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Role of transversal size of bunches in soft X-ray Smith-Purcell radiation

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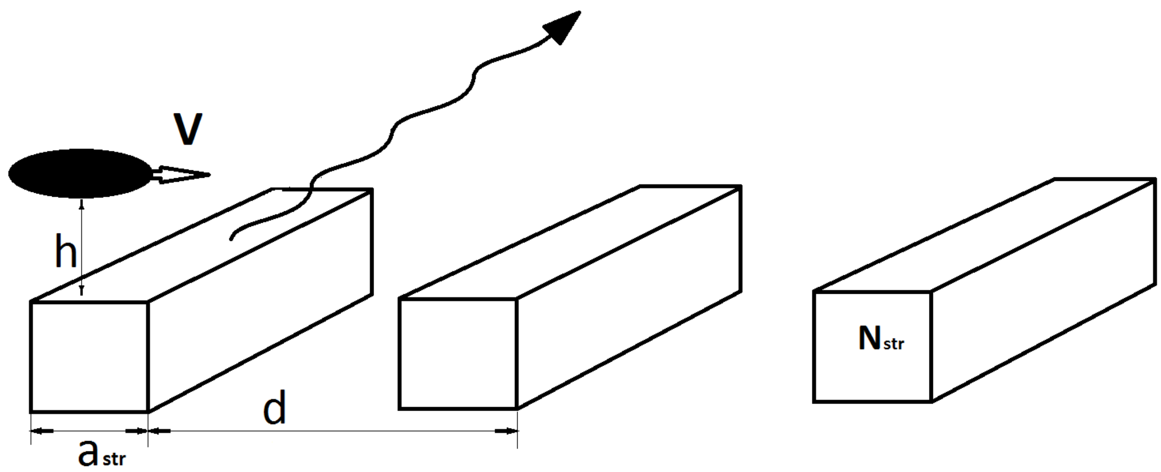
General of Smith-Purcell radiation

The theory of X-ray and UV radiation from the bunch!



Possibility of noninvasive submicron bunch diagnostics

A.A. Tishchenko, A.P. Potylitsyn, M.N. Strikhanov, Phys. Rev. E (2004)

A.A. Tishchenko, A.P. Potylitsyn, M.N. Strikhanov, Phys. Lett. A (2006)

Polarization current method

The polarization current density:

$$\mathbf{j}(\mathbf{r},\omega) = \frac{\omega}{4\pi i}\left(\varepsilon(\omega)-1\right)\sum_{n=1}^{N}\mathbf{E}_0^n(\mathbf{r},\omega)$$

The radiation field:

$$\mathbf{E}(\mathbf{r},\omega) = \frac{i\omega}{c^2}\frac{e^{ikr}}{r}\left[\mathbf{n},\left[\mathbf{n},\int d^3r'\,\mathbf{j}(\mathbf{r}',\omega)e^{-i\mathbf{k}\mathbf{r}'}\right]\right]$$

$$\Downarrow$$

$$\frac{d^2W_N(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{d^2W_1(\mathbf{n},\omega)}{d\Omega d\omega}F$$

A.P. Potylitsyn, M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko, *Diffraction Radiation from Relativistic Particles*, Springer, 2010.

Spectral- angular distribution

$$\frac{d^2W_N(\mathbf{n},\omega)}{d\Omega d\omega}=\frac{d^2W_1(\mathbf{n},\omega)}{d\Omega d\omega}F$$

$$\frac{d^2W_1(\mathbf{n},\omega)}{d\Omega d(\hbar\omega)}=\left(\frac{\varepsilon(\omega)-1}{v\varphi}\right)^2\frac{\alpha c^2}{(4\pi)^2}e^{-2\rho h}4\sin^2\left(\frac{a_{str}\varphi}{2}\right)\frac{\sin^2\left(\frac{N_{str}d\varphi}{2}\right)}{\sin^2\frac{d\varphi}{2}}\frac{\omega^2}{c^2}\frac{1-n_z^2+\frac{\mathbf{A}^2-(\mathbf{nA})^2}{\rho^2}}{\beta^{-2}\gamma^{-2}+\varepsilon n_y^2+\varepsilon n_z^2}$$

$$\boxed{F=\left\langle\left|\sum_{n=1}^{N}e^{-i\xi x_n}e^{-ik_y y_n}e^{-\rho z_n}\right|^2\right\rangle}$$

$$\varphi=\frac{\omega}{v}-k_x,\ \ \xi=\frac{\omega}{v},\ \ \rho^2=\left(\frac{\omega}{c\beta\gamma}\right)^2+k_y^2,\ \ \mathbf{A}=\frac{\omega}{c\beta\gamma^2}\mathbf{e}_x+k_y\mathbf{e}_y,\ \ \mathbf{k}=\sqrt{\varepsilon}\frac{\omega}{c}\mathbf{n}$$

Form-Factor

$$F = N + N(N-1)G_{coh}$$

$$\boxed{F = NG_{incoh} + N(N-1)G_{coh}}$$

cylindrical cross section of the bunch

$$G_{incoh} = 2\frac{I_1(2\rho r_0)}{2\rho r_0}$$

$$G_{coh} = \frac{4\sin^2\left(\frac{\omega l}{2v}\right)}{\left(\frac{\omega l}{2v}\right)^2}\frac{I_1^2\left(\frac{\omega r_0}{c\beta\gamma}\right)}{\left(\frac{\omega r_0}{c\beta\gamma}\right)^2}$$

rectangular cross section of the bunch

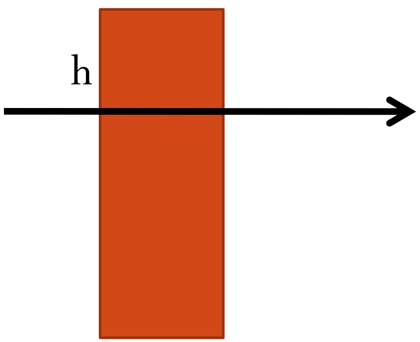
$$G_{incoh} = \frac{sh(a\rho)}{a\rho}$$

$$G_{coh} = \frac{\sin^2\frac{\omega l}{2v}}{\left(\frac{\omega l}{2v}\right)^2}\frac{\sin^2\left(\frac{bk_y}{2}\right)}{\left(\frac{bk_y}{2}\right)^2}\frac{sh^2\left(\frac{a\rho}{2}\right)}{\left(\frac{a\rho}{2}\right)^2}$$

$$\rho^2 = \left(\frac{\omega}{c\beta\gamma}\right)^2 + k_y^2$$

D.Yu. Sergeeva, A.A. Tishchenko, M.N. Strikhanov, Nucl. Instr. and Meth.B, 2013 (in print).

Transition radiation



$$G_{coh} = \frac{4}{\left(\gamma^{-2}\beta^{-2} + \varepsilon\sin^2\theta_m\right)^2} \frac{\sin^2\left(\frac{\omega l}{2v}\right)}{\left(\frac{\omega l}{2v}\right)^2} \times \left(4Q_1 \frac{J_1^2\left(k_z r_0\right)}{k_z^2 r_0^2} + Q_2 e^{-2\rho' h} \frac{I_1^2\left(\rho' r_0\right)}{\left(\rho' r_0\right)^2} - \right.$$

$$\left. -4e^{-h\rho'}\left(Q_1\cos\left(k_z h\right) + Q_3 \sin\left(k_z h\right)\right) \frac{I_1\left(\rho' r_0\right)}{\rho' r_0} \frac{J_1\left(k_z r_0\right)}{k_z r_0} \right)$$

$$G_{incoh} = \frac{1}{\left(\gamma^{-2}\beta^{-2} + \varepsilon\sin^2\theta_m\right)^2} \left(4Q_1 + 2e^{-2\rho' h} Q_2 \frac{I_1\left(2\rho' r_0\right)}{2\rho' r_0} - \right.$$

$$-4e^{-h\rho'}\left(\frac{I_1\left(Z_+\right)}{Z_+} + \frac{I_1\left(Z_-\right)}{Z_-}\right)\left[Q_1\cos\left(k_z h\right) - Q_3\sin\left(k_z h\right)\right] -$$

$$\left. -4e^{-h\rho'} i \left(\frac{I_1\left(Z_+\right)}{Z_+} - \frac{I_1\left(Z_-\right)}{Z_-}\right)\left[Q_3\cos\left(k_z h\right) + Q_1\sin\left(k_z h\right)\right] \right)$$

$$\rho' = \frac{\omega}{c\beta\gamma}, \ \mathbf{k} = \sqrt{\varepsilon}\,\frac{\omega}{c}\mathbf{n}, \ Z_\pm = r_0\left(\rho' \pm i k_z\right),$$

$$Q_1 = \frac{\omega^2}{c^2}\sin^2\theta_m\left(\gamma^{-2}\beta^{-1} - \sqrt{\varepsilon}\cos\theta_m\right)^2,$$

