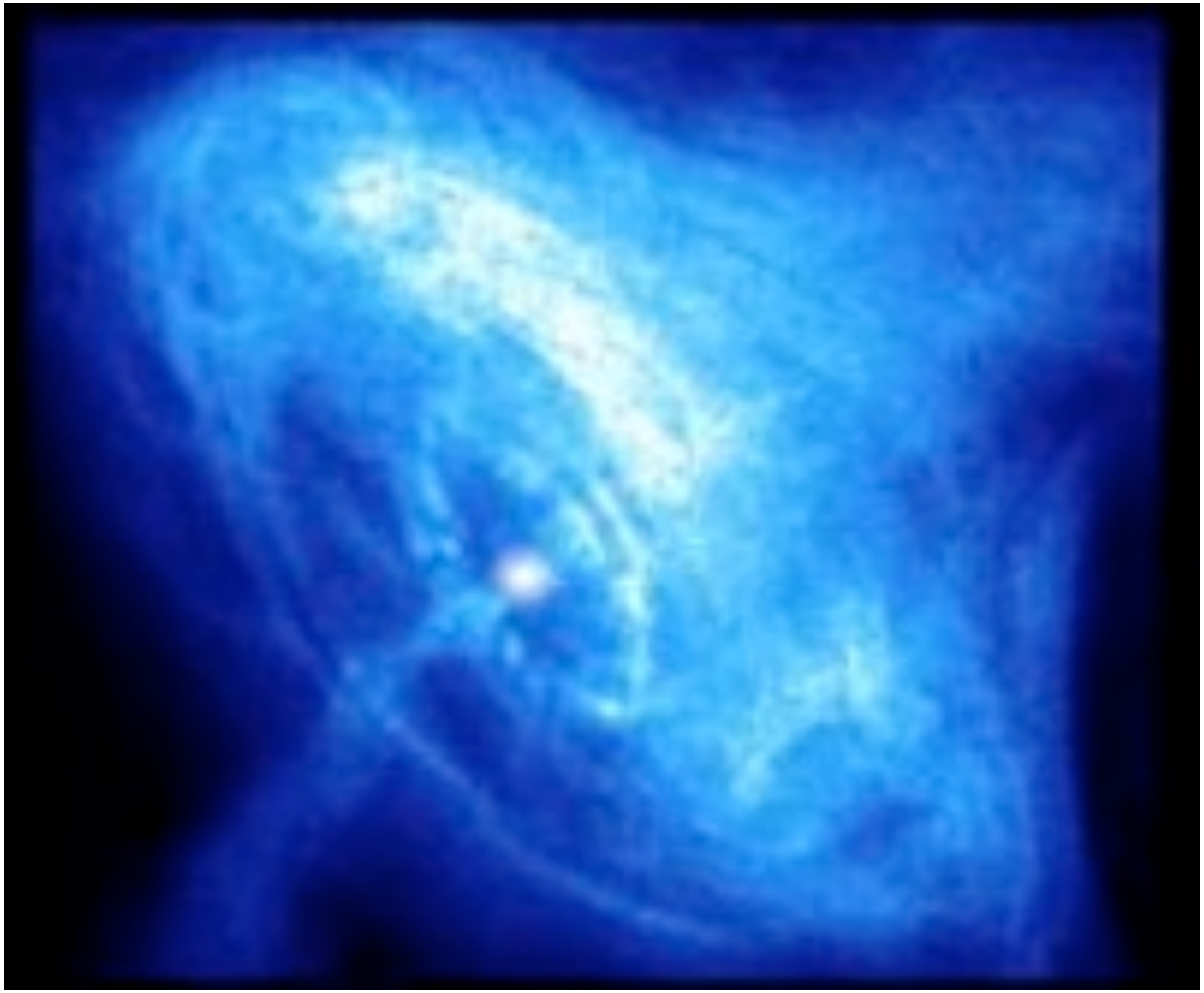
Transverse Beam Dynamics III

12) … let´s talk about acceleration

Errors in Field and Gradient:
Liouville during Acceleration
The Δp/p ≠0 problem
Dispersion
Chromaticity



