Properties of Inverse Compton Radiation

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Outline

What to report

- Compton inverse scattering: model simplifications $=$ Thomson radiation
- Practical parameters for acceleratists and users
- “Quasimonochromatizm” of Compton sources ?

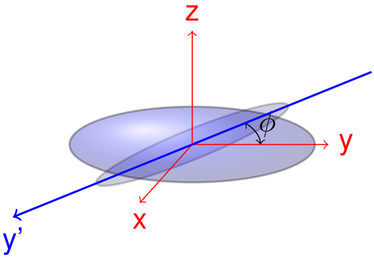
Hierarchy of Practical Parameters for Users

depend on user's specific requirements

- Spectral density – number of x– or gamma–ray photons within 0.1 % energy interval (sometimes within 1 keV or 10 keV) per second
- Spectral-angular density: same as above per unit of solid angle (usually 1 μsterad)
- Spectral-angular brightness: same as above, from unit of emitting surface (usually from 1 mm^2)

YIELD – Major figure of merit

Pulse and Bunch – 3D Gaussian Distribution



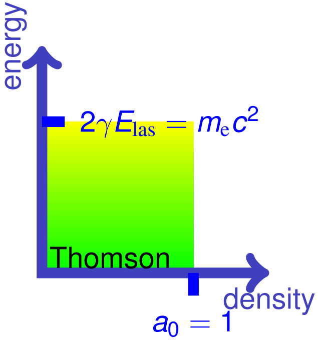
Yield per crossing

$$Y = \frac{N_{\mathrm{las}} N_e \sigma_{\mathrm{C}}}{2\pi\sqrt{{\sigma'}_z^2 + \sigma_z^2}} \times \frac{1}{\sqrt{\sigma_x^2 + {\sigma'}_x^2 + \left(\sigma_y^2 + {\sigma'}_y^2\right)\tan^2\phi/2}}$$

$\sigma'_{x,y,z}$, $\sigma_{x,y,z}$ are rms dimensions of the laser pulse and the electron bunch

Model: Linear Thomson Scattering (no recoil)

$\sigma_C = \frac{8}{3} r_0^2$ **Thomson cross section**



Energy (frequency) of photons

Wavelength in the electron rest frame equal to the electron Compton wavelength

$E_e \ll 125$ GeV for $E_{\text{las}} = 1$ eV

Density of photons (field strength)

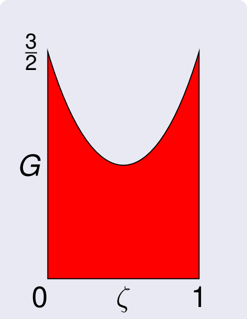
$$a_0^2 = \frac{3}{4\pi\alpha} N_T \approx 32.7 \times N_T \ .$$

N_T # photons (Thomson) scattered per laser wavelength:

$N_T \ll 3$ quanta scattered off per 100 laser wavelengths

Linear Thomson, Small–Angle Approximation

No recoil, small angle of scattered photons from the trajectory



Energy vs. angle

$$\gamma_x = \gamma_{\mathrm{las}} \frac{1+\beta\cos\phi}{1-\beta\cos\psi} \approx \gamma_{\mathrm{las}} \frac{2\gamma^2(1+\cos\phi)}{1+\gamma^2\psi^2}$$

Spectral density (normalized)

$$G(\zeta) = 3[1/2 - \zeta(1-\zeta)]\,; \quad 0 \le \zeta \le 1$$

$$\zeta \equiv \gamma_x \Big/ \gamma_x^{(\mathrm{max})}\,;\ \gamma_x^{(\mathrm{max})} = 2(1+\cos\phi)\gamma^2\gamma_{\mathrm{las}}.$$