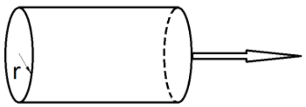
$$Q_2 = \frac{\omega^2}{c^2}\left(-\gamma^{-2}\sin^2\theta_m + \gamma^{-2}\beta^{-2} + \gamma^{-2}\varepsilon\sin^4\theta_m + \varepsilon\sin^2\theta_m\cos^2\theta_m\right)$$

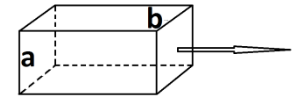
$$Q_3 = \frac{\omega^2}{c^2}\left(-\sqrt{\varepsilon}\gamma^{-1}\beta^{-1}\sin\theta_m\cos^2\theta_m + \gamma^{-3}\beta^{-1}\sqrt{\varepsilon}\sin^3\theta_m + \gamma^{-1}\varepsilon\sin^3\theta_m\cos\theta_m - \gamma^{-3}\beta^{-2}\sin\theta_m\cos\theta_m\right)$$

$h \to \infty$

G.M. Garibian and Y. Shi
X-ray Transition Radiation, 1983

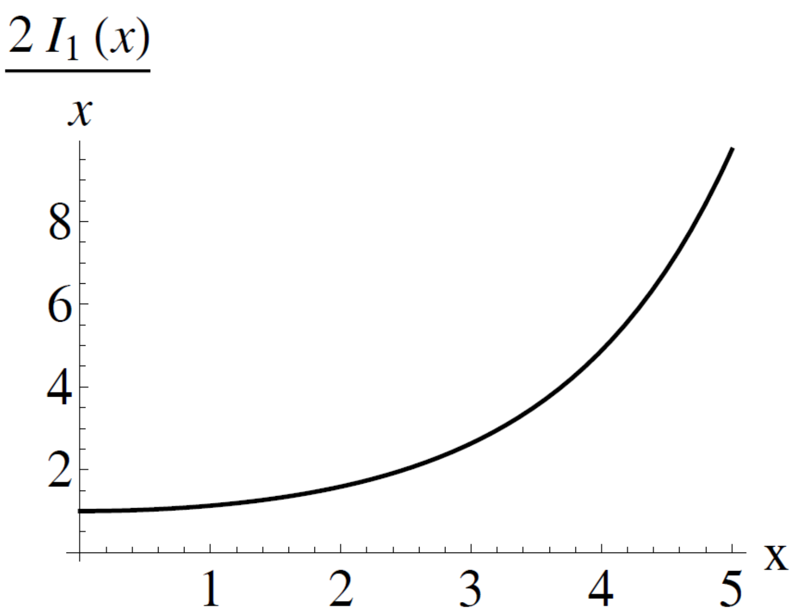
Diffraction radiation: incoherent form-factor

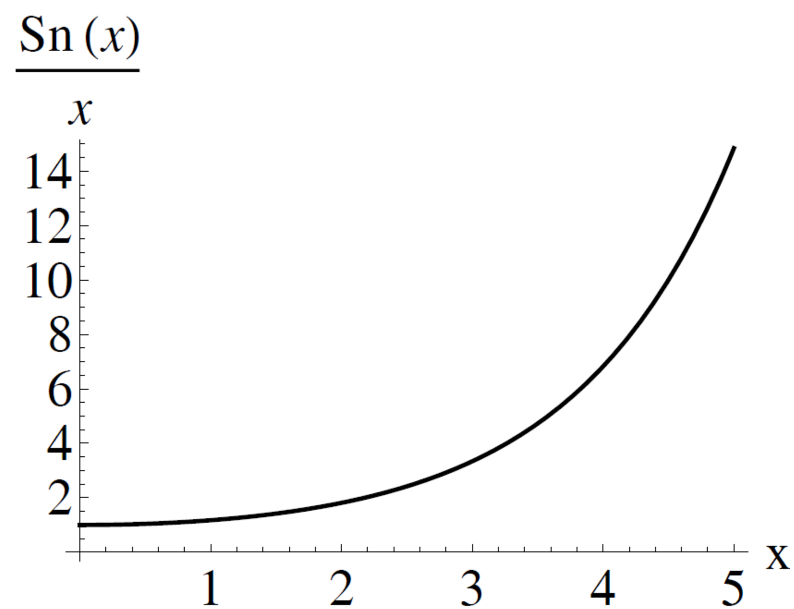




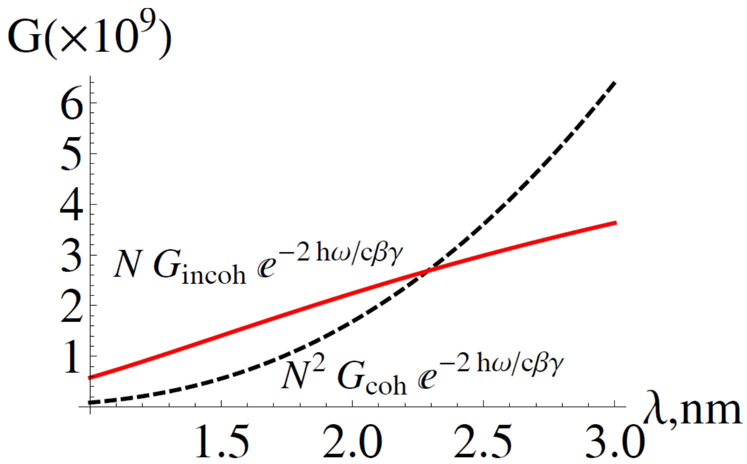
$$\frac{dW_{incoh}}{d\omega d\Omega} \propto N \frac{2I_1\left(\frac{2\omega}{c\beta\gamma} r_0\right)}{\frac{2\omega}{c\beta\gamma} r_0} e^{-\frac{2\omega h}{c\beta\gamma}}$$

$$\frac{dW_{incoh}}{d\omega d\Omega} \propto N \frac{Sh\left(\frac{2\omega}{c\beta\gamma} a\right)}{\frac{\omega}{c\beta\gamma} a} e^{-\frac{2\omega h}{c\beta\gamma}}$$



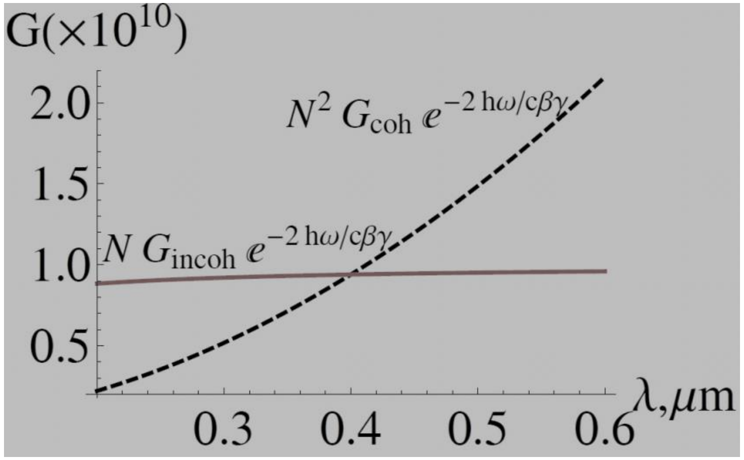


Role coherent and incoherent form-factor



(Fig. 1) $l = 50\,\mu m,\ r_0 = 5\,\mu m, N = 10^{10},\ h = 10\,\mu m$

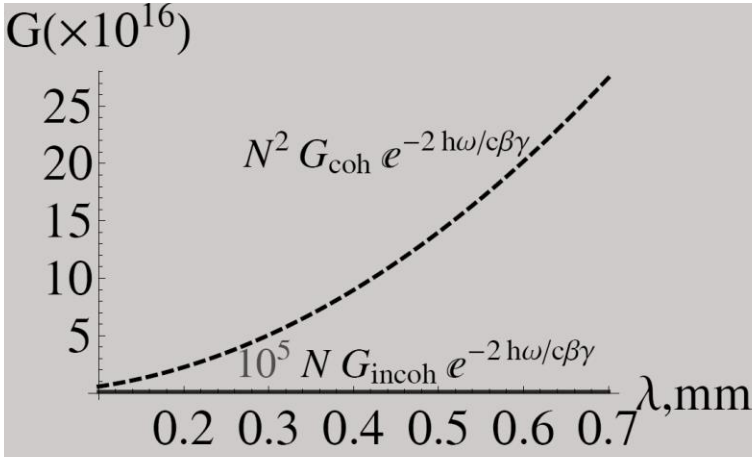
$\gamma = 4 \cdot 10^4$ *(energy of SLAC)*