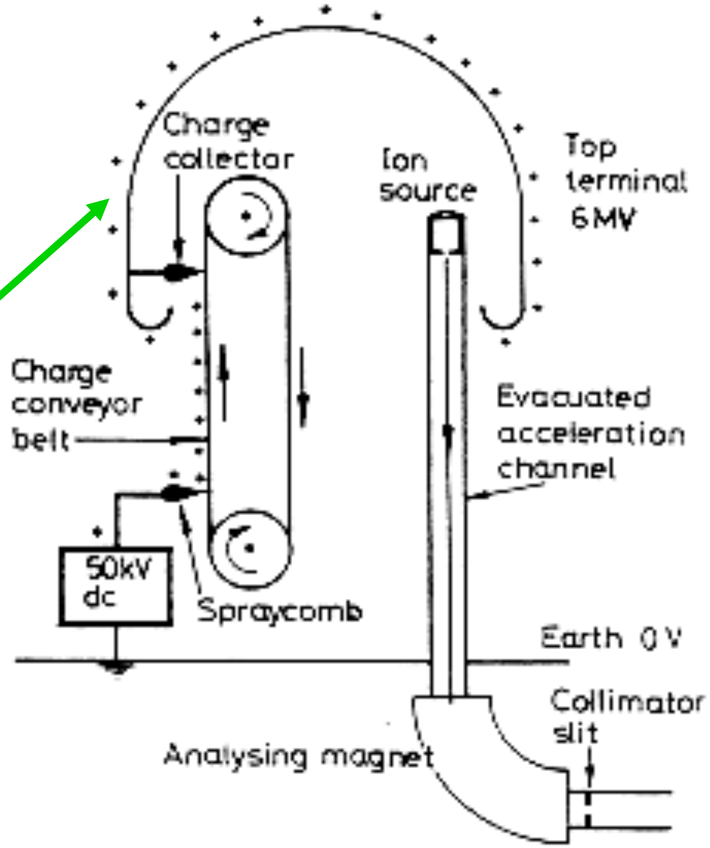
crab nebula,

burst of charged particles E = 10^{20} eV

Electrostatic Machines

(Tandem –) van de Graaff Accelerator

creating high voltages by mechanical transport of charges

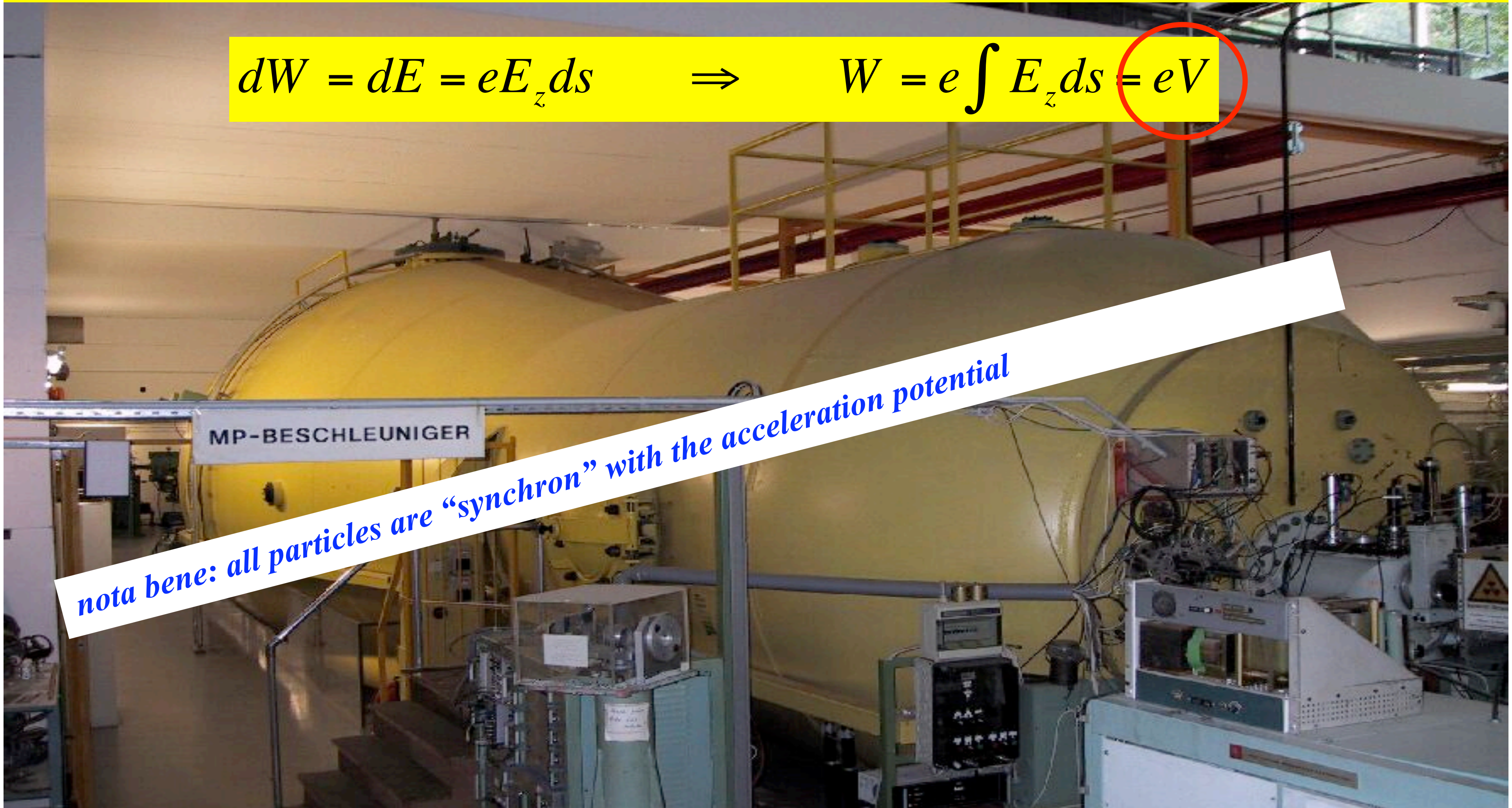


** Terminal Potential: U ≈ 12 ...28 MV*
using high pressure gas to suppress discharge (SF_6)

*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

The „Tandem principle“: Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$

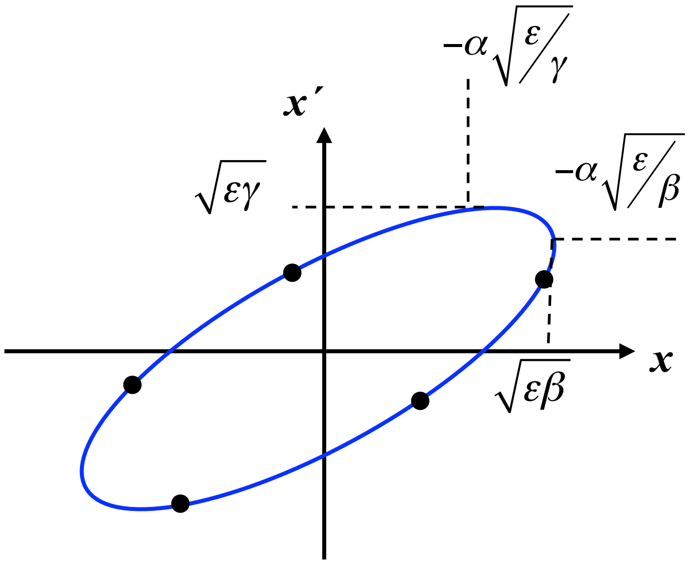


nota bene: all particles are "synchron" with the acceleration potential

Electro Static Accelerator: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

13.) Liouville during Acceleration

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$$



***Beam Emittance** corresponds to the area covered in the x, x´ Phase Space Ellipse*

***Liouville:** Area in phase space is constant.*

But so sorry ... $\varepsilon \neq const$!

Classical Mechanics:

phase space = diagram of the two canonical variables
position & momentum
x p_x

$$p_j = \frac{\partial L}{\partial \dot{q}_j} \quad ; \quad L = T - V = kin.\,Energy - pot.\,Energy$$

According to Hamiltonian mechanics:
phase space diagram relates the variables q and p

$$q = position = x$$
$$p = momentum = \gamma m v = m c \gamma \beta_x$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad ; \quad \beta_x = \frac{\dot{x}}{c}$$

Liouvilles Theorem: $\int p\,dq = const$

for convenience (i.e. because we are lazy bones) we use in accelerator theory:

$$x' = \frac{dx}{ds} = \frac{dx}{dt}\frac{dt}{ds} = \frac{\beta_x}{\beta}$$

where $\beta_x = v_x / c$

$$\int p\,dq = mc\int \gamma \beta_x dx$$

$$\int p\,dq = mc\gamma\beta \underbrace{\int x' dx}_{\varepsilon}$$

$$\Rightarrow \quad \varepsilon = \int x' dx \propto \frac{1}{\beta\gamma}$$

the beam emittance shrinks during acceleration $\varepsilon \sim 1/\gamma$

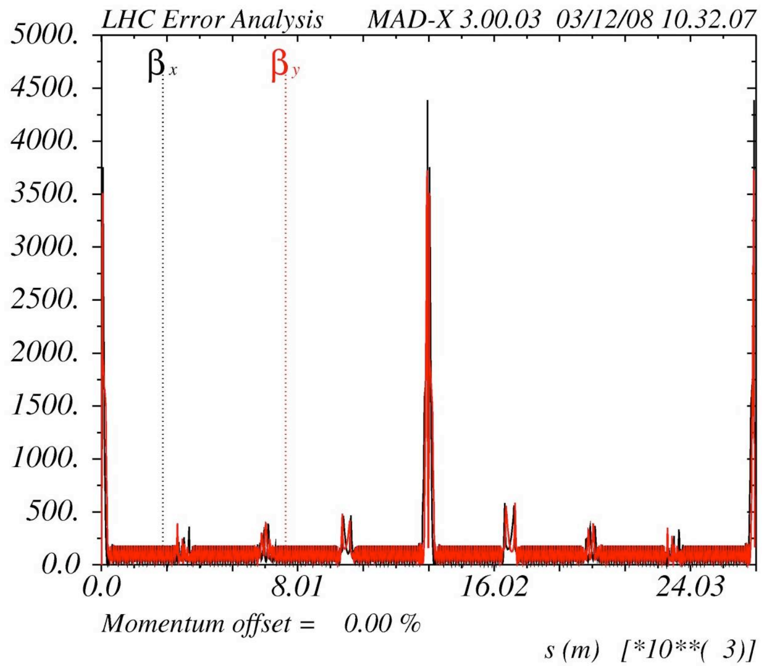
Nota bene:

1.) A proton machine … or an electron linac … needs the highest aperture at injection energy !!!
as soon as we start to accelerate the beam size shrinks as $\gamma^{-1/2}$ in both planes.

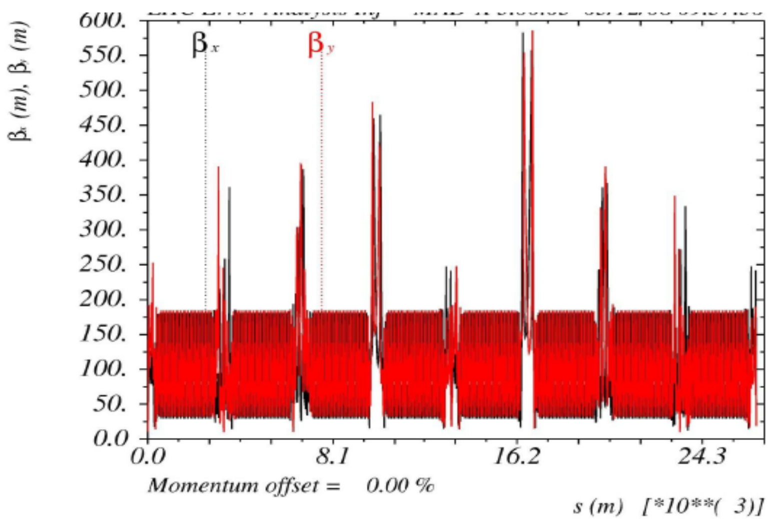
$$\sigma = \sqrt{\varepsilon\beta}$$

2.) At lowest energy the machine will have the major aperture problems,
→ here we have to minimise $\hat{\beta}$

3.) we need different beam optics adopted to the energy:
A Mini Beta concept will only be adequate at flat top.



