

MINLO

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[arXiv:1206.3572](https://arxiv.org/abs/1206.3572) and [arXiv:1212.4504](https://arxiv.org/abs/1212.4504)

Preliminary remark on scales

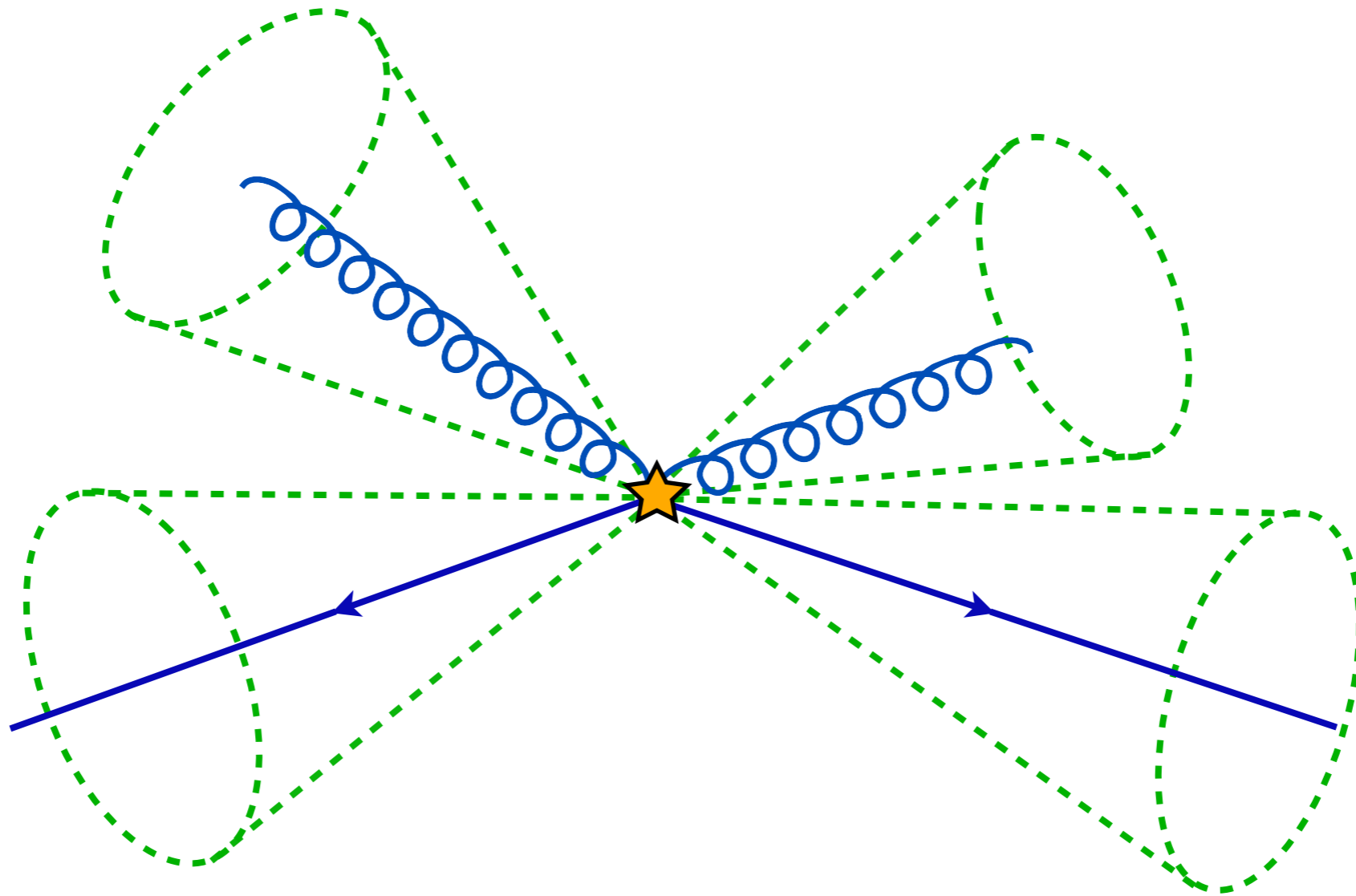
- ‘Good scales’ are commonly found retrospectively requiring NLO corrs be small or sensitivity minimized
- ‘Bad scales’ → big scale logs → big corr^s & sensitivity
- Big corrections can have real physical origins: new production channels, I.R. logs, big colour factors, big gluon lumi ...
- Adjusting scale to make corr^s / sensitivity small can effectively ‘eat’ unrelated physics in scale choice

MiNLO: Multiscale improved NLO

- MiNLO only addresses processes with jets
- MiNLO recipe doesn't aim at minimizing μ sensitivity
- It's more about getting a better central value
- It's a priori i.e. there's not so much you can 'tune'
- The MiNLO scheme is just the same one used for the matrix elements in MCs using ME+PS merging ...
- with a couple of easy-to-do modificatⁿs to keep NLO NLO
- It therefore accounts for potentially big Sudakov logs, beyond NLO, that can turn predictⁿs to garbage

Reminder of ME+PS methods

1. Take an ME generator and generate events ...

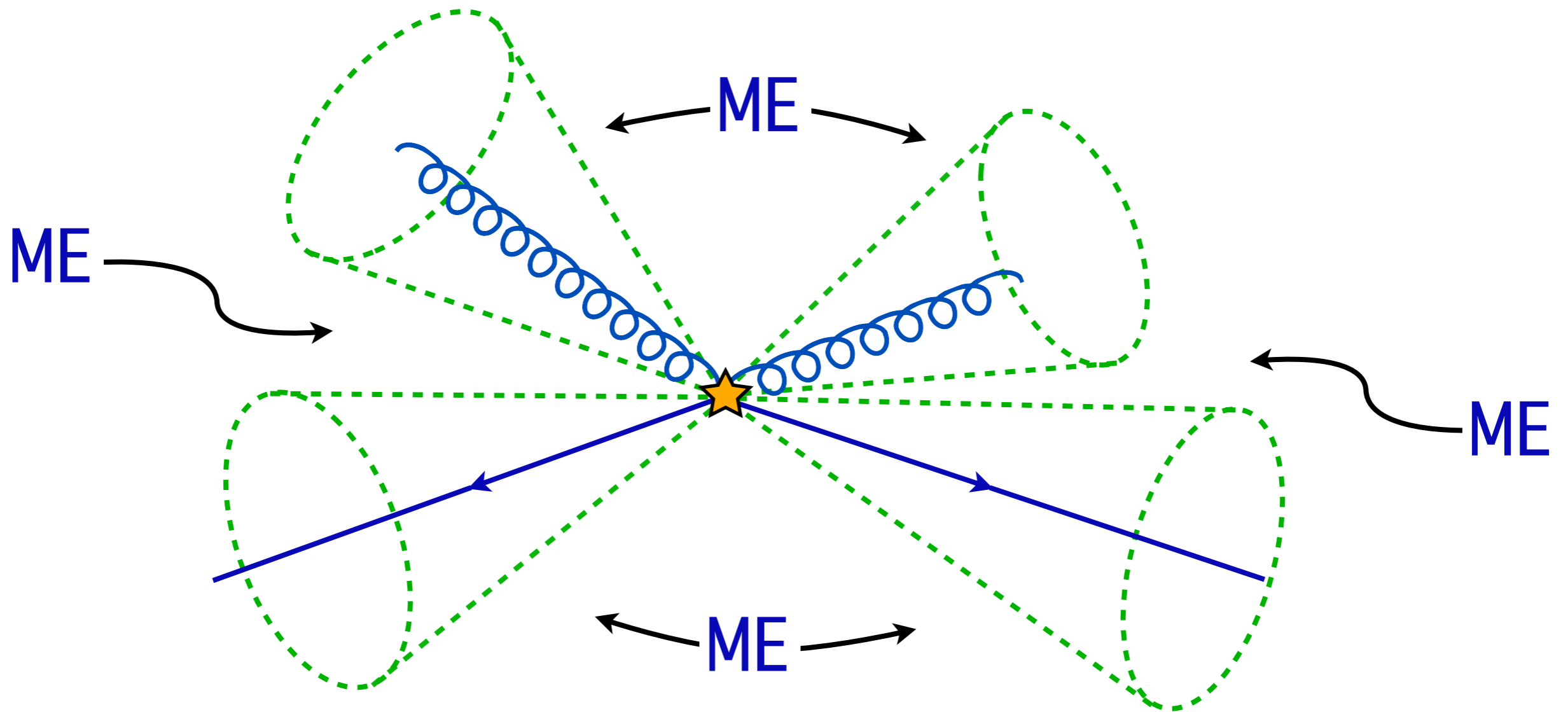


... with a cut defined in terms of a jet measure: q_0

$$\alpha_s = \alpha_s(q_0), \quad \mu_F = q_0$$

Reminder of ME+PS methods

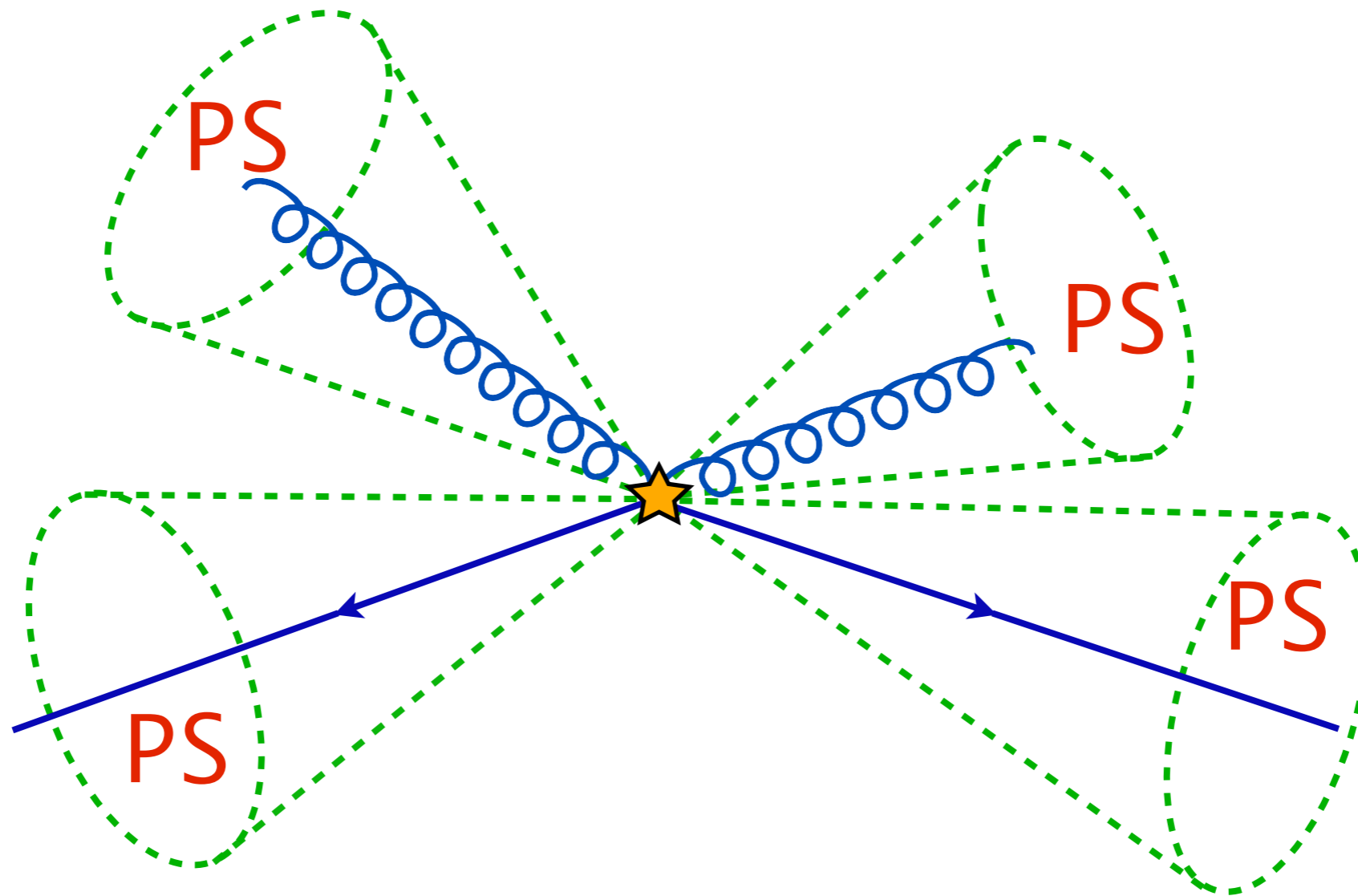
This partitions phase space into the ME region ...



$$[y_{ij} \geq q_0 \quad \forall \text{ partons } i, j]$$

Reminder of ME+PS methods

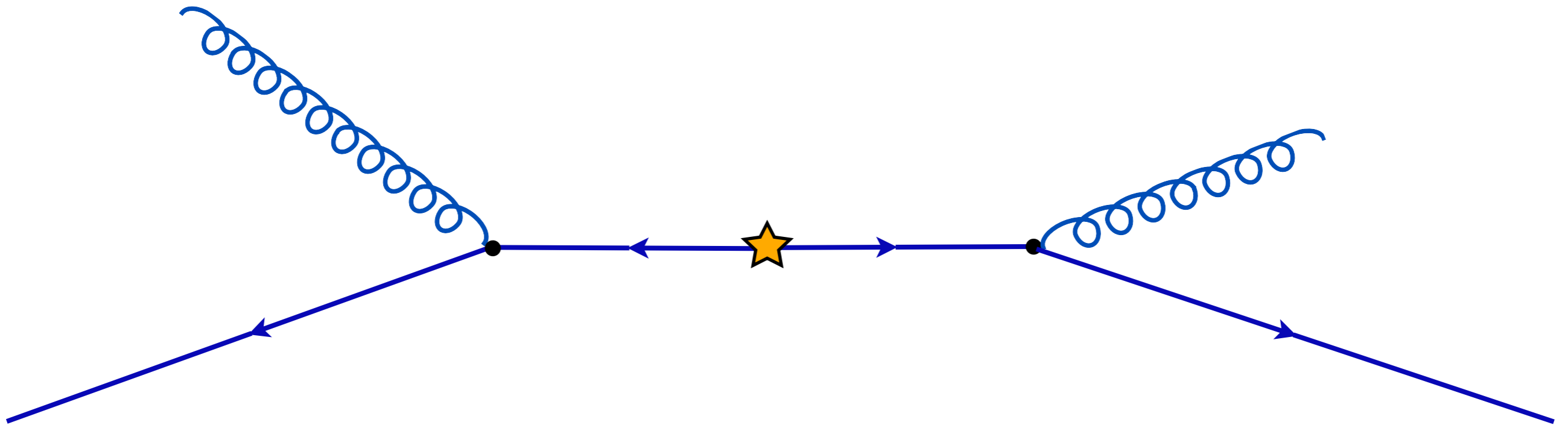
... and the **PS regions** of phase space.



