Beam-Beam Interactions

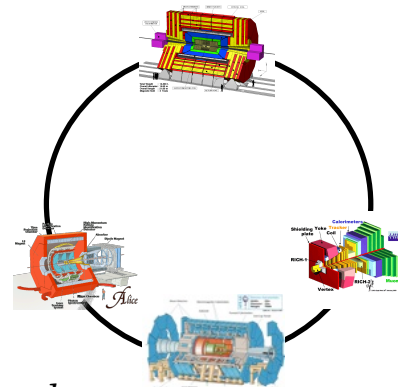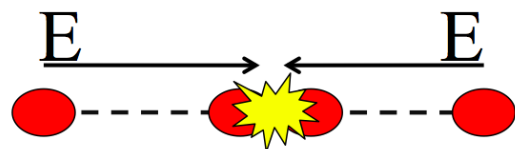Tatiana Pieloni (BE-ABP-ICE)

Thanks to W. Herr

Hadron Circular Colliders

$$E^* \approx 2 \times E$$

$$N_{event/s} = L \cdot \sigma_{event}$$

$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$

Bunch intensity: $\qquad N_p = 1.15 - 1.65 \cdot 10^{11} \; ppb$

Transverse Beam size: $\qquad \sigma_{x,y} = 16 - 30 \; \mu m$

Number of bunches $\qquad 1370 - 2808$

Revolution frequency $\qquad 11 \; kHz$

When do we have beam-beam effects?

➤ They occur when two beams get closer and collide

➤ Two types
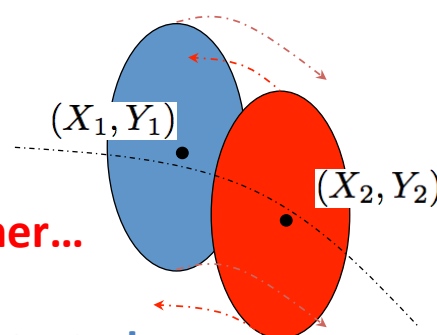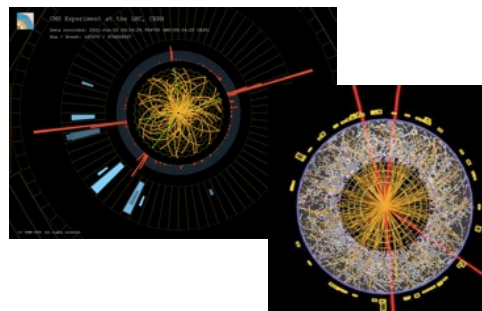
  ➤ High energy collisions between two particles (wanted)
  ➤ Distortions of beam by electromagnetic forces (unwanted)

➤ **Unfortunately: usually both go together...**
➤ **0.001% (or less) of particles collide**
➤ **99.999% (or more) of particles are distorted**

(X_1, Y_1)

(X_2, Y_2)

Beam-beam effects: overview

➤ **Circular Colliders**: interaction occurs at every turn

  • Many effects and problems
  • Try to understand some of them

  • Overview of effects (single particle and multi-particle effects)
  • Qualitative and physical picture of effects
  • Observations from the LHC
  • Mathematical derivations and more info in References or at
  Beam-beam webpage http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/
 **And CAS Proceedings**

References:

[1] http://cern.ch/Werner.Herr/CAS2009/proceedings/bb_proc.pdf

[2] V. Shiltsev et al, "Beam beam effects in the Tevatron", Phys. Rev. ST Accel. Beams 8, 101001 (2005)

[3] Lyn Evans "The beam-beam interaction", CERN 84-15 (1984)

[4] Alex Chao "Lie Algebra Techniques for Nonlinear Dynamics" SLAC-PUB-9574 (2002)

[5] J. D. Jackson, "Classical Electrodynamics", John Wiley & Sons, NY, 1962.

[6] H. Grote, F. Schmidt, L. H. A. Leunissen,"LHC Dynamic Aperture at Collision", LHC-Project-Note 197, (1999).

[7] W. Herr,"Features and implications of different LHC crossing schemes", LHC-Project-Note 628, (2003).

[8] A. Hofmann,"Beam-beam modes for two beams with unequal tunes", CERN-SL-99-039 (AP) (1999) p. 56.

[9] Y. Alexahin, "On the Landau damping and decoherence of transverse dipole oscillations in colliding beams ", Part. Acc. 59, 43 (1996).
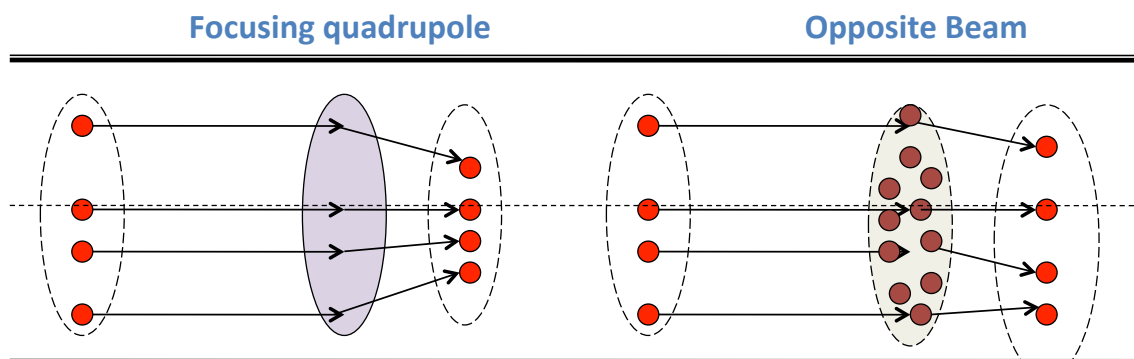
…much more on the LHC Beam-beam webpage:

http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/

Beams EM potential

➤ **Beam is a collection of charges**
➤ **Beam is an electromagnetic potential for other charges**
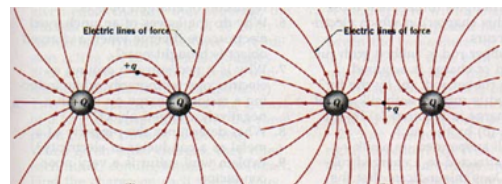
Force on itself (space charge) and opposing beam (beam-beam effects)

Single particle motion and whole bunch motion distorted

Focusing quadrupole	Opposite Beam

A beam acts on particles like an electromagnetic lens, but…

Beam-beam Mathematics

General approach in electromagnetic problems Reference[5] already applied to beam-beam interactions in Reference[1,3, 4]

$$\Delta U = -\frac{1}{\epsilon_0}\rho(x,y,z)$$

Derive potential from Poisson equation for charges with distribution ρ

Solution of Poisson equation

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z) = \frac{1}{4\pi\epsilon_0}\int\int\int\frac{\rho(x_0,y_0,z_0)dx_0dy_0dz_0}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}}$$

$$\vec{E} = -\nabla U(x,y,z,\sigma_x,\sigma_y,\sigma_z)$$

Then compute the fields

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

From Lorentz force one calculates the force acting on test particle with charge q

Making some assumptions we can simplify the problem and derive analytical formula for the force…

Round Gaussian distributions:

Gaussian distribution for charges:
Round beams:
Very relativistic, Force has only radial component :

$$\sigma_x = \sigma_y = \sigma$$
$$\beta \approx 1 \qquad r^2 = x^2 + y^2$$

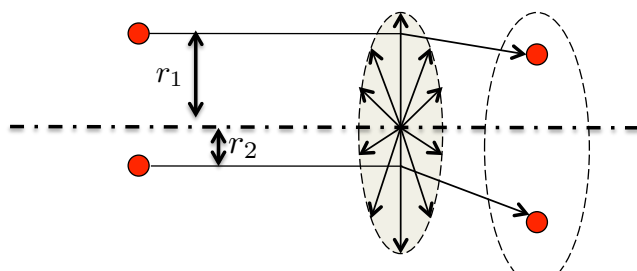$$F \propto \frac{N_p}{\sigma}\cdot\frac{1}{r}\cdot\left[1-e^{-\frac{r^2}{2\sigma^2}}\right]$$

Beam-beam Force

$$\Delta r' = \frac{1}{mc\beta\gamma}\int F_r(r,s,t)\,dt$$

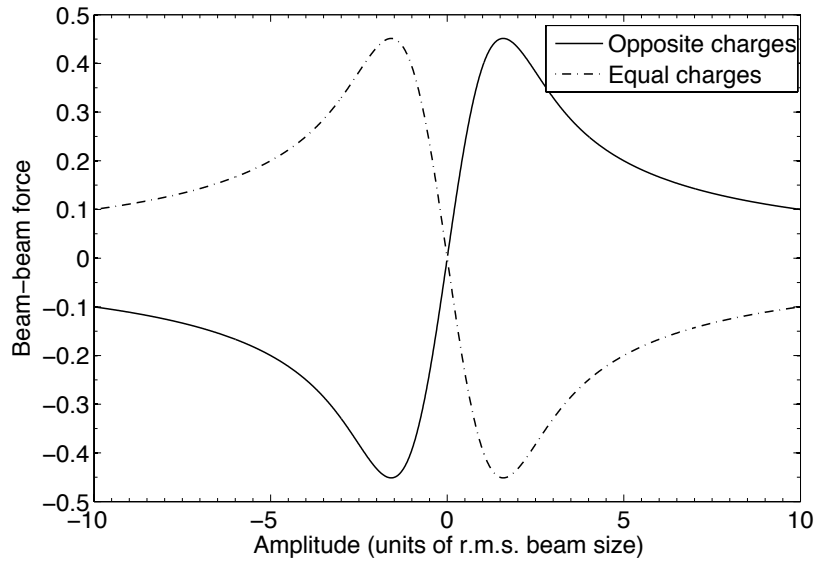**Beam-beam kick obtained integrating the force over the collision (i.e. time of passage)**

$$\Delta r' = -\frac{N_p r_0}{r}\cdot\frac{r}{r^2}[1-e^{-\frac{r^2}{2\sigma^2}}]$$

Only radial component in relativistic case

How does this force looks like?

Beam-beam Force

$$F_r(r) = \pm \frac{ne^2(1 + \beta_{rel}^2)}{2\pi\epsilon_0} \frac{1}{r}\left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

Why do we care?

Pushing for luminosity means stronger beam-beam effects

$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

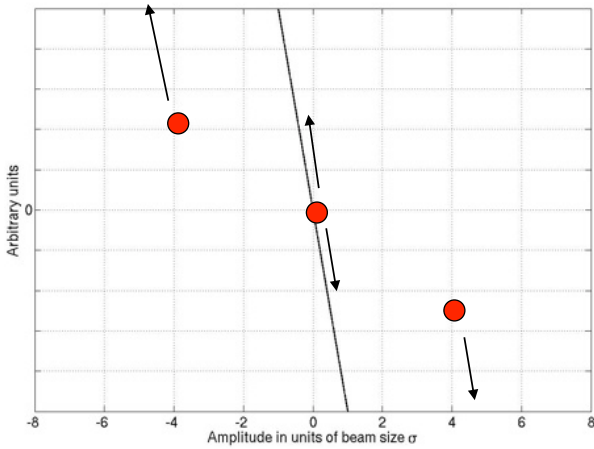$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

Physics fill lasts for many hours 10h – 24h

Strongest non-linearity in a collider YOU CANNOT AVOID!