LHC mini beta optics at 7000 GeV

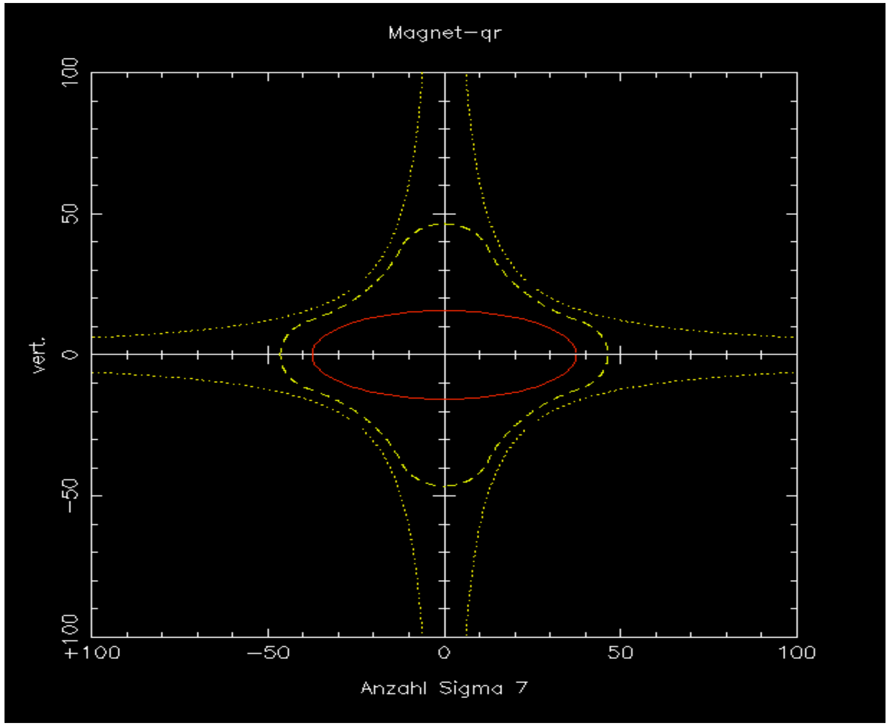


LHC injection optics at 450 GeV

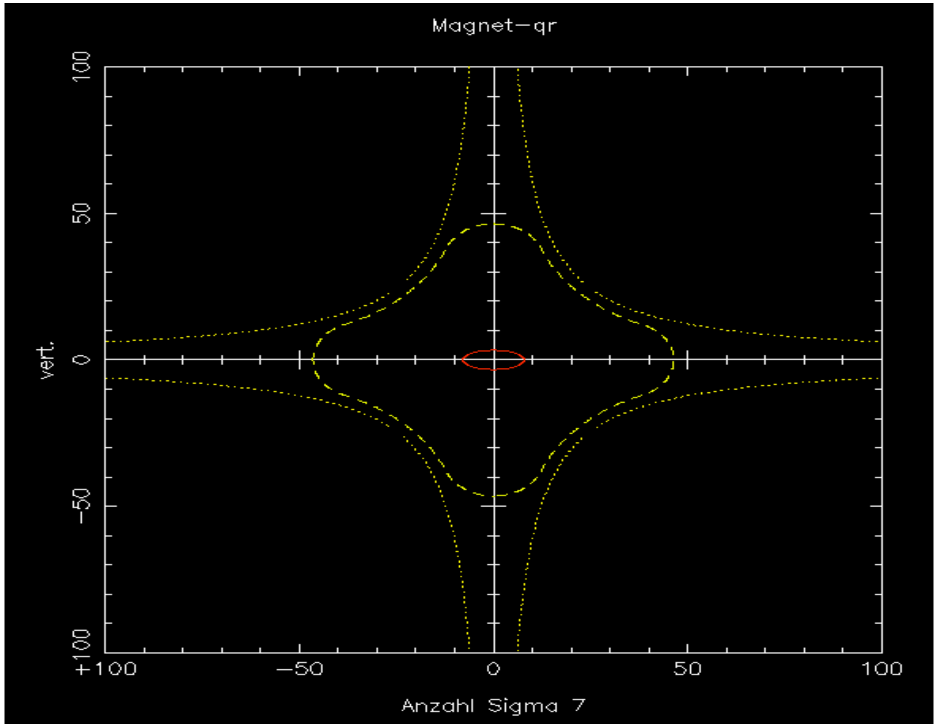
Example: HERA proton ring

injection energy: 40 GeV $\gamma = 43$
flat top energy: 920 GeV $\gamma = 980$

emittance ε *(40GeV)* $= 1.2 * 10^{-7}$
ε *(920GeV)* $= 5.1 * 10^{-9}$



7 σ beam envelope at E = 40 GeV



… and at E = 920 GeV

The „ not so ideal world “

14.) The „ $\Delta p / p \neq 0$" Problem

ideal accelerator: ***all particles will see the same accelerating voltage.***

→ $\Delta p / p = 0$

„nearly ideal“ accelerator: Cockroft Walton or van de Graaf

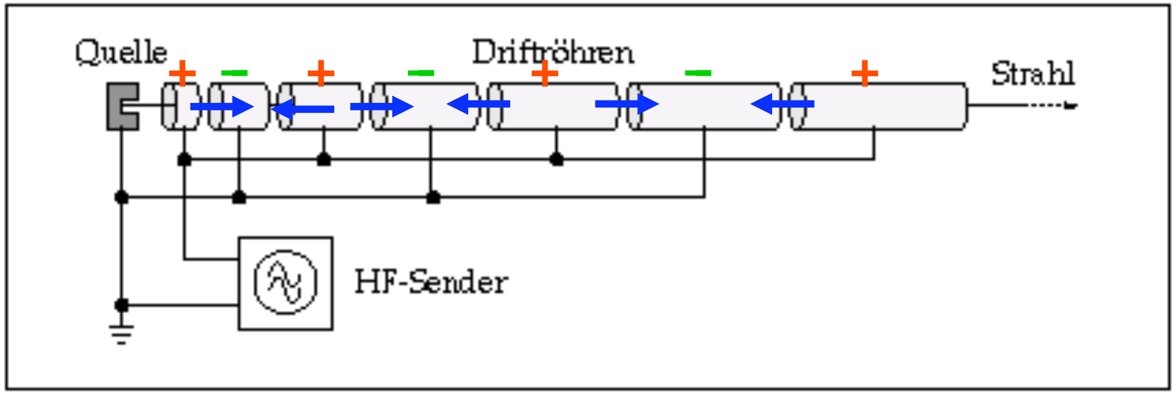
$\Delta p / p \approx 10^{-5}$

Vivitron, Straßbourg, inner structure of the acc. section

MP Tandem van de Graaf Accelerator at MPI for Nucl. Phys. Heidelberg

RF Acceleration

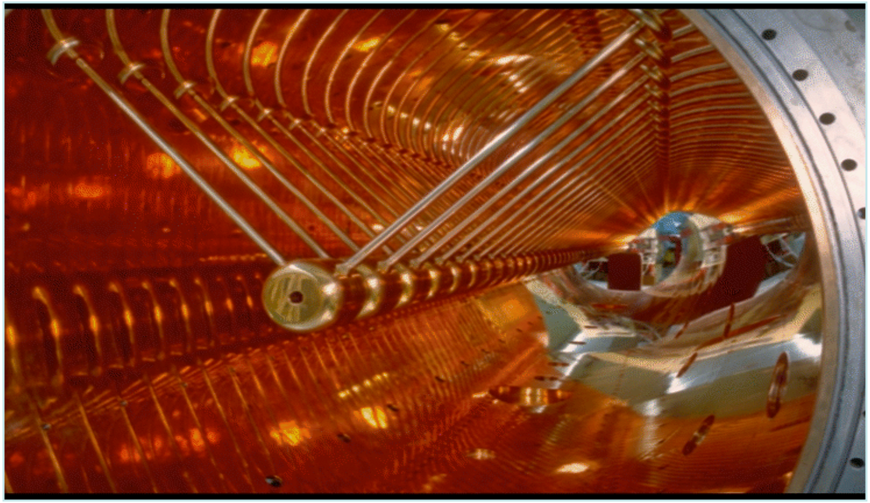
1928, Wideroe



Energy Gain per „Gap":

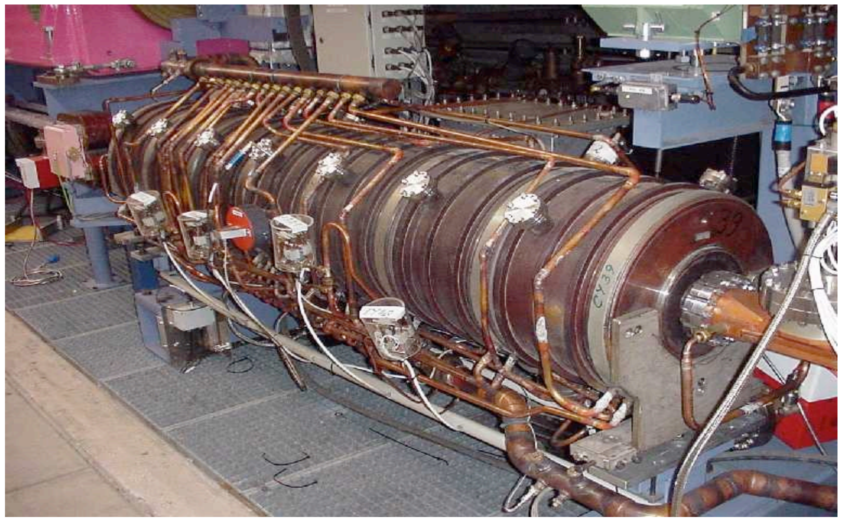
$$W = n * q U_0 \sin\omega_{RF} t$$

drift tube structure at a proton linac (GSI Unilac)



n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

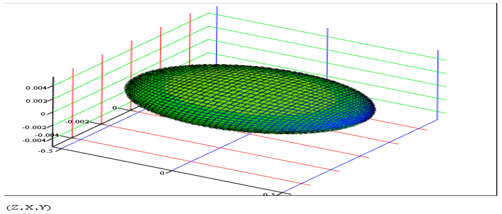
500 MHz cavities in an electron storage ring



* *RF Acceleration:* ***multiple application of the same acceleration voltage; brillant idea to gain higher energies***

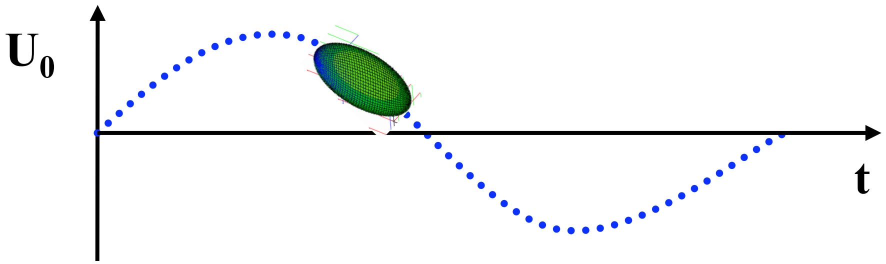
RF Acceleration-Problem: panta rhei !!!

(Heraklit: 540-480 v. Chr.)



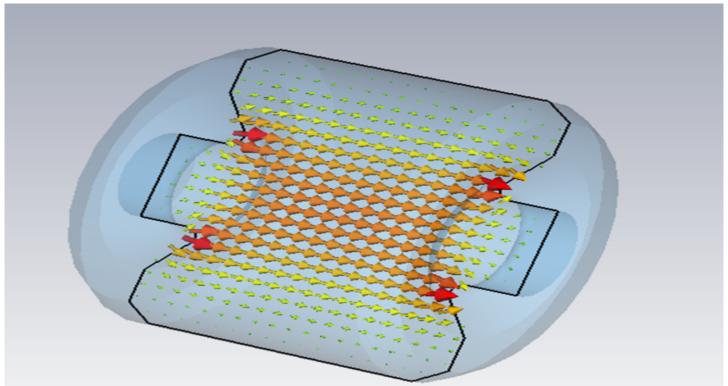
Bunch length of Electrons ≈ 1cm

just a stupid (and nearly wrong) example)



$$\lambda = 75\ cm$$

$$\left.\begin{array}{l} \nu = 400\,MHz \\ c = \lambda\,\nu \end{array}\right\} \quad \lambda = 75\ cm$$



$$\sin(90^o) = 1$$

$$\sin(84^o) = 0.994$$

$$\frac{\Delta U}{U} = 6.0\ \ 10^{-3}$$

typical momentum spread of an electron bunch:

$$\frac{\Delta p}{p} \approx 1.0\ \ 10^{-3}$$

Dispersive and Chromatic Effects: $\Delta p/p \neq 0$

Are there any Problems ???