Head-on Collision, Large Beam Size

$\epsilon_{x,z}$ normalized emittance, $s_{x,z}$ angular spreads

- Spectral density
  $\mathcal{F} \times (\sigma_x \sigma_z)^{-1} = \mathcal{F}\gamma\,(\epsilon_x \beta_x \epsilon_z \beta_z)^{-1/2} \approx 1.5 \times 10^{-3} \times Y$
- Spectral–angular density
  $\mathcal{F} \times (\sigma_x \sigma_z s_x s_z)^{-1} = \mathcal{F}\gamma^2\,(\epsilon_x \epsilon_z)^{-1} \approx 1.5 \times 10^{-9} \times Y/(s_x s_z)$
- Spectral–angular brightness $\mathcal{F} \times (\sigma_x \sigma_z \sigma_x^a \sigma_z^a \sigma'_x \sigma'_z)^{-1} =$
  $\mathcal{F}\gamma^2\,(\epsilon_x \epsilon_z \sigma'_x \sigma'_z)^{-1} \approx 1.5 \times 10^{-9} \times Y/(s_x s_z \sigma'_x \sigma'_z[\mathrm{mm}^2])$

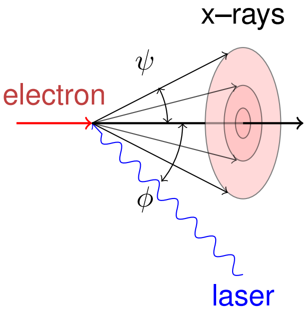
Non-head-on: $\sigma'_x \to \sigma_x$

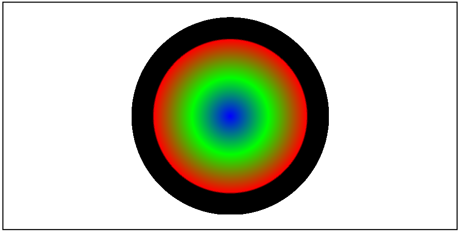
Monochromaticity of Compton Sources

Numerous papers: *. . . quasi-mono-energetic radiation in the X-ray and gamma ray region of the electromagnetic spectrum . . .*

To what limits collimation narrows the spectrum ?

Compton radiation quasi monochromatic:
monochromatization obtained by collimation

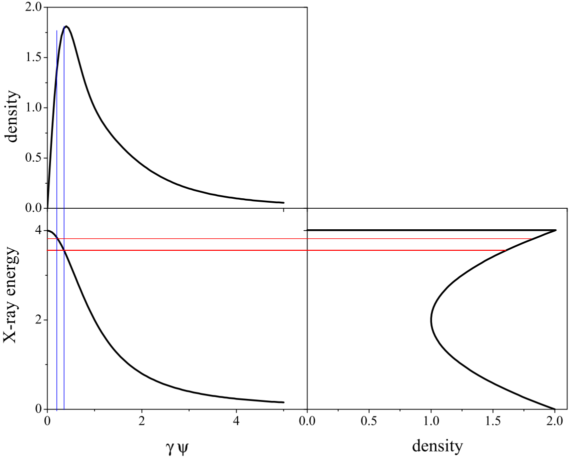




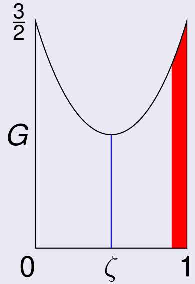
X–ray photon with a definite energy scattered off at the definite angle from electron's trajectory

Spectral–Angular Density Function

Ideal: zero emittances and spread



Spectrum Passed Collimator



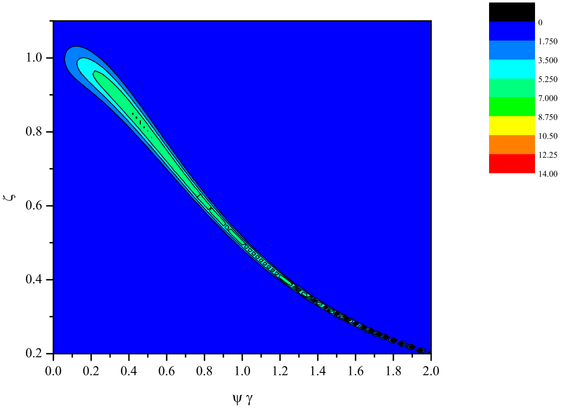
Collimated Ideal Spectrum

$$G(\zeta) = 3[1/2 - \zeta(1-\zeta)]\times \left[\mathrm{H}\left(\zeta - \frac{1}{1+\chi_f^2}\right) - \mathrm{H}\left(\zeta - \frac{1}{1+\chi_i^2}\right)\right]$$