Two main questions:
What happens to a single particle?
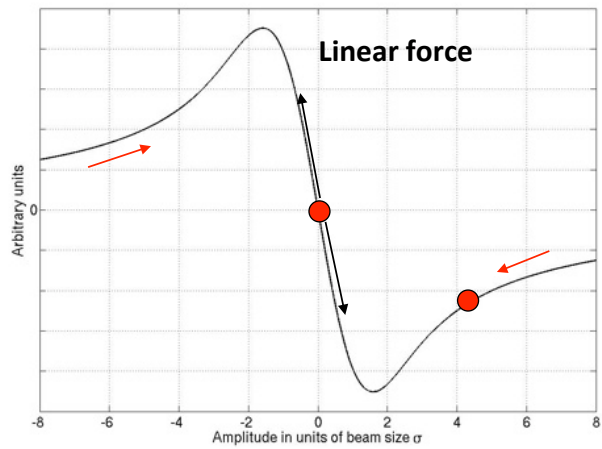**What happens to the whole beam?**

Beam-Beam Force: single particle…

Lattice defocusing quadrupole

Beam-beam force

Linear force

Arbitrary units

Amplitude in units of beam size σ

Arbitrary units

Amplitude in units of beam size σ

$$F = -k \cdot r$$

$$\mathrm{F} \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}} \right]$$

For small amplitudes: linear force
For large amplitude: very non-linear
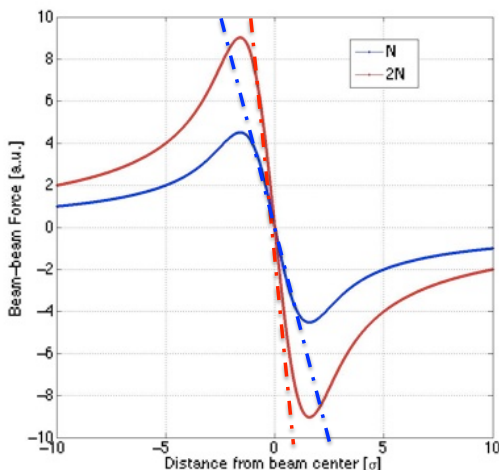**The beam will act as a strong non-linear electromagnetic lens!**

Can we quantify the beam-beam strenght?

Quantifies the strength of the force but does NOT reflect the nonlinear nature of the force
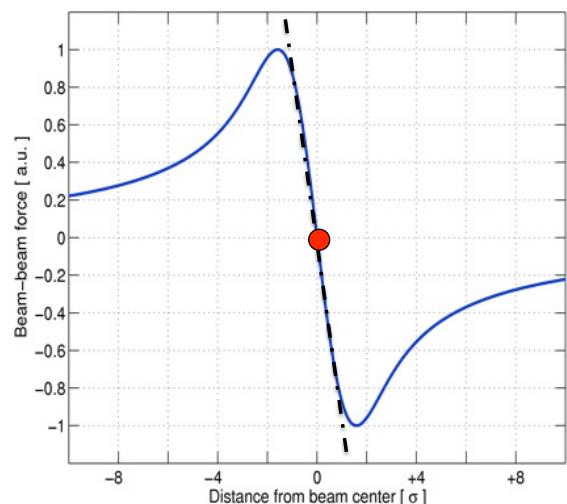**For small amplitudes: linear force**

Beam-beam force

$$F \propto -\xi \cdot r$$

The slope of the force gives you the beam-beam parameter ξ

Beam–beam force [a.u.]

Distance from beam center [σ]

Beam-beam Force [a.u.]

Distance from beam center [σ]

N
2N

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}} \right]$$

$$\Delta r' = \frac{2 N_p r_0}{\gamma} \cdot \frac{1}{r} \cdot \left[1 - \left(1 - \frac{r^2}{2\sigma^2} + ... \right) \right]$$

Colliders:

For round beams:

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2}$$

For non-round beams:

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi \gamma \sigma_{x,y}(\sigma_x + \sigma_y)}$$

Examples:

Parameters	LHC nominal	LHC 2012
Intensity $N_{p,e}$/bunch	$1.15 \ 10^{11}$	$1.6 \ 10^{11}$
Energy GeV	7000	4000
Beam emittance	3.75 µmrad	2.2-2.5 µmrad
Crossing angle (µrad)	285	290
$\beta_{x,y}^*$ (m)	1.25-0.05	0.60-0.60
Luminosity	$1 \ 10^{34}$	$7.6 \ 10^{33}$
ξ_{bb}	**0.0034**	**0.006**

Linear Tune shift

For small amplitudes beam-beam can be approximated as linear force as a quadrupole

$$F \propto -\xi \cdot r$$

Focal length:

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*}$$

Beam-beam matrix:

$$\begin{pmatrix} 1 & 0 \\ -\frac{\xi \cdot 4\pi}{\beta^*} & 1 \end{pmatrix}$$

Perturbed one turn matrix with perturbed tune ΔQ and beta function at the IP β*:

$$\begin{pmatrix} cos(2\pi(Q+\Delta Q)) & \beta^* sin(2\pi(Q+\Delta Q)) \\ -\frac{1}{\beta^*} sin(2\pi(Q+\Delta Q)) & cos(2\pi(Q+\Delta Q)) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} cos(2\pi Q) & \beta_0^* sin(2\pi Q) \\ -\frac{1}{\beta_0^*} sin(2\pi Q) & cos(2\pi Q) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix}$$

Linear tune

Solving the one turn matrix one can derive the tune shift ΔQ and the perturbed beta function at the IP β^*:

Tune is changed

$$cos(2\pi(Q + \Delta Q)) = cos(2\pi Q) - \frac{\beta_0^* \cdot 4\pi\xi}{\beta^*} sin(2\pi Q)$$
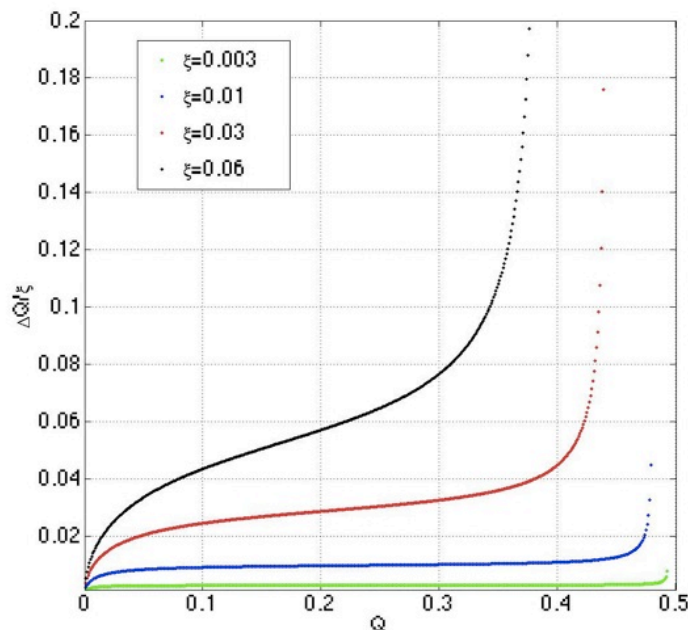
β-function is changed:

$$\frac{\beta^*}{\beta_0^*} = \frac{sin(2\pi Q)}{sin(2\pi(Q + \Delta Q))}$$

…how do they change?

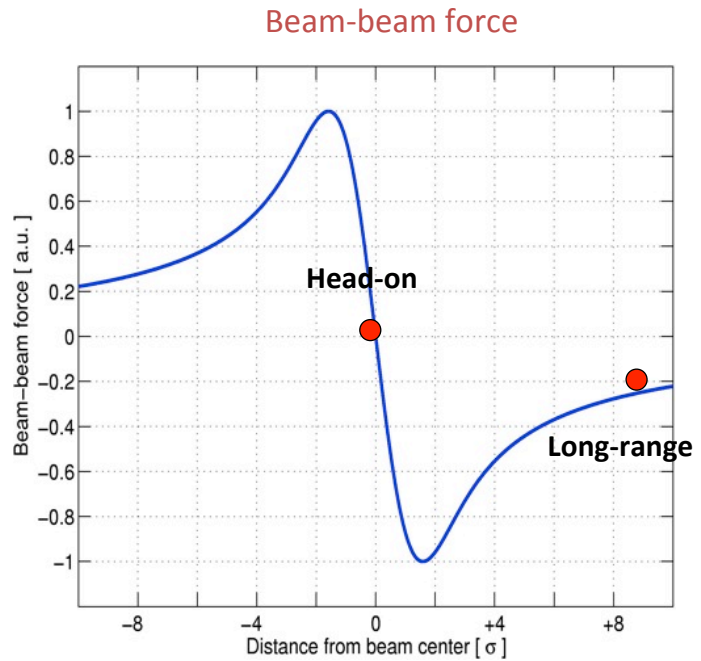
Tune dependence of tune shift and dynamic beta

Tune shift as a function of tune

Larger ξ ➡ **Strongest variation with Q**

Head-on and Long-range interactions

Beam-beam force

$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$

Other beam passing in the center force: HEAD-ON beam-beam interaction

Other beam passing at an offset of the force: LONG-RANGE beam-beam interaction

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Multiple bunch Complications