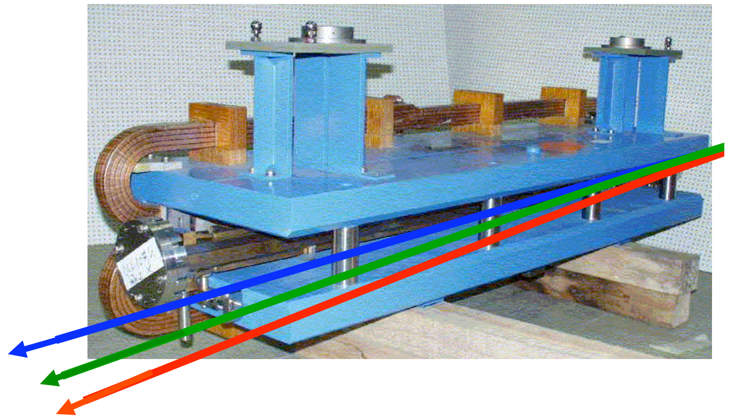
Sure there are !!!

font colors due to pedagogical reasons

15.) Dispersion and Chromaticity:
Magnet Errors for Δp/p ≠ 0

Influence of external fields on the beam: *prop. to magn. field & prop. zu 1/p*

dipole magnet $\alpha = \frac{\int B\,dl}{p/e}$



$$x_D(s) = D(s)\,\frac{\Delta p}{p}$$

focusing lens $k = \frac{g}{p/e}$

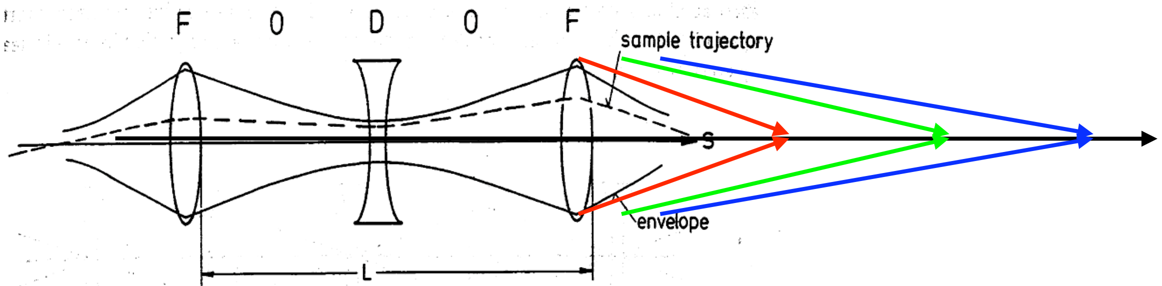


Figure 29: FODO cell

particle having ...

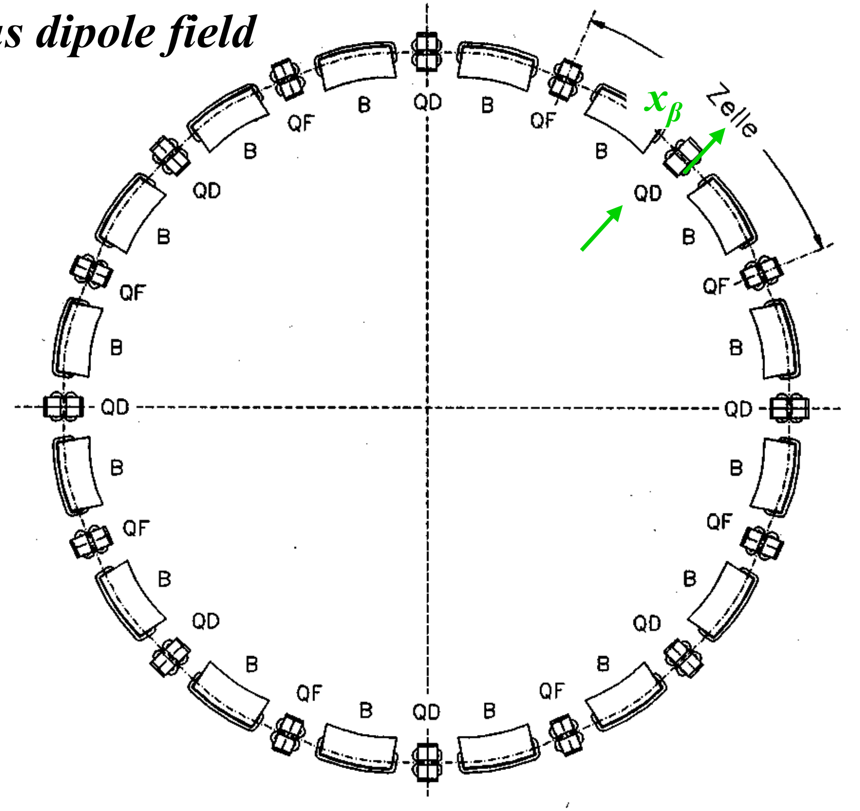
to high energy

to low energy

ideal energy

Dispersion

Example: homogeneous dipole field



ıit for Δp/p > 0

$: D(s) \cdot \frac{\Delta p}{p}$

Matrix formalism:

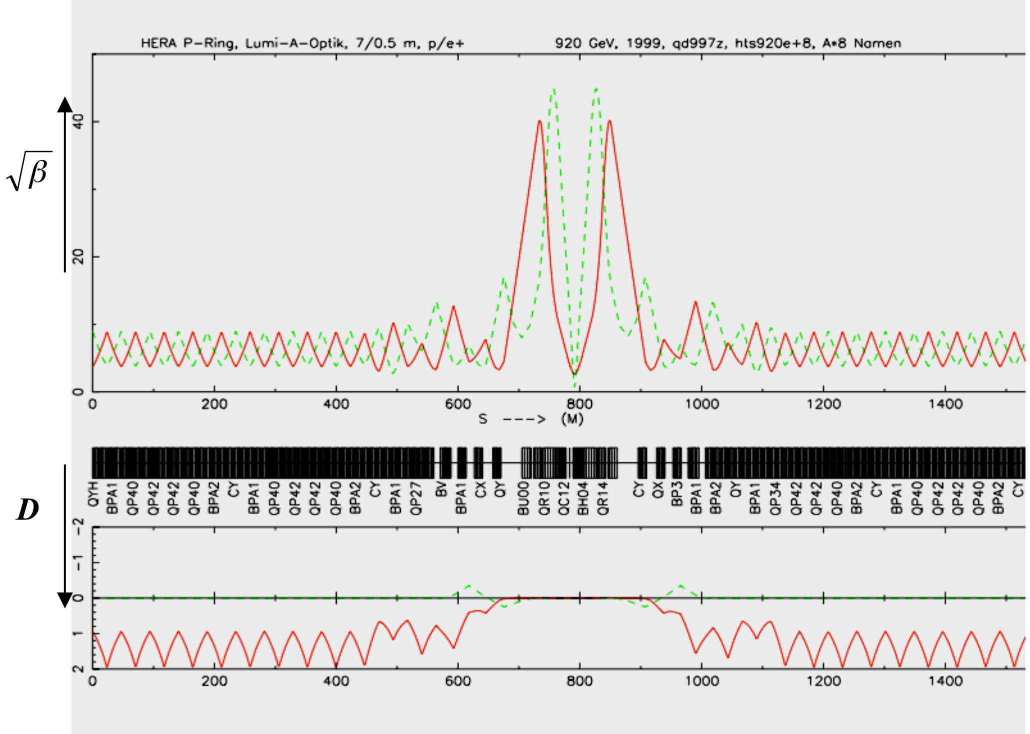
$$x(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}$$

$$x(s) = C(s) \cdot x_0 + S(s) \cdot x_0' + D(s) \cdot \frac{\Delta p}{p}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0 + \frac{\Delta p}{p} \begin{pmatrix} D \\ D' \end{pmatrix}_0$$

or expressed as 3x3 matrix

$$\begin{pmatrix} x \\ x' \\ \Delta p / p \end{pmatrix}_s = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ \Delta p / p \end{pmatrix}_0$$



Example

$$x_\beta = 1 \dots 2\ mm$$
$$D(s) \approx 1 \dots 2\ m$$
$$\Delta p / p \approx 1 \cdot 10^{-3}$$

Amplitude of Orbit oscillation

contribution due to Dispersion ≈ beam size

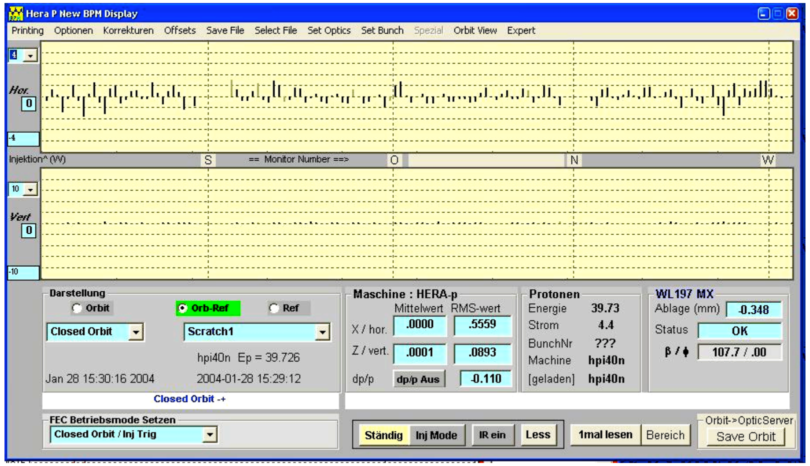
→ Dispersion must vanish at the collision point

!