(Fig. 2) $\gamma = 2500,\ l = 0.9\,cm,\ a = 7\,\mu m,$

$N = 10^{10},\ h = 5\,\mu m$ *(energy of KEK ATF)*

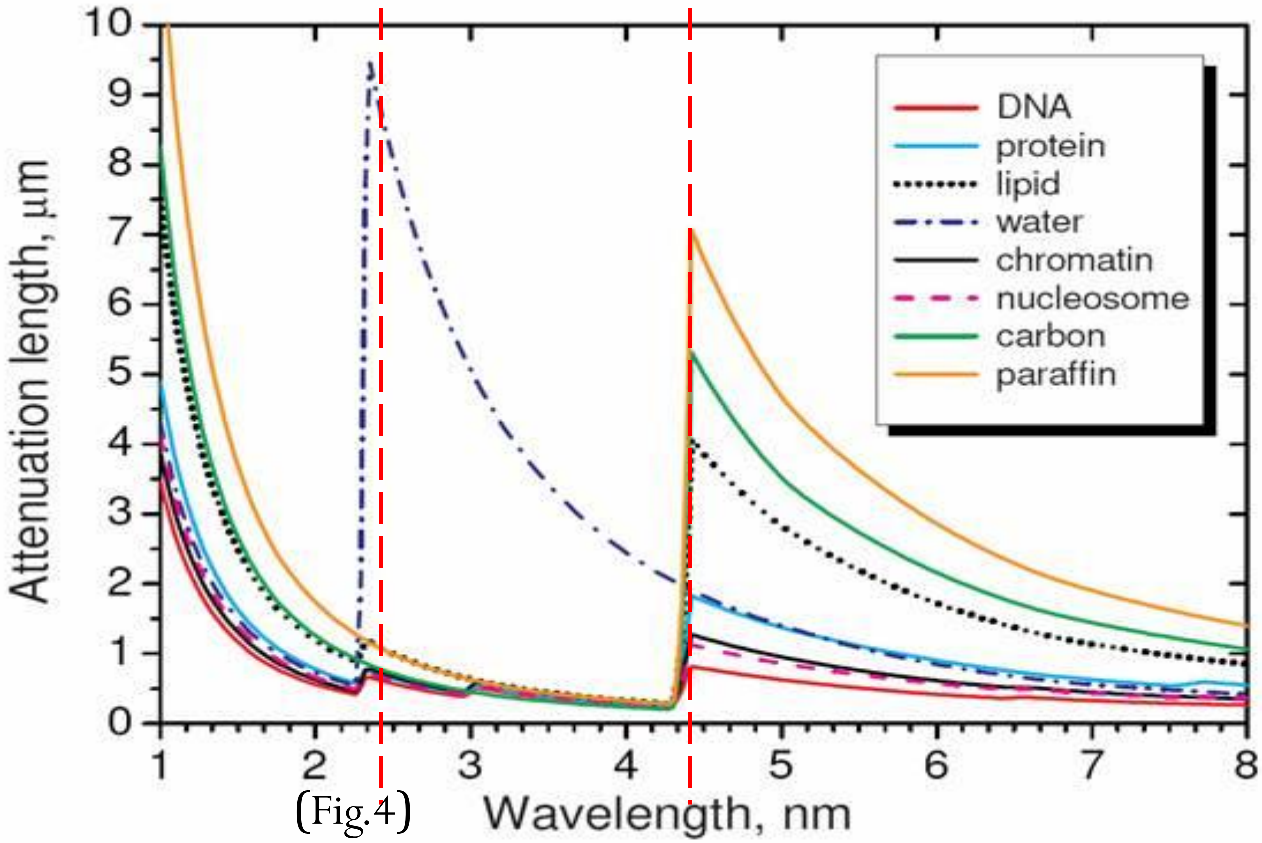
$\lambda^* \square\ l$



(Fig. 3) $\gamma = 15,\ l = 0.3\,cm,\ r_0 = 50\,\mu m,\ \ N = 10^{10}\,electrons,\ h = 55\,\mu m$

(energy of LUCX)

Penetration depth in water and in biological materials



(Fig.4)

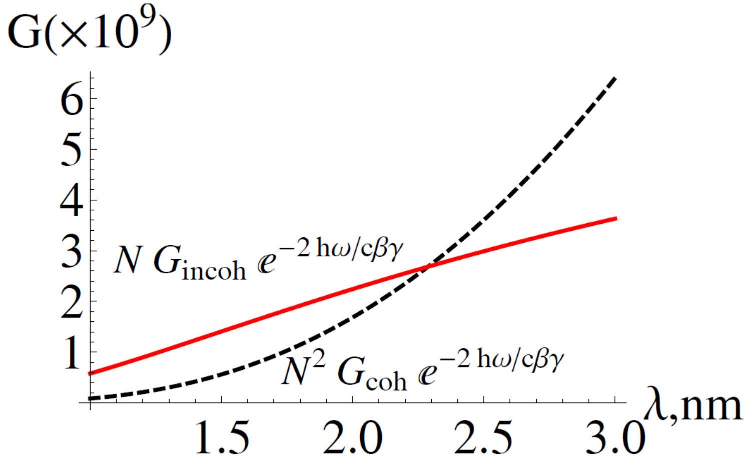
Soft X-ray
1 - 4 nm (1000 –300 eV)

EUV
4 – 60 nm (300 –20 eV)

Water Window
4.4 - 2.3 nm (0.28 -0.53 keV)

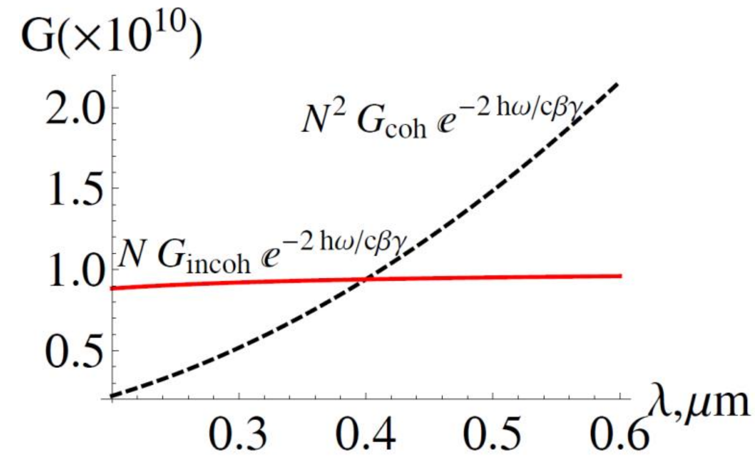
Carbon Window
5 - 4.4 nm (0.25 -0.28 keV)

Role coherent and incoherent form-factor



(Fig.1) $l = 50\,\mu m,\ r_0 = 5\,\mu m, N = 10^{10},\ h = 10\,\mu m$

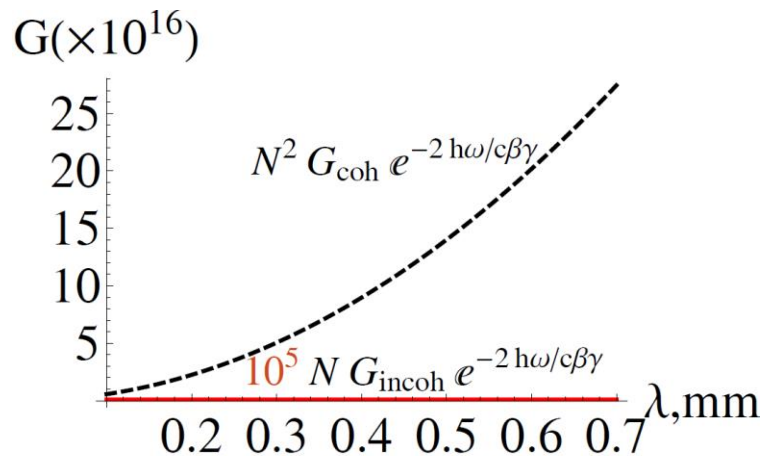
$\gamma = 4 \cdot 10^4$ *(energy of SLAC)*