$$[y_{ij} \geq q_0 \quad \forall \text{ partons } i, j]$$

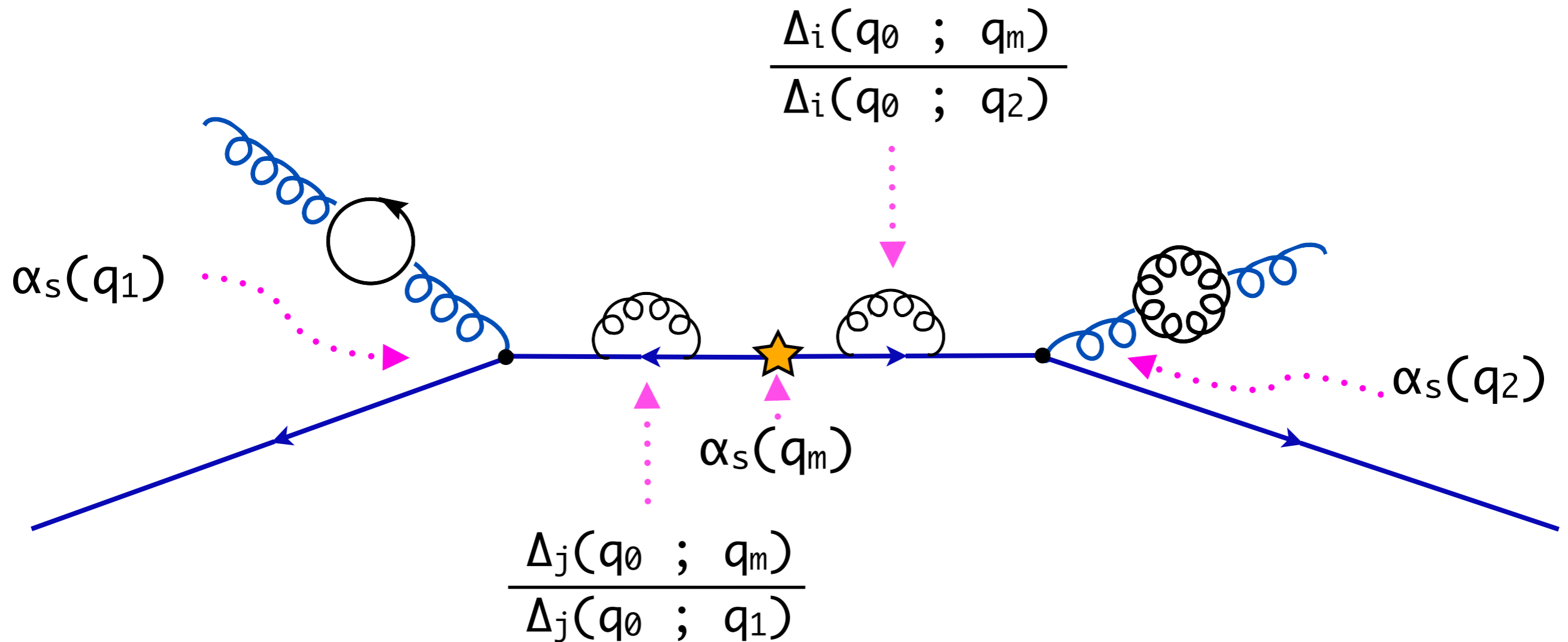
Reminder of ME+PS methods

2. Apply k_T jet-algo to get a **shower history**



Reminder of ME+PS methods

3. Include **LL infrared effects** [like a PS does]

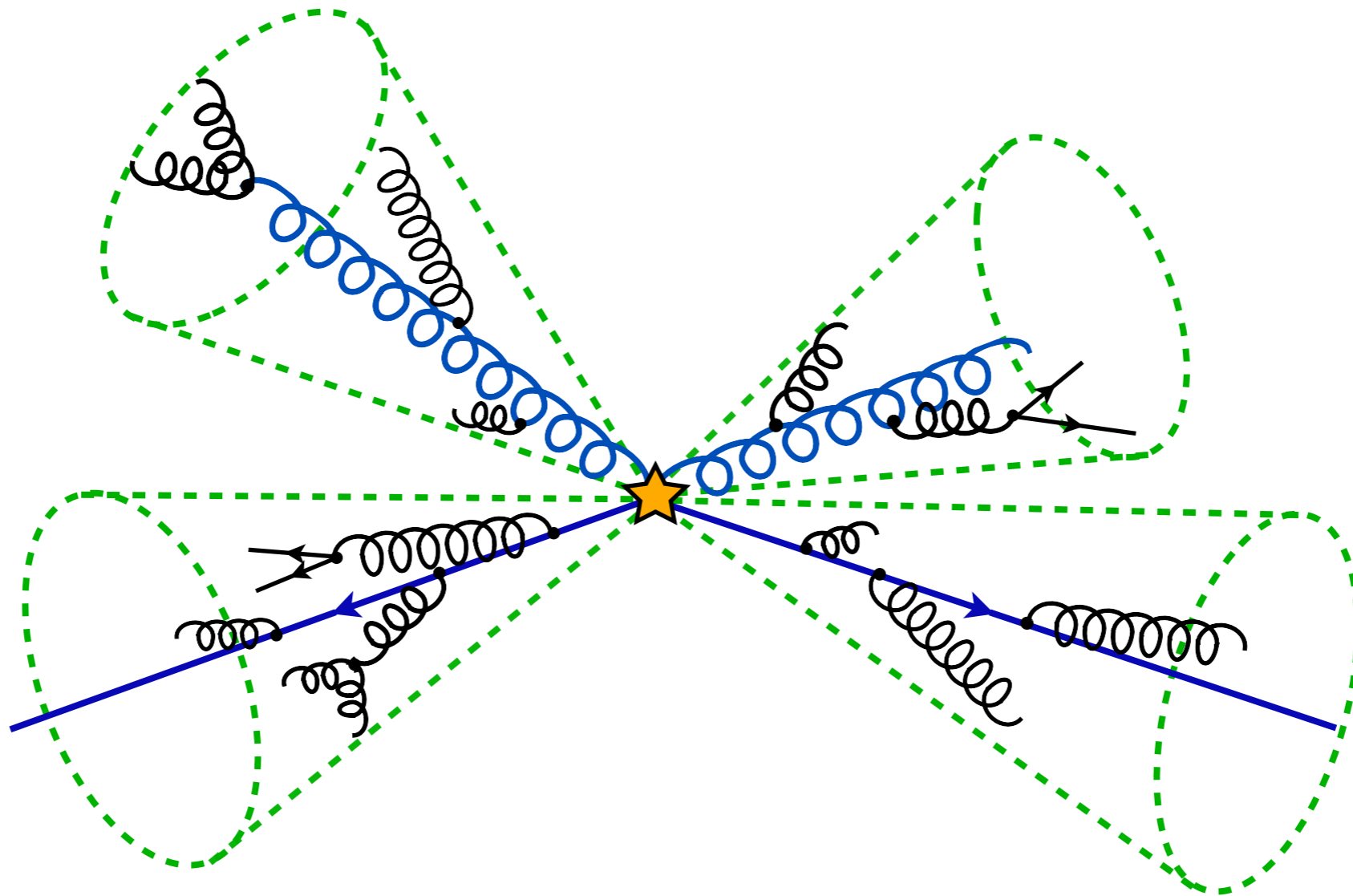


- Coupling constant weighting for branching vertices

- Sudakov suppression factors for no further emission in ME region

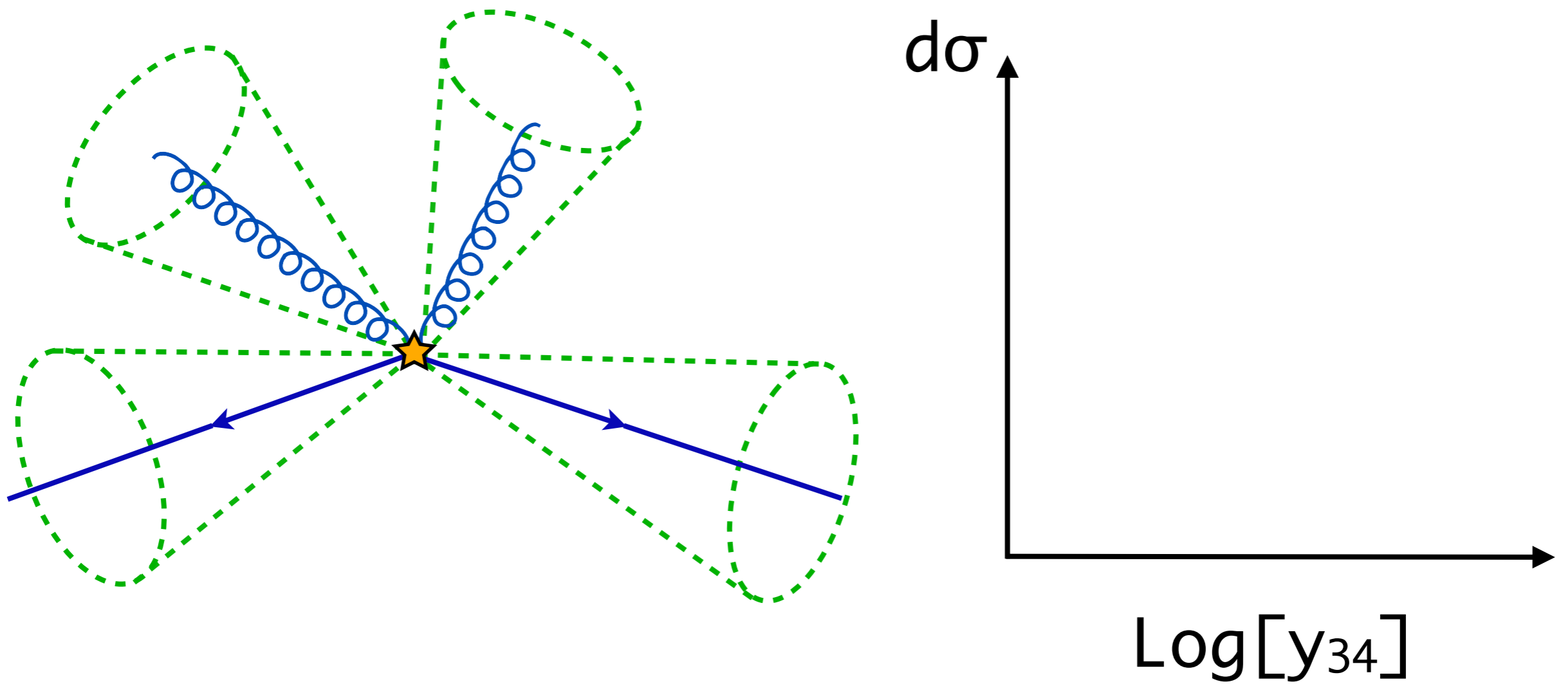
Reminder of ME+PS methods