Calculate D, D´: ... takes a couple of sunny Sunday evenings !

$$D(s) = S(s)\int_{s0}^{s1} \frac{1}{\rho} C(\tilde{s})\,d\tilde{s} - C(s)\int_{s0}^{s1} \frac{1}{\rho} S(\tilde{s})\,d\tilde{s}$$

Dispersion is visible



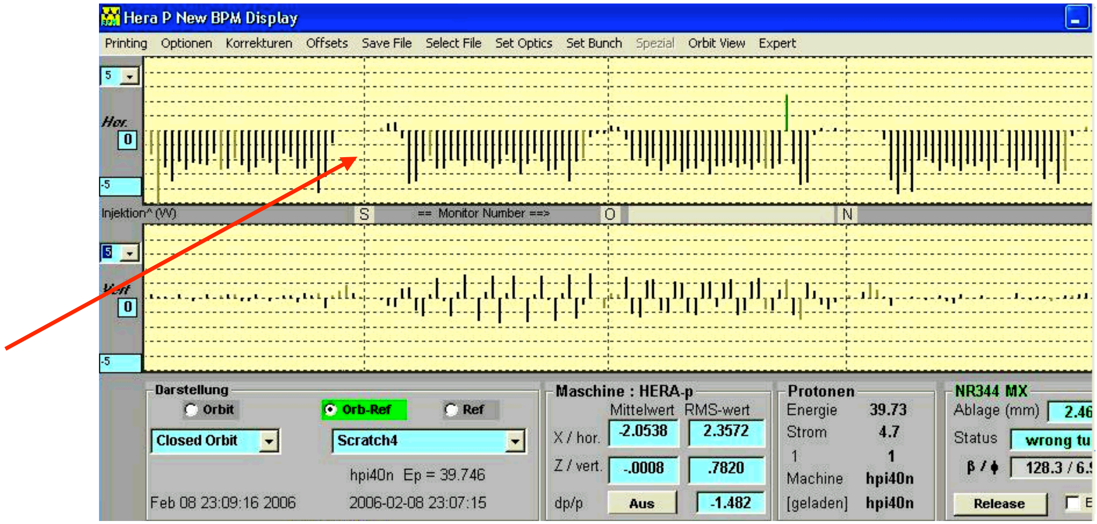
HERA Standard Orbit

HERA Dispersion Orbit

dedicated energy change of the stored beam

→ closed orbit is moved to a dispersions trajectory

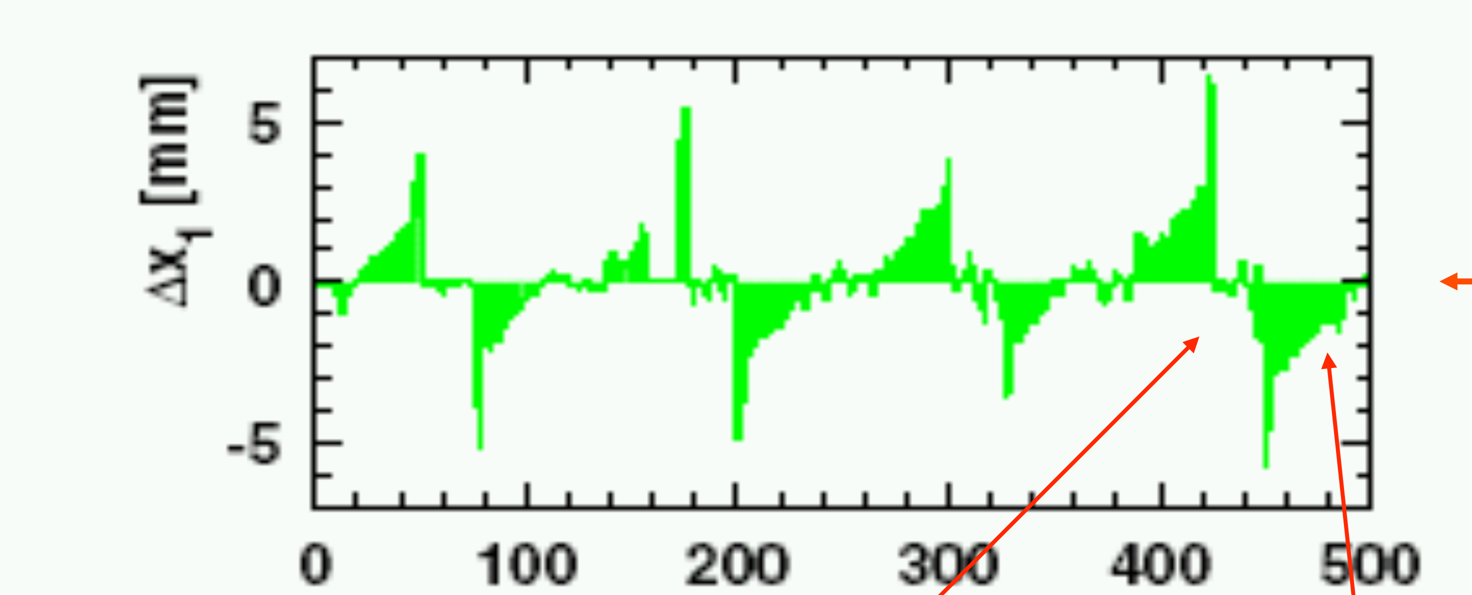
$$x_D = D(s) * \frac{\Delta p}{p}$$



Attention: at the Interaction Points we require D=D′= 0

Periodic Dispersion:

„Sawtooth Effect“ at LEP (CERN)



Electron course

BPM Number

In the straight sections they are accelerated by the rf cavities so much that they „overshoot“ and reach nearly the outer side of the vacuum chamber.

In the arc the electron beam loses so much energy in each octant that the particle are running more and more on a dispersion trajectory.

16.) Chromaticity:
A Quadrupole Error for Δp/p ≠ 0

Influence of external fields on the beam: *prop. to magn. field & prop. zu 1/p*

focusing lens $k = \frac{g}{p/e}$

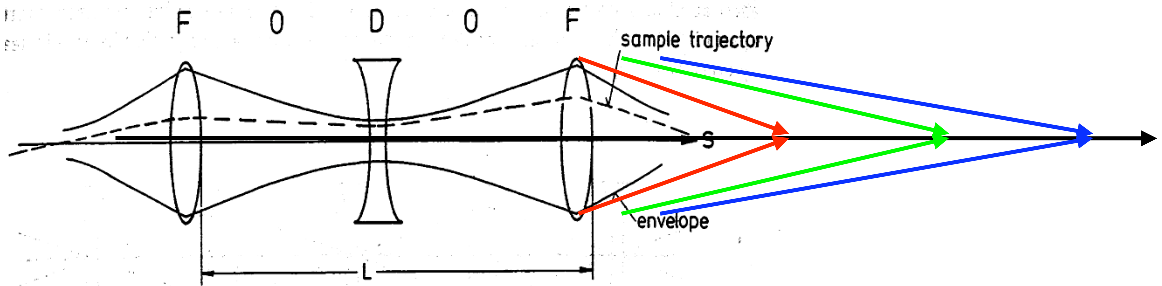


Figure 29: FODO cell

particle having ...
to high energy
to low energy
ideal energy

... which acts like a quadrupole error in the machine
and leads to a tune spread:

$$\Delta Q = -\frac{1}{4\pi}\frac{\Delta p}{p_0}k_0\beta(s)ds$$

definition of chromaticity:

$$\Delta Q = Q' * \frac{\Delta p}{p}$$

… what is wrong about Chromaticity:

Problem: chromaticity is generated by the lattice itself !!

Q' is a number indicating the size of the tune spot in the working diagram,
Q' is always created if the beam is focussed
→ it is determined by the focusing strength k of all quadrupoles

$$Q' = -\frac{1}{4\pi}\oint k(s)\beta(s)\,ds$$

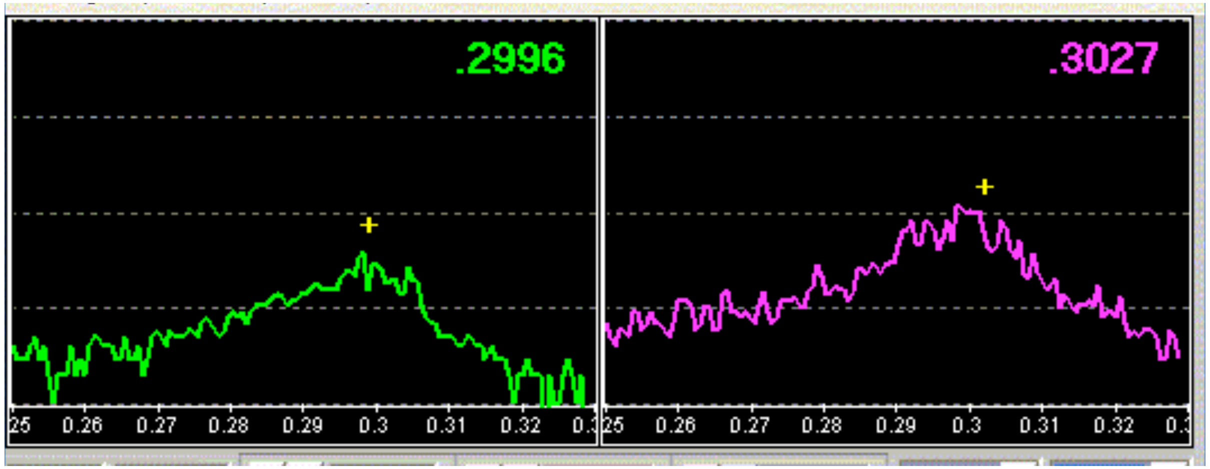
k = quadrupole strength
β = betafunction indicates the beam size … and even more the sensitivity of the beam to external fields