(Fig.2) $\gamma = 2500,\ l = 0.9\,cm,\ a = 7\,\mu m,$

$N = 10^{10},\ h = 5\,\mu m$ *(energy of KEK ATF)*

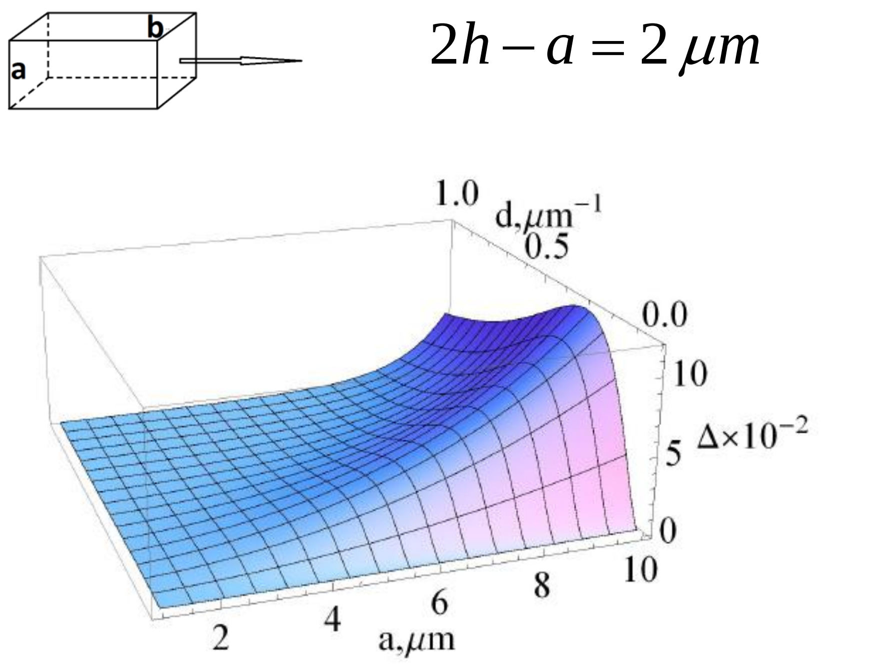
$\lambda^{*}\ \square\ l$



(Fig.3) $\gamma = 15,\ l = 0.3\,cm,\ r_0 = 50\,\mu m,\ \ N = 10^{10}\,electrons,\ h = 55\,\mu m$

(energy of LUCX)

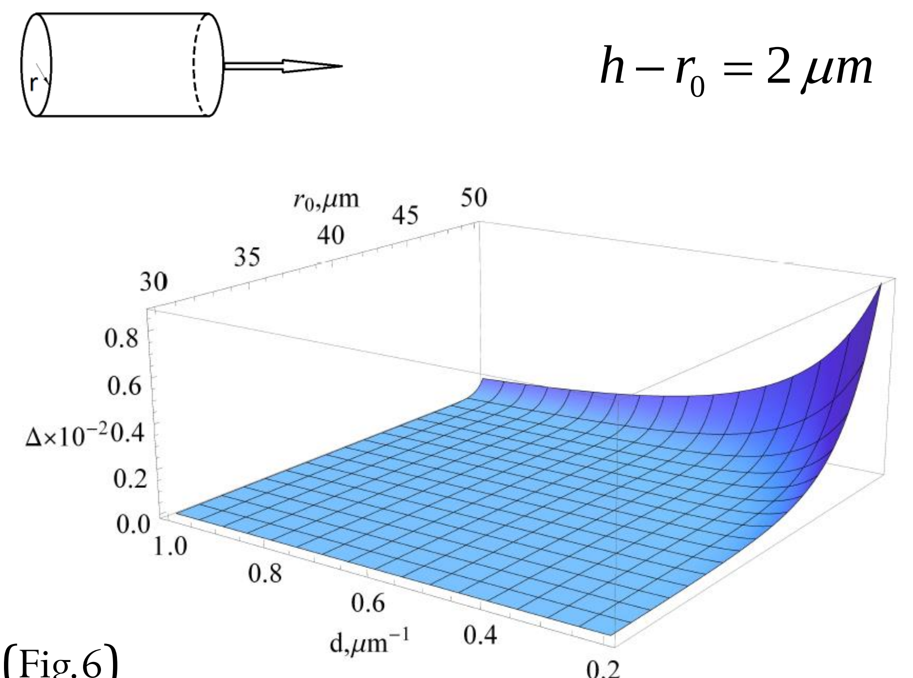
Incoherent form-factor



(Fig.5)

$$\Delta = G_{incoh} e^{-2dh} - e^{-2dh}$$

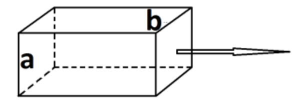
$$d = \frac{\omega}{c\beta\gamma}$$

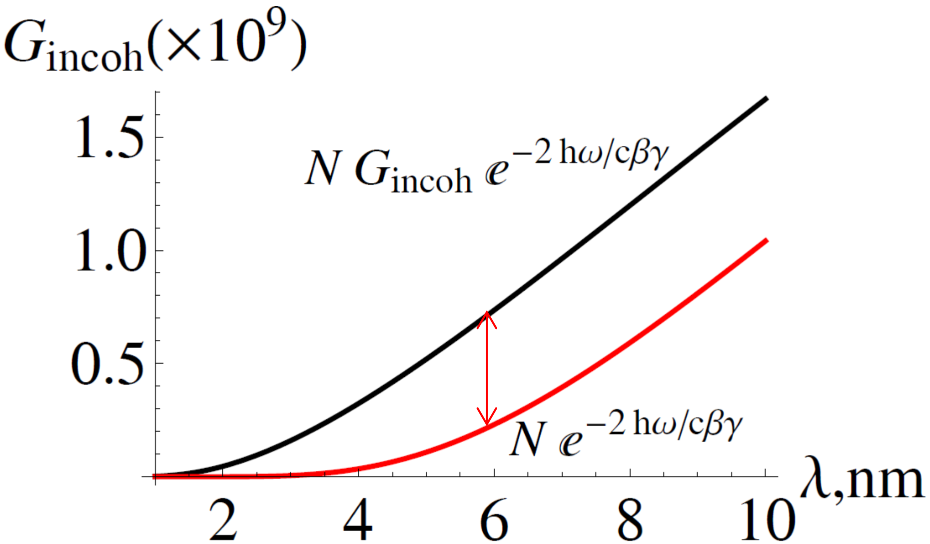


(Fig.6)

Incoherent form-factor

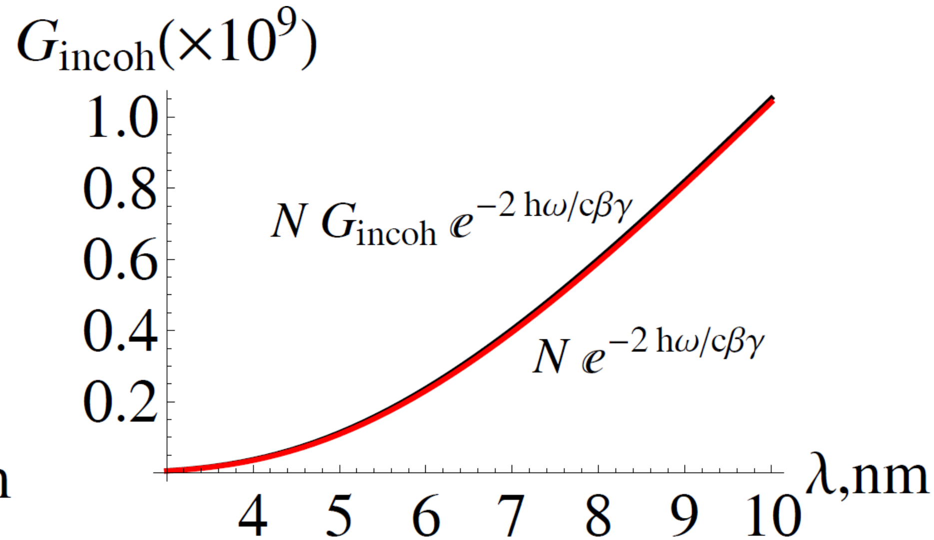
rectangular cross section of the bunch





(Fig.7) $\gamma = 2500,\ l = 0.9\,cm,\ a = 7\,\mu m,$

$N = 10^{10},\ h = 4.5\,\mu m$



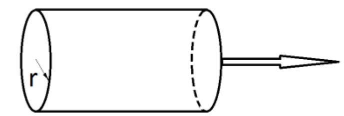
(Fig.8) $\gamma = 2500,\ l = 0.9\,cm,\ a = 1\,\mu m,$