4. Fill below the q_0 cut with **vetoed** showers



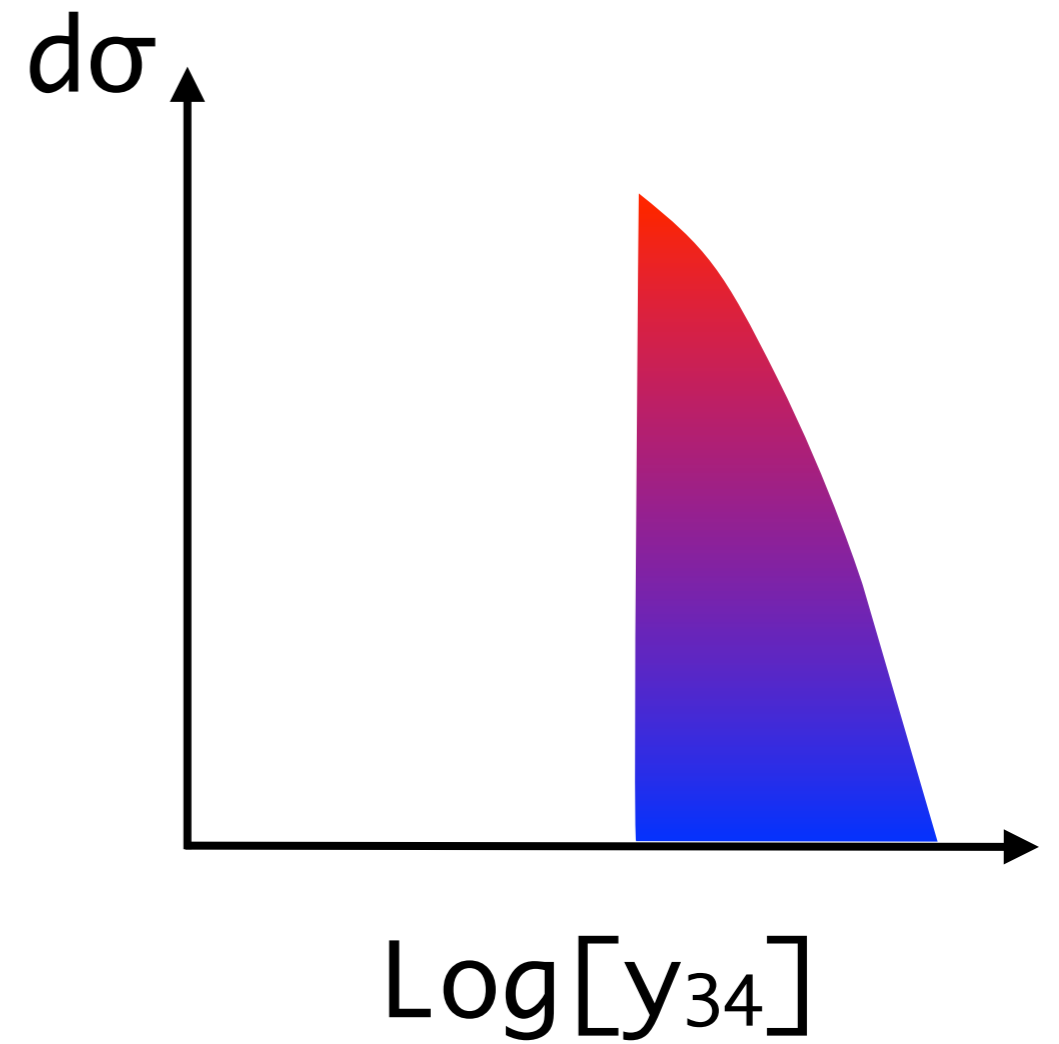
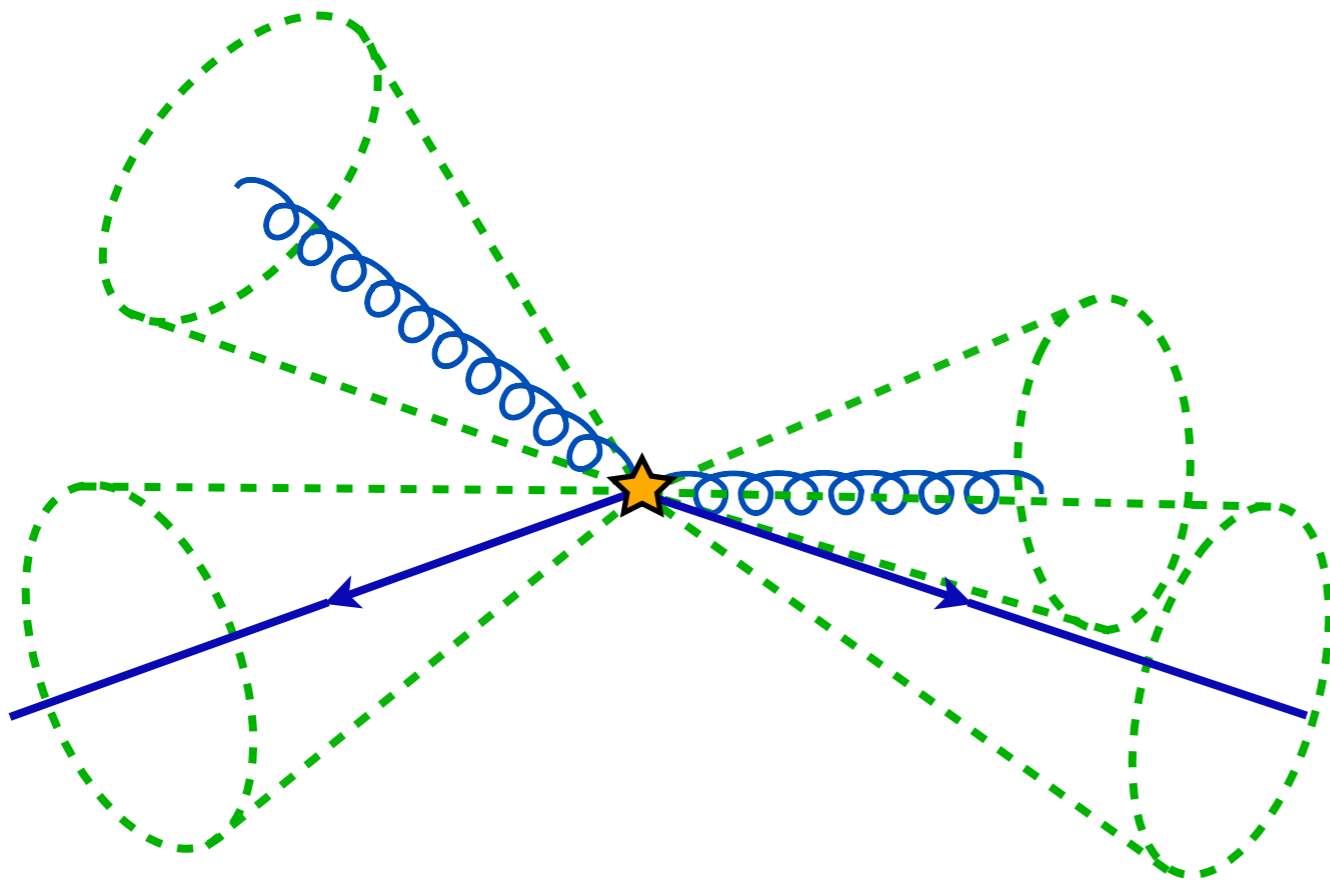
Reminder of ME+PS methods

Gives smooth behaviour as pseudopartons get close ...



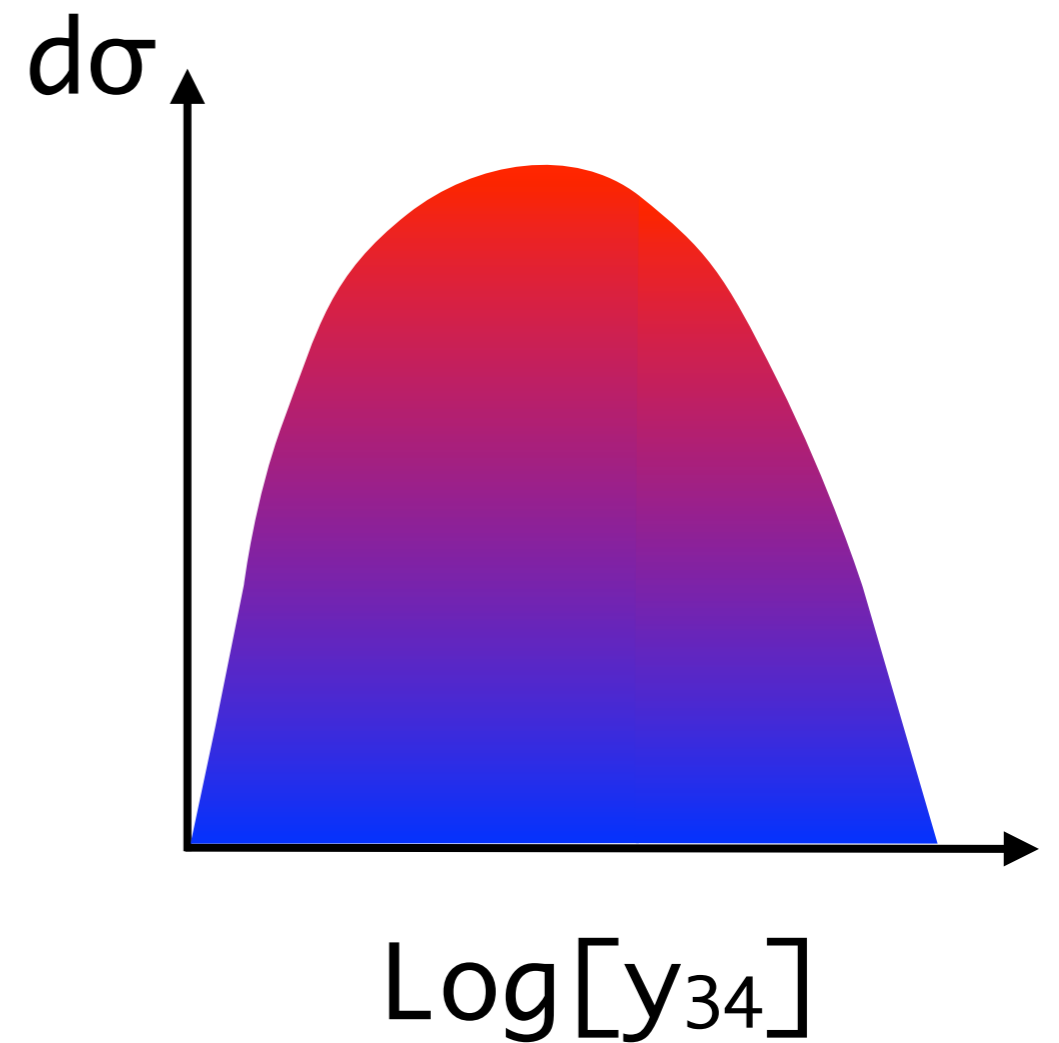
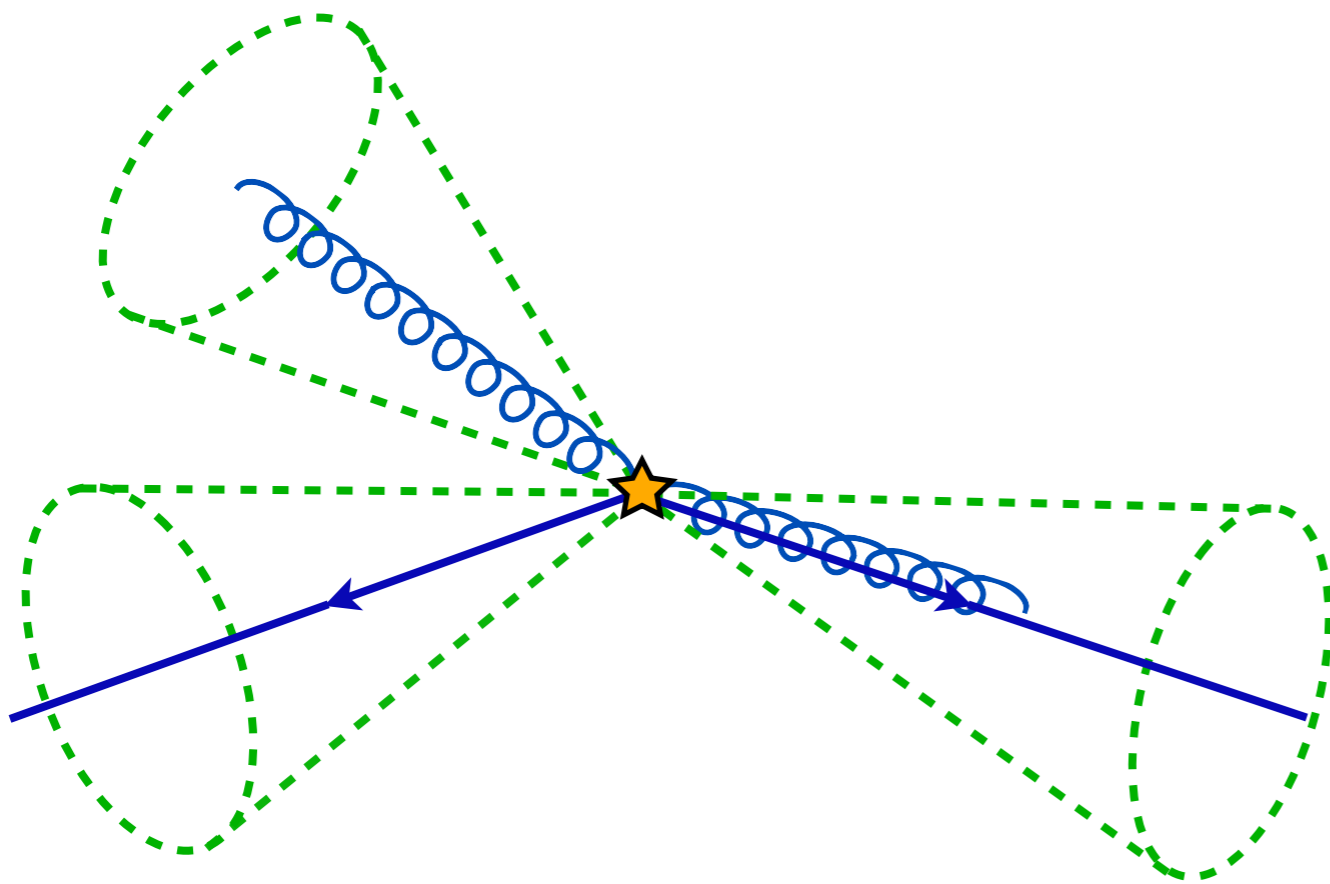
Reminder of ME+PS methods

Gives smooth behaviour as pseudopartons get close ...



Reminder of ME+PS methods

Gives smooth behaviour as pseudopartons get close ...



