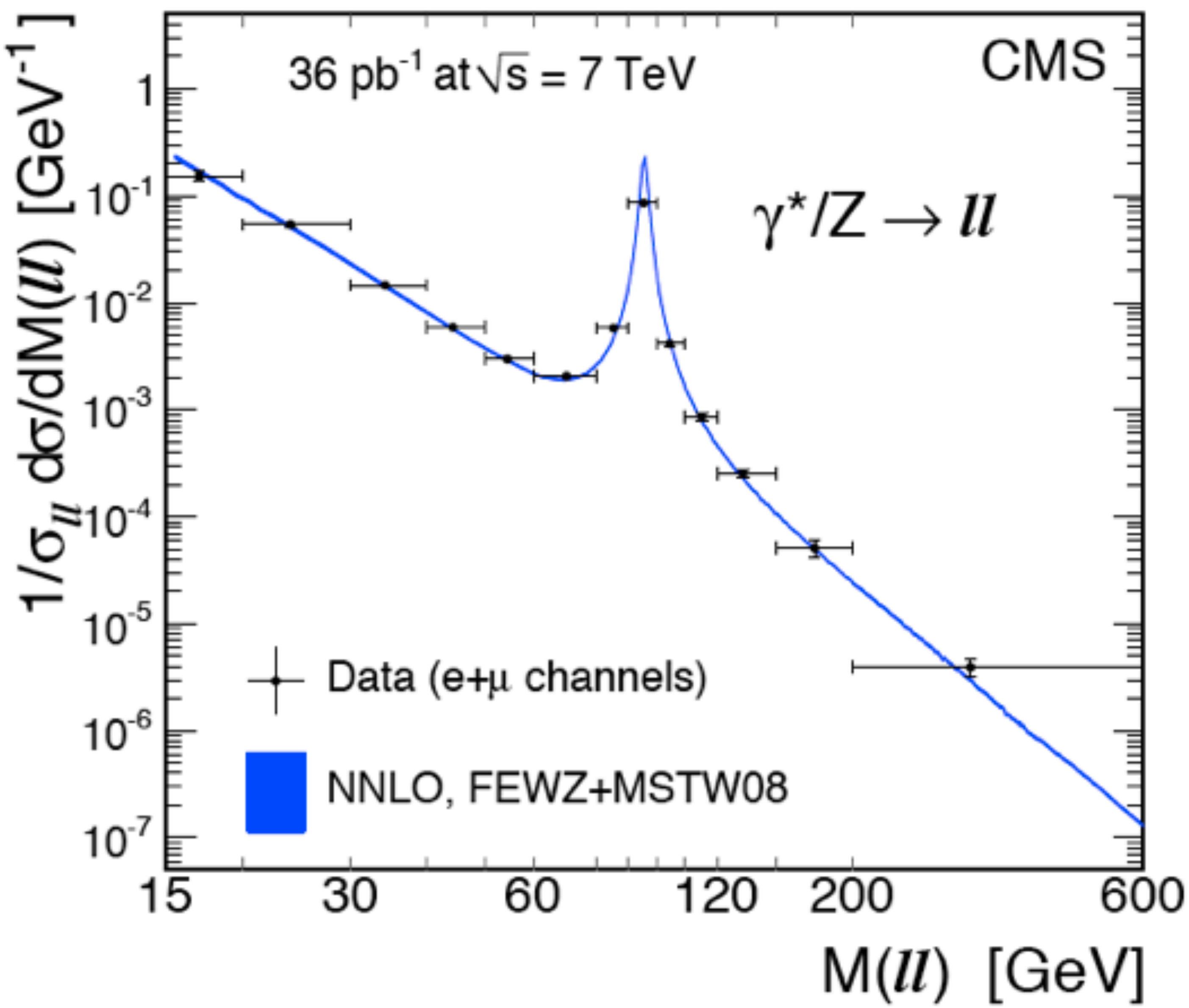
- We need to understand the difference at low mass between FEWZ and POWHEG
- $FEWZ_{NLO}$ and $FEWZ_{NNLO}$ are very similar

Low-mass Drell-Yan

- From internal CMS talk:



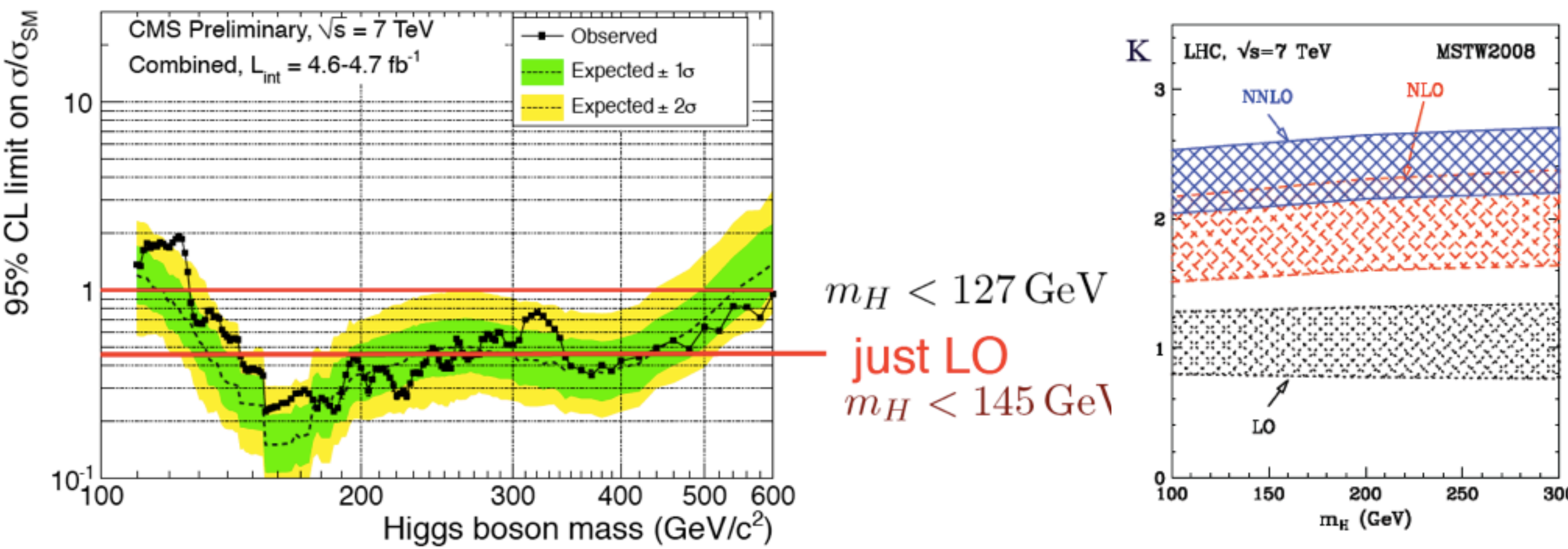
Low-mass Drell-Yan



- Double muon trigger: p_{T1}>16 GeV, p_{T2}>7 GeV
- For M=[15,20] GeV: NLO→LO, NNLO→NLO, need a hard jet to generate this configuration
- α_S(15 GeV)≈0.17, K-factor≈1.9 when going from 'N'LO→'N'NLO
- Corrections to POWHEG acceptance of ≈1.5-2
- CMS defines a FEWZ/POWHEG correction factor for the low bins
- Maybe in [15,20] could use NLO for DY+jet, but in 20-30 bin need NNLO to account for all regions

The Higgs in gluon-fusion

- Can't rely upon LO or even NLO for Higgs production in gluon-fusion...



from de Florian, Higgs Magnificent Mile 2012

The Higgs in gluon-fusion

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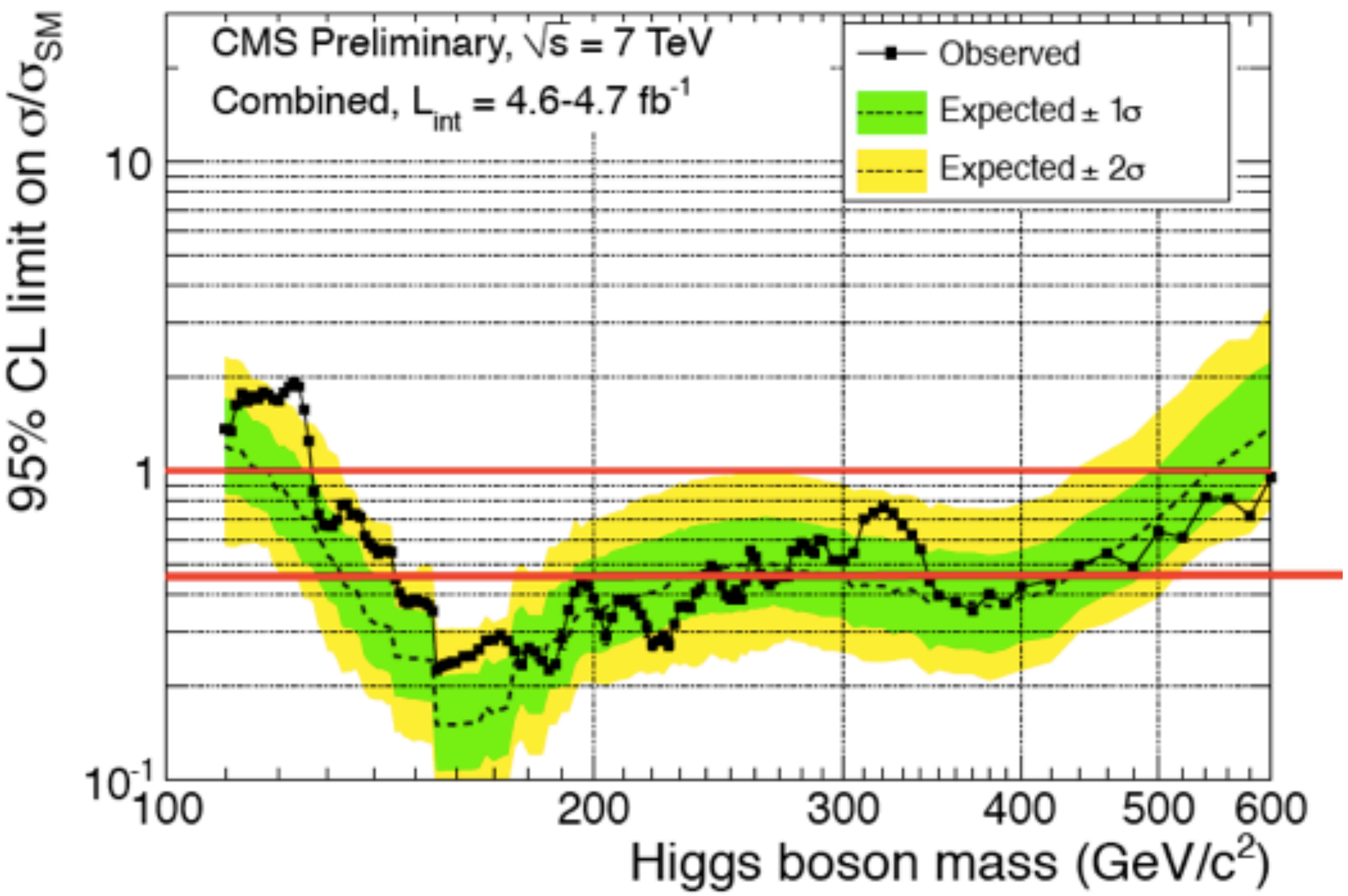