Example: LHC

$Q' = 250$
$\Delta p/p = +/- 0.2 * 10^{-3}$
$\Delta Q = 0.256 \dots 0.36$

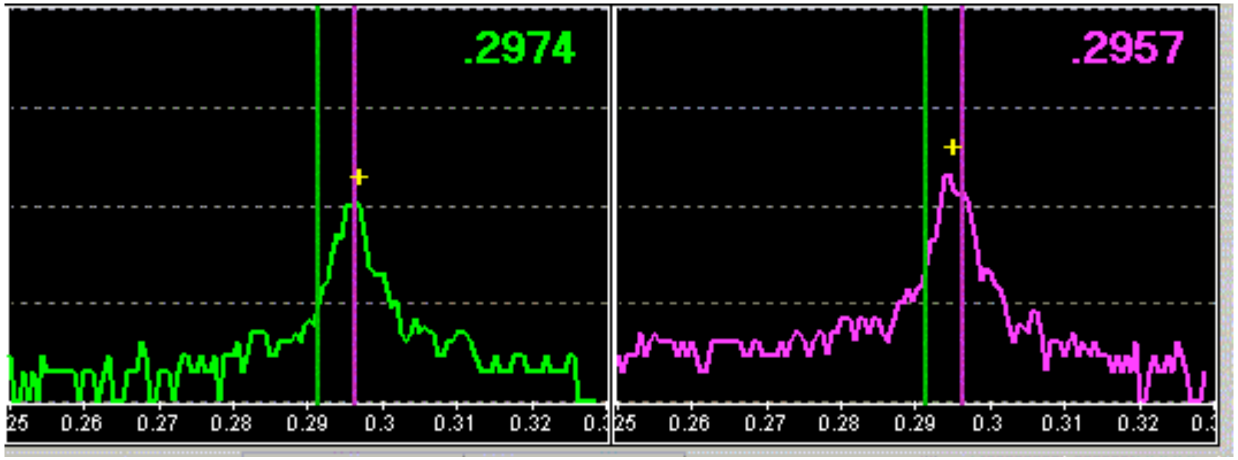
→ Some particles get very close to resonances and are lost

in other words: the tune is not a point it is a pancake



Tune signal for a nearly uncompensated cromaticity (Q' ≈ 20)

Ideal situation: cromaticity well corrected, (Q' ≈ 1)

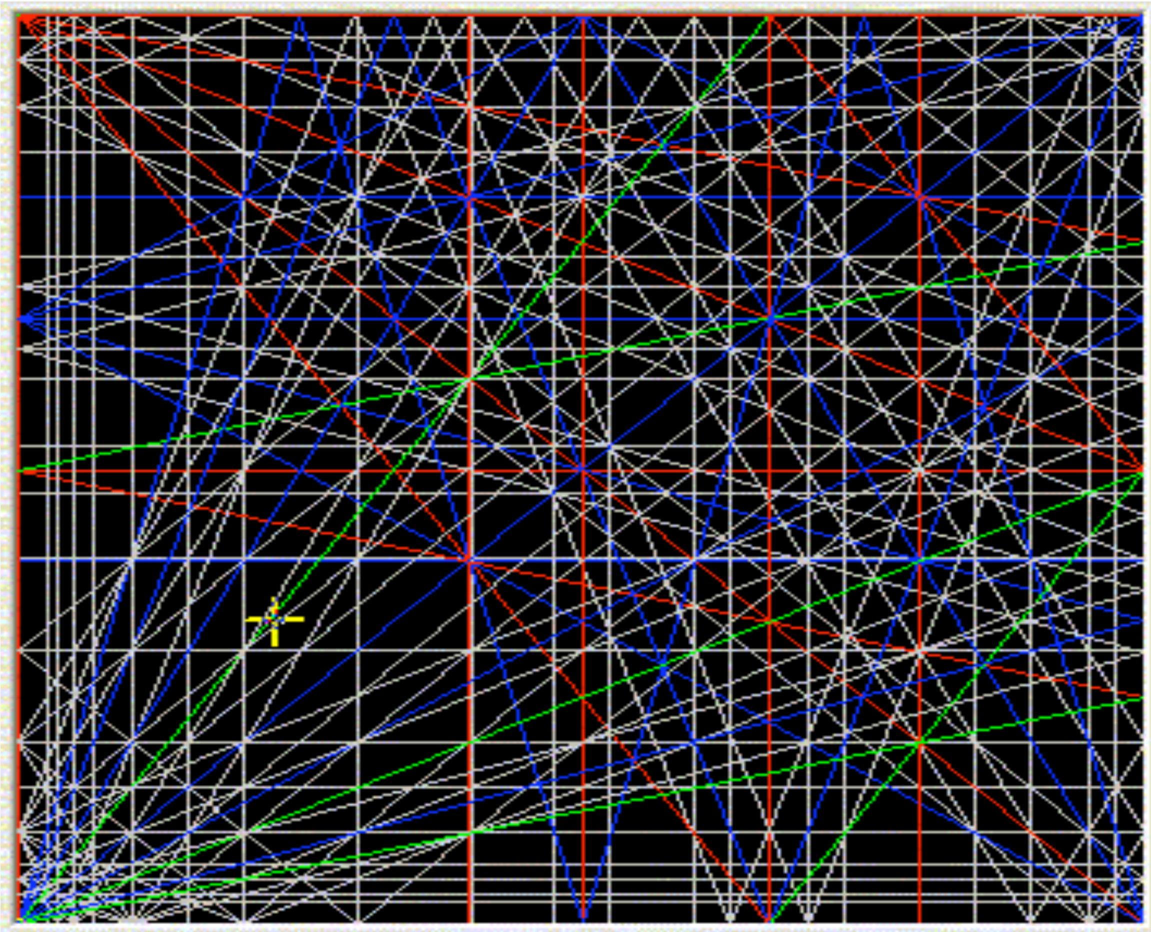


Tune and Resonances

$$m*Q_x+n*Q_y+l*Q_s = integer$$

Tune diagram up to 3rd order

… and up to 7th order



Homework for the operateurs: find a nice place for the tune where against all probability the beam will survive

Chromaticity Correction:

We need a magnetic field that focuses stronger those individual particles that have larger momentum and focuses weaker those with lower momentum.

... but that does not exist.

Trick: 1.) sort the particle trajectories according to their energy

2.) introduce magnetic fields that increase stronger than linear with the distance Δx to the centre

3.) calculate these fields (sextupoles) in a way that the lack of focusing strength is exactly compensated.

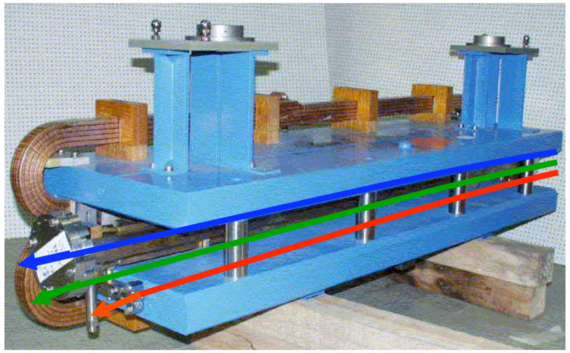
we use the dispersion to do the job

Correction of Q':

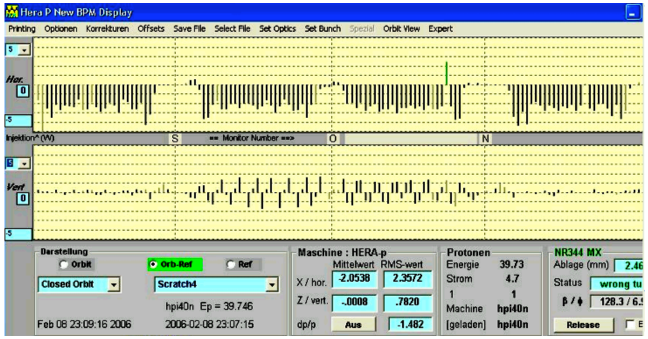
Need: additional quadrupole strength for each momentum deviation Δp/p

1.) sort the particles acording to their momentum

$$x_D(s) = D(s)\frac{\Delta p}{p}$$



… using the dispersion function



***2.) apply a** magnetic field that rises quadratically **with x** (sextupole field)*

$$\left.\begin{aligned} B_x &= \tilde{g}xz \\ B_z &= \frac{1}{2}\tilde{g}(x^2 - z^2) \end{aligned}\right\} \qquad \frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x} = \tilde{g}x$$

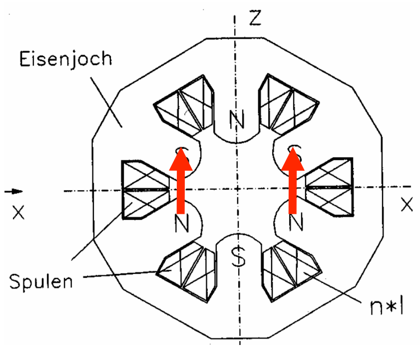
linear rising „gradient“:

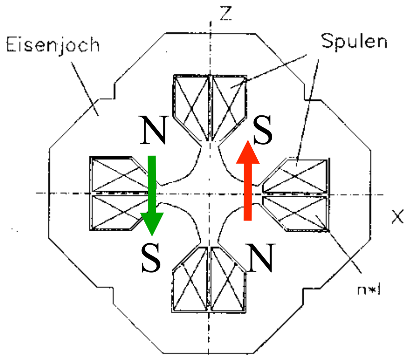
Correction of Q':

k_1 normalised quadrupole strength

k_2 normalised sextupole strength

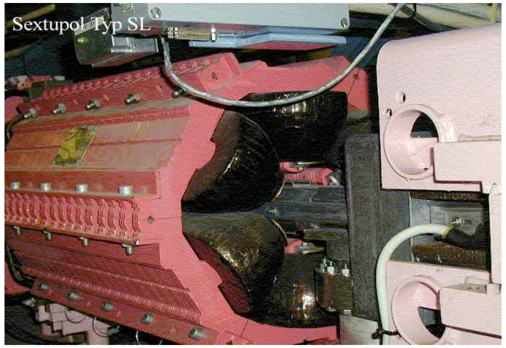
Sextupole Magnets:





$$k_1(sext) = \frac{\tilde{g}\, x}{p/e} = k_2 * x$$

$$k_1(sext) = k_2 * D * \frac{\Delta p}{p}$$



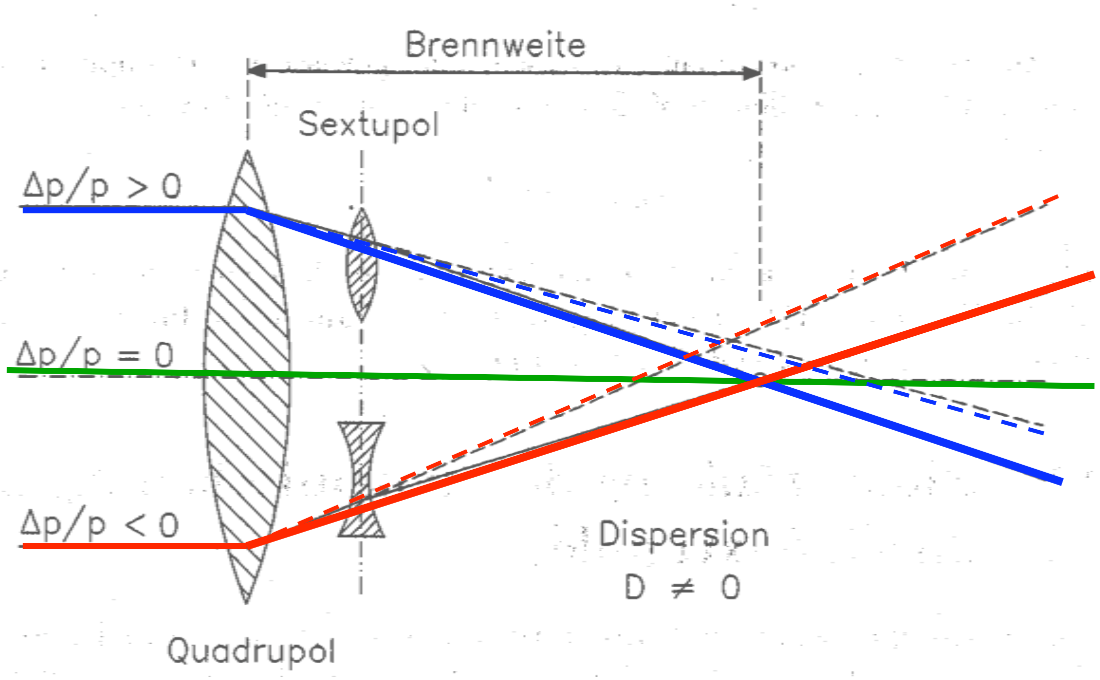
corrected chromaticity

considering a single cell:

$$Q'_{cell\_x} = -\frac{1}{4\pi}\left\{k_{qf}\hat{\beta}_x l_{qf} - k_{qd}\breve{\beta}_x l_{qd}\right\} + \frac{1}{4\pi}\sum_{F\,sext} k_2^F l_{sext} D_x^F \beta_x^F - \frac{1}{4\pi}\sum_{D\,sext} k_2^D l_{sext} D_x^D \beta_x^D$$

$$Q'_{cell\_y} = -\frac{1}{4\pi}\left\{-k_{qf}\breve{\beta}_y l_{qf} + k_{qd}\hat{\beta}_y l_{qd}\right\} + \frac{1}{4\pi}\sum_{F\,sext} k_2^F l_{sext} D_x^F \beta_x^F - \frac{1}{4\pi}\sum_{D\,sext} k_2^D l_{sext} D_x^D \beta_x^D$$

Chromatizitätskorrektur:

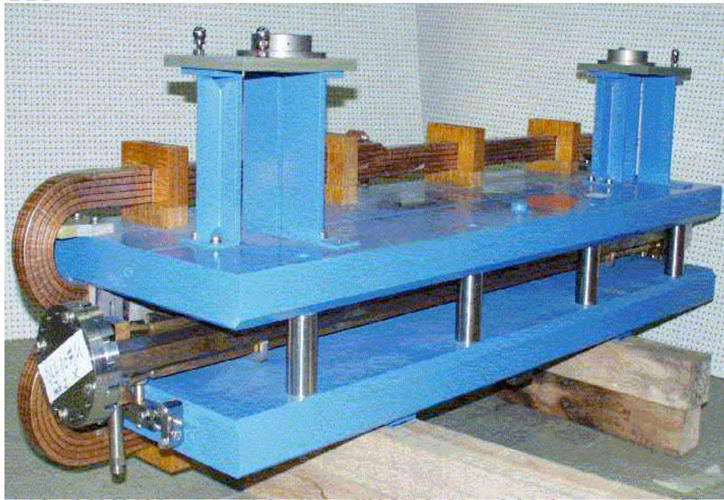


Einstellung am Speicherring:

Sextupolströme so variieren, dass $\xi \approx +1 \ldots +2$

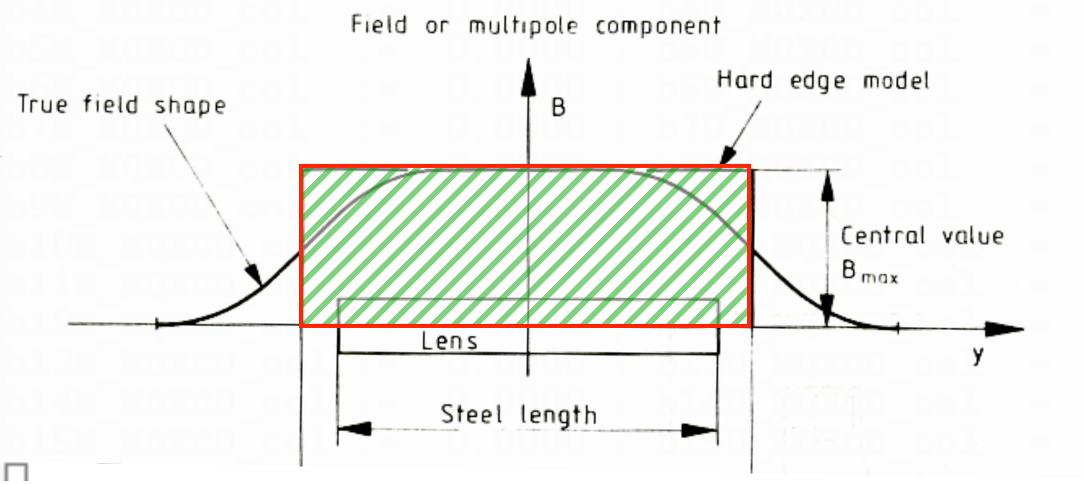
A word of caution: keep non-linear terms in your storage ring low.

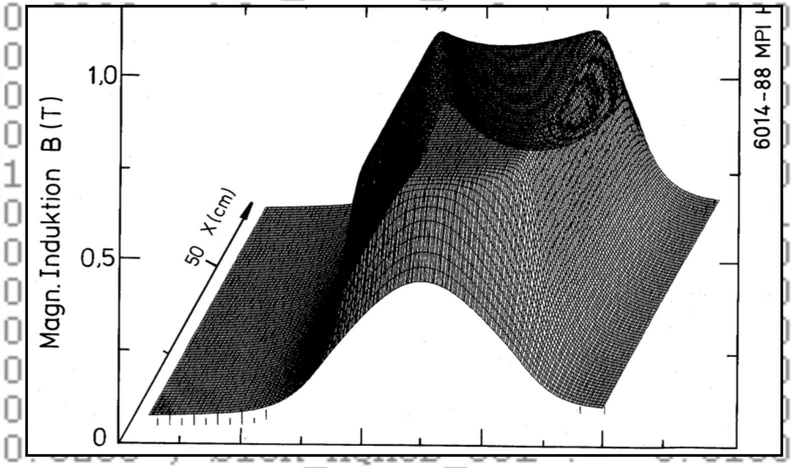
$$B_y + iB_x = B_{ref} * \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{r_0} \right)^{n-1}$$



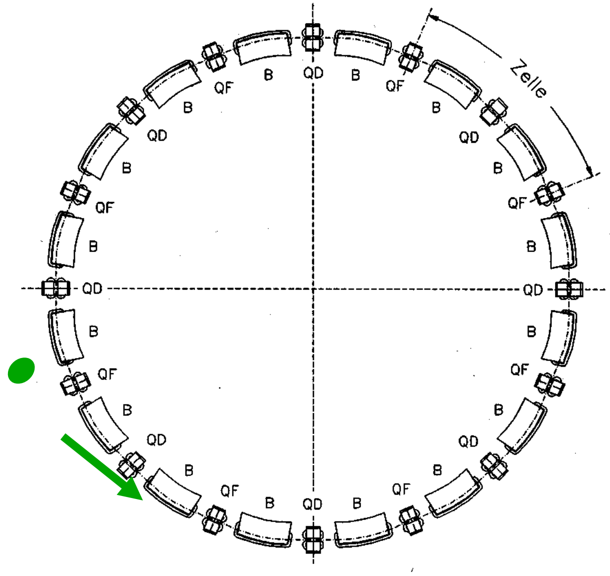
"effective magnetic length"

$$B * l_{eff} = \int_0^{l_{mag}} B\,ds$$





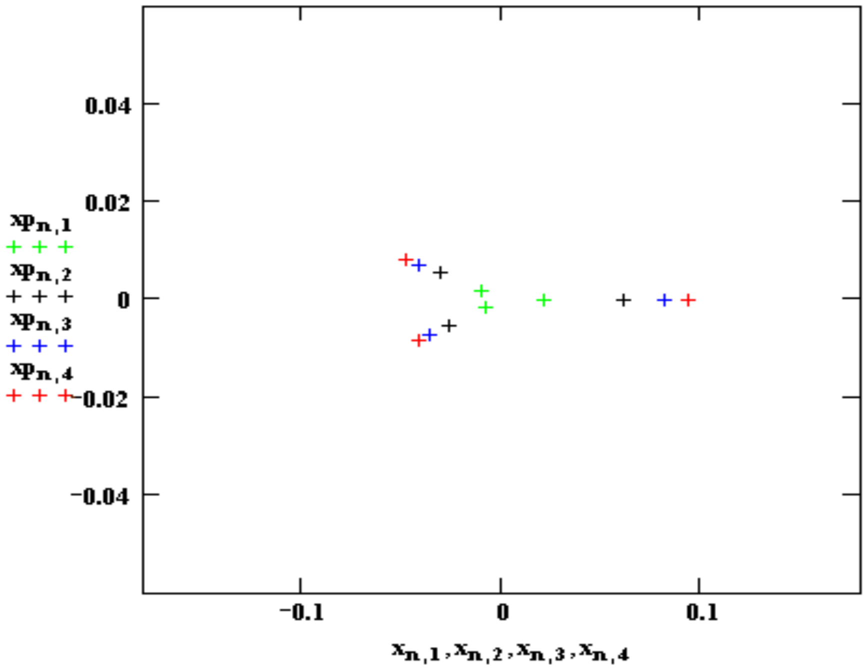
Clearly there is another problem ...
... if it were easy everybody could do it