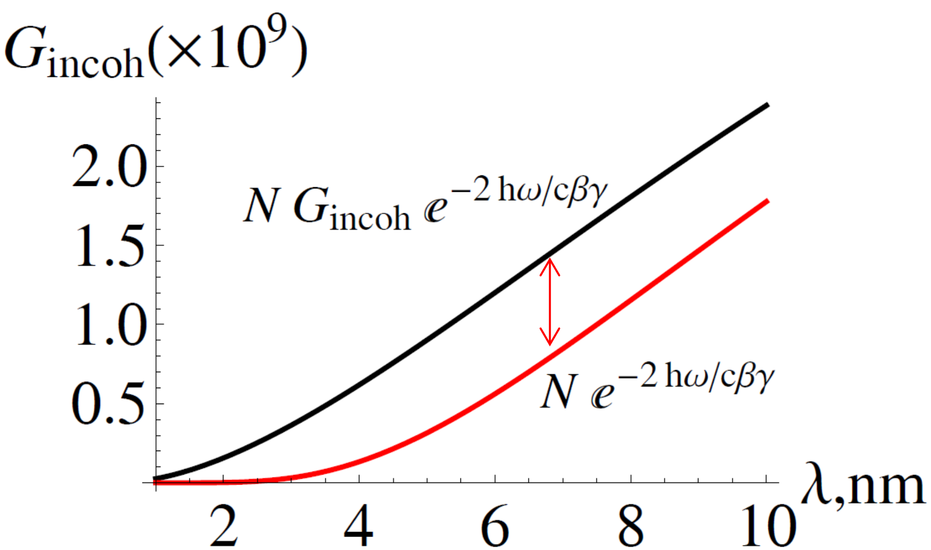
$N = 10^{10},\ h = 4.5\,\mu m$

(energy of KEK ATF)

Incoherent form-factor

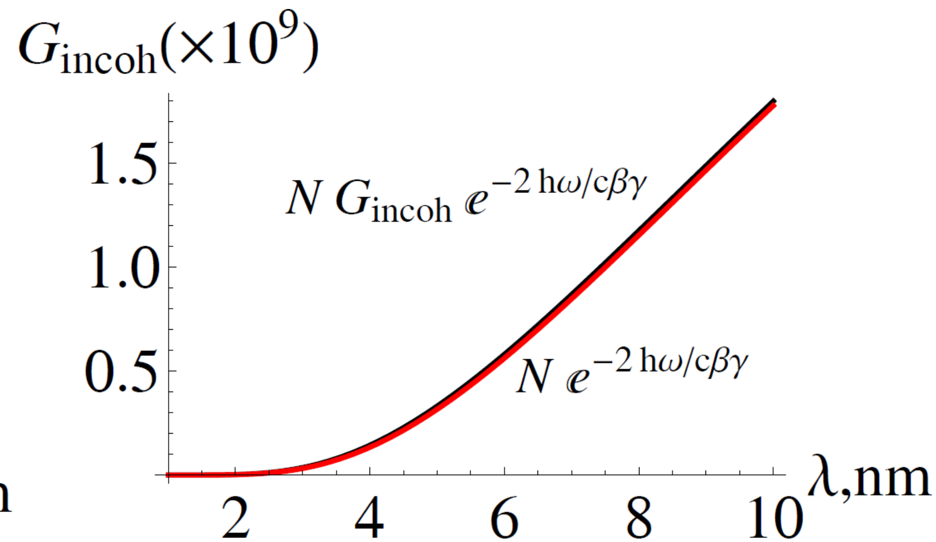
cylindrical cross section of the bunch





(Fig.9) $\gamma = 4\cdot 10^4,\ r_0 = 50\,\mu m,\ \ N = 10^{10},$

$h = 55\,\mu m$



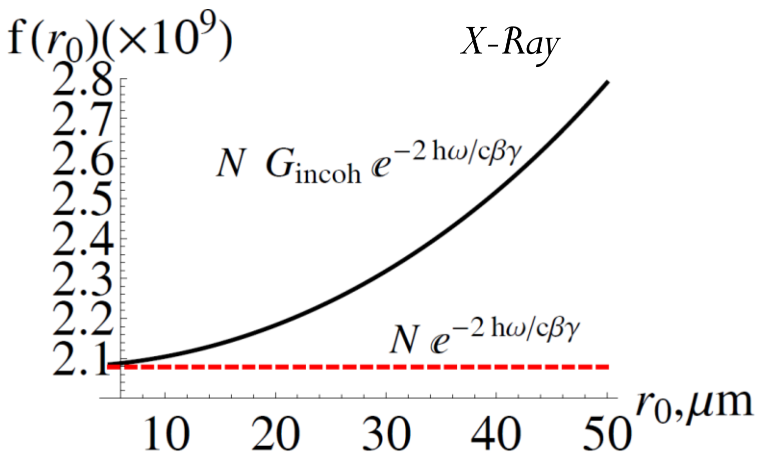
(Fig.10) $\gamma = 4\cdot 10^4,\ r_0 = 10\,\mu m,$

$N = 10^{10},\ h = 55\,\mu m$

(energy of SLAC)

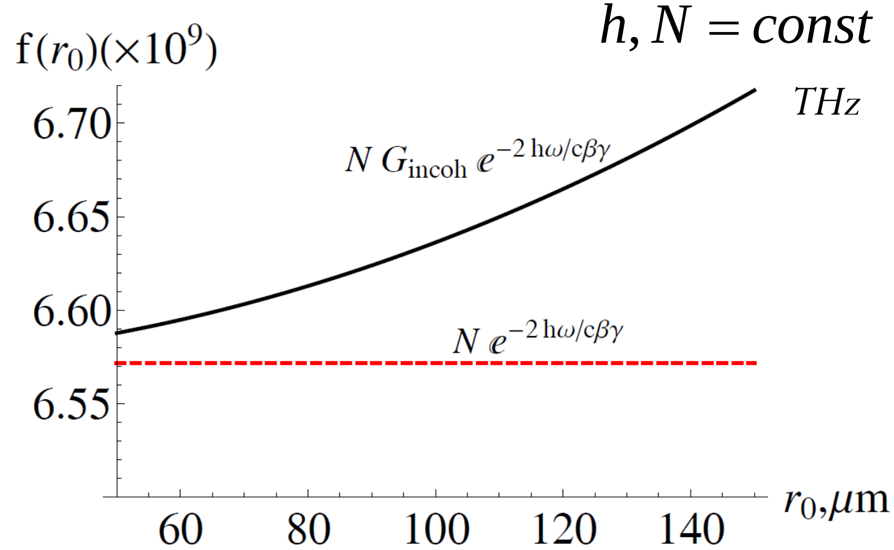
Role of the bunch transverse size for incoherent form-factor

$h, N = const$



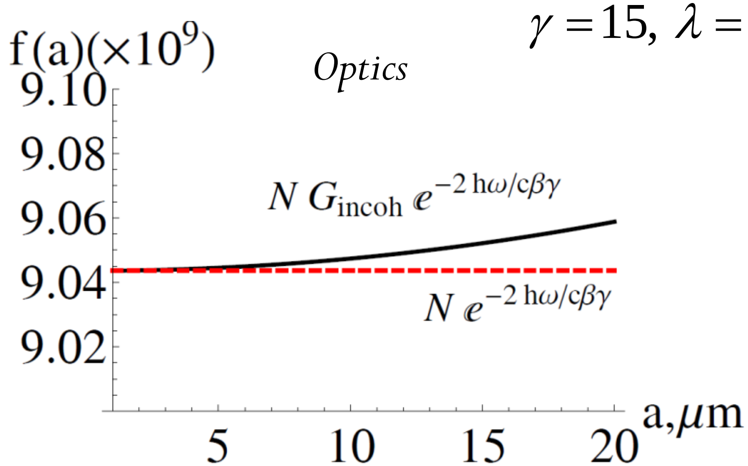
(Fig.11)

$\gamma = 4\cdot 10^4,\ \lambda = 10\,nm,\ h = 50\,\mu m,\ \ N = 10^{10}$



(Fig.12)