Extension 1: don't break scale compensation

- NLO x-secs have the generic form:

$$\frac{d\sigma}{d\Phi} = \alpha_S^N(\mu_R) B + \alpha_S^{N+1}(\mu_R) \left[V + N b_0 \log \frac{\mu_R^2}{Q^2} B \right] + \alpha_S^{N+1}(\mu_R) R$$

- Vary $\mu_R \rightarrow \mu_R'$ in Born & you get back Born + $O(\alpha_S^{N+1})$

$$\alpha_S^N(\mu_R) B + N b_0 \alpha_S^{N+1}(\mu_R) \log \frac{\mu_R^2}{\mu_R'^2} B + \mathcal{O}(\alpha_S^{N+2})$$

- Vary $\mu_R \rightarrow \mu_R'$ in virtual & you get back virtual + $O(\alpha_S^{N+1})$

$$\alpha_S^{N+1}(\mu_R) \left[V + N b_0 \log \frac{\mu_R^2}{Q^2} B \right] - N b_0 \alpha_S^{N+1}(\mu_R) \log \frac{\mu_R^2}{\mu_R'^2} B + \mathcal{O}(\alpha_S^{N+2})$$

- The net variation is $O(\alpha_S^{N+2}) \rightarrow$ scale compensation

Extension 1: don't break scale compensation

- For this to hold when using multiple different scales in Born α_s 's input a fancy choice of μ_R to the virtuals:

$$\mu_R = \left(\prod_{i=1}^N \mu_i \right)^{\frac{1}{N}}$$

- This way virt. $\mu \rightarrow \mu'$ $O(\alpha_s^{N+1})$ variatⁿ cancels that of Born

$$\alpha_S^{N+1}(\mu_R) \left[V + N b_0 \log \frac{\left(\prod_{i=1}^N \mu_i^2 \right)^{1/N}}{Q^2} B \right] - \underbrace{b_0 \alpha_S^{N+1}(\mu_R) \sum_{i=1}^N \log \frac{\mu_i^2}{\mu_i'^2} B}_{\text{dashed red line}} + \mathcal{O}(\alpha_S^{N+2})$$

- Equivalently, use some fixed μ & subtract 'by hand' a [scale] compensating term derived from the α_s^N wghts
- Latter can be viewed as undoing the spurious NLO effect coming from reweighting the Born with MEPS α_s 's

Extension 2: don't overcount Sudakov logs

- Multiplication of Born by Sudakovs generates NLO IR logs

$$\Delta_f(Q_0, Q) = 1 + \Delta_f^{(1)}(Q_0, Q) + \mathcal{O}(\alpha_s^2),$$
$$\Delta_f^{(1)}(Q_0, Q) = -\frac{C_f}{\pi} \alpha_s \left[\frac{1}{4} \log^2 \frac{Q^2}{Q_0^2} - \log \frac{Q^2}{Q_0^2} B_f \right]$$

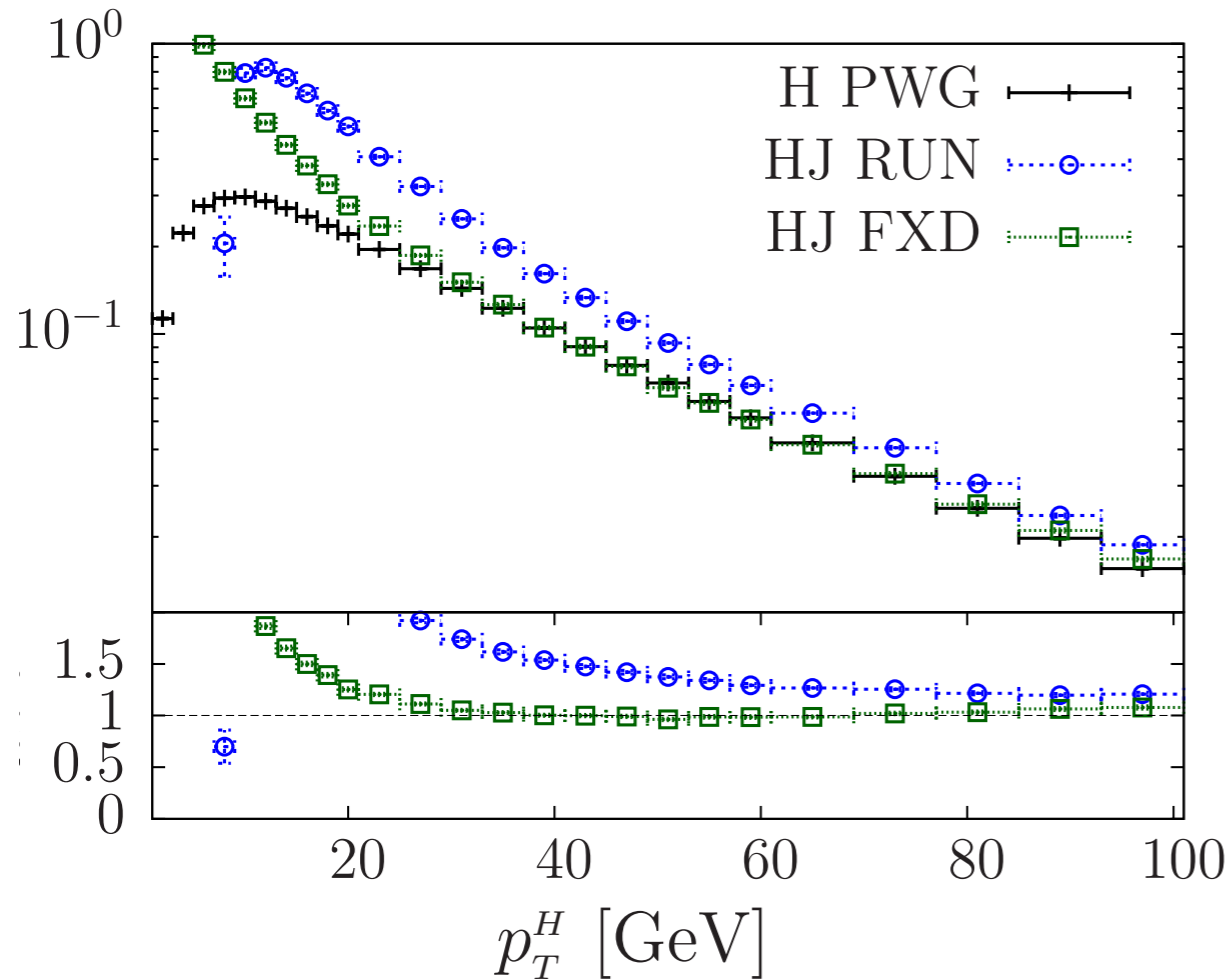
- But NLO was NLO so it already had them in it
- So as well as deleting NLO terms generated by α_s wghts we delete $\mathcal{O}(\alpha_s)$ expansion of all Sudakov wghts * Born

Properties of MiNLO

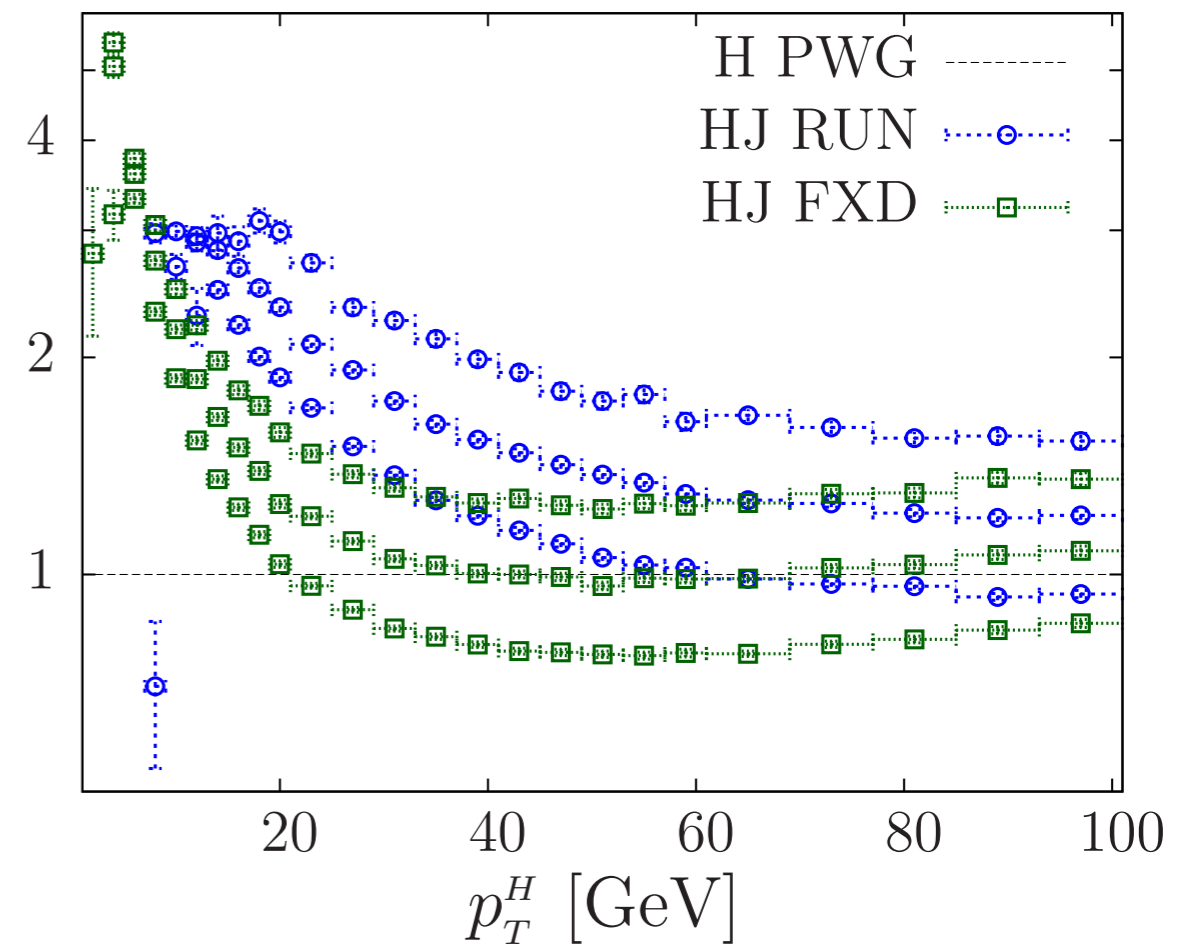
- MiNLO is always NLO accurate - same as NLO up to NNLO
- For sufficiently inclusive observables MiNLO is also NLL accurate
- When used as starting point for Powheg / MC@NLO the scope of the resummation greatly extended ; multiple emissions are explicitly accounted for.

Case study: NLO Higgs + 1 jet

$d\sigma/dp_T^H$ [pb/GeV]