MANY INTERACTIONS

$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

Num. of bunches : $n_b = 2808$

3.7 m

r **Long range**
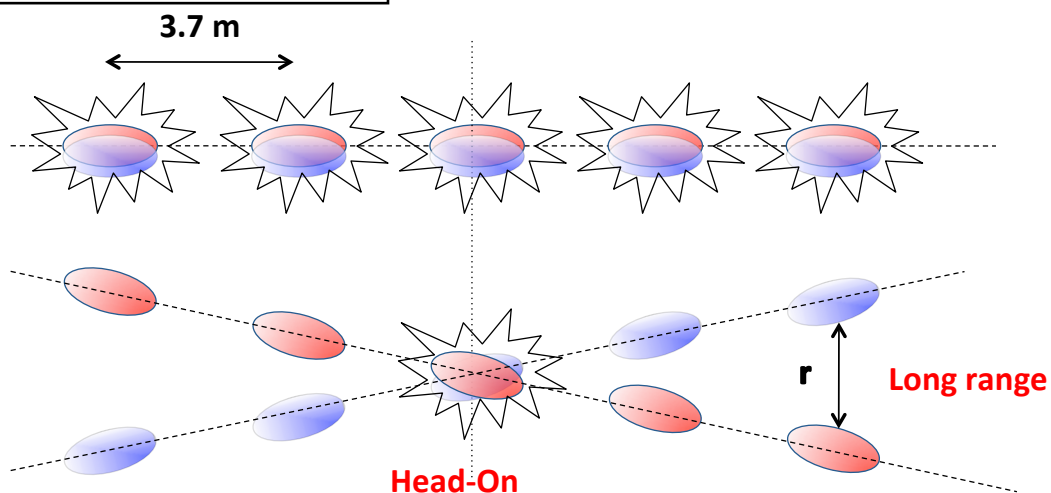
Head-On

For 25ns case 124 BBIs per turn: 4 HO and 120 LR

LHC, KEKB… colliders

- **Crossing angle operation**
- **High number of bunches in train structures**

Head-On

Long Range
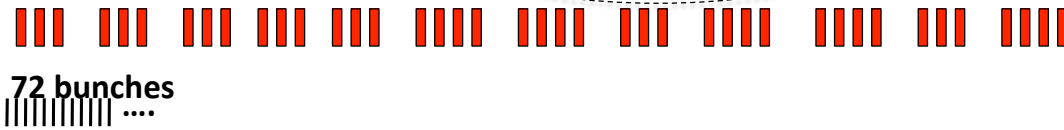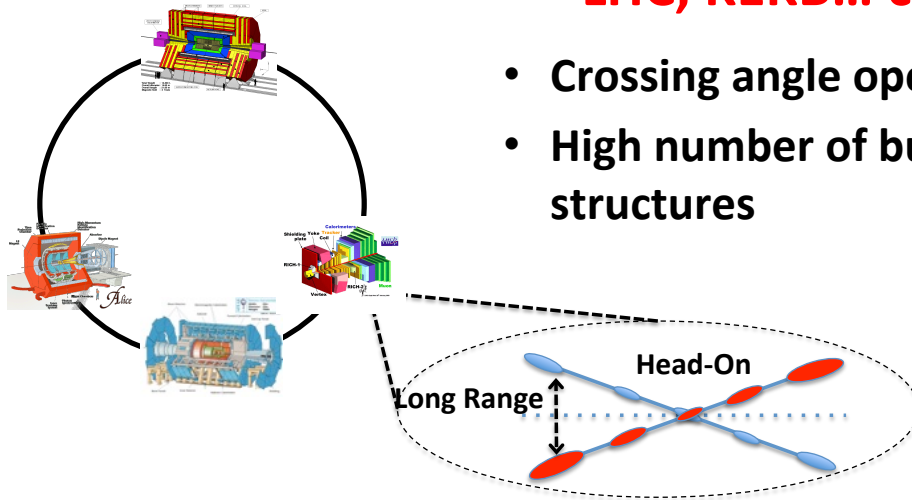
72 bunches

	SppS	Tevatron	RHIC	LHC
Number Bunches	6	36	109	2808
LR interactions	9	70	0	120/40
Head-on interactions	3	2	2	4

A beam is a collection of particles
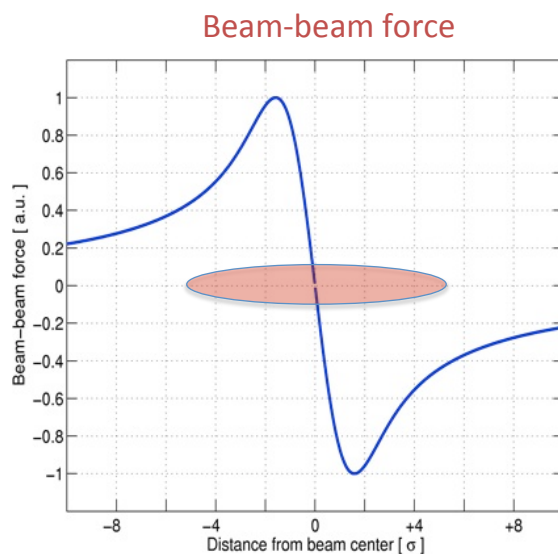
Beam-beam force

Beam 2 passing in the center of force produce by Beam 1
Particles of Beam 2 will experience different ranges of the beam-beam forces

Tune shift as a function of amplitude (detuning with amplitude or tune spread)

A beam will experience all the force range

Beam-beam force
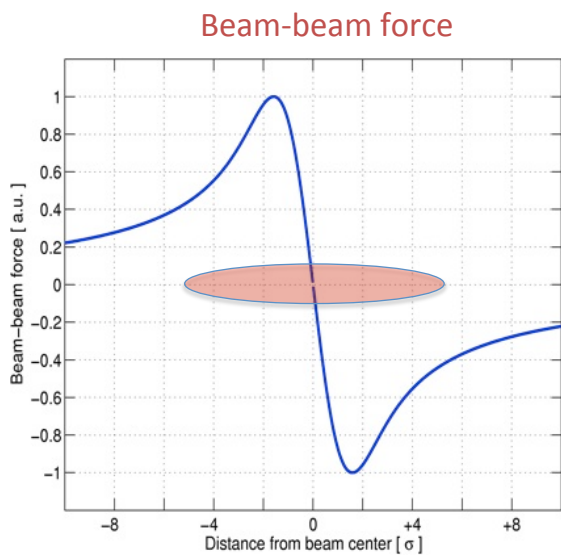
Beam-beam force

Second beam passing in the center
HEAD-ON beam-beam interaction
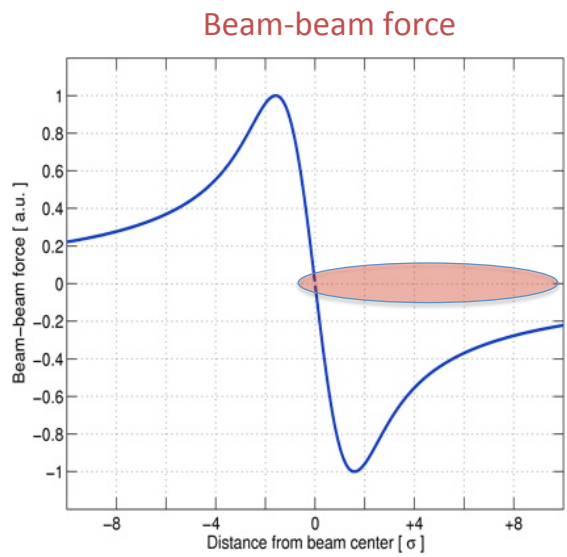
Second beam displaced offset
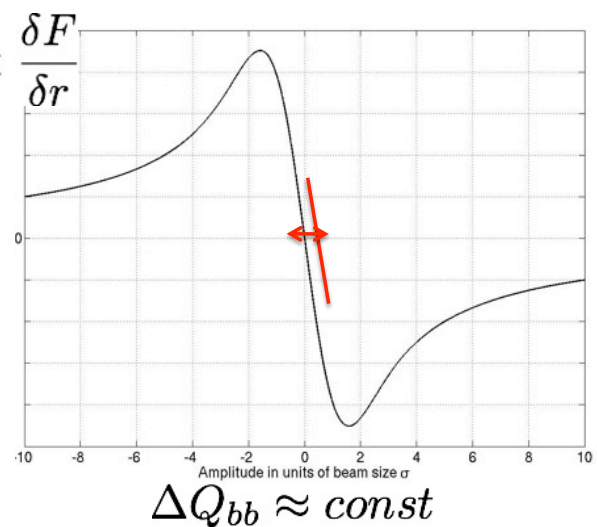**LONG-RANGE** beam-beam interaction

Different particles will see different force

Detuning with Amplitude for head-on

Instantaneous tune shift of test particle when it crosses the other beam
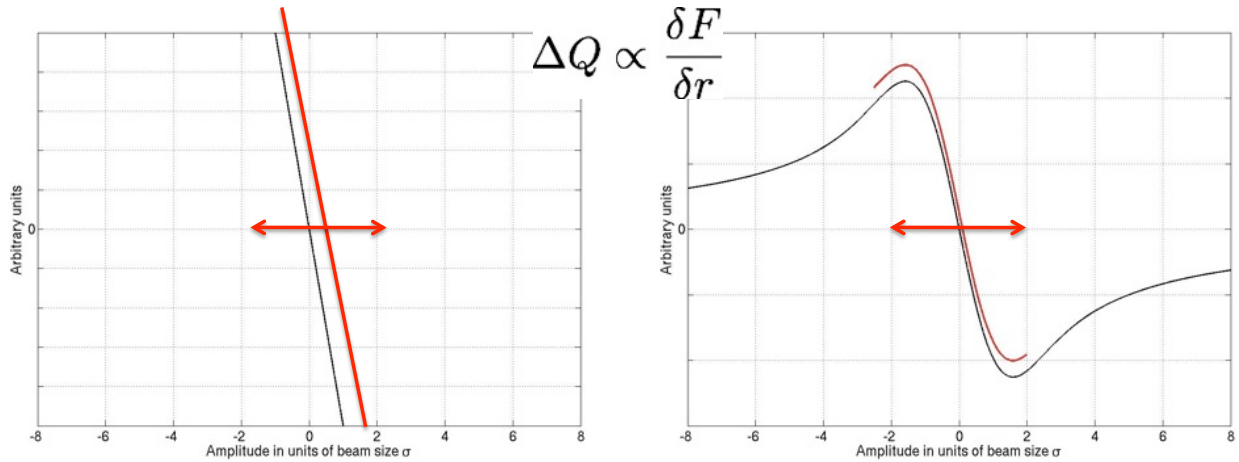is related to the derivative of the force with respect to the amplitude

$$\Delta Q \propto \frac{\delta F}{\delta r}$$

$$\Delta Q_{quad} = const$$
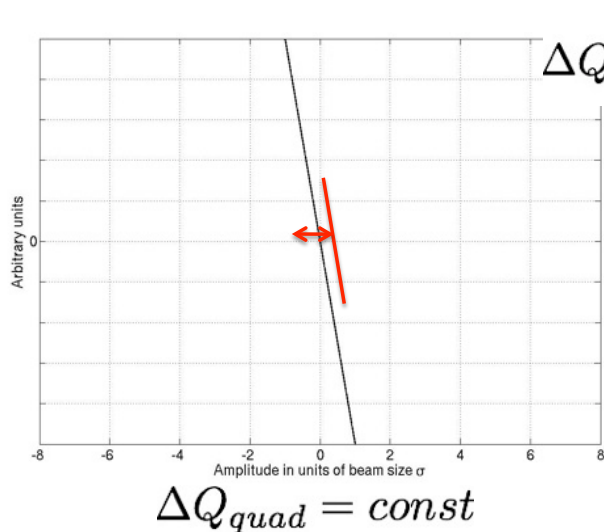
$$\Delta Q_{bb} \approx const$$

**For small amplitude test particle
linear tune shift**

$$\lim_{r \to 0} \Delta Q(r) = -\frac{N r_0 \beta^*}{4\pi\gamma\sigma^2} = \xi$$

Detuning with Amplitude for head-on

Beam with many particles this results in a tune spread

$$\Delta Q \propto \frac{\delta F}{\delta r}$$

$$\Delta Q_{quad} = const \qquad \Delta Q_{bb} \neq const$$

$$\Delta Q(x) = \frac{N r_0 \beta}{4\pi\gamma\sigma^2} \cdot \frac{1}{(\frac{x}{2})^2} \cdot \left(\exp -(\frac{x}{2})^2 I_0(\frac{x}{2})^2 - 1\right)$$