$\gamma = 15,\ \lambda = 0.3\,mm,\ \ h = 150\,\mu m, N = 10^{10}$

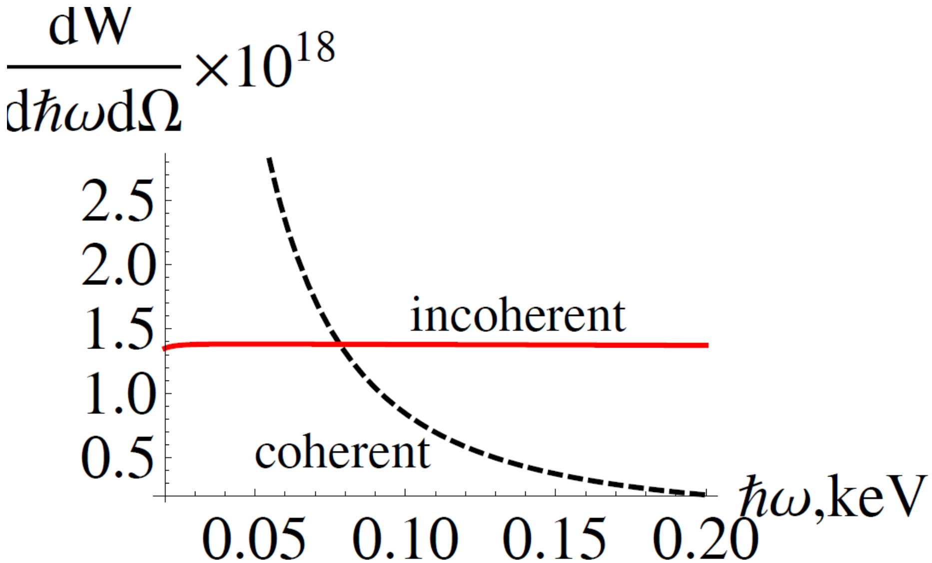


(Fig.13) $\gamma = 2500,\ \lambda = 0.5\,\mu m,\ \ h = 10\,\mu m,\ \ N = 10^{10}$

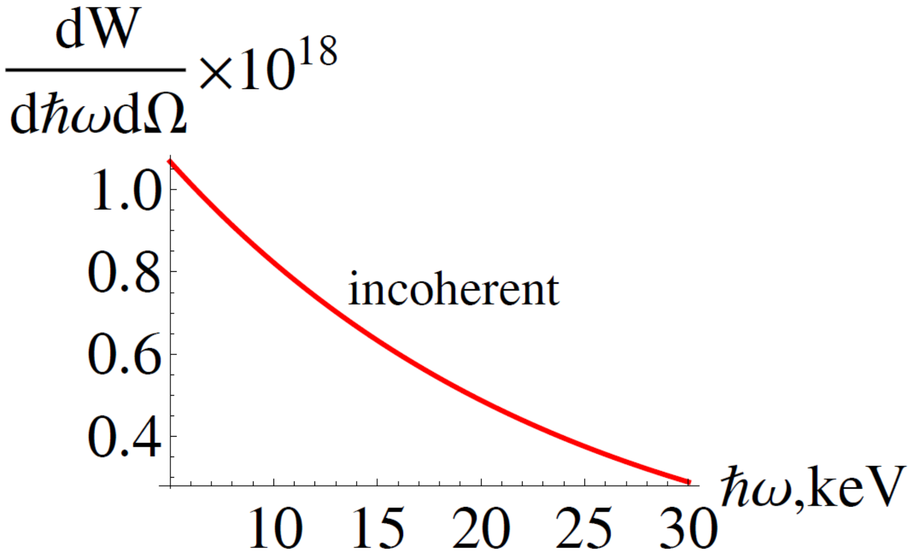
Possibility of noninvasive submicron bunch diagnostics

ILC

$$\gamma = 10^6$$



(Fig. 14)



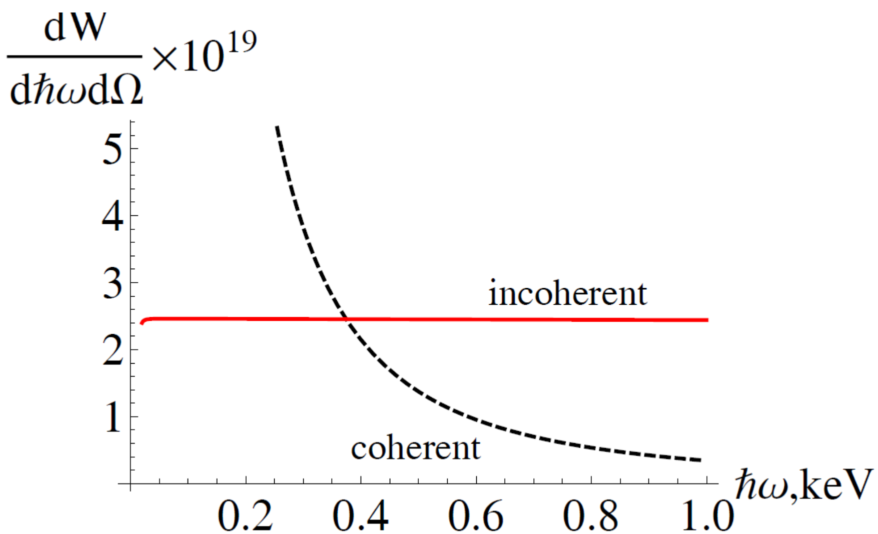
(Fig. 15)

$$l = 0.3\,nm,\ a = 5\,nm,\ \ N = 0.75 \cdot 10^{10},\ h = 5\,\mu m$$

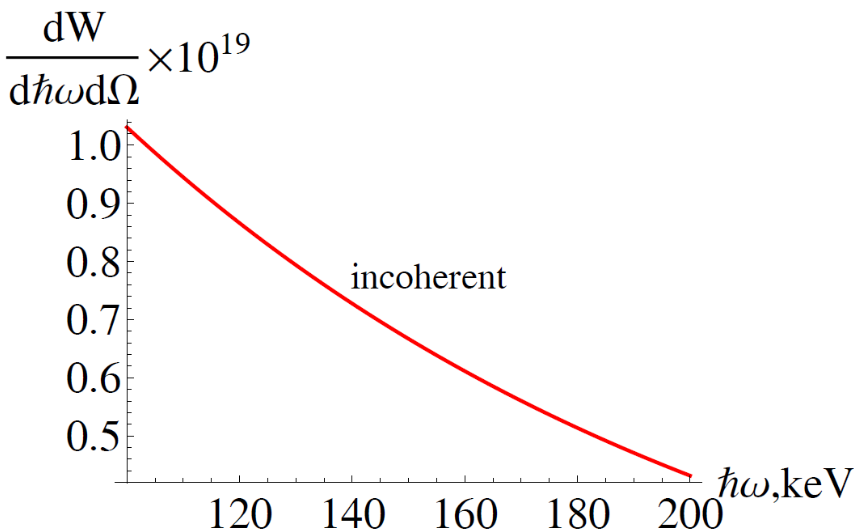
Possibility of noninvasive submicron bunch diagnostics

CLIC

$$\gamma = 6 \cdot 10^6$$



(Fig. 16)



(Fig. 17)

$$l = 44\,\mu m,\; a = 1\,nm,\; N = 0.37 \cdot 10^{10},\; h = 5\,\mu m$$

Summary

- Incoherent form-factor is proved to exist; uniform distribution is explored in detail and it is shown that parameters range is rather narrow;