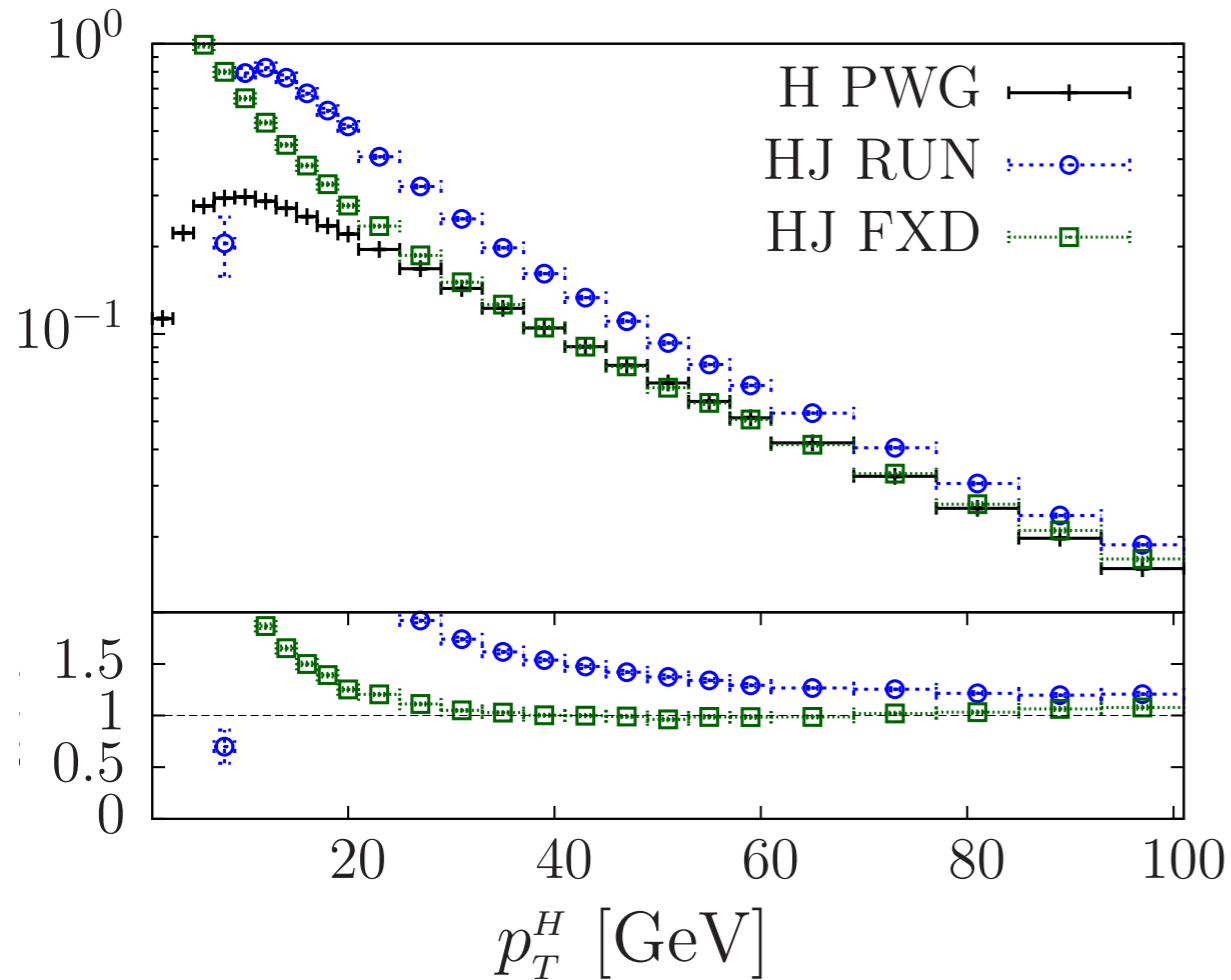
Ratio



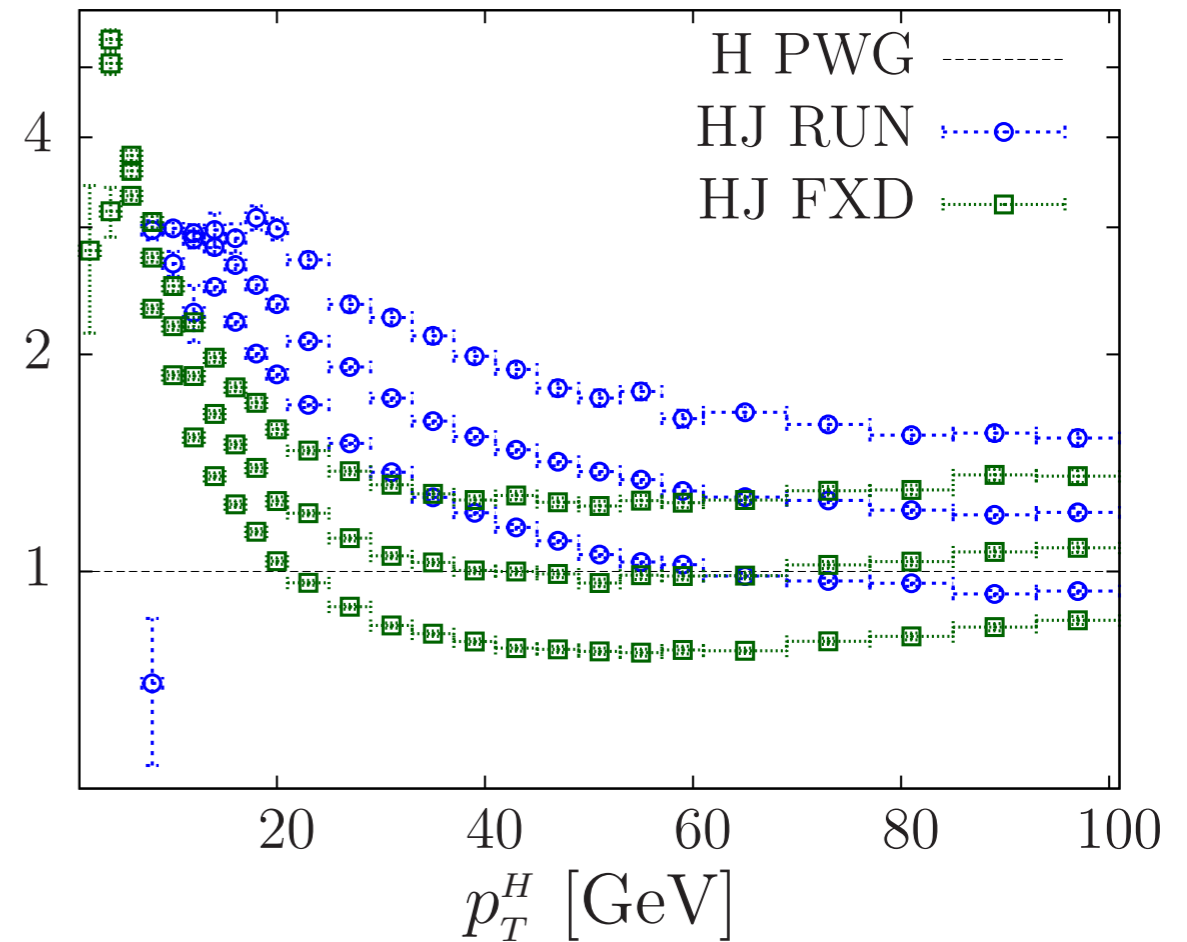
- H PWG: $gg \rightarrow$ Higgs at NLO Powheg+Pythia
- HJ RUN: NLO H+1 jet with $\mu_R = \mu_F = p_{T,H}$
- HJ FXD: NLO H+1 jet with $\mu_R = \mu_F = M_H$
- The ref. line for ratios is NLO $gg \rightarrow$ H Powheg+Pythia

Case study: NLO Higgs + 1 jet

$d\sigma/dp_T^H$ [pb/GeV]

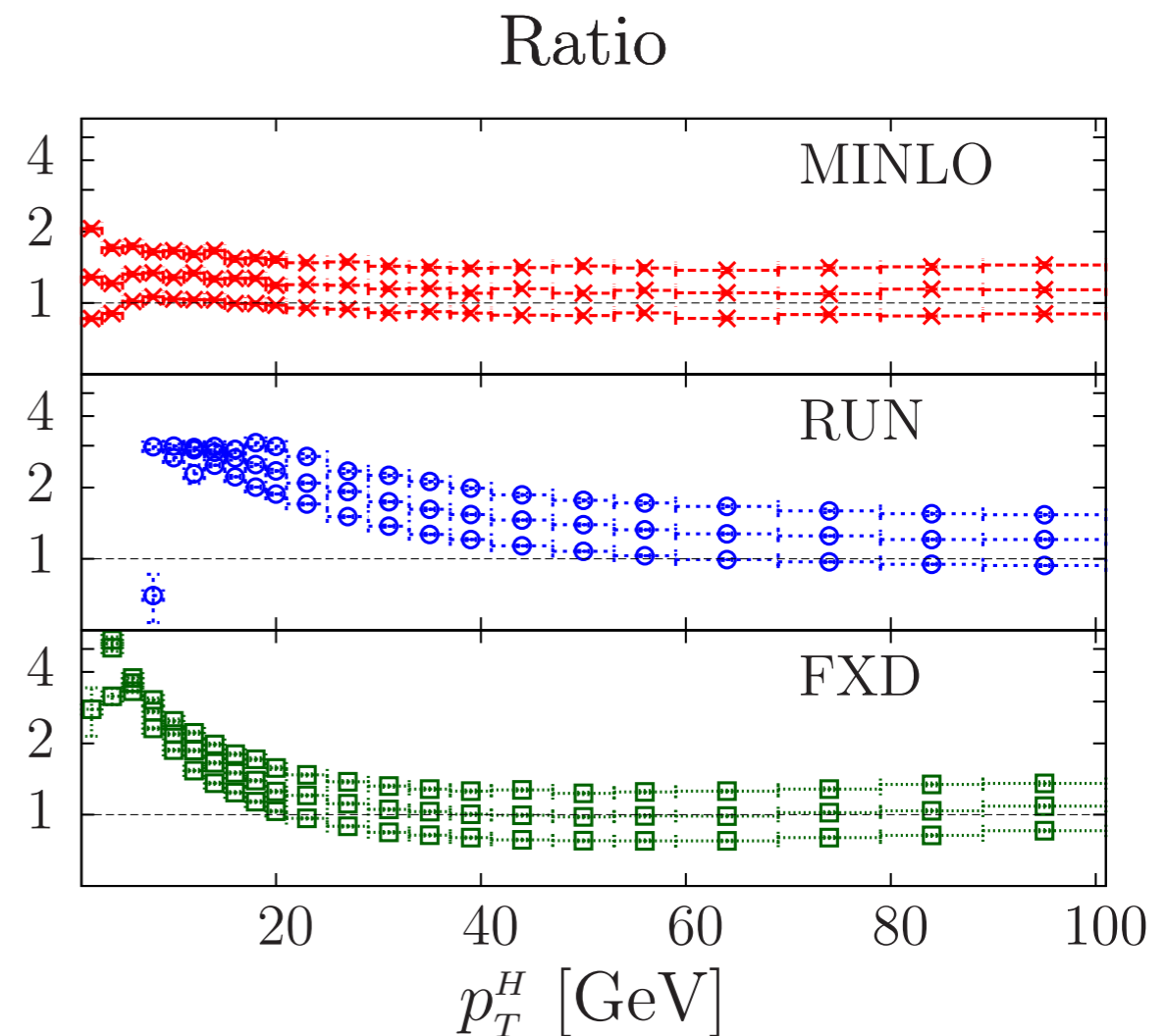
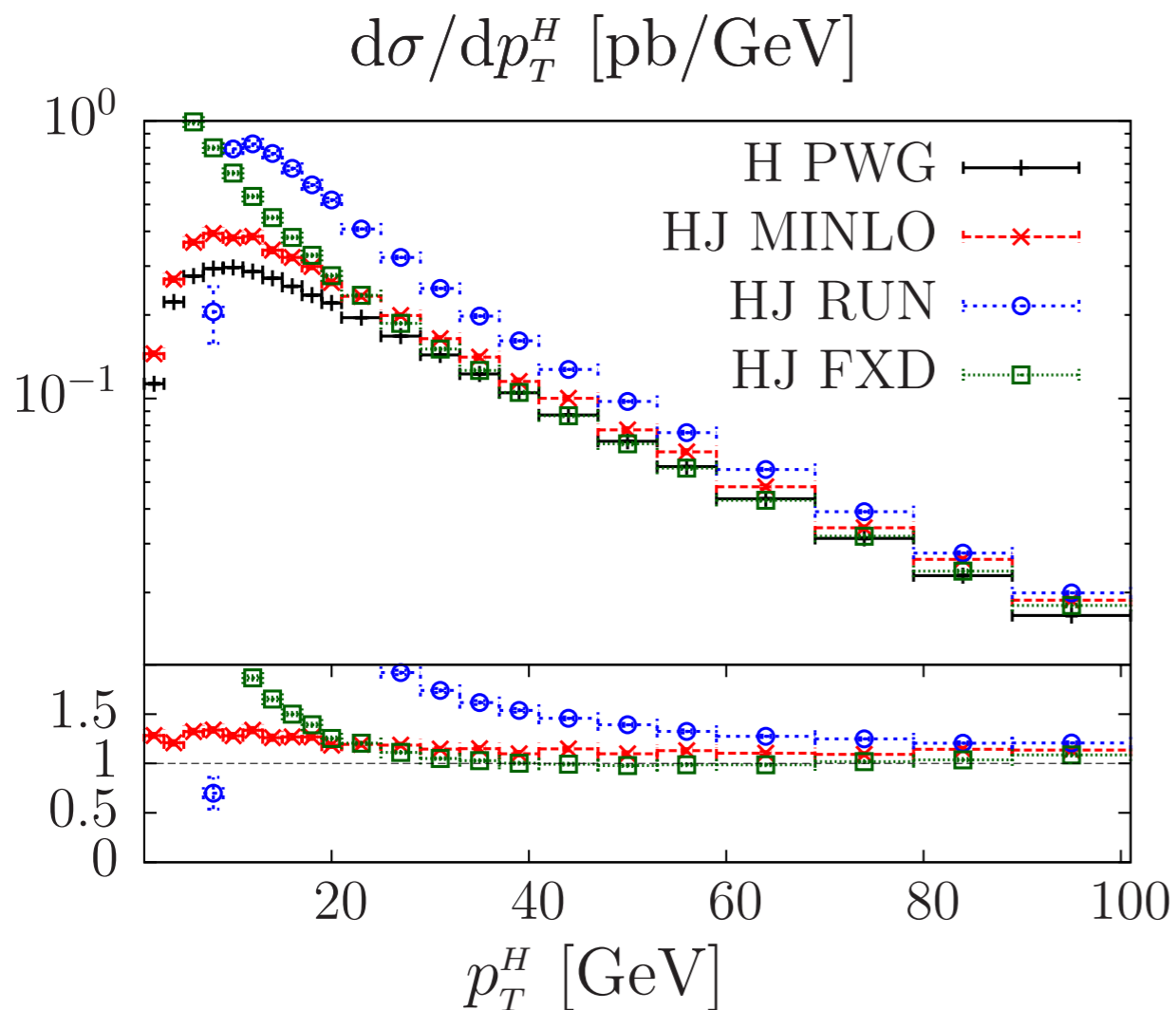


Ratio



- HJ RUN above HJ FXD : $\alpha_s(p_T) > \alpha_s(m_H)$
- NLO H+1 jet calcs outside each other's envelopes by 60 GeV
- HJ RUN [$\mu_R = \mu_F = p_{T,H}$] departs from resummed H PWG at 60 GeV
- HJ FXD's high μ_R makes up for missing Sudakov a bit longer
- Uncertainty envelopes shrink on way down from 40-60 GeV :-/

Case study: NLO Higgs + 1 jet



- MiNLO agrees w. other H+1 jet NLOs at high p_T as promised
- MiNLO within 40% of H PWG in deep Sudakov region
- MiNLO's scale uncertainty does not shrink towards low p_T
- 'Normal' bands shrink to 0 by having 1st Sudakov log only
- Shrinking envelope as $p_T \rightarrow 0$ is surely a bad sign

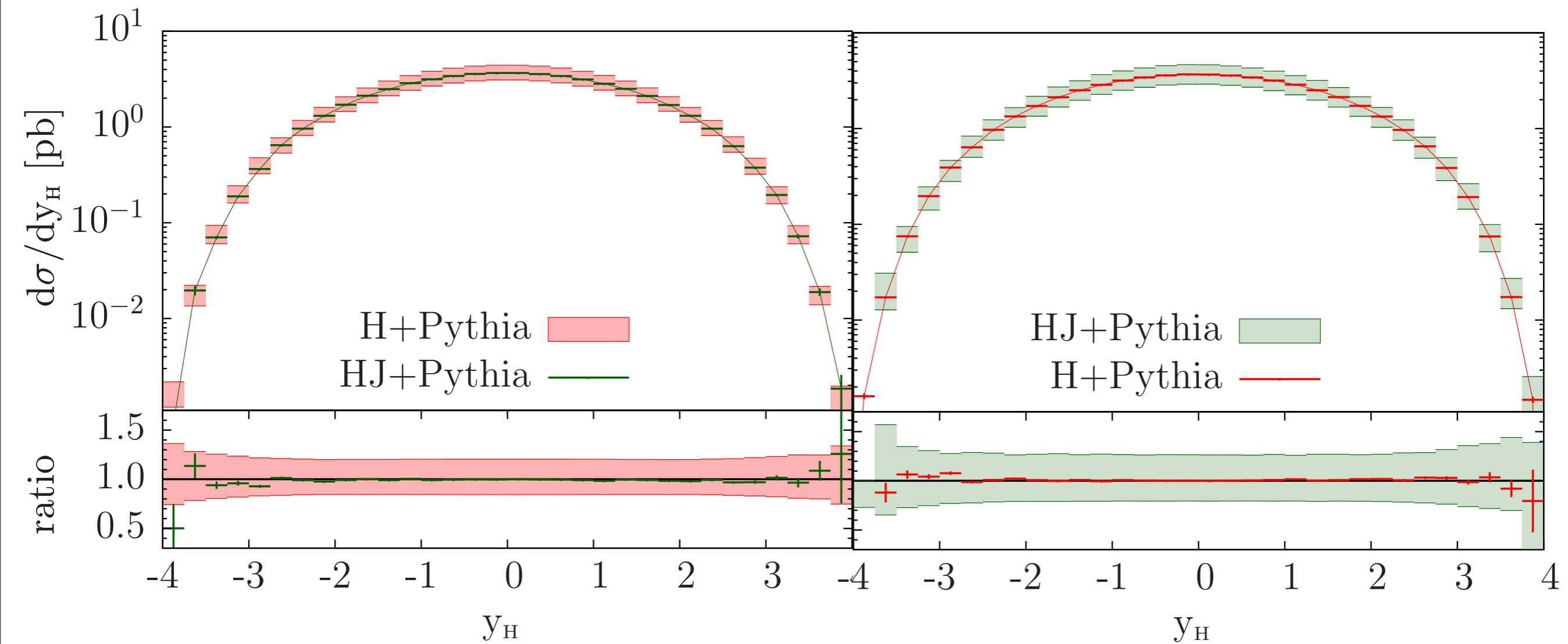
MiNLO in NLO & NLOPS

- Theoretically well motivated scale setting recipe for procs with light jets, based on that used in ME+PS
- Unlike std. NLO it doesn't break when Born kinematics approach soft / collinear configurations
- Scale uncertainties are also more reliable
- The same as conventional NLO up to NNLO terms
- Agrees better with conventional NLO using higher scales e.g. $H_T/2$ [... but not H_T]
- It's a prerequisite for merging NLO+PS's together
- It's pretty simple to implement ...