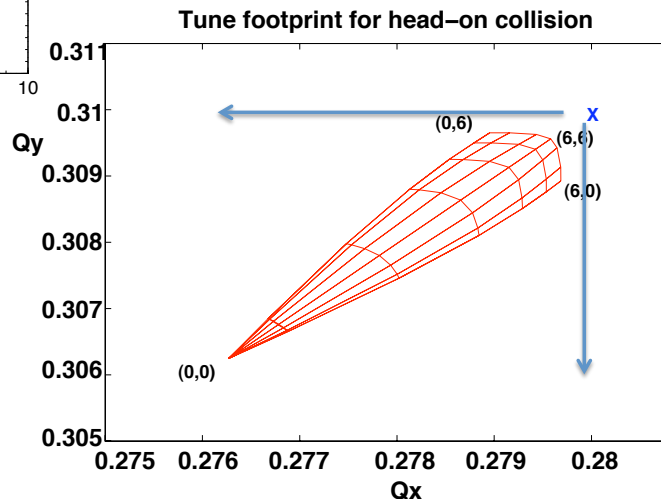Mathematical derivation in Ref [3] using Hamiltonian formalism and in Ref [4] using Lie Algebra

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Head-on detuning with amplitude and footprints

1-D plot of detuning with amplitude
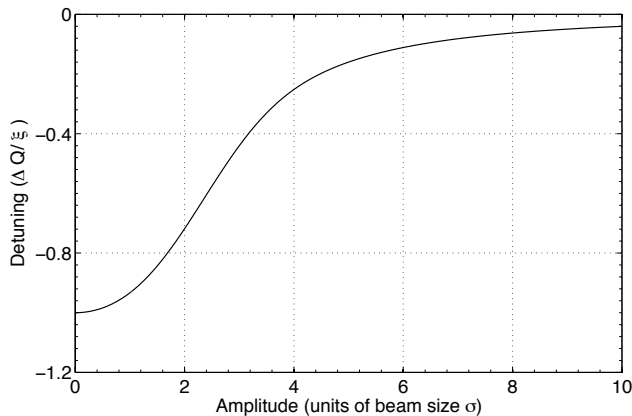
And in the other plane?
THE SAME DERIVATION
same tune spread

FOOTPRINT
2-D mapping of the detuning with amplitude of particles

Tune footprint for head-on collision

And for long-range interactions?
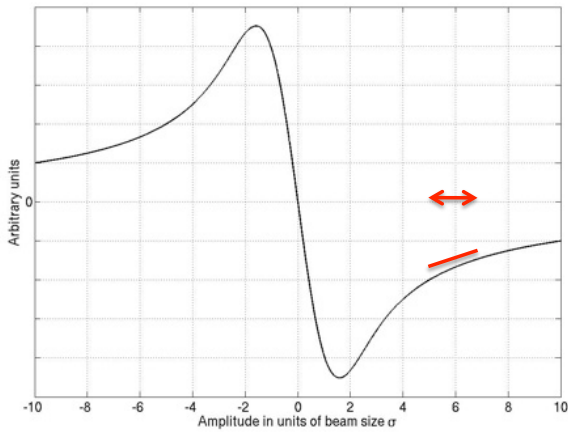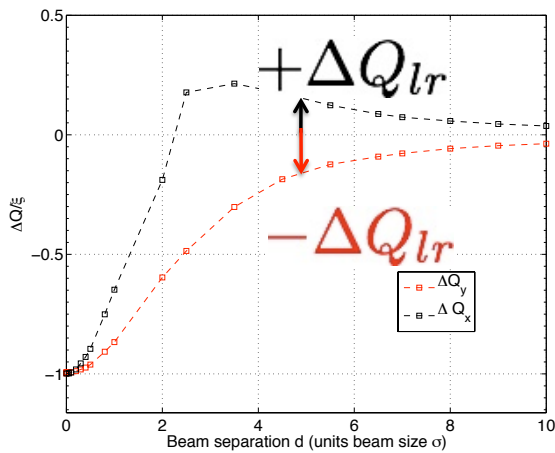

Second beam centered at d (i.e. 6σ)
- Small amplitude particles **positive tune shifts**
- Large amplitude can go to **negative tune shifts**
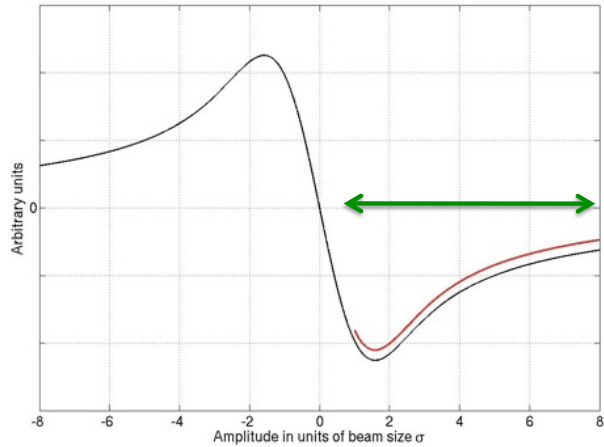
**Long range tune shift scaling for distances** $d > 6\sigma$

$$\Delta Q_{lr} \propto -\frac{N}{d^2}$$

Long-range footprints

The picture is more complicated
now the LARGE amplitude particles see the second beam and have larger tune shift

Separation in vertical plane!
And in horizontal plane?
The test particle is centered with the opposite beam
tune spread more like for head-on at large amplitudes

footprint from long range interactions
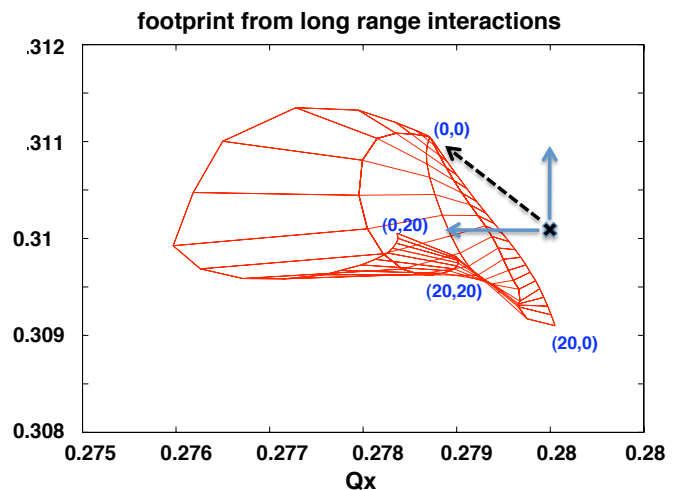
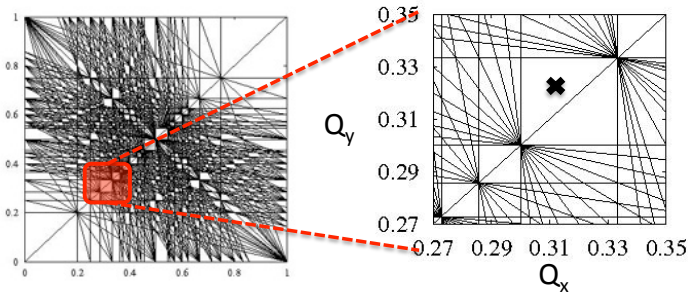# Beam-beam tune shift and spread

(0,6)

(0,0)

(0,6)
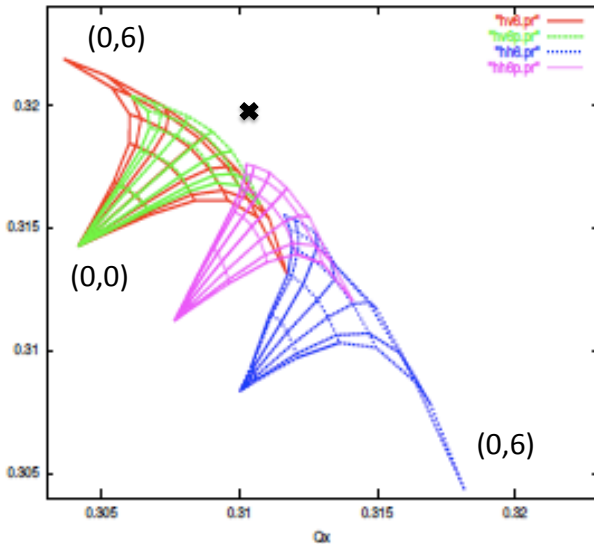
Footprints depend on:
- number of interactions
- Type (Head-on and long-range)
- Plane of interaction

When long-range effects become important footprint wings appear and alternating crossing important

Aim to reduce the area as much as possible!

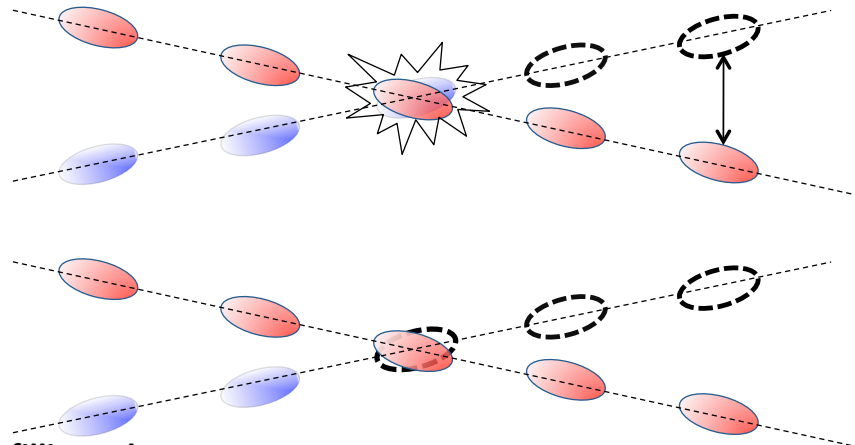Passive compensation of tune shift Ref[7]
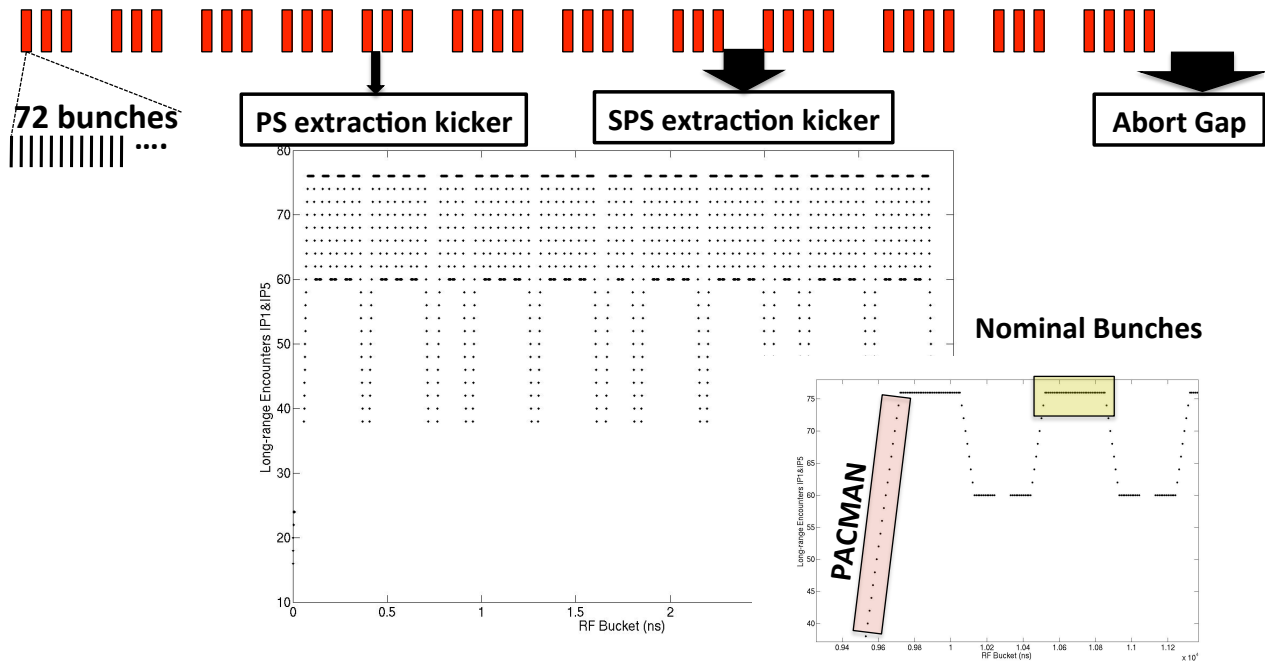