Again: the phase space ellipse

for each turn write down – at a given position „s" in the ring – the single partilce amplitude x and the angle x´... and plot it.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

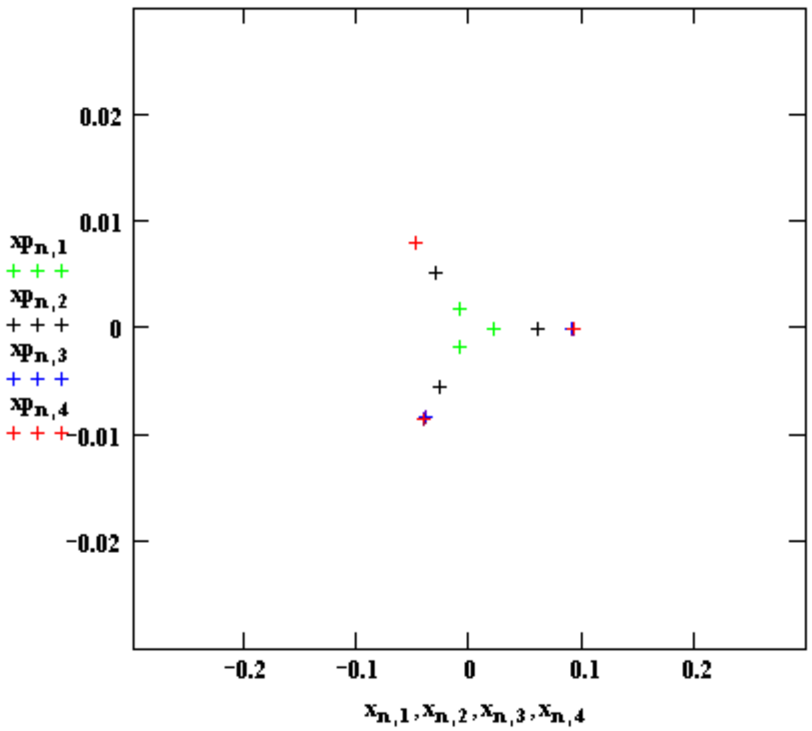


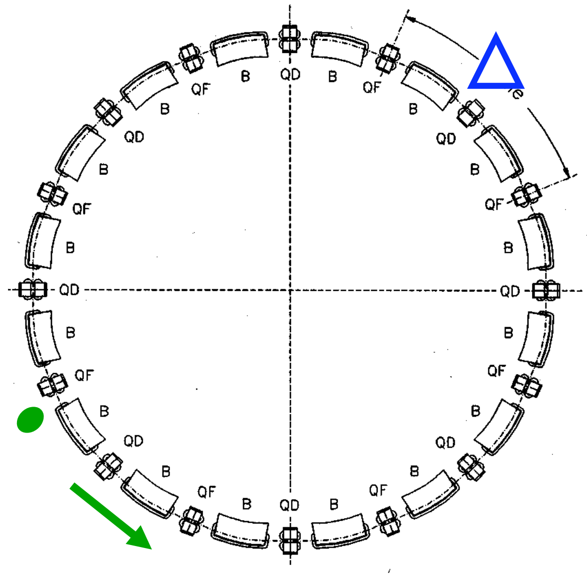
A beam of 4 particles

– each having a slightly different emittance:

Installation of a weak (!!!) sextupole magnet

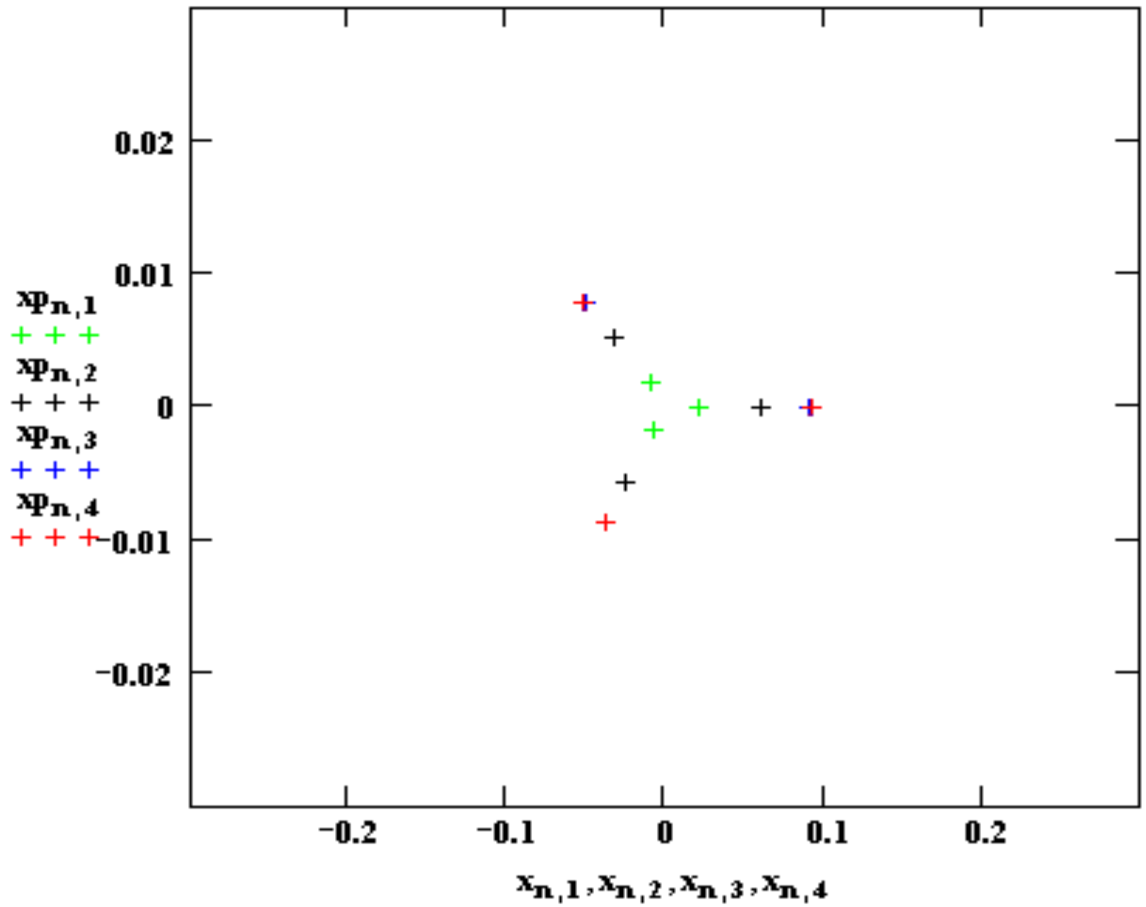
The good news: sextupole fields in accelerators cannot be treated analytically anymore.
→ no equatiuons; instead: Computer simulation „ particle tracking "

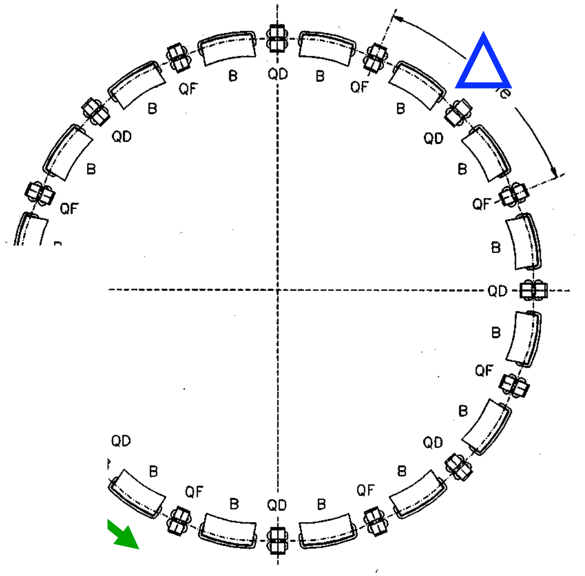




Effect of a strong (!!!) Sextupole …

→ Catastrophy !





„dynamic aperture“

Bibliography:

1.) Edmund Wilson: Introd. to Particle Accelerators
Oxford Press, 2001

2.) Klaus Wille: Physics of Particle Accelerators and Synchrotron
Radiation Facilicties, Teubner, Stuttgart 1992

3.) Peter Schmüser: Basic Course on Accelerator Optics, CERN Acc.
School: 5th general acc. phys. course CERN 94-01

4.) Bernhard Holzer: Lattice Design, CERN Acc. School: Interm.Acc.phys course,
http://cas.web.cern.ch/cas/ZEUTHEN/lectures-zeuthen.htm

5.) Herni Bruck: Accelerateurs Circulaires des Particules,
presse Universitaires de France, Paris 1966 (english / francais)

6.) M.S. Livingston, J.P. Blewett: Particle Accelerators,
Mc Graw-Hill, New York,1962

7.) Frank Hinterberger: Physik der Teilchenbeschleuniger, Springer Verlag 1997

8.) Mathew Sands: The Physics of e+ e- Storage Rings, SLAC report 121, 1970

9.) D. Edwards, M. Syphers : An Introduction to the Physics of Particle
Accelerators, SSC Lab 1990