$$\chi_f \geq \chi_i\,; \qquad \chi \equiv \gamma\psi$$

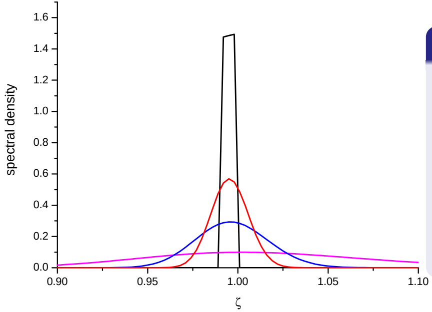
the collimation angle scales as $1/\gamma$
e.g. 0.1 % attained at $\psi \approx 31.6/\gamma$ mrad

Spectral–Angular Density Function
Zero emittances, finite energy spread



Spectral angular density smeared off vertically

Spectrum Passed Collimator



Collimated Spectrum

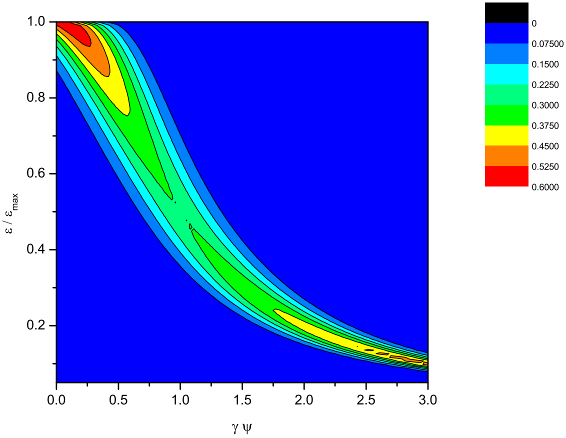
$$\mathcal{G}_\gamma(\zeta, x_\mathrm{i}, x_\mathrm{f}) = \frac{3}{4}\left[1 - 2\zeta\left(1-\zeta\right)\right] \times \left(\mathrm{Erf}\left(\eta_\mathrm{i}\right) - \mathrm{Erf}\left(\eta_\mathrm{f}\right)\right) \ ,$$

where $\eta_{\mathrm{i,f}} = -\left(1/\zeta - 1 - x_{\mathrm{i,f}}^2\right)/2\sqrt{2}s_\gamma$, $\mathrm{Erf}(z)$ the error integral function.

For pin-hole collimation energy spread of gammas is the doubled spread of electrons

Spectral–Angular Density Function

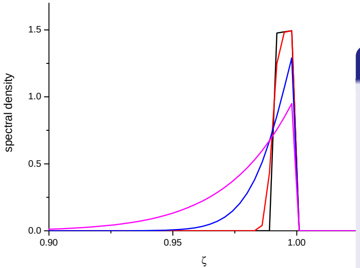
Zero spread, finite emittances



Spectral angular density smeared off horizontally

Spectrum Passed Collimator

1D case (flat beam)



Collimated Spectrum

$$\mathcal{G}_\psi(\zeta, x_\mathrm{i}, x_\mathrm{f}) = \frac{3\left[\zeta^2 + \left(1 - \zeta^2\right)^2\right]}{2\sqrt{2\pi}s_\psi} \times$$

$$\left(\mathrm{erf}\left(\eta_\mathrm{f}^-\right) + \mathrm{erf}\left(\eta_\mathrm{f}^+\right) - \mathrm{erf}\left(\eta_\mathrm{i}^-\right) - \mathrm{erf}\left(\eta_\mathrm{i}^+\right)\right) \quad ,$$

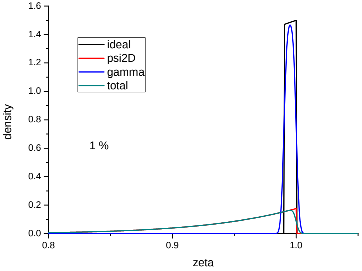
with $\eta_\mathrm{i,f}^\pm = \left(x_\mathrm{i,f} \pm \sqrt{1/\zeta - 1}\right) / \sqrt{2}s_\psi$.