Complications
PACMAN and SUPER PACMAN bunches

72 bunches
||||||||||| ....

Pacman:

miss long range BBI

Super Pacman:

miss head-on BBI

IP2 and IP8 depending on filling scheme

Different bunch families: Pacman and Super Pacman

LHC Complications: filling schemes

72 bunches |||||||||| ....

PS extraction kicker

SPS extraction kicker

Abort Gap

Nominal Bunches

PACMAN

Pacman Bunche: different number of long-range interactions

Pacman and Super-pacman

LHCb+ALICE(LR)

LHCb

ATLAS+CMS+LHCb+ALICE(LR)

ATLAS+CMS

ATLAS+CMS+LHCb

LHC

Legend
- B1b0
- B1b922
- B1b816
- B1b995
- B1b1816
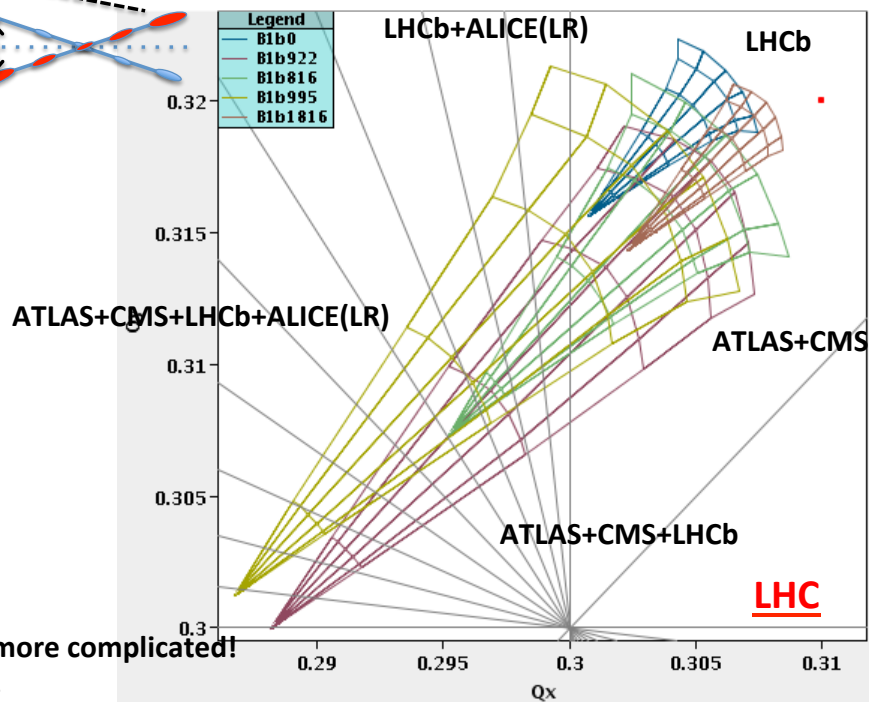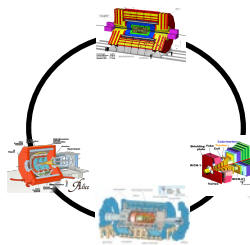
...operationally it is even more complicated!
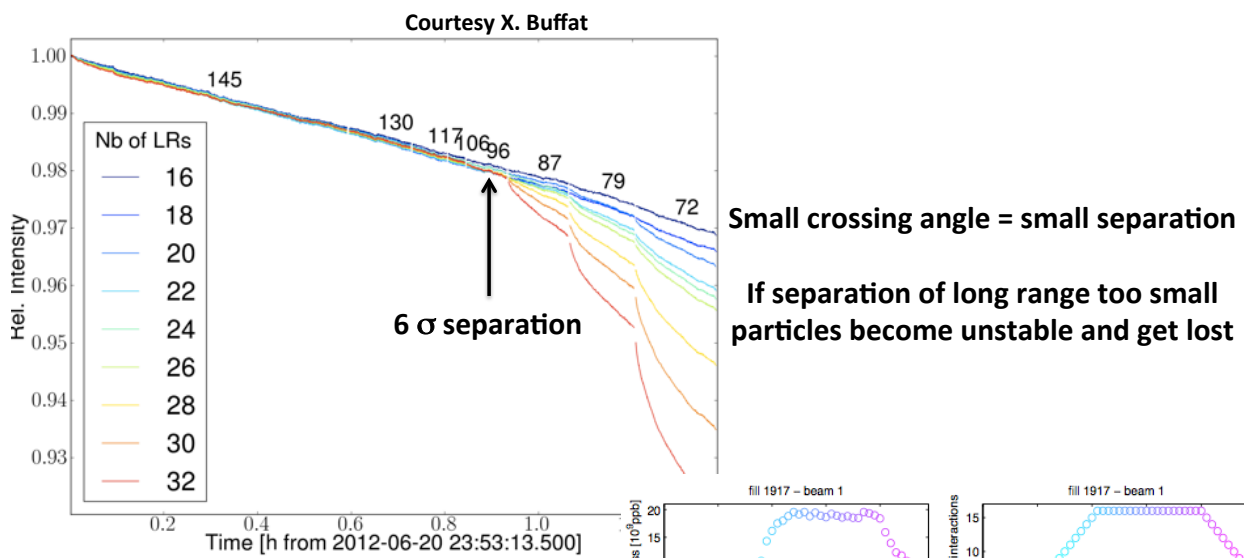...intensities, emittances...

Particle Losses

Dynamic Aperture: area in amplitude space with stable motion

Stable area of particles depends on beam intensity and crossing angle

$$F_{bb} \propto N_p$$

$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$

10-12 σ separation

Ref [6]
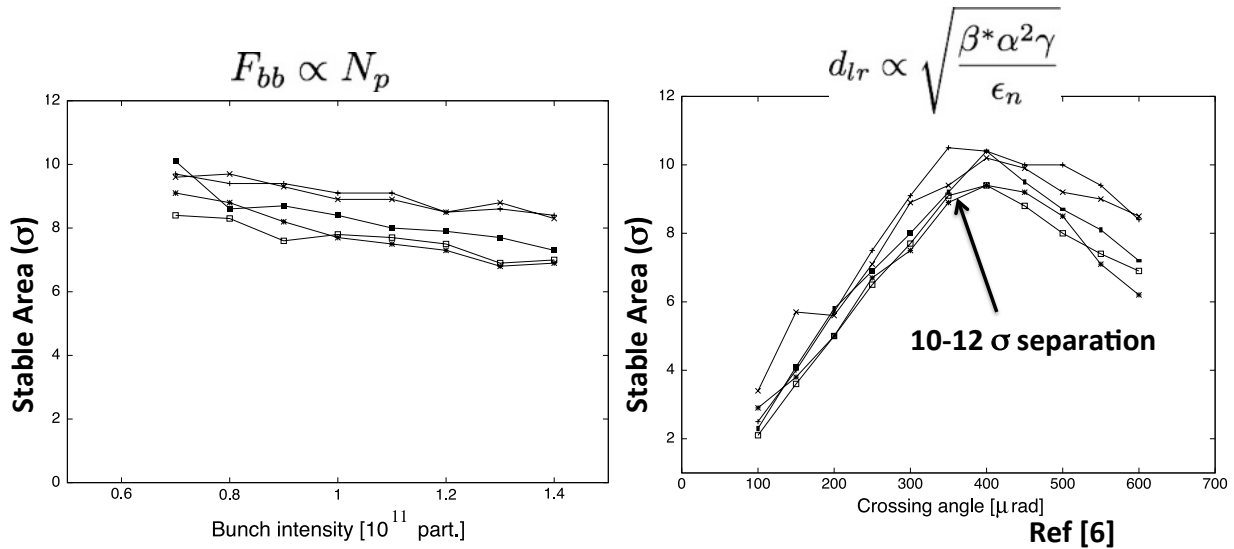
Stable area depends on beam-beam interactions therefore the choice of running parameters (crossing angles, β*, intensity) is the result of careful study of different effects!

DO we see the effects of LR in the LHC?