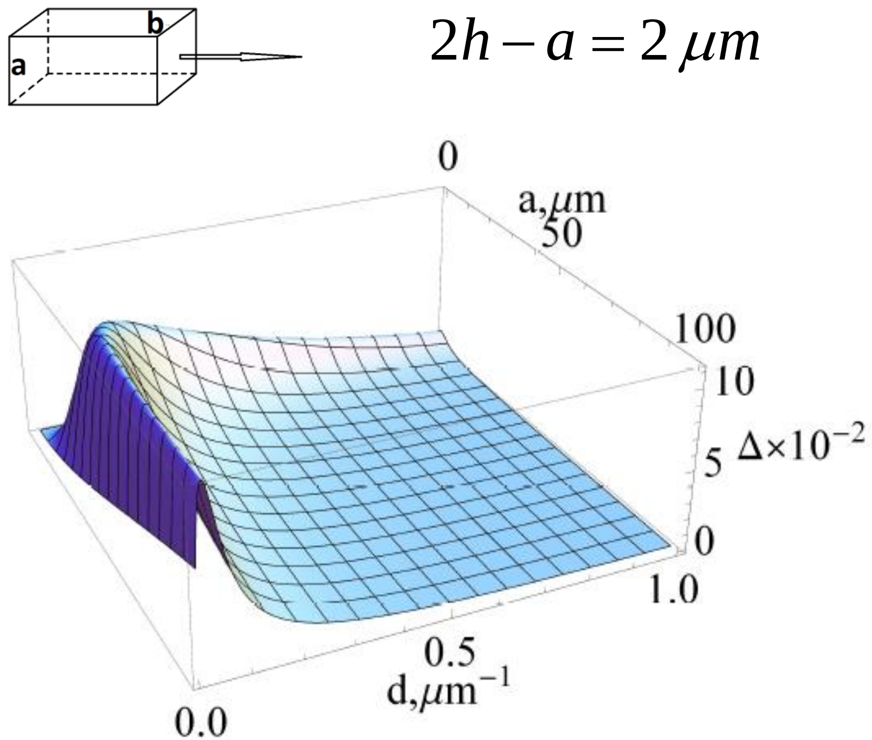
- Coherent form-factor depends on transversal size for $\lambda \square l$.

- The theory of Smith-Purcell radiation is developed for UV and X-ray frequency ranges, which is very important for noninvasive submicron beam diagnostics.

Thank you for your attention!

Incoherent form-factor

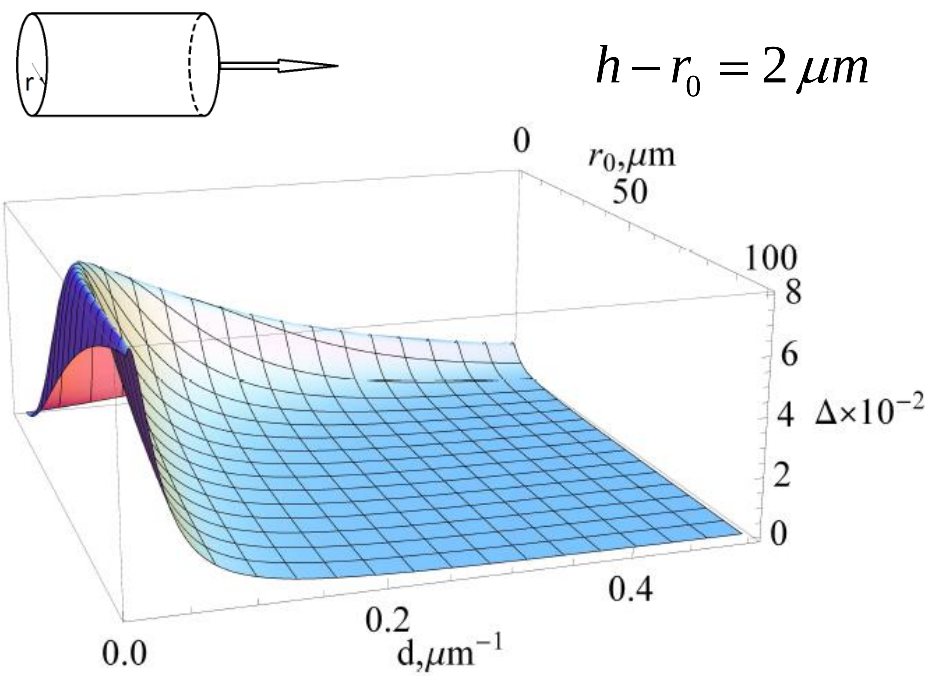
$$2h - a = 2\,\mu m$$



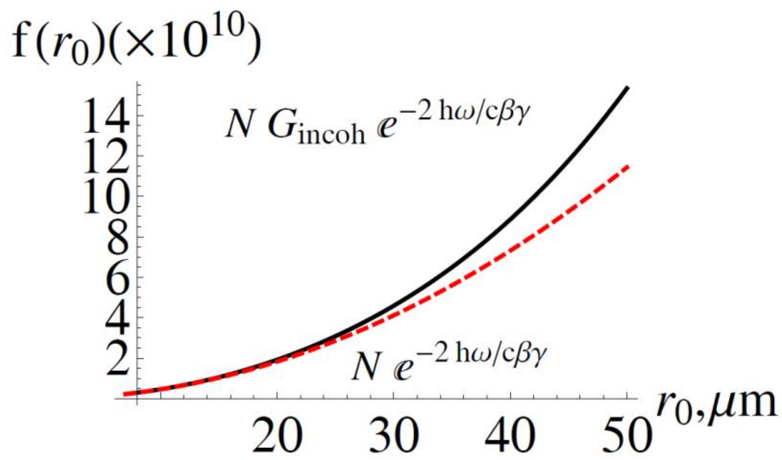
$$\Delta = G_{incoh} e^{-2dh} - e^{-2dh}$$

$$d = \frac{\omega}{c\beta\gamma}$$

$$h - r_0 = 2\,\mu m$$

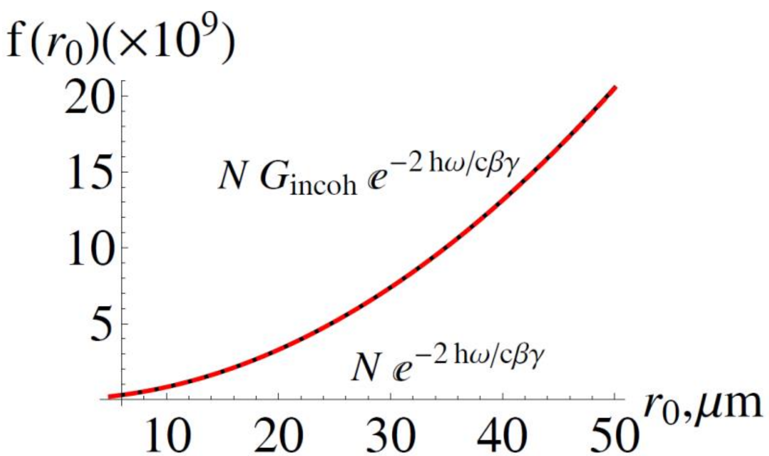


Incoherent form-factor and transverse size of the bunch

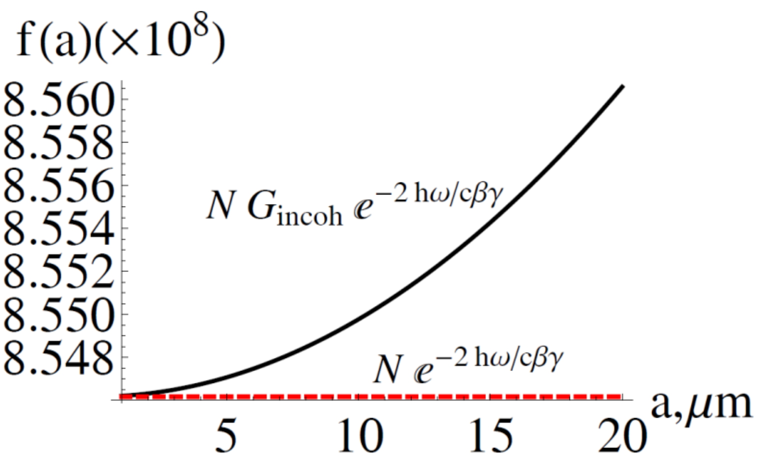


$h, \rho = const$

$$\gamma = 4 \cdot 10^4,\ \lambda = 10\, nm,\ \ h = 50\, \mu m,\ N = \rho \pi r_0^2 l,\ \ l = 70\, \mu m, \rho = 10^{18}$$