MiNLO Mk2

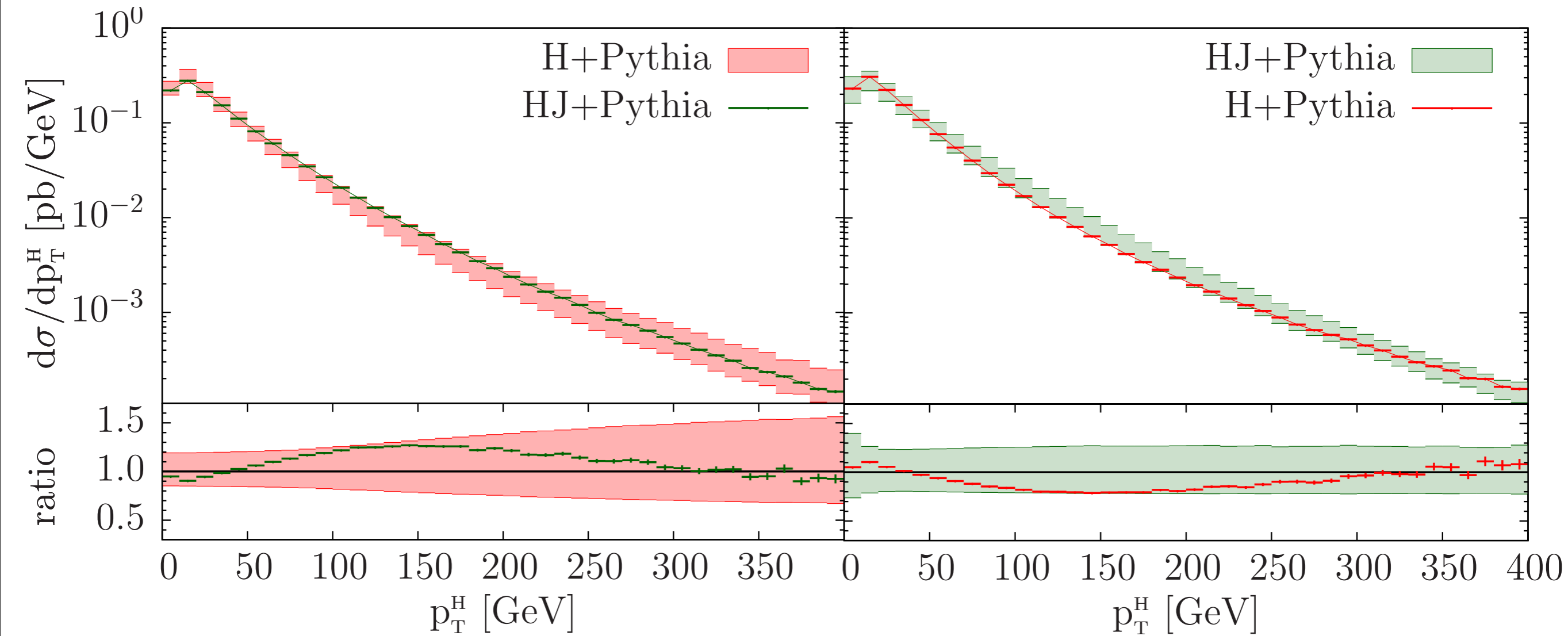
- First MiNLO paper claims MiNLO Boson+jet is LO accurate for inclusive Boson predictions
- Rigorous investigatⁿ in arXiv:1212.4504 [Nason et al.]
- Reveals claim to be true ...
- It improved MiNLO s.t. NLO Boson+jet alone also gives NLO for incl. boson observables
- Like CKKW, at NLO level, but w/o any actual merging

Case study: NLO H vs MiNLO Mk2 HJ



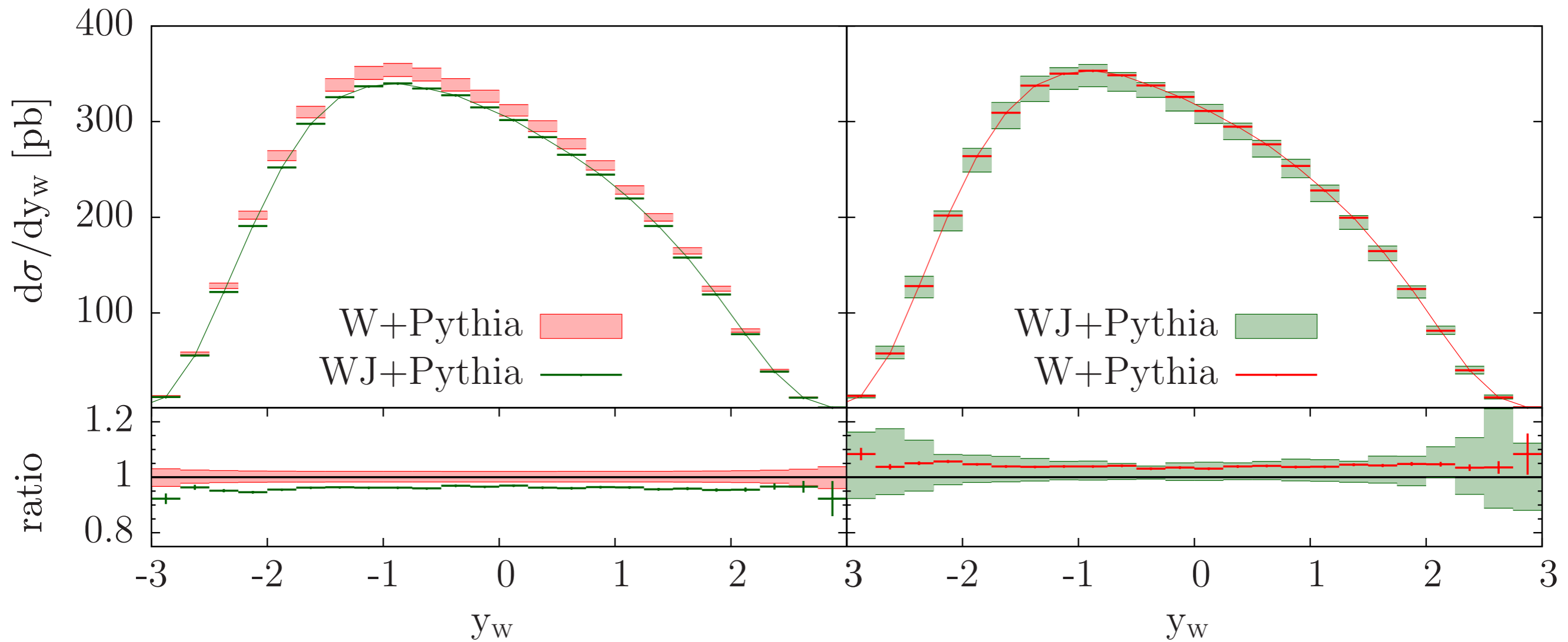
- Left NLO H PWG uncertainty w. MiNLO HJ inset as green +'s
- Right MiNLO HJ uncertainty w. NLO H inset in red +'s
- Both 7 pt independent μ_R , μ_F scale variation bands

Case study: NLO H vs MiNLO Mk2 HJ



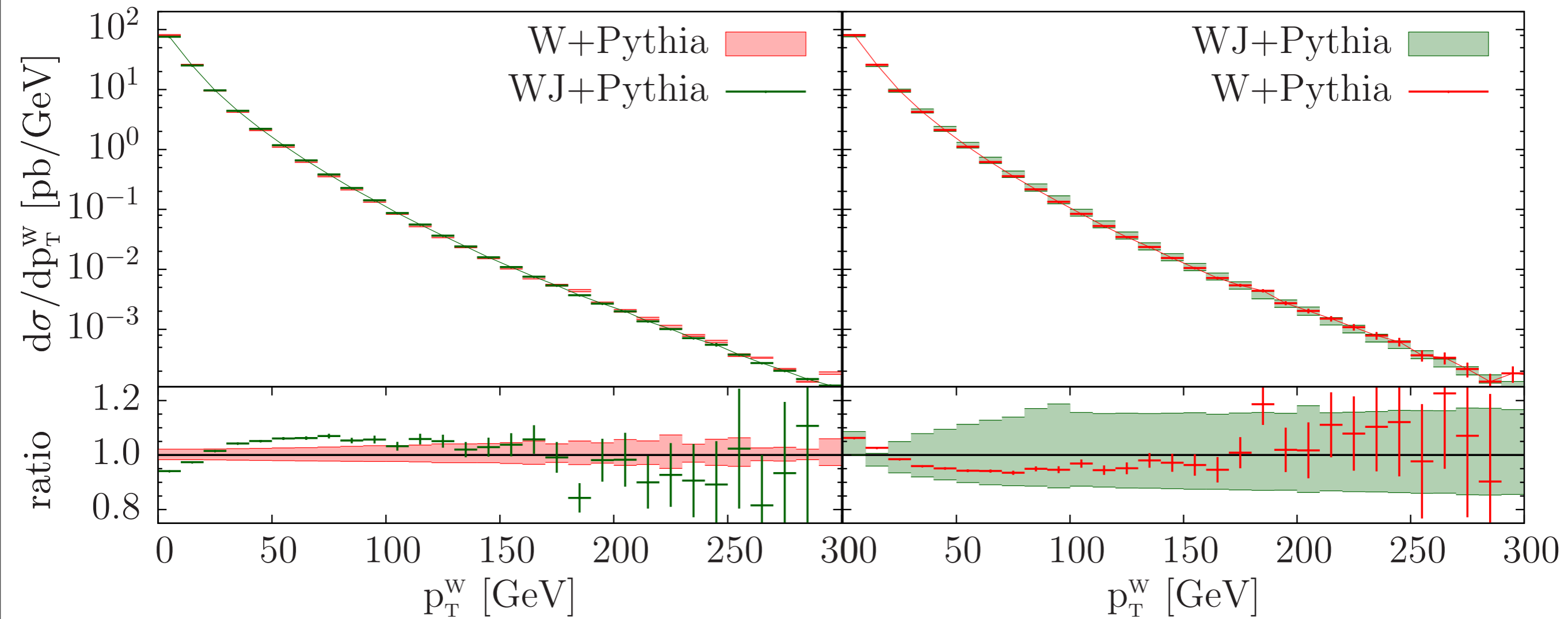
- Again central values in good agreement
- MiNLO HJ is NLO for H+jet and H inclusive
- Powheg H only NLO for H inclusive
- Hence MiNLO HJ band is expectedly smaller at high p_T

Case study: NLO W vs MiNLO Mk2 WJ



- W⁻ @ Tevatron with 3-pt. symmetric scale unc. bands
- MiNLO WJ low w.r.t Powheg W by 4-5%, band larger by ~2% in central region and gets wider toward large y_W
- Powheg W uncertainty is pretty small < 3% ...
- NO shape differences