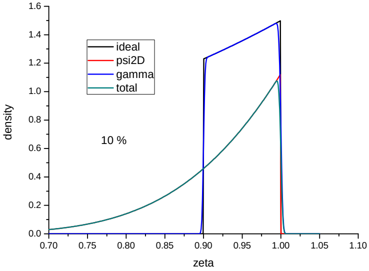
X-ray energy spectra for angular spread 0.01, 0.1, 0.5, 1.0 into collimating range 0...0.1.

2D beam (round) produces wider spectrum

Integral Spectra Passed Collimator

Energy spread 0.001, angular spread $s\gamma = 0.2$



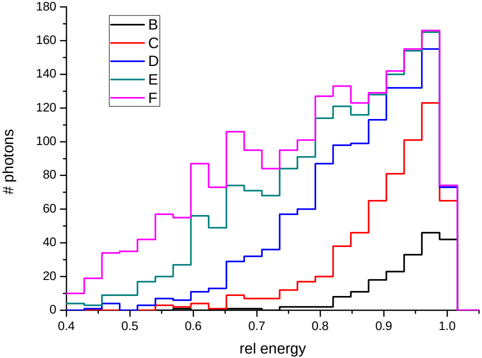


fwmh $\approx 5.6\,\%$

Angular spread dominant

fwmh $\approx 8.4\,\%$

Integral Spectra Passed Collimator – Simulation (similar to LUCX)



$\beta_x = 0.200\,\text{m}$
$\beta_z = 0.014\,\text{m}$
$\varepsilon_x = 2.00 \times 10^{-7}\,\text{m}$
$\varepsilon_z = 2.00 \times 10^{-7}\,\text{m}$
$E_{\text{las}} = 1.164\,\text{eV}$
$\phi = 0.34907$
$E_{\text{e}} = 40.0\,\text{MeV}$
$E_{\text{x}}^{(\text{max})} = 27670.3\,\text{eV}$

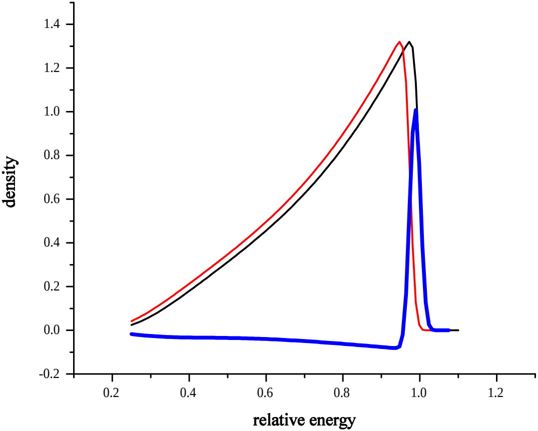
$\psi\gamma = 0.2, 0.4, 0.6, 0.8, 1.0$ (B to F histogram)

Summary

- Main endusers parameters of Compton sources were derived
- Angular spread of electron trajectories dominantly contributes to width of collimated spectrum
- Smaller emittance of electron beam better in any case
- Quasimonochromatizm doubtful, dependent on personal imagination ability

‘Electronic’ monochromatization

Basis – steep high-energy edge



‘Electronic’ monochromatization

spectral width $\geq 2\times$ beam energy spread

Outlook

- Electronically subtracted spectra (medicine ?)
- Increase in yield - by array of laser resonators preferable (Urakawa et al)