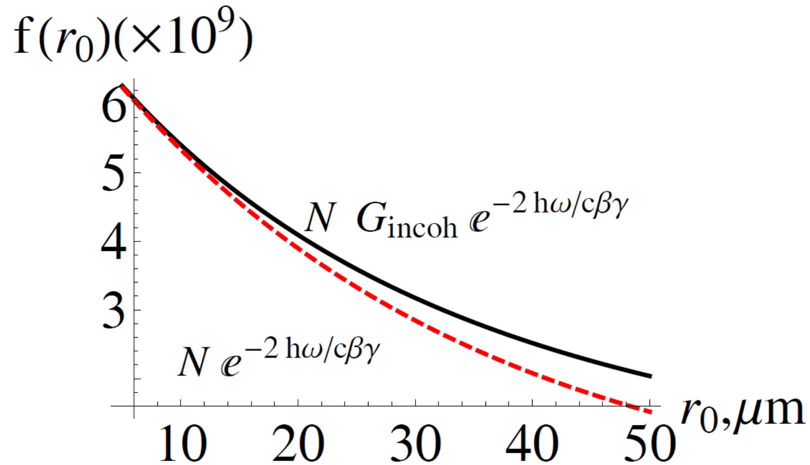
$$\gamma = 15,\ \lambda = 0.3\, mm,\ \ h = 50\, \mu m,\ N = \rho \pi r_0^2 l,$$
$$l = 0.3\, cm, \rho = 10^{16}\, cm^{-3}$$



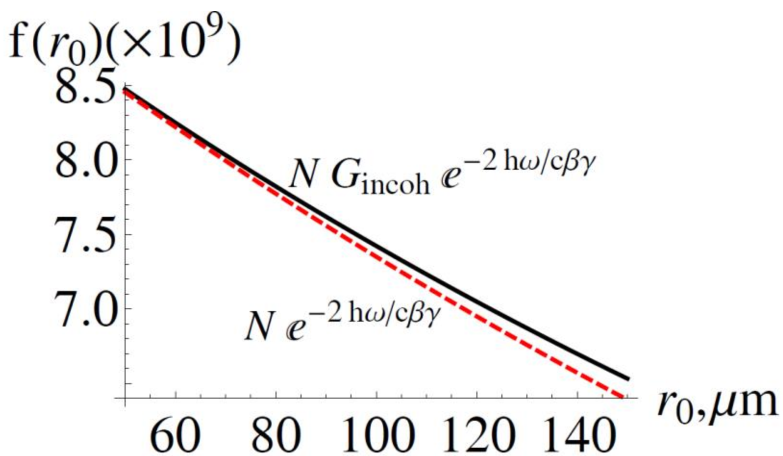
$$\gamma = 2500,\ \lambda = 0.5\, \mu m,\ \ h = 5 \mu m,\ N = \rho \pi r_0^2 l,$$
$$l = 0.9 cm, \rho = 10^{18}$$

Incoherent form-factor and transverse size of the bunch

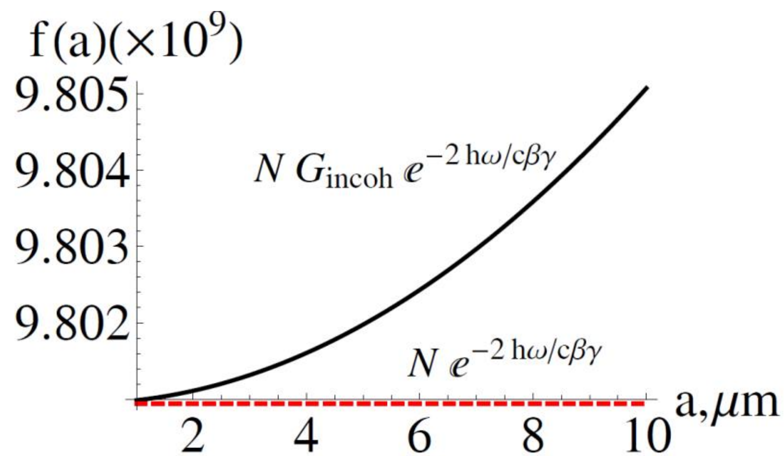
$N, \Delta = const$



$\gamma = 4 \cdot 10^4,\ \lambda = 10\,nm,\ \Delta = h - r_0 = 10\,\mu m,\ N = 10^{10}$

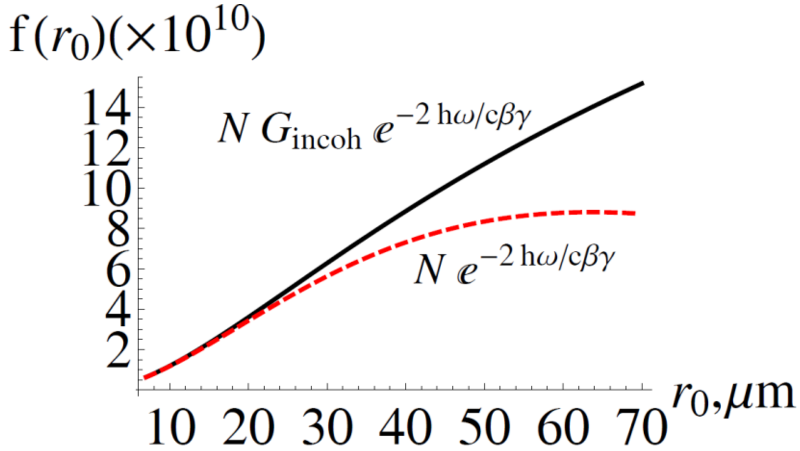


$\gamma = 15,\ \lambda = 0.3\,mm,\ \Delta = h - r_0 = 10\,\mu m,\ N = 10^{10}$



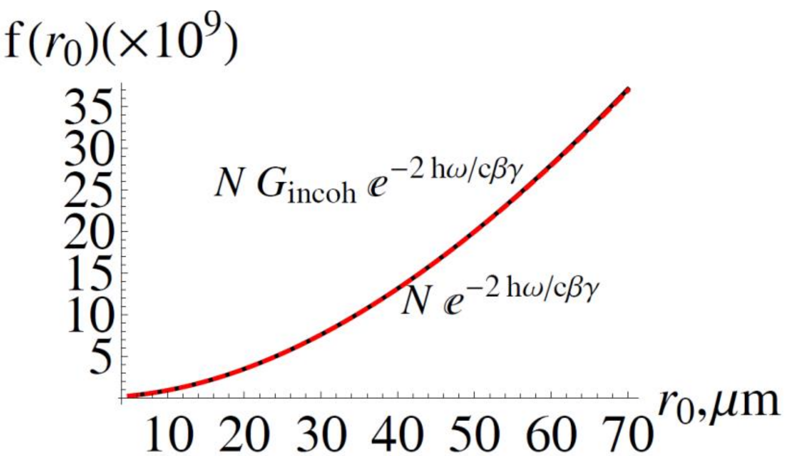
$\gamma = 2500,\ \lambda = 0.5\,\mu m,\ \Delta = h - \frac{a}{2} = 2\,\mu m,\ N = 10^{10}$

Incoherent form-factor and transverse size of the bunch

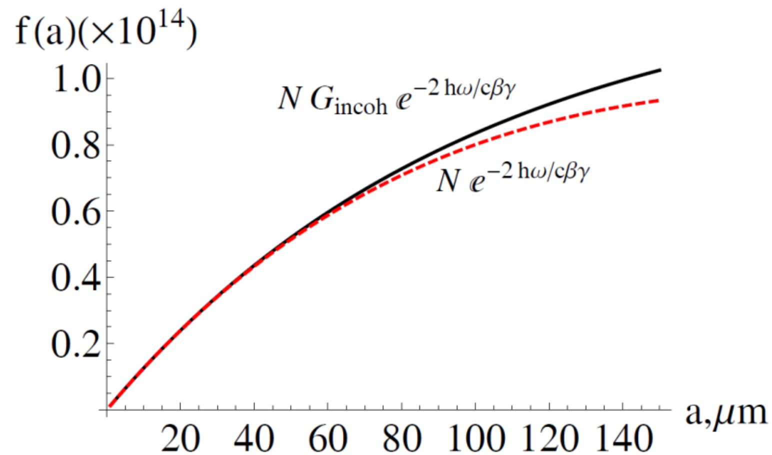


$\Delta, \rho = const$

$$\gamma = 4\cdot 10^4,\ \lambda = 10\,nm,\ \Delta = h - r_0 = 10\,\mu m,\ N = \rho\pi r_0^2 l,\ l = 70\,\mu m,\ \rho = 10^{18}$$



$$\gamma = 15,\ \lambda = 0.3\,mm,\ \Delta = h - r_0 = 10\,\mu m,\ N = \rho\pi r_0^2 l,$$
$$l = 0.3\,cm,\ \rho = 10^{16}\,cm^{-3}$$



$$\gamma = 2500,\ \lambda = 0.5\,\mu m,\ \Delta = h - \frac{a}{2} = 2\,\mu m,\ N = \rho\pi r_0^2 l,$$
$$l = 0.9\,cm,\ \rho = 10^{18}$$