Case study: NLO W vs MiNLO Mk2 WJ

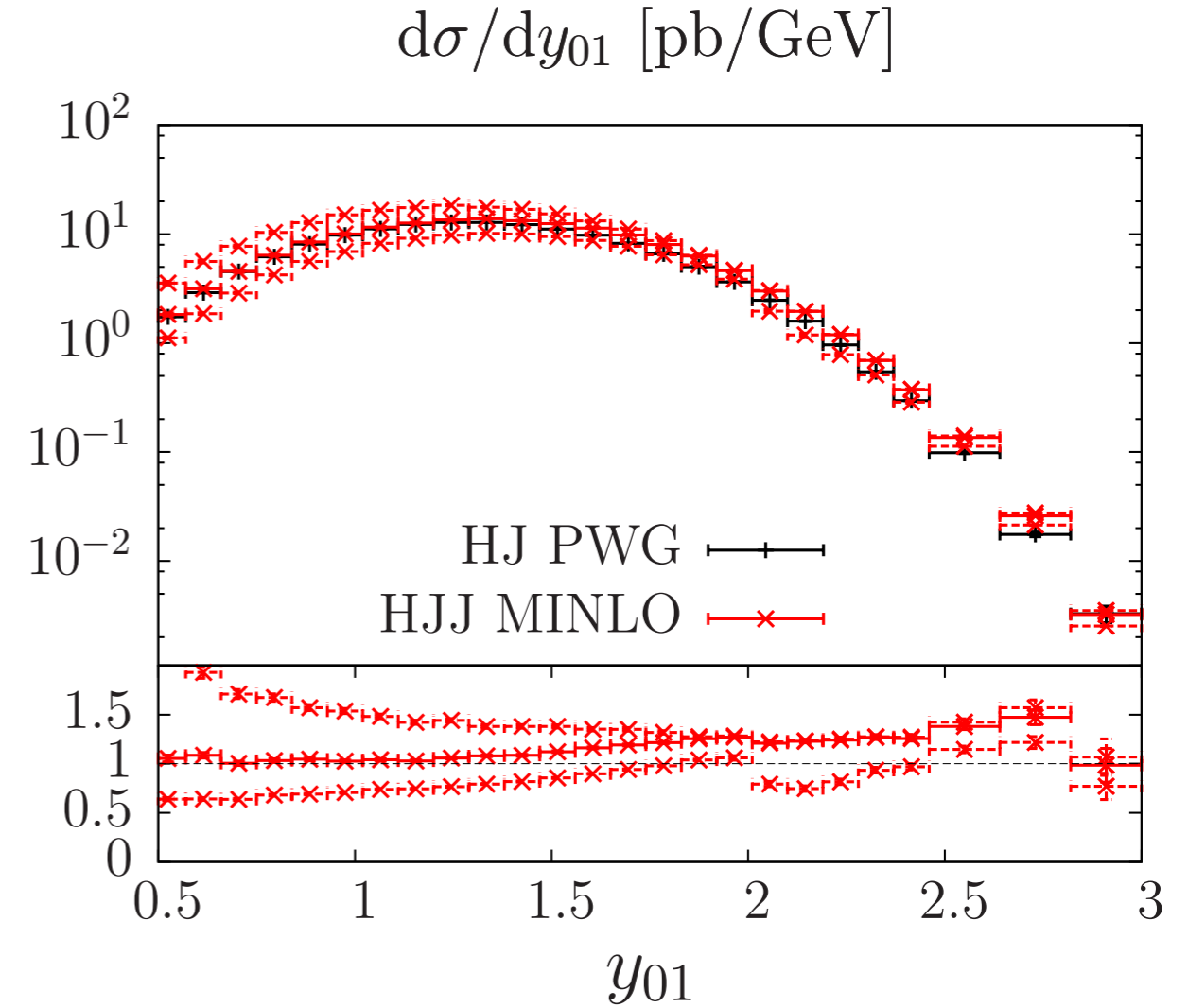
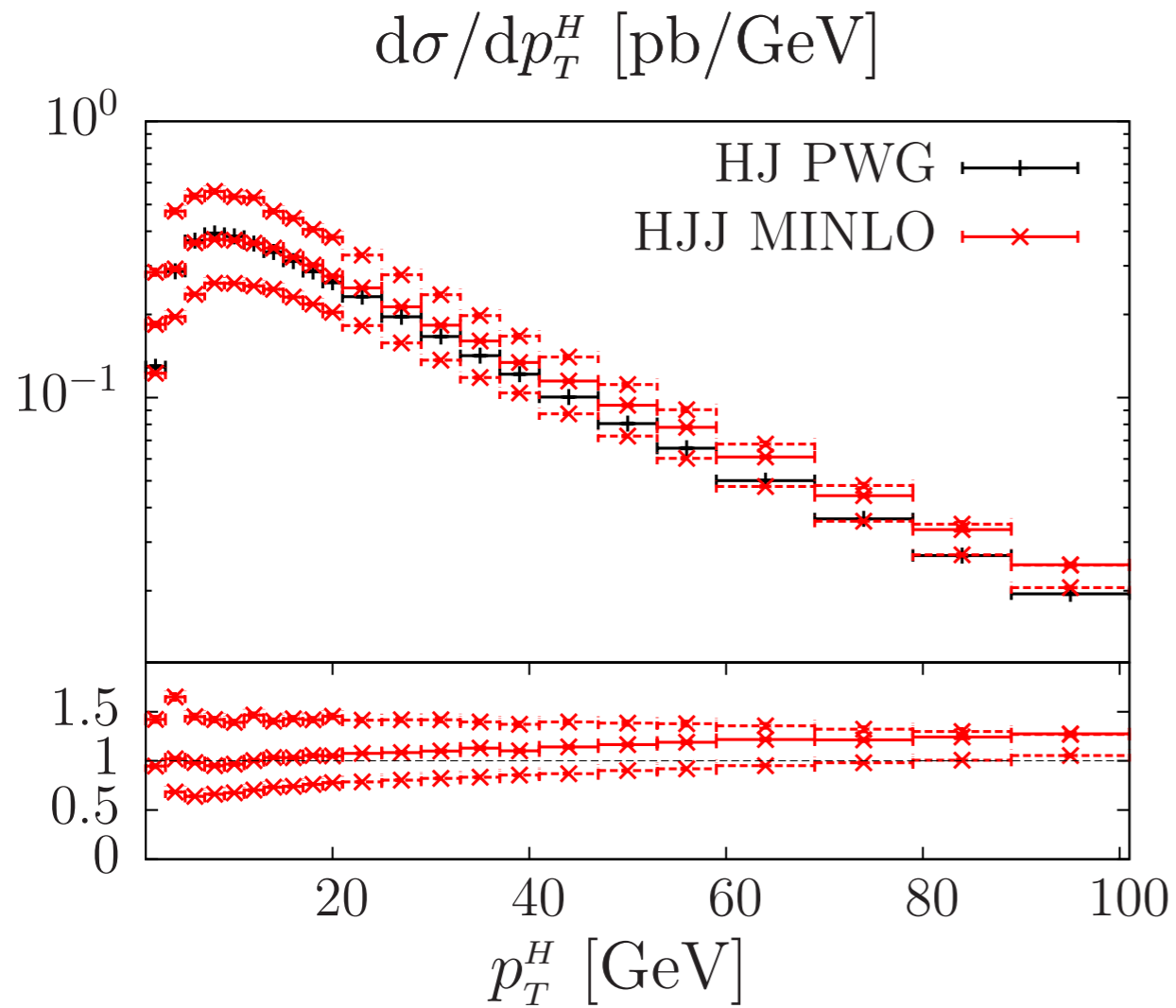


- As with Higgs case differences in Sudakov f.f.s manifest in the low p_T part of the spectrum
- Powheg W error band is highly spurious for $p_T > \sim m_W$ [L0]
- MiNLO WJ band looks pretty reasonable [NLO]

Conclusions on MiNLO Mk2

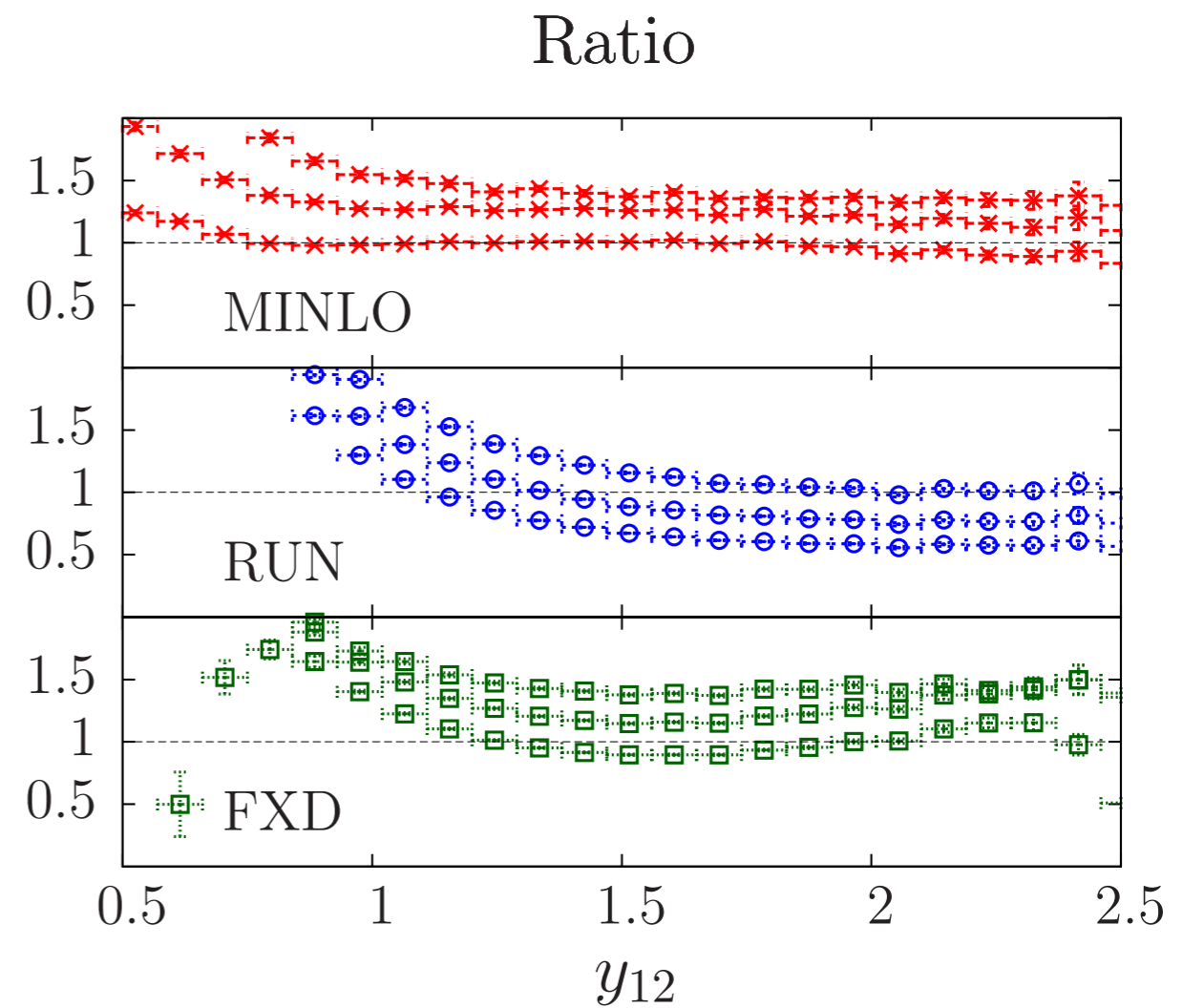
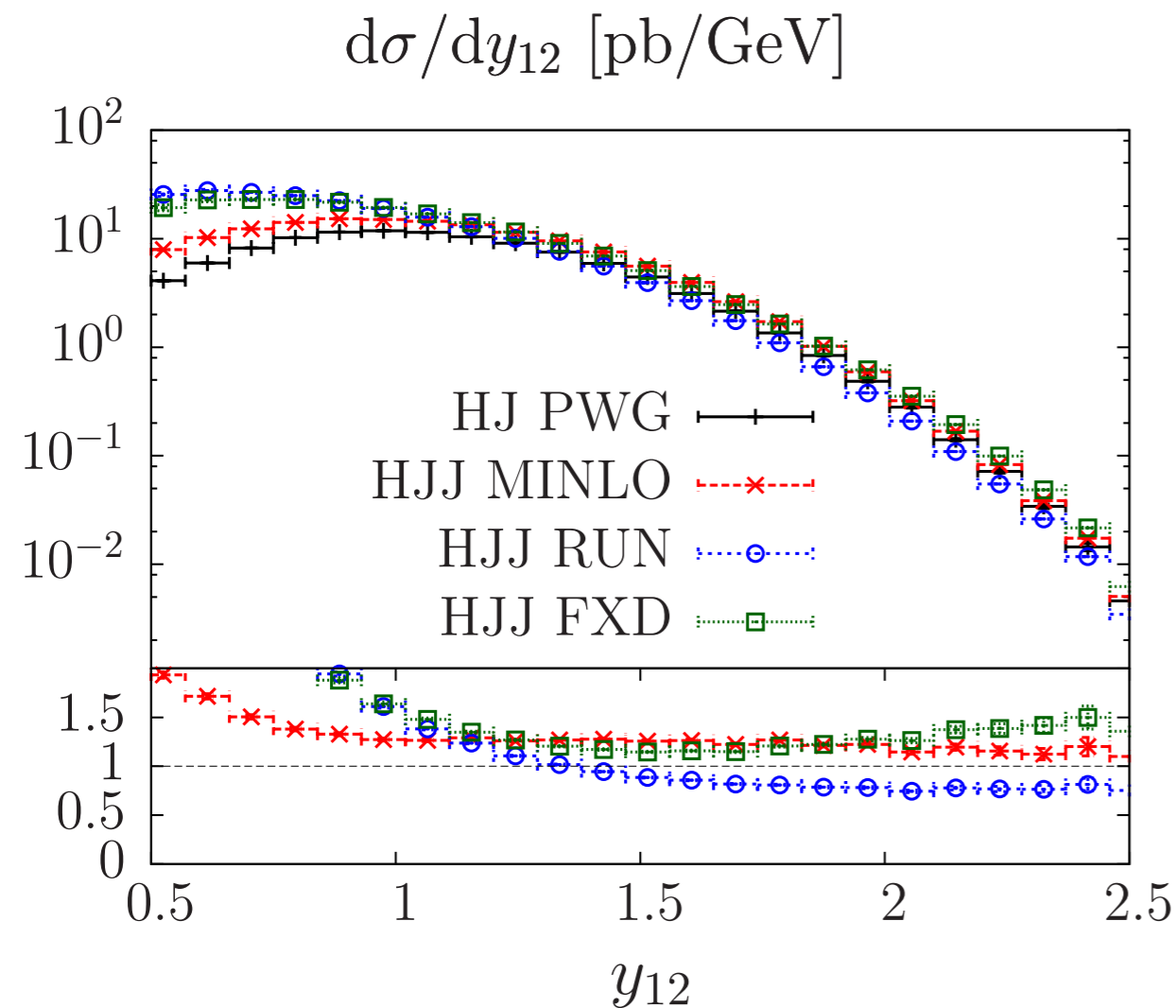
- MiNLO for NLO boson+jet, alone, refined to return, simultaneously, NLO predictions for incl. boson prodⁿ
- Log accuracy is the same as before
- ‘CKKW at NLO’ without actual merging
- Trivial rwgt of events [NNLO ÷ MiNLO] gives NNLO+PS
- Work needed to clarify relⁿ of scale variation in conventional NLO inclusive w.r.t MiNLO for inclusive
- Applicatⁿ to other white-stuff+jet goes the same way
- Applicatⁿ to higher jet multiplicities requires we learn more resummation technology [0(yearS)]

