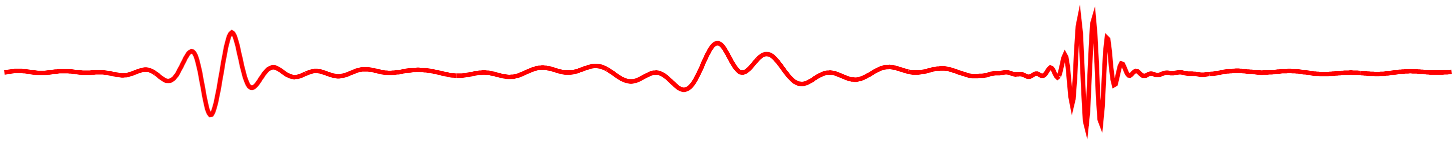
An introduction to linear imperfections



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CERN Accelerator School,

November 2013

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- ⋆ Some CERN lattices
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- ⋆ β-beating in the CERN synchrotrons
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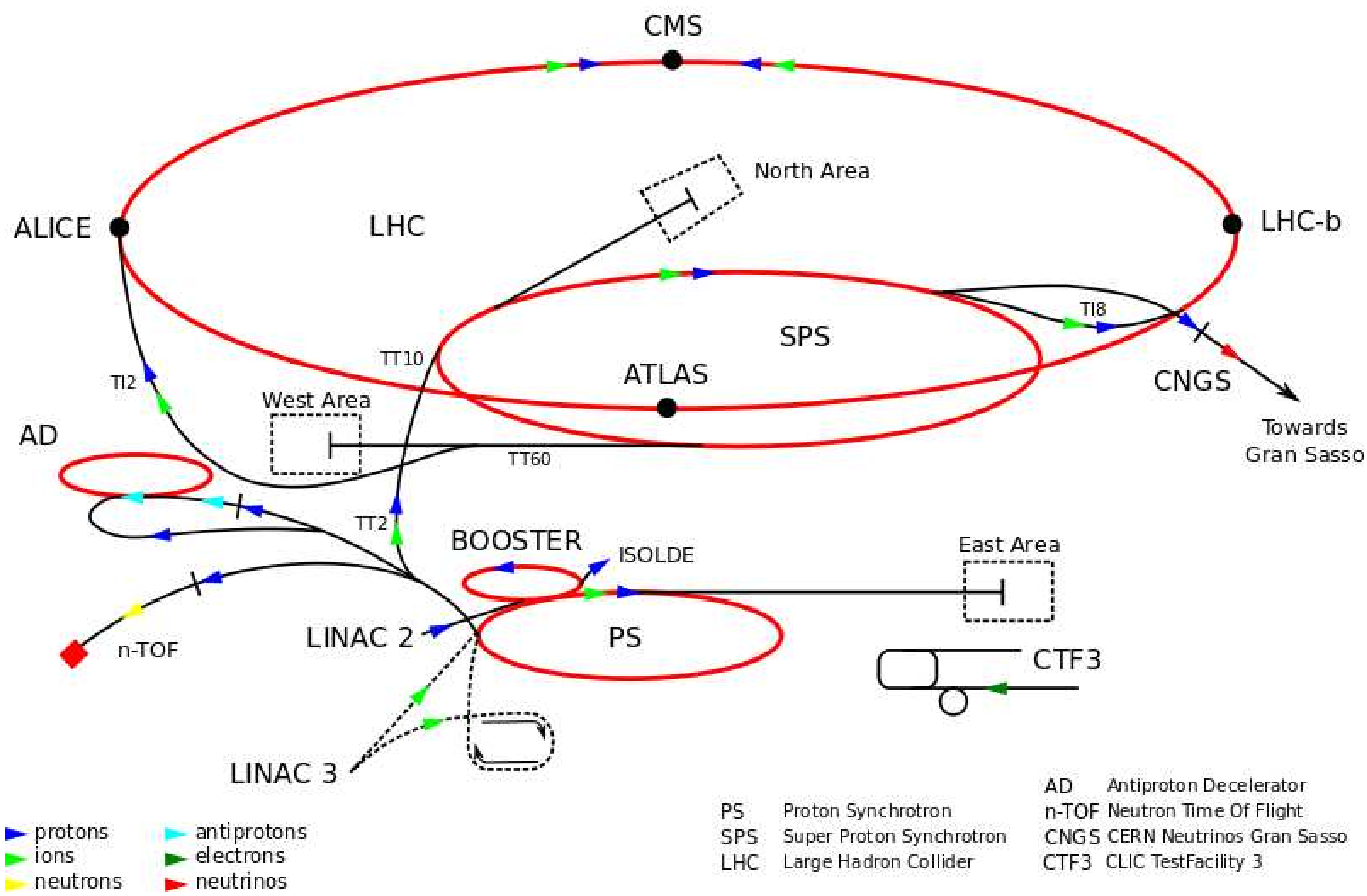
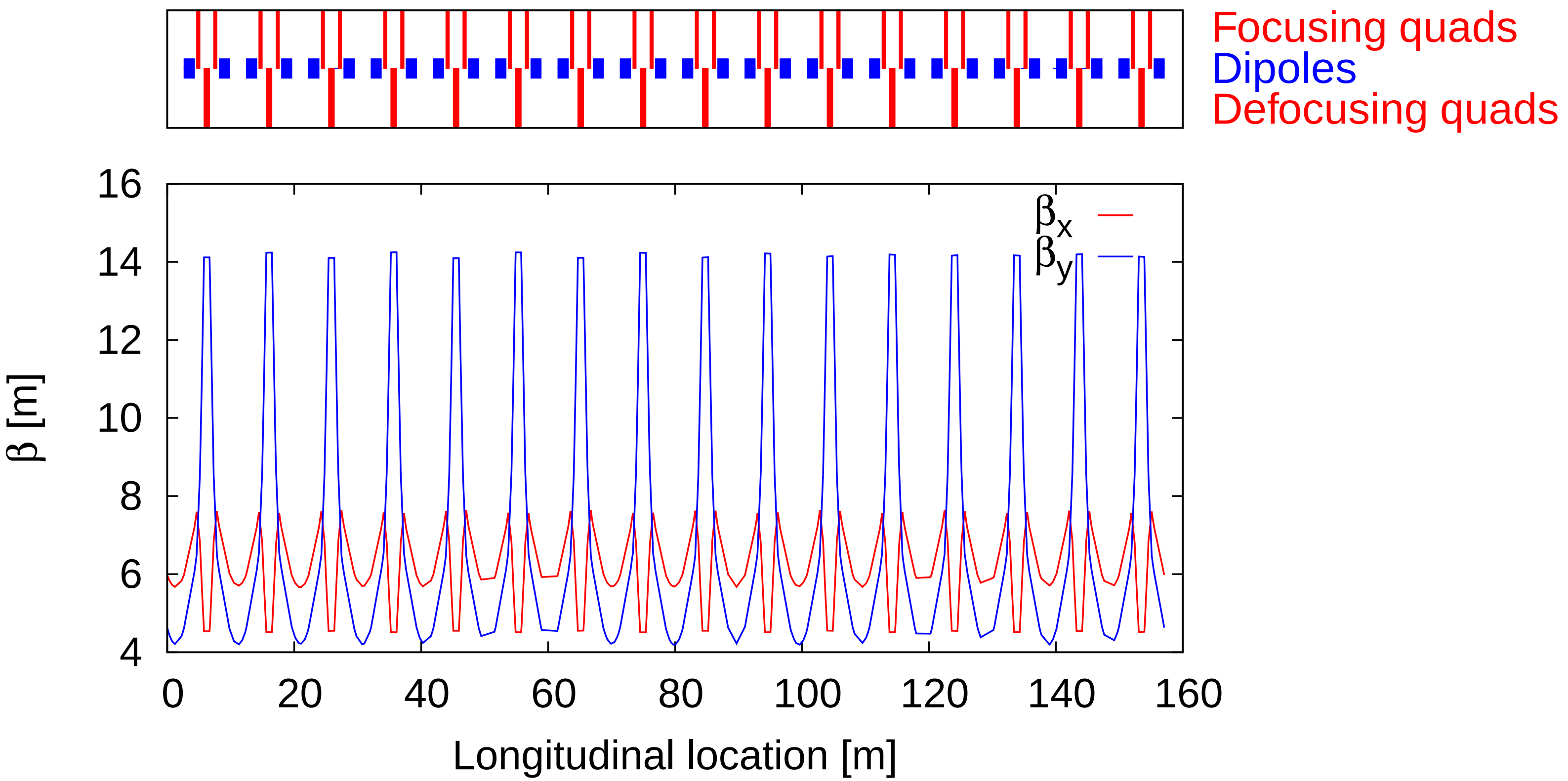
- ⋆ Mismatched injection

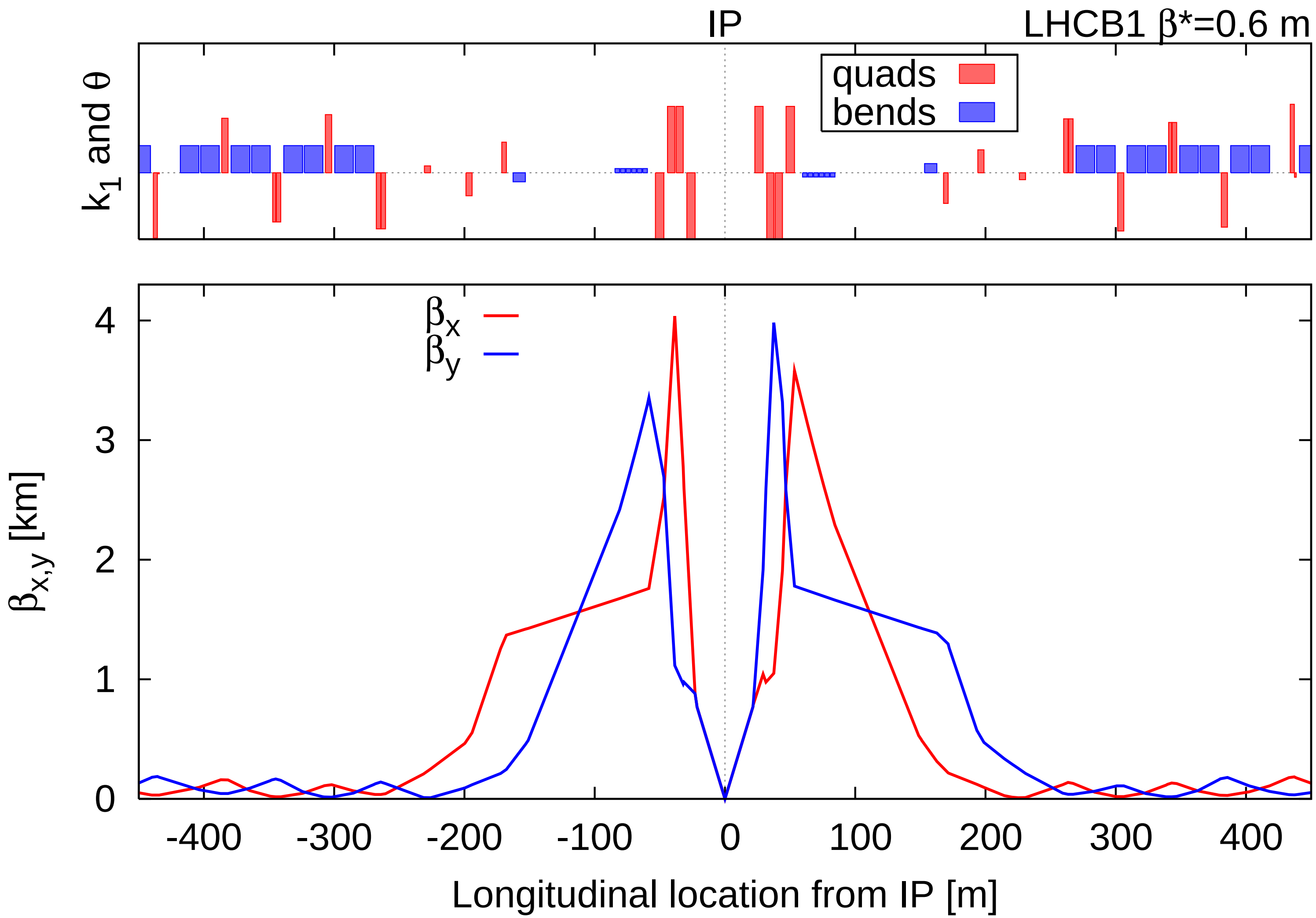
CERN accelerator complex