Courtesy X. Buffat

Small crossing angle = small separation

If separation of long range too small particles become unstable and get lost

6 σ separation

Courtesy G. Papotti

Particle losses follow number of Long range interactions

Nominal LHC will have twice the number of interactions

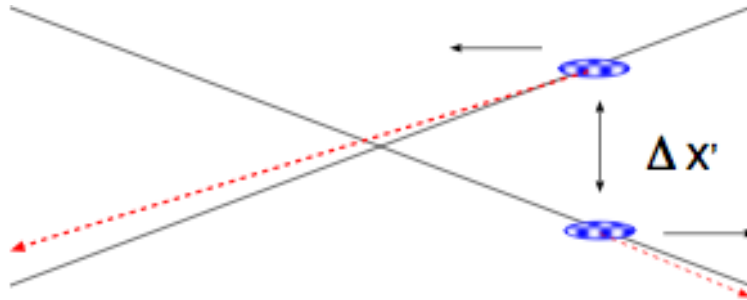# Long-range BB and Orbit Effects

Long Range Beam-beam interactions lead to orbit effects

Long range kick
$$\Delta x'(x+d,y,r) = -\frac{2Nr_0}{\gamma}\frac{(x+d)}{r^2}[1 - \exp(-\frac{r^2}{2\sigma^2})]$$

For well separated beams $\qquad d \gg \sigma$

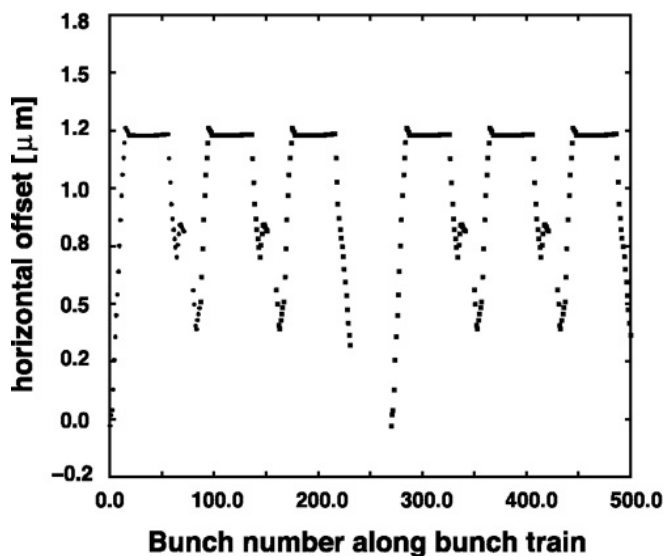**The force has an amplitude independent contribution: ORBIT KICK**

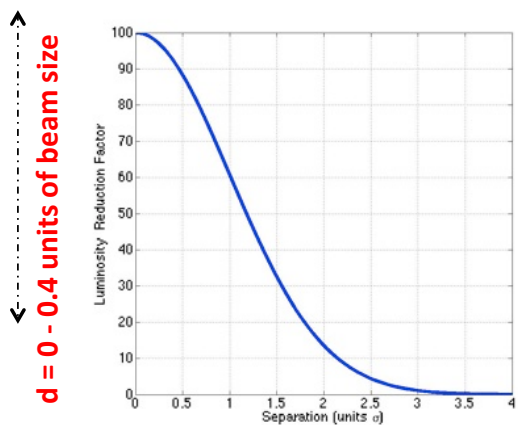$$\Delta x' = \frac{const}{d}[1 - \frac{x}{d} + O(\frac{x^2}{d^2}) + \dots]$$

$\Delta x'$

Orbit can be corrected but we should remember PACMAN effects

LHC orbit effects

Orbit effects different due to Pacman effects and the many long-range add up giving a non negligible effect
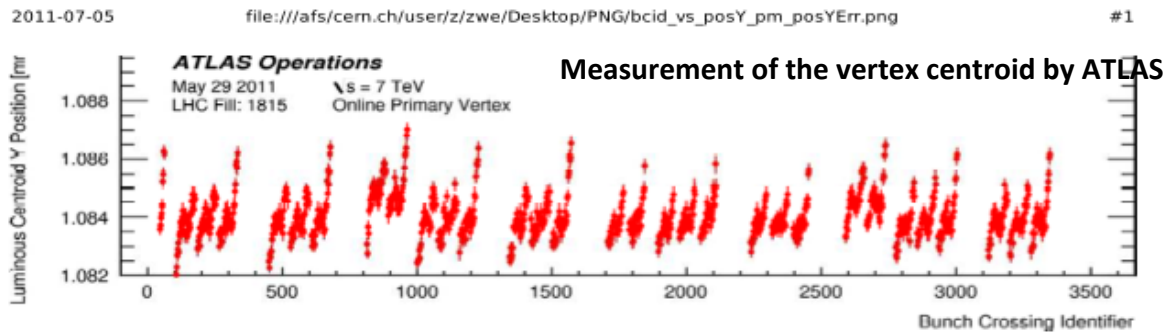
$$L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}}$$

horizontal offset [μm] vs Bunch number along bunch train

d = 0 - 0.4 units of beam size

Luminosity Reduction Factor vs Separation (units σ)

Ref [7]

Long range orbit effect

Long range interactions leads to orbit offsets at the experiment a direct consequence is deterioration of the luminosity
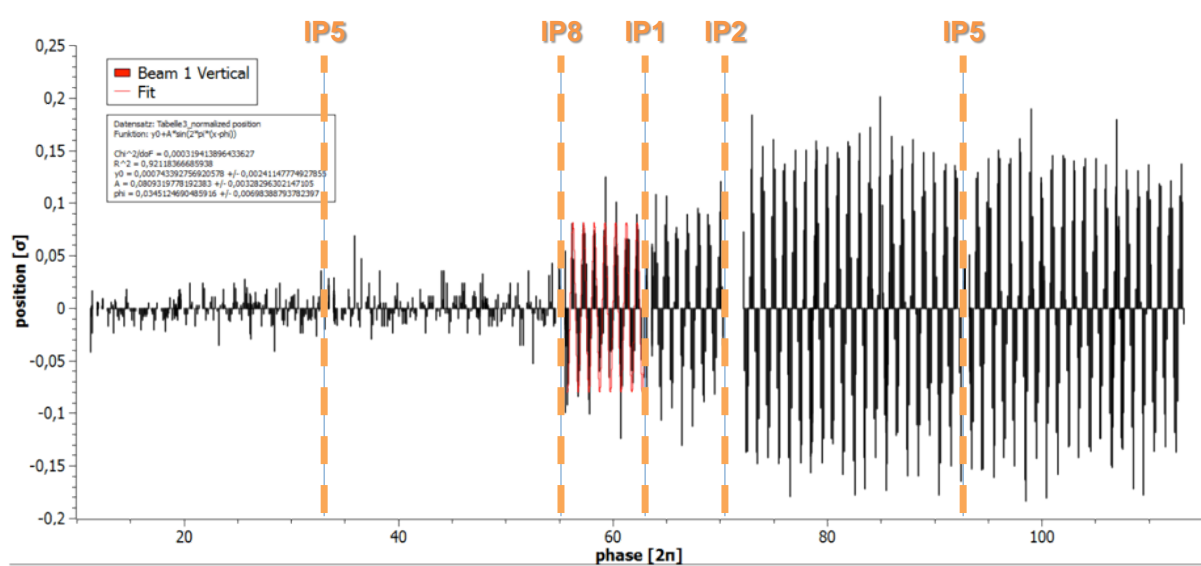

Measurement of the vertex centroid by ATLAS

Courtesy W. Kozanecki

Calculations for nominal LHC
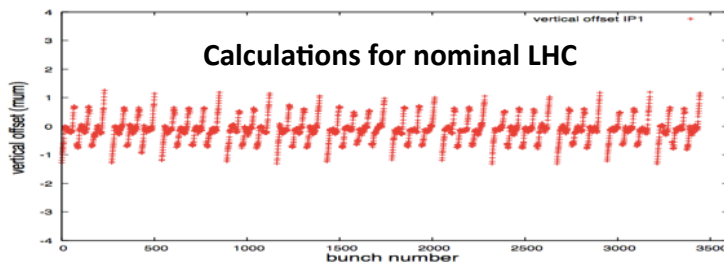
Effect is already visible with reduced number of interactions

Ref [7]

Long range orbit effect observations:

Courtesy T. Baer

Vertical oscillation starts when one beam is ejected and dumped

Coherent dipolar beam-beam modes

Coherent beam-beam effects arise from the forces which an exciting bunch exerts on a whole test bunch during collision

We study the collective behaviour of all particles of a bunch

Coherent motion requires an organized behaviour of all particles of the bunch

Coherent beam-beam force

- Beam distributions Ψ_1 **and** Ψ_2 mutually changed by interaction
- Interaction depends on distributions
    - Beam 1 Ψ_1 solution depends on beam 2 Ψ_2
    - Beam 2 Ψ_2 solution depends on beam 1 Ψ_1
- Need a self-consistent solution

Coherent beam-beam effects
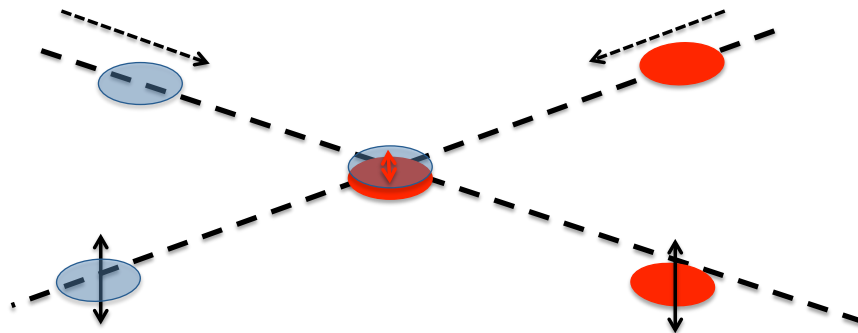
- Whole bunch sees a kick as an entity (coherent kick)
- Coherent kick seen by full bunch different from single particle kick
- Requires integration of individual kick over particle distribution

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{4\sigma^2}}\right]$$

- Coherent kick of separated beams can excite coherent dipolar oscillations
- All bunches couple because each bunch "sees" many opposing bunches(LR): many coherent modes possible!
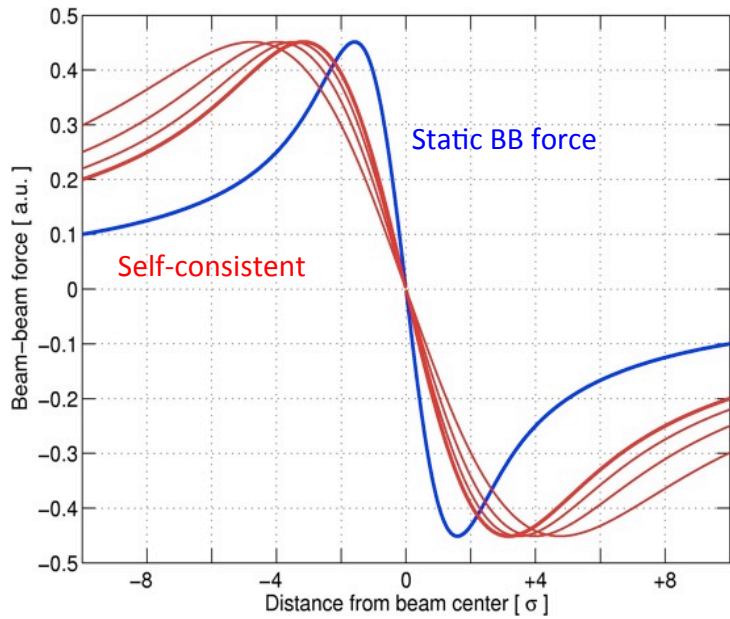
Coherent effects
Self-consistent treatment needed