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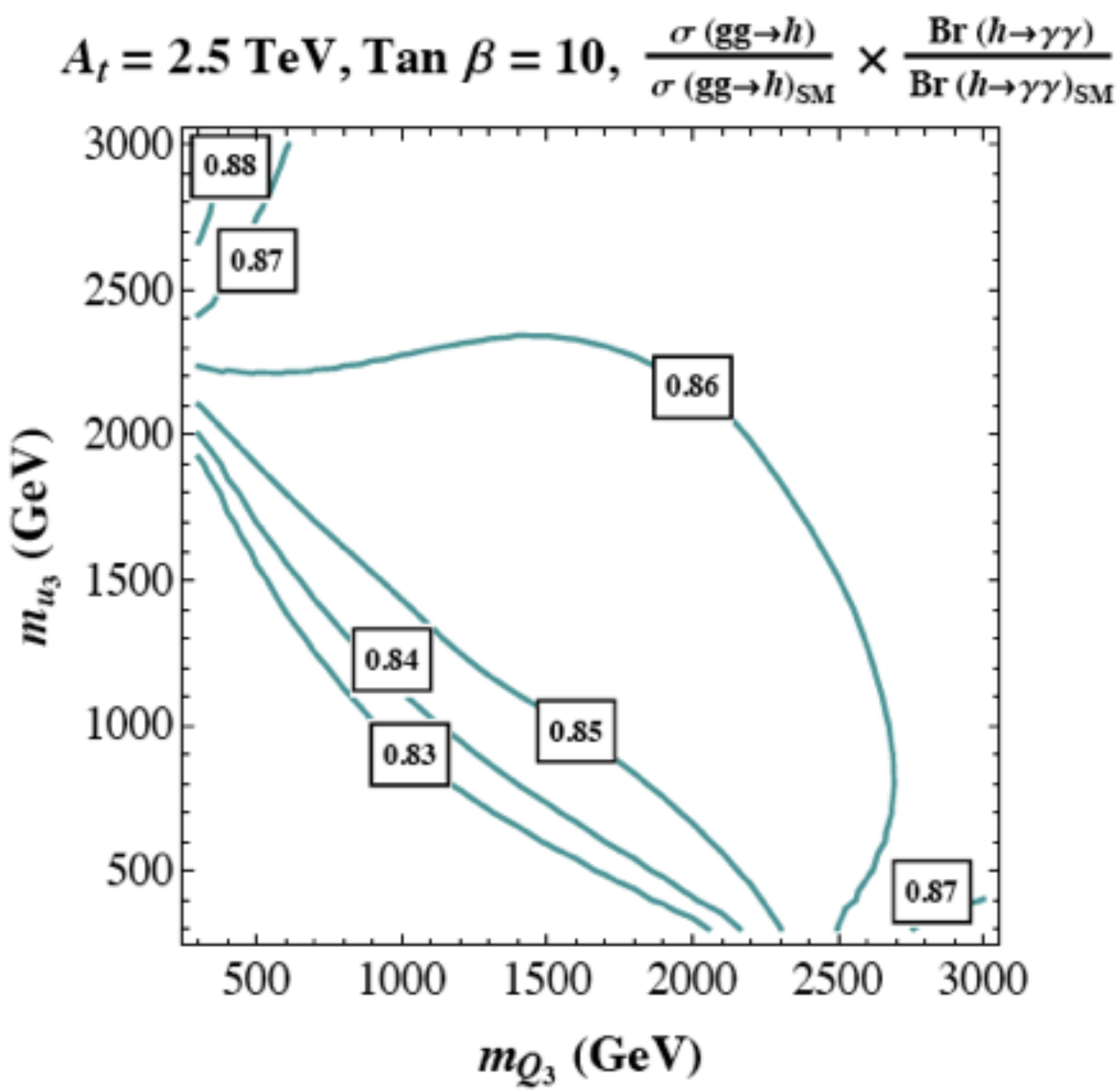
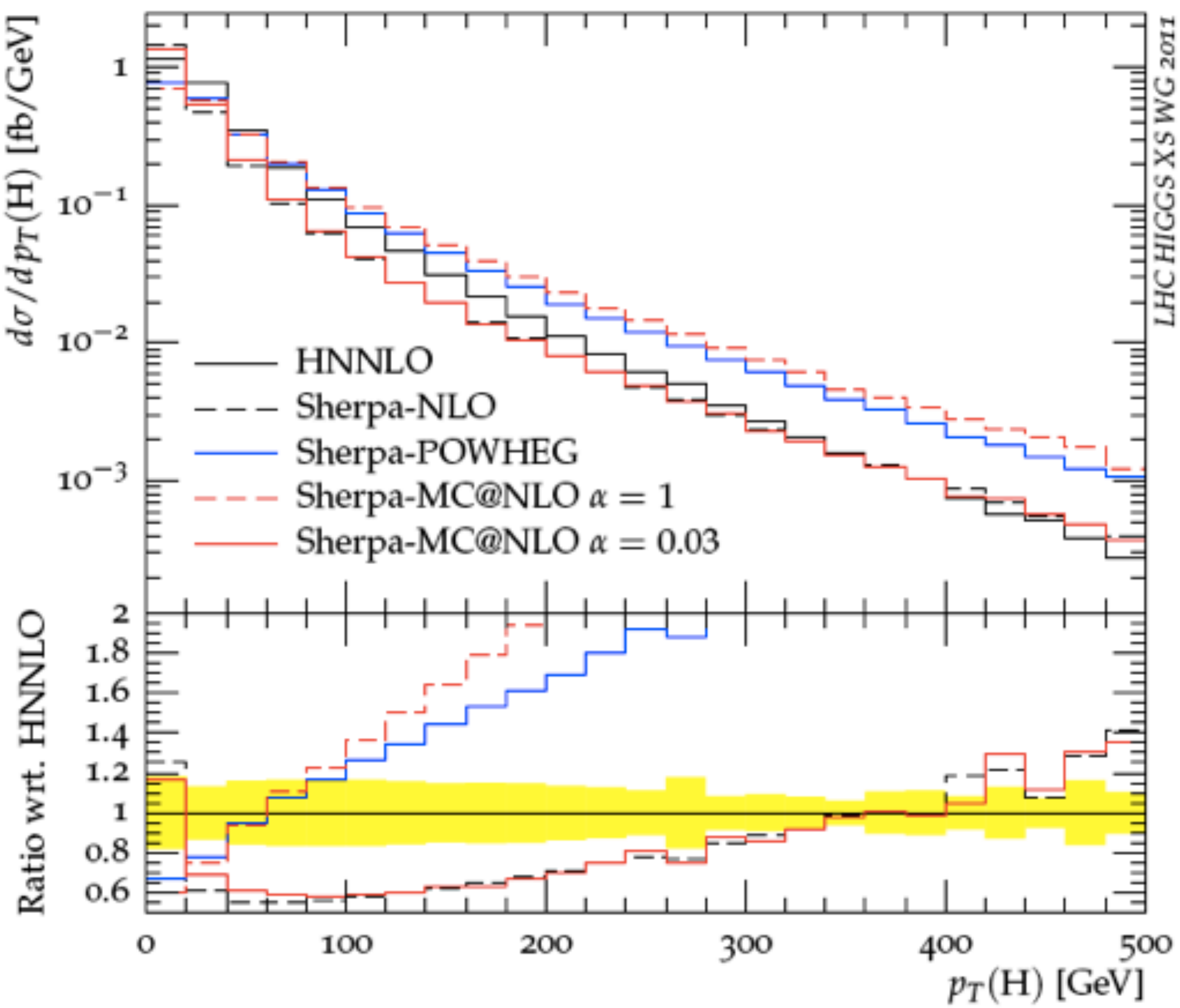


Carena, Gori, Shah, Wagner 2011

The Higgs in gluon-fusion

- ... and NLO+parton-shower tools can have very large uncertainties for exactly the interesting variables



Grazzini, Hoeth, Krauss, FP, Schoenherr, Siegert 2012

- What exactly is stuck up in the exponent in the various codes modifies the pT spectrum
- Can (very roughly) give some indication of uncalculated NNLO terms to Higgs+jet

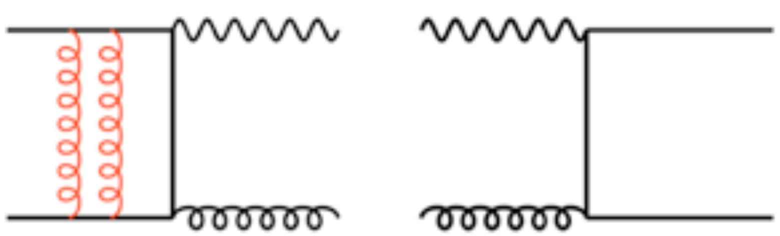
Lessons

- Many other examples to give (ttbar, dijet cross sections for gluon PDF, $e^+e^- \rightarrow 3$ jets for α_s extraction)
- Moral: Need NNLO for most interesting processes at the LHC, too much potential interplay between QCD and analysis cuts for LO/NLO.
- Only a special class of observables currently computed: at NNLO colorless final state (W, Z, Higgs, WH, $\gamma\gamma$)
- Need at least the capability for $2 \rightarrow 2$ with colored final states; would like a method in principle extendable to higher multiplicities

Structure of NNLO cross section

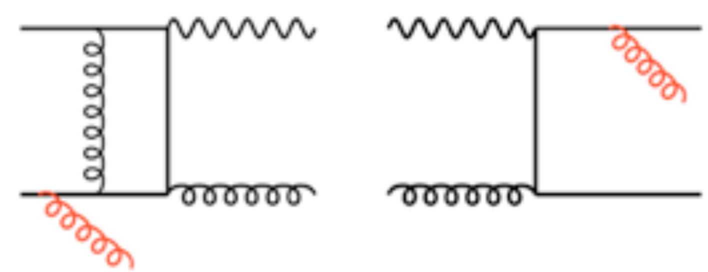
- Need the following ingredients for a NNLO cross section

2-loop matrix elements, m partons



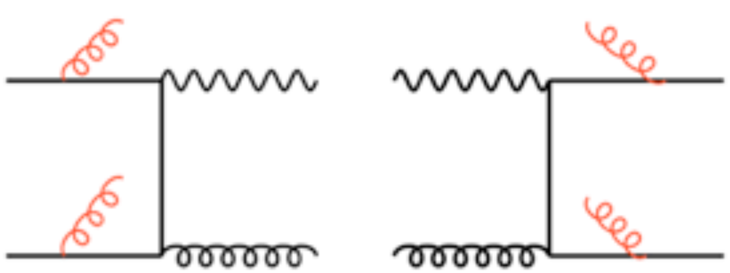
- Explicit IR poles from loop integrals

1-loop matrix elements, m+1 partons



- Explicit IR poles from loops
- Implicit IR poles from single unresolved radiation

Tree level matrix elements, m+2 partons



Implicit IR poles from double unresolved radiation

- IR singularities cancel in the sum of real and virtual corrections and mass factorization counterterms but only after phase space integration for real radiations
- Need a procedure to extract poles before phase-space integration to allow for differential observables

How to calculate at NLO

- Well-honed techniques for calculating and combining real+virtual at NLO
- Virtual corrections with Feynman diagrams or new unitarity techniques
- To deal with IR singularities of real emission, dipole subtraction (Catani, Seymour 1996)

$$d\sigma_{NLO} = \int_{d\Phi_{m+1}} \left(d\sigma^R_{NLO} - d\sigma^S_{NLO} \right) + \left[\int_{d\Phi_{m+1}} d\sigma^S_{NLO} + \int_{d\Phi_m} d\sigma^V_{NLO} \right]$$

Approximates real-emission matrix elements in all singular limits so this difference is numerically integrable

Simple enough to integrate analytically so that 1/ε poles can be cancelled against virtual corrections

What's known at NNLO

- Two-loop amplitudes for dijet, γ+jet, H+jet, V+jet, known, some for over 10 years
- One-loop corrections to real emission known
- Singular limits of double-real emission known for over 10 years
- The problem is how to use the singular limits of the double-real emission

Singular limits

- An example to illustrate the difficulties: $q_a + q_b \to V + g_1 + g_2 + g_3$
- One possible NNLO singularity: triple-collinear, $g_1 || g_2 || g_3$

$$|\mathcal{M}_{a_1,\ldots,a_m,\ldots}(p_1,\ldots,p_m,\ldots)|^2 \simeq \left(\frac{8\pi\mu^{2\epsilon}\alpha_S}{s_{1\ldots m}}\right)^{m-1} \mathcal{T}^{ss'}_{a,\ldots}(xp,\ldots)\, \hat{P}^{ss'}_{a_1\ldots a_m}$$

$$\begin{aligned}
\hat{P}^{\mu\nu}_{g_1g_2g_3} = {} & C_A^2 \left\{ \frac{(1-\epsilon)}{4s_{12}^2}\left[-g^{\mu\nu}t^2_{12,3} + 16 s_{123}\frac{z_1^2 z_2^2}{z_3(1-z_3)}\left(\frac{\tilde{k}_2}{z_2} - \frac{\tilde{k}_1}{z_1}\right)^\mu \left(\frac{\tilde{k}_2}{z_2} - \frac{\tilde{k}_1}{z_1}\right)^\nu\right]\right. \\
& - \frac{3}{4}(1-\epsilon)g^{\mu\nu} + \frac{s_{123}}{s_{12}}g^{\mu\nu}\frac{1}{z_3}\left[\frac{2(1-z_3)+4z_3^2}{1-z_3} - \frac{1-2z_3(1-z_3)}{z_1(1-z_1)}\right] \\
& + \frac{s_{123}(1-\epsilon)}{s_{12}s_{13}}\left[2z_1\left(\tilde{k}_2^\mu\tilde{k}_2^\nu\frac{1-2z_3}{z_3(1-z_3)} + \tilde{k}_3^\mu\tilde{k}_3^\nu\frac{1-2z_2}{z_2(1-z_2)}\right)\right. \\
& + \frac{s_{123}}{2(1-\epsilon)}g^{\mu\nu}\left(\frac{4z_2z_3 + 2z_1(1-z_1) - 1}{(1-z_2)(1-z_3)} - \frac{1-2z_1(1-z_1)}{z_2z_3}\right) \\
& \left.\left. + \left(\tilde{k}_2^\mu\tilde{k}_3^\nu + \tilde{k}_3^\mu\tilde{k}_2^\nu\right)\left(\frac{2z_2(1-z_2)}{z_3(1-z_3)} - 3\right)\right]\right\} + (5 \text{ permutations}) \; .
\end{aligned}$$

Catani, Grazzini 1999

- Another limit: g_1 soft, $g_2 || g_3$

$$\begin{aligned}
|\mathcal{M}_{g,a_1,a_2,\ldots,a_n}(q,p_1,p_2,\ldots,p_n)|^2 \simeq {} & -\frac{2}{s_{12}}\,(4\pi\mu^{2\epsilon}\alpha_S)^2 \\
& \cdot \langle\, \mathcal{M}_{a,\ldots,a_n}(p,\ldots,p_n)\,|\, \hat{\boldsymbol{P}}_{a_1a_2}\left[\boldsymbol{J}^\dagger_{(12)\mu}(q)\,\boldsymbol{J}^\mu_{(12)}(q)\right] |\, \mathcal{M}_{a,\ldots,a_n}(p,\ldots,p_n)\,\rangle
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{J}^\dagger_{(12)\mu}(q)\,\boldsymbol{J}^\mu_{(12)}(q) = {} & \sum_{i,j=3}^{n} \boldsymbol{T}_i\cdot\boldsymbol{T}_j\,\frac{p_i\cdot p_j}{(p_i\cdot q)(p_j\cdot q)} + 2\sum_{i=3}^{n}\boldsymbol{T}_i\cdot\boldsymbol{T}_{(12)}\,\frac{p_i\cdot(p_1+p_2)}{(p_i\cdot q)(p_1+p_2)\cdot q} \\
& + \boldsymbol{T}^2_{(12)}\,\frac{(p_1+p_2)^2}{((p_1+p_2)\cdot q)^2} \qquad (122)
\end{aligned}$$

Antenna subtraction

- One problem is that there are numerous other singular limits: two particles soft, one soft and two collinear, triple-collinear, double-collinear, ...
- People moved to instead constructing "antennae" from actual physical processes that should reproduce singular limits, but are simple enough to handle Glover, Gehrmann, Gehrmann-deRidder 2000's
- For example: Higgs decay to gluons at NNLO encodes singular final-state emissions from gluons
- Called antenna subtraction, used to obtain $e^+e^- \to 3$ jets at NNLO
- Very difficult to construct the required antennae for hadronic scattering
- Also requires some integration over final-state phase space, kinematic information is lost
- Recent results for part of all-gluon dijet production Gehrmann et al. 2013

Entangled singularities

- Why doesn't one first partition the phase space so that in a certain region only the appropriate singular limit is used?

$$\langle \hat{P}_{\bar{q}'_1 q'_2 q_3} \rangle = \frac{1}{2} C_F T_R \frac{s_{123}}{s_{12}} \left[-\frac{t^2_{12,3}}{s_{12} s_{123}} + \frac{4 z_3 + (z_1 - z_2)^2}{z_1 + z_2} + (1 - 2\epsilon) \left(z_1 + z_2 - \frac{s_{12}}{s_{123}} \right) \right]$$

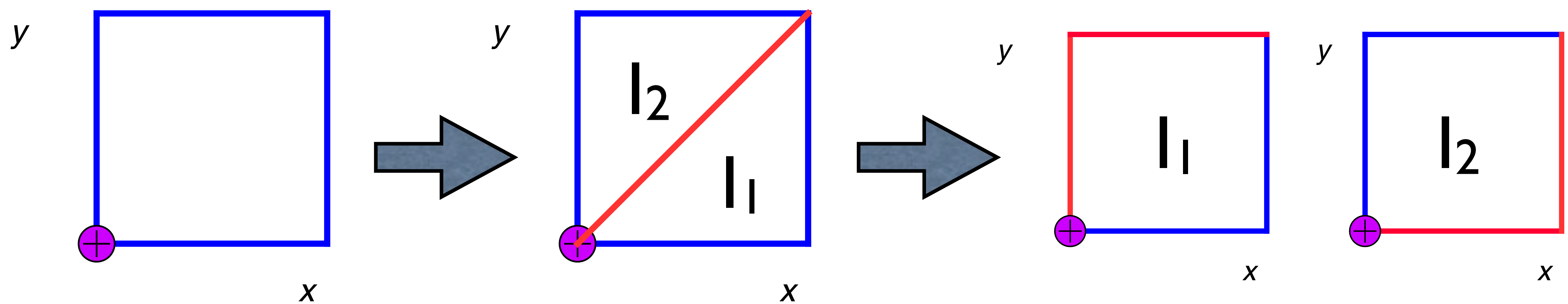
- $s_{123} \sim E_1 E_2 (1-c_{12}) + E_1 E_3 (1-c_{13}) + E_2 E_3 (1-c_{23})$
- What goes to zero quicker? $E_1, E_2, E_3, (1-c_{12}), (1-c_{13})$, or $(1-c_{23})$?
- Need to order the limits, since singularities must be extracted from integrals of the schematic form:

$$\int_0^1 dx dy \frac{x^\epsilon y^\epsilon}{(x+y)^2} F_J(x,y)$$

- Need a systematic technique for ordering limits, too many of such issues appear

Sector decomposition

- Can define a systematic procedure to order limits



$$I = \int_0^1 \mathrm{d}x \mathrm{d}y \frac{x^\epsilon y^\epsilon}{(x+y)^2}$$

$$I_2 = \int_0^1 \mathrm{d}y \int_0^y \mathrm{d}x \frac{x^\epsilon y^\epsilon}{(x+y)^2}$$

$$I_1 = \int_0^1 \mathrm{d}x \mathrm{d}y \frac{x^{-1+2\epsilon} y^\epsilon}{(1+y)^2}$$

$$I_2 = \int_0^1 \mathrm{d}x \mathrm{d}y \frac{y^{-1+2\epsilon} x^\epsilon}{(1+x)^2}$$

$$y^{-1-\epsilon} = -\frac{\delta(y)}{\epsilon} + \left[\frac{1}{y}\right]_+ - \epsilon \left[\frac{\ln y}{y}\right]_+ + \mathcal{O}(\epsilon^2)$$

Binoth, Heinrich; Anastasiou, Melnikov, FP 2003-2005

Sector decomposition

- Give up on the idea of analytic cancellation of poles; calculate the coefficients of $1/\varepsilon^n$ Laurent expansion numerically

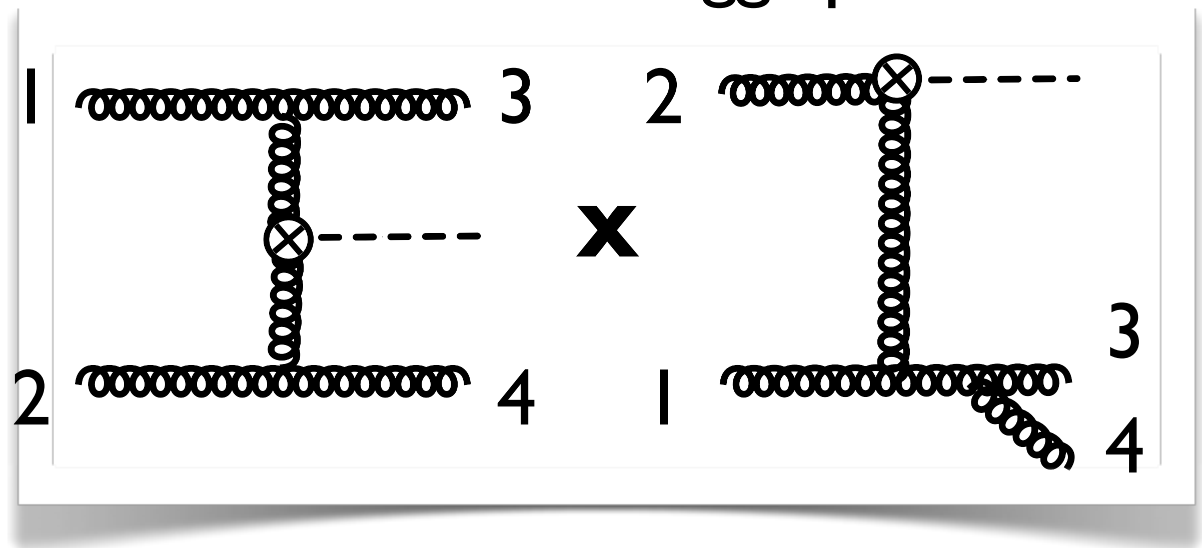
- Apply directly to each interference of diagrams which appears

- Used for the first differential NNLO calculations at hadron colliders: Higgs, W/Z Anastasiou, Melnikov, FP; Melnikov, FP 2005-2006

- The drawback: originally used a global phase-space parameterization for a given interference

Higgs production

- To illustrate the drawbacks, use Higgs production as an example



- Invariants that occur in this topology : s_{13}, s_{24}, s_{134}, s_{34}. These contain collinear singularities $p_1||p_3$, $p_2||p_4$, $p_3||p_4$, $p_1||p_3||p_4$

- Can only have: $p1||p3$ & $p2||p4$ or $p1||p3||p4$. Not all invariants above can have collinear singularities simultaneously

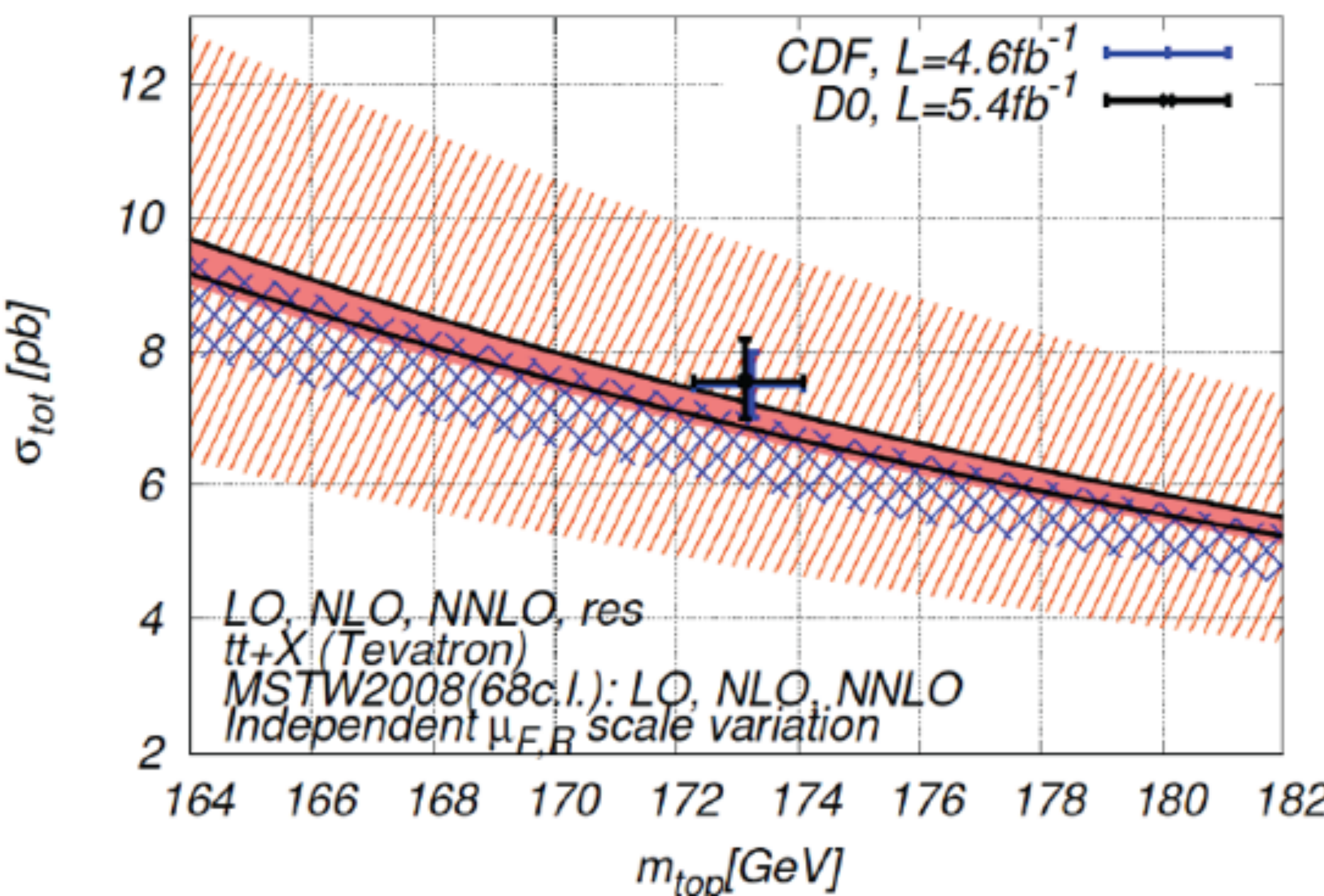
- Would need to start over with entirely new parameterization for Higgs+jet

not easy to disentangle all singularities!

$$
\begin{aligned}
s_{23} &= -\lambda_2\lambda_3(1-z)\left[1+\lambda_1(1-rt)/r/(r+t)\right], \\
s_{24} &= -(1-\lambda_2)\lambda_3(1-z)\left[1+\lambda_1(1-rt)/r/(r+t)\right], \\
s_{34} &= \lambda_1\lambda_3(1-\lambda_3)(1-z)^2(1+u)^2/K_p/r, \\
s_{13} &= -\frac{(1-\lambda_3)(1-z)}{K_p r\left[1+\lambda_1(1-rt)/r/(r+t)\right]}\left[A_1+A_2+2(2\lambda_4-1)\sqrt{A_1A_2}\right]
\end{aligned}
$$

Partitioning+decomposing

- A suggestion recently that removes drawback of previous slide: pre-partitioning of the phase space leads to a phase-space parameterization applicable to NNLO real-radiation corrections for any process, regardless of multiplicity (Czakon, 2010)
- Partition the phase space such that in each partition only a subset of particles leads to singularities, and only one triple collinear or one double collinear singularity can occur



- First NNLO results for colored final-states at hadron colliders Baernreuther, Czakon, Mitov 2012

Extensions and future applications

- Top quark pair production doesn't have final-state collinear singularities, so some extension still needed. Worked out in a simple test case recently (Boughezal, Melnikov, FP 2012)
- First results expected for Higgs+jet at NNLO shortly (Boughezal, Caola, Melnikov, FP, Schulze)

Source (0-jet)	Signal (%)	Bkg. (%)
Inclusive ggF signal ren./fact. scale	13	-
1-jet incl. ggF signal ren./fact. scale	10	-
PDF model (signal only)	8	-
QCD scale (acceptance)	4	-
Jet energy scale and resolution	4	2
W+jets fake factor	-	5
WW theoretical model	-	5
Source (1-jet)	**Signal (%)**	**Bkg. (%)**
1-jet incl. ggF signal ren./fact. scale	26	-
2-jet incl. ggF signal ren./fact. scale	15	-
Parton shower/ U.E. model (signal only)	10	-
b-tagging efficiency	-	11
PDF model (signal only)	7	-
QCD scale (acceptance)	4	2
Jet energy scale and resolution	1	3
W+jets fake factor	-	5
WW theoretical model	-	3

- Theoretical error in H+jet already becoming a limiting factor in LHC measurements
- NNLO needed to improve recent efforts at resumming logarithms associated with the jet-veto in the 1-jet bins; want to match the NNLL+NNLO precision achieved for 0-jets

systematics in the WW channel, from J. Qian

Conclusions

- Many examples from LHC data motivate the development of a technique for the calculation of NNLO corrections to 2→2+n scattering processes
- We've studied such a method for real-radiation corrections that is simply extendable to higher multiplicities. Already has led to the first NNLO results for 2→2 scattering at hadron colliders (Baernreuther, Czakon, Mitov ttbar, 2012)
- First NNLO calculations of 2→2 jet cross sections at the LHC upcoming... stay tuned