Case study: NLO Higgs + 2 jets



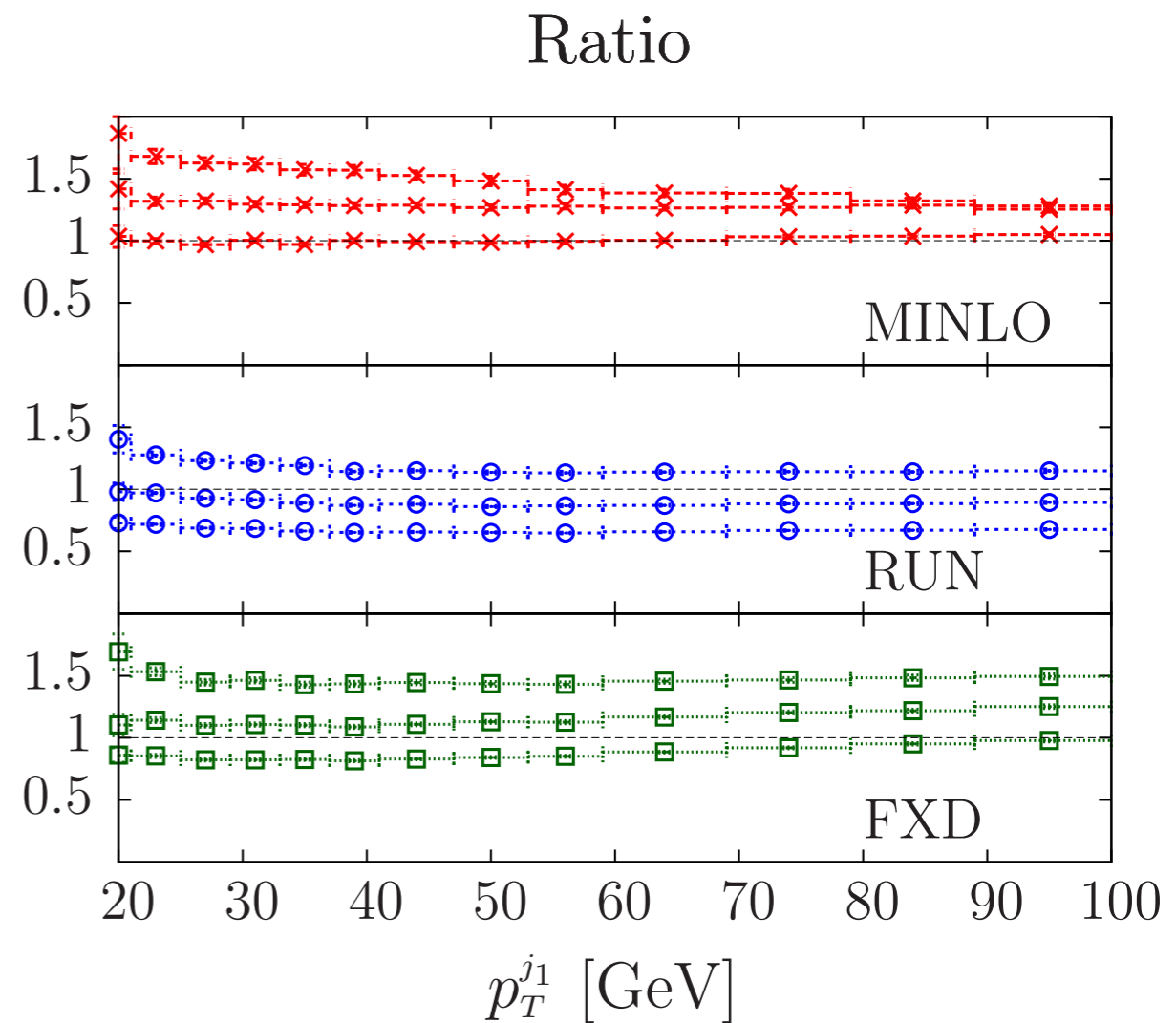
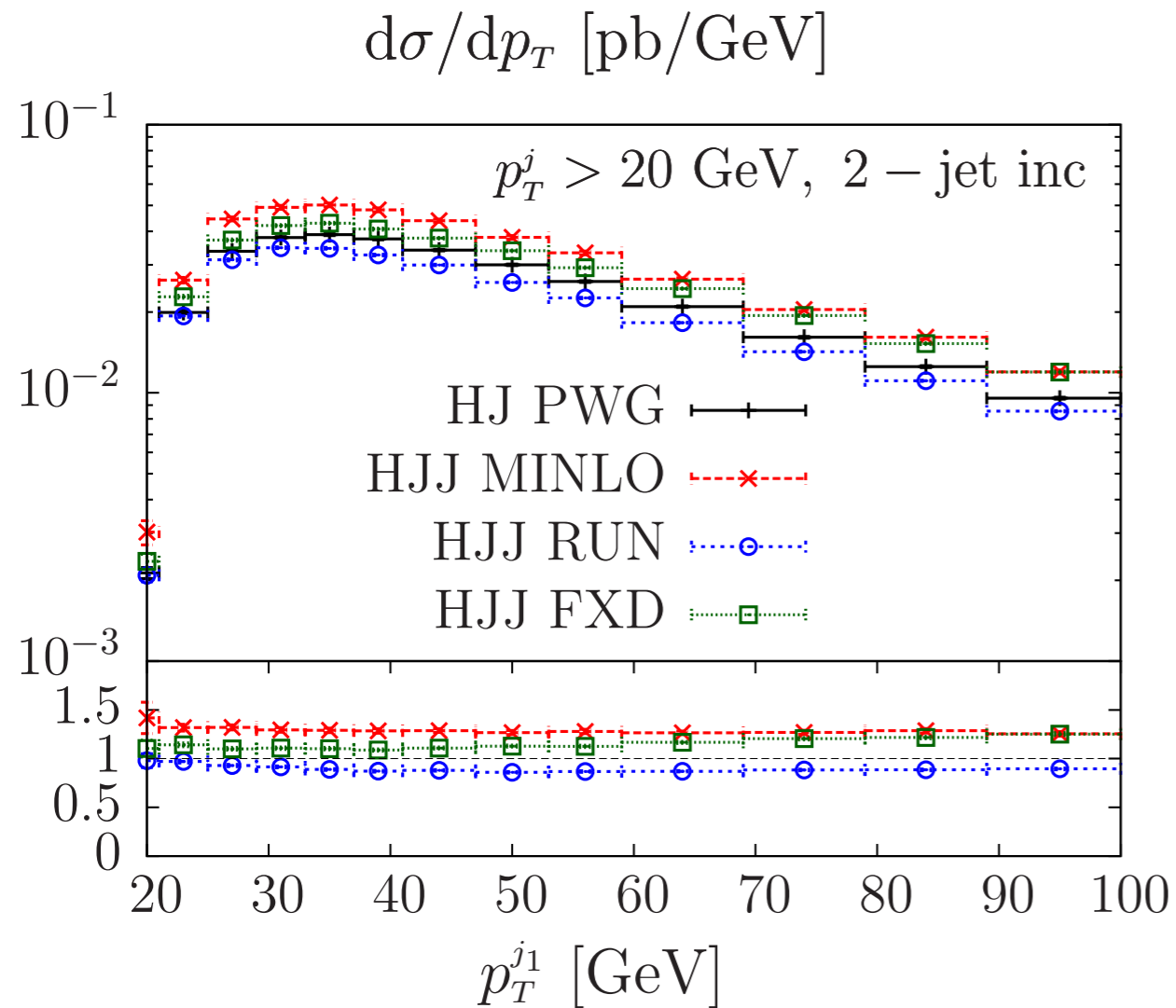
- HJ PWG MiNLO Higgs + 1 jet feeding Powheg+Pythia
- HJJ MINLO MiNLO H+2 jets
- As before conventional NLO returns nonsense towards low p_T
- HJJ MINLO follows MiNLO H+1 jet [w.shower] down to $p_T = 0$

Case study: NLO Higgs + 2 jets



- HJ PWG MiNLO Higgs+1 jet feeding Powheg+Pythia [ref line]
- HJJ MINLO MiNLO H+2 jets
- HJJ RUN NLO H+2 jets with $\mu_R = \mu_F = H_T$
- HJJ FXD NLO H+2 jets with $\mu_R = \mu_F = M_H$

Case study: NLO Higgs + 2 jets



- Conventional NLO with $\mu_R = \mu_F = H_T$ outside MiNLO envelope
- Conventional NLO with $\mu_R = \mu_F = H_T/2$ in better agreement
- $H_T/2$ is a preferred choice nowadays in multi-jet NLO, apparently giving increased scale stability
- [So far] MiNLO HJJ & ZJJ results agree OK with $H_T/2$

MINLO Mk2 3-slide scant explanation

- LO $d\sigma / dy dp_T$ x-sec of Boson+jet is $O(\alpha_s)$
- It has a $p_T \rightarrow 0$ finite bit which naturally integrates to $O(\alpha_s)$ over p_T
- It also has a $p_T \rightarrow 0$ singular bit

$$\frac{d\sigma_S}{dy dp_T^2} = \frac{\hat{\sigma}_0}{p_T^2} \left[\alpha_S A \ln \frac{m^2}{p_T^2} f_i f_j + \alpha_S B f_i f_j + p_T^2 \frac{d}{dp_T^2} (f_i f_j) \right]$$

- Now do MINLO [“@LO”] i.e. multiply by a Sudakov and take scale in PDFs and α_s to be p_T

$$\Delta(p_T) = \exp \left[- \int_{p_T^2}^{m^2} \frac{dq^2}{q^2} \alpha_S(p_T) \left[A \ln \frac{m^2}{q^2} + B \right] \right]$$

MINLO Mk2 3-slide scant explanation

- Using $\frac{d\Delta}{dp_T^2} = \Delta \frac{d \ln \Delta}{dp_T^2}$ we can write exactly

$$\Delta \frac{d\sigma_S}{dy dp_T^2} = \hat{\sigma}_0 \frac{d\Delta}{dp_T^2} f_i f_j + \hat{\sigma}_0 \Delta \frac{d}{dp_T^2} (f_i f_j) = \hat{\sigma}_0 \frac{d}{dp_T^2} (\Delta f_i f_j)$$

- Integrating from $p_T = 0$ [$\Delta = 0$] to $p_T = m$ [$\Delta = 1$] gives the LO Boson rapidity distⁿ:

$$\frac{d\sigma_S}{dy} = \hat{\sigma}_0 f_i f_j$$

- Imagine we forgot the B-term in the Sudakov, we would not be able to make the exact differential and get

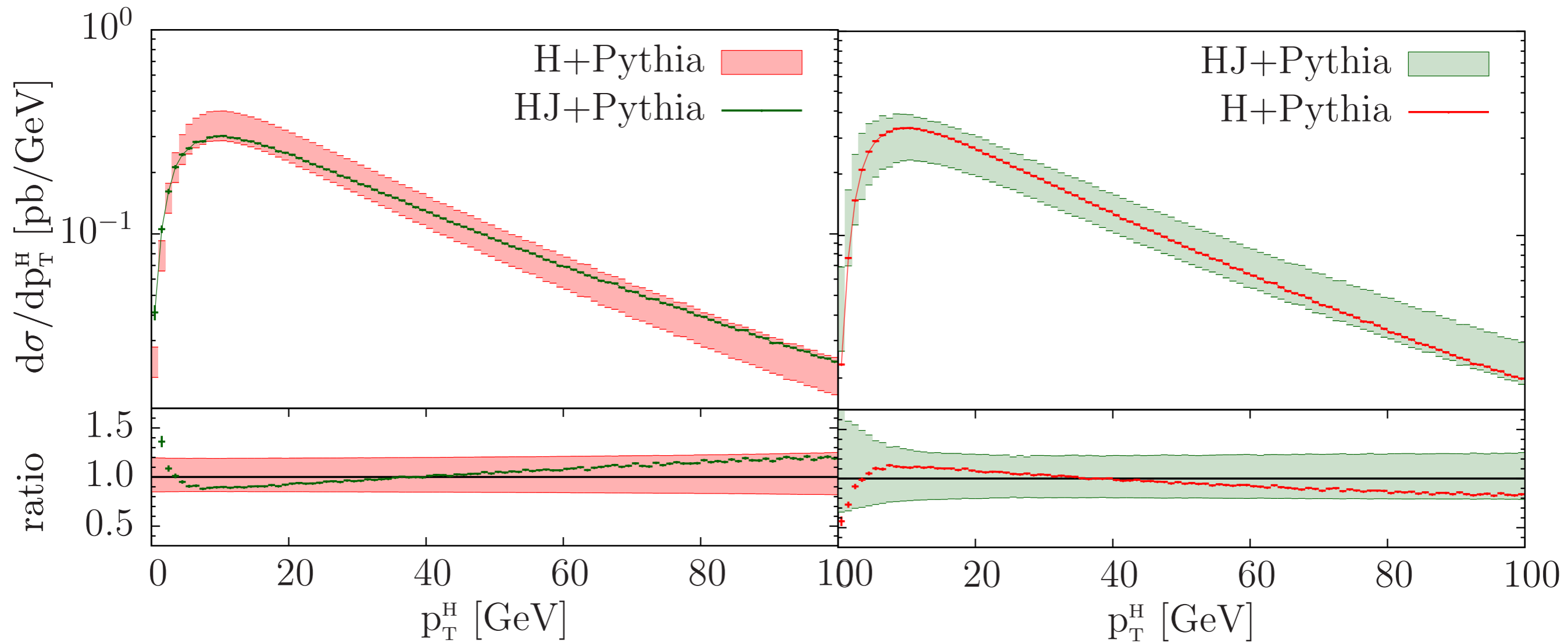
$$\frac{d\sigma_S}{dy} \simeq \hat{\sigma}_0 f_i f_j + \int dp_T^2 \frac{\hat{\sigma}_0}{p_T^2} \Delta \alpha_S B f_i f_j$$

- $1/p_T^2$ factor promotes integral from $0(\alpha_S)$ to $0(\sqrt{\alpha_S})$

MiNLO Mk2 3-slide scant explanation

- By not having the B term in the Sudakov you get $L0+O(\sqrt{\alpha_s})$: this is not L0 accuracy, $L0 + O(\alpha_s)$ is.
- Message : for L0 B+jet to give L0 B-incl. Sudakov exponent must have same $f_i f_j$ singularities as what it multiplies, or you get leftover Sudakov junk in B-incl.
- Same thing holds at NLO level : mandates original MiNLO formulation be refined to include the NLO correction to the B term in the Sudakov.
- In MiNLO B+jet, on p_T integration, the Sudakov logs and PDF evol^n terms disappear leaving behind NLO B

Case study: NLO H vs MiNLO Mk2 HJ



- MiNLO HJ band widens at p_T ; approaching strong coupling
- H band not realistic as $p_T \rightarrow 0$; reflects tot. x-sec unc.
- Difference in shape as $p_T \rightarrow 0$ due to different Sudakovs: extra NNLL terms in MiNLO HJ, finite ones in Powheg H