Perturbative methods

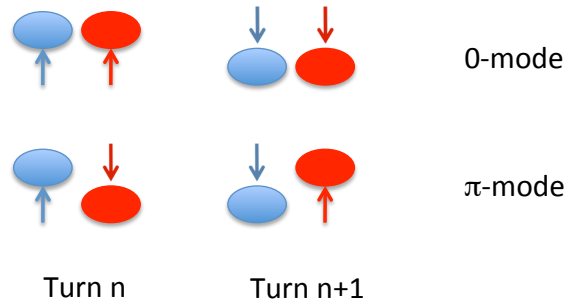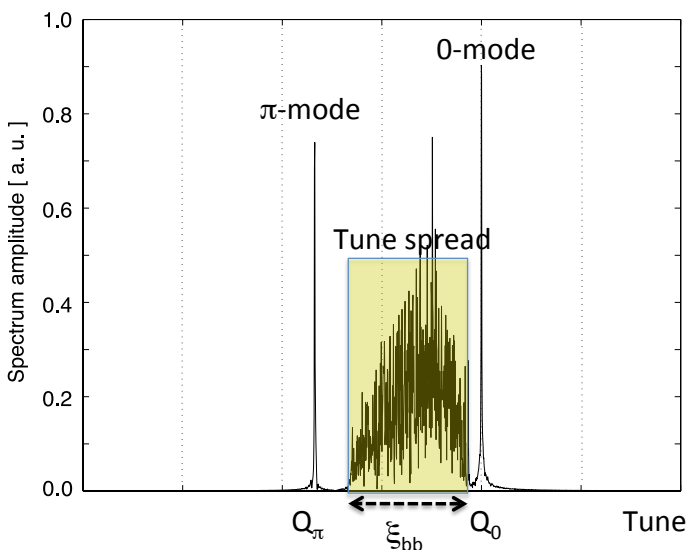static source of distortion:
example magnet

Self-consistent method

source of distortion changes
as a result of the distortion

For a complete understanding of BB effect a self-consistent treatment should be used
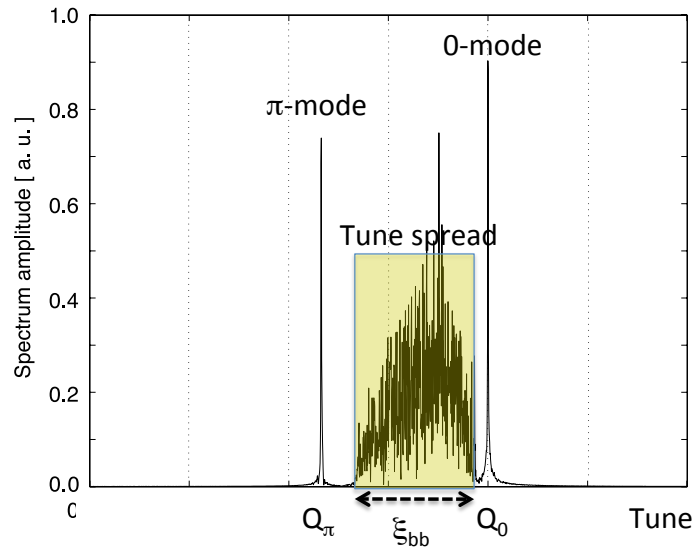

Simple case:
one bunch per beam

0-mode

π-mode

Turn n   Turn n+1

MOVIE

0-mode at unperturbed tune Q_0

π-mode is shifted at $Q_\pi = 1.1\text{-}1.3\ \xi_{bb}$

Incoherent tune spread range $[0, -\xi]$

$$\Delta Q = Y \cdot \xi$$

- Coherent mode: two bunches are "locked" in a coherent oscillation
- 0-mode is stable (mode with NO tune shift)
- π-mode can become unstable (mode with largest tune shift)
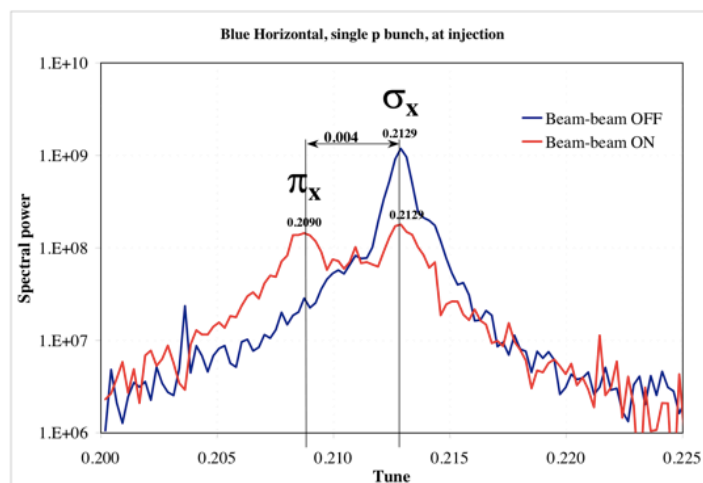
Simple case: one bunch per beam and Landau damping

Incoherent tune spread is the Landau damping region any mode with frequency laying in this range should not develop
• π-mode has frequency out of tune spread (Y) so it is not damped!
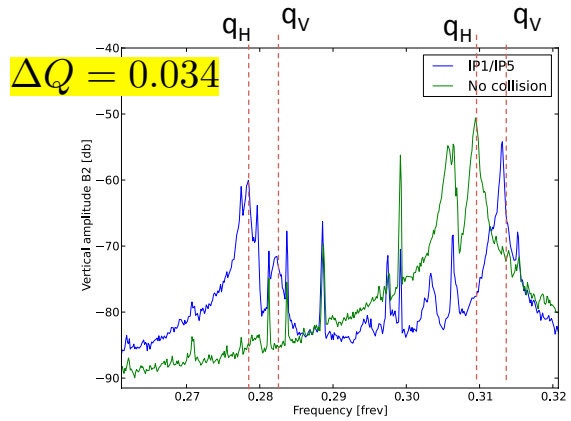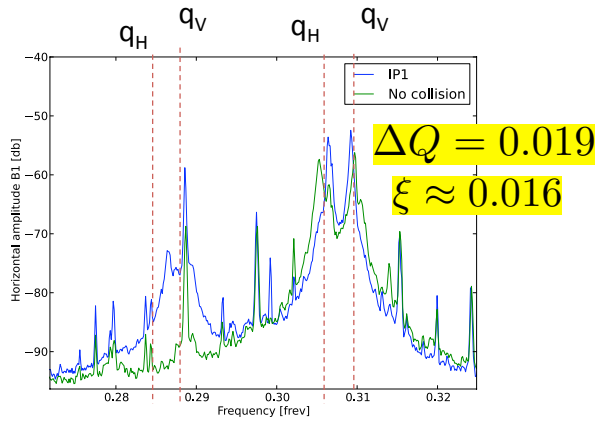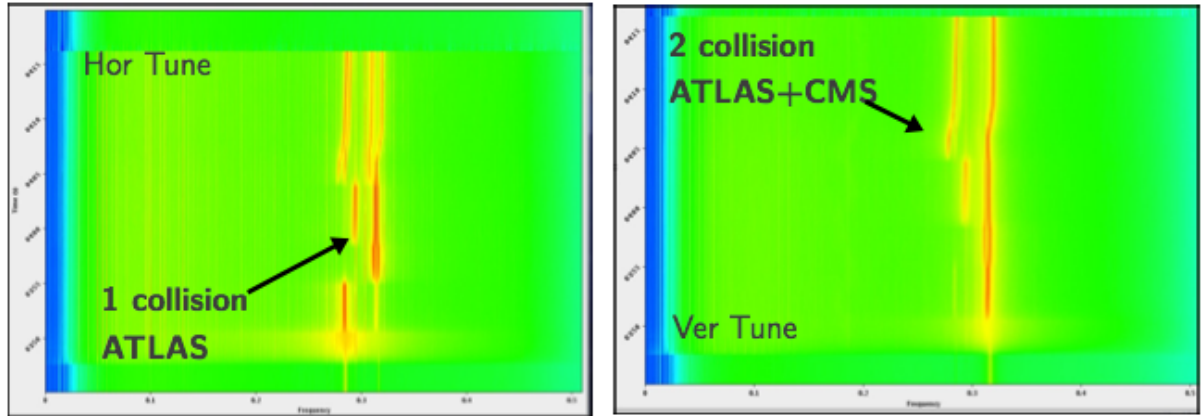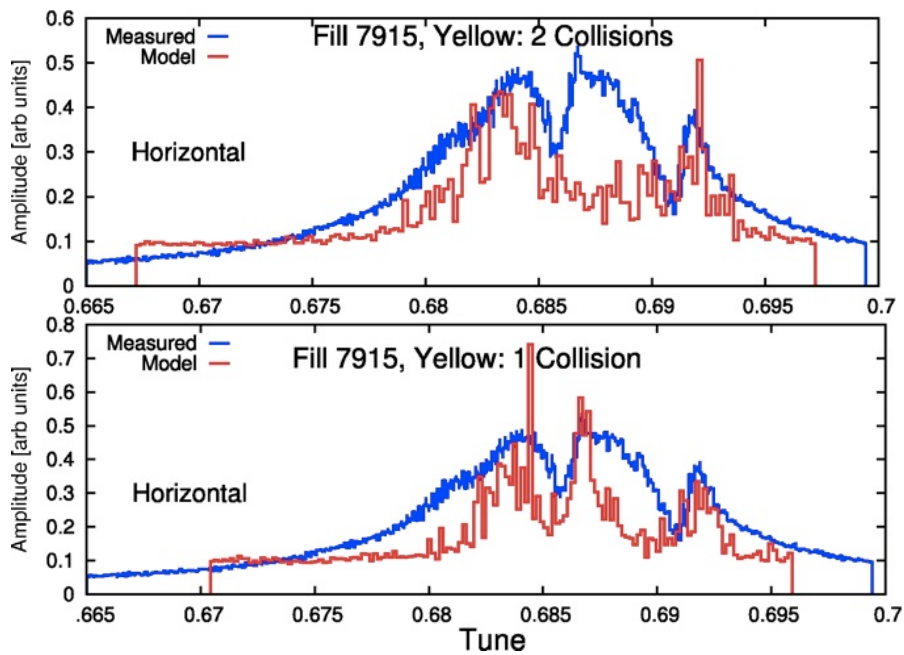
Coherent modes at RHIC

Courtesy W. Fischer (BNL)

Tune spectra before collision and in collision two modes visible
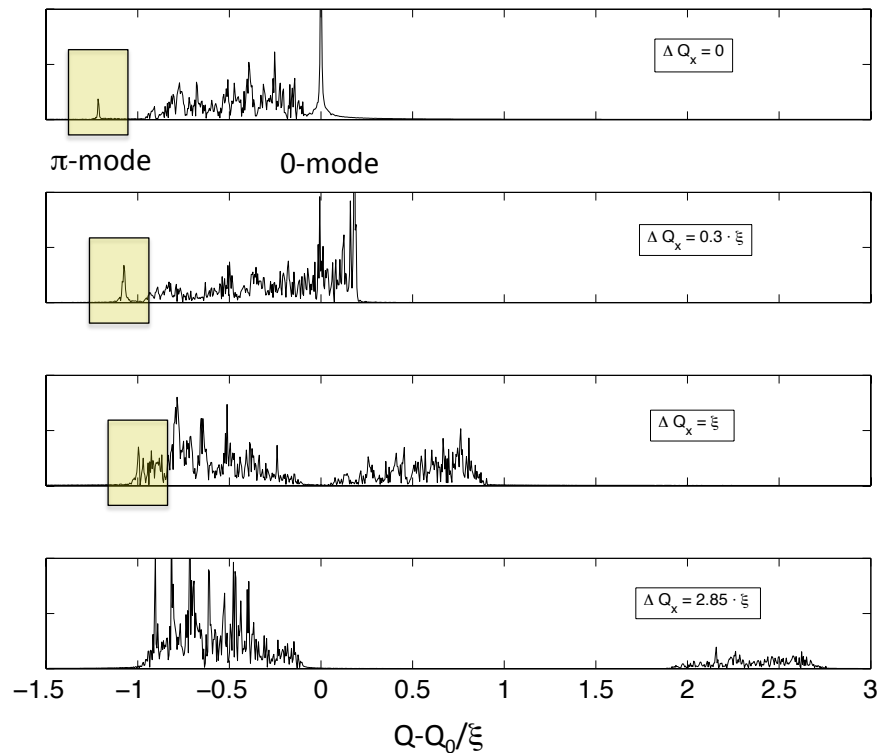
Head-on beam-beam coherent mode: LHC

$$\Delta Q = 0.019$$
$$\xi \approx 0.016$$

$$\Delta Q = 0.034$$

Beam-beam coherent modes and Landau Damping

Pacman effect on coherent modes
Single bunch diagnostic so important

Different Tunes

π-mode   0-mode

$\Delta Q_x = 0$

$\Delta Q_x = 0.3 \cdot \xi$

$\Delta Q_x = \xi$

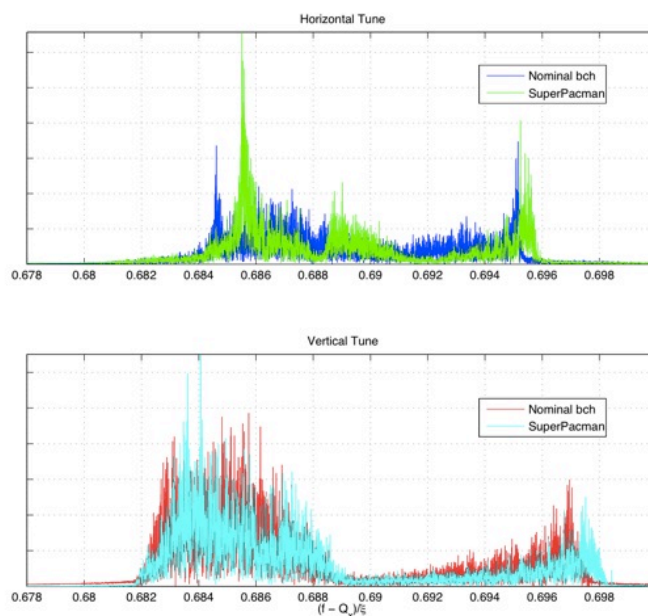$\Delta Q_x = 2.85 \cdot \xi$

$Q - Q_0 / \xi$

Tune split breaks symmetry and coherent modes disappear

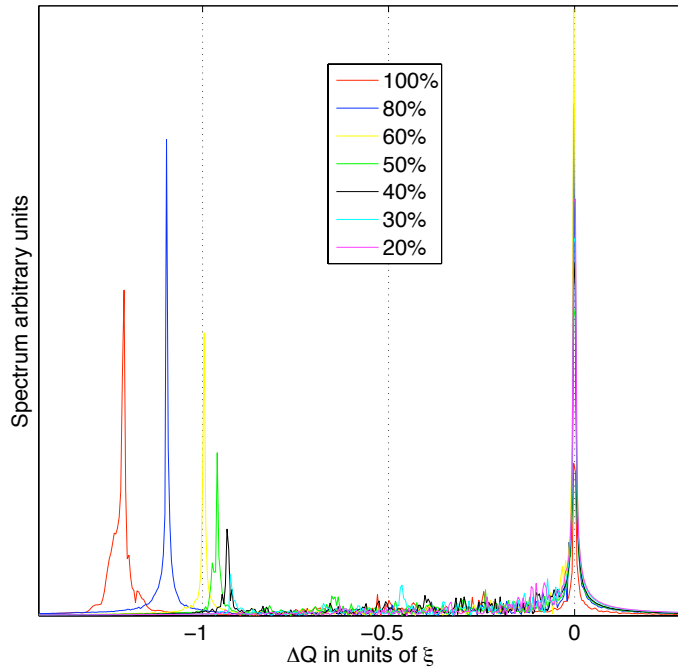Analytical calculations in Reference [8]

Different tunes or intensities

RHIC running with mirrored tune for years to break coherent oscillations

LHC has used a tune split to suppress coherent BB modes
**2010 Physics Run**

Different bunch intensities

For two bunches colliding head-on in one IP the coherent mode disappears if intensity ratio between bunches is 55% **Reference[9]**

We assumed:
- equal emittances
- equal tunes
- NO PACMAN effects
(bunches will have different tunes)

For coherent modes the key is to break the simmetry in your coupled system…(tunes, intensities, collision patters…)
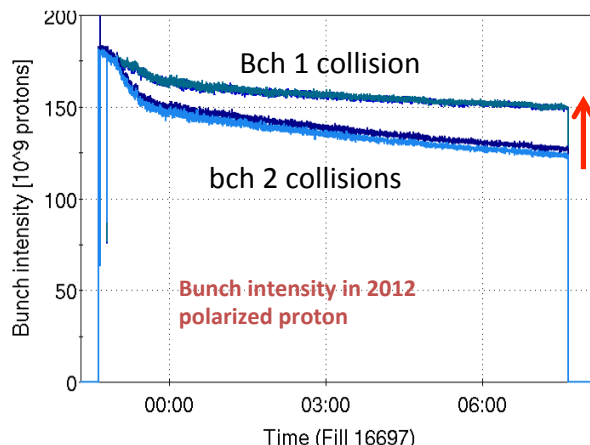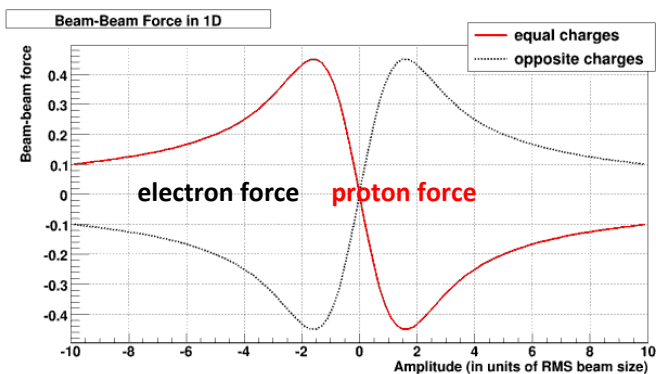
And Long range interactions?
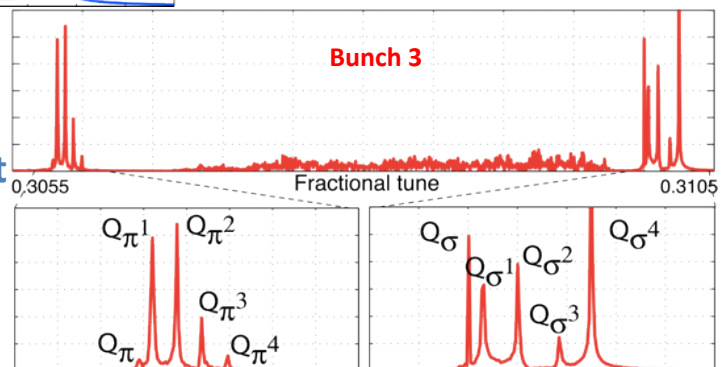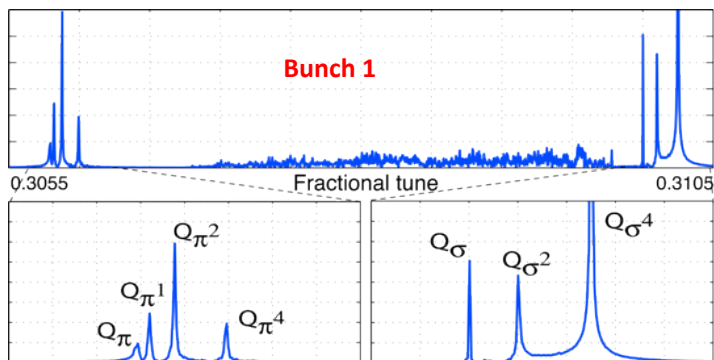
- **Each bunch will have different number of modes and tune spectra**
- **No Landau damping of long-range coherent modes**

Single bunch diagnostic can make the difference

Beam-beam compensations:

Head-on

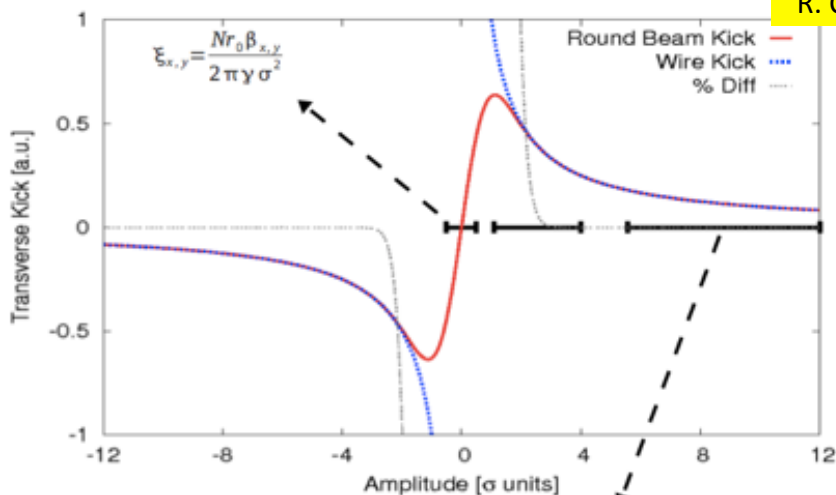- Linear e-lens, suppress shift
- Non-linear e-lens, suppress tune spread

- Past experience: at Tevatron linear and non-linear e-lenses, also hollow…
- Present: test for half compensation at RHIC with non-linear e-lens

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Beam-beam compensations: long-range

Beam-beam wire compensation

$$\xi_{x,y} = \frac{N r_0 \beta_{x,y}}{2 \pi \gamma \sigma^2}$$

$$\sigma << d: \qquad \Delta x'(x,d) = -\frac{K}{d} \cdot \left(1 + \frac{x}{d} + \frac{x^2}{d^2} + \dots\right)$$

- Past experience: at RHIC several tests till 2009…
- Present: simulation studies on-going for possible use in HL-LHC…

…not covered here…

- *Linear colliders special issues*
- *Asymmetric beams effects*
- *Coasting beams*
- *Beamstrahlung*
- *Synchrobetatron coupling*
- *Beam-beam experiments*
- *Beam-beam and impedance*
- *…*

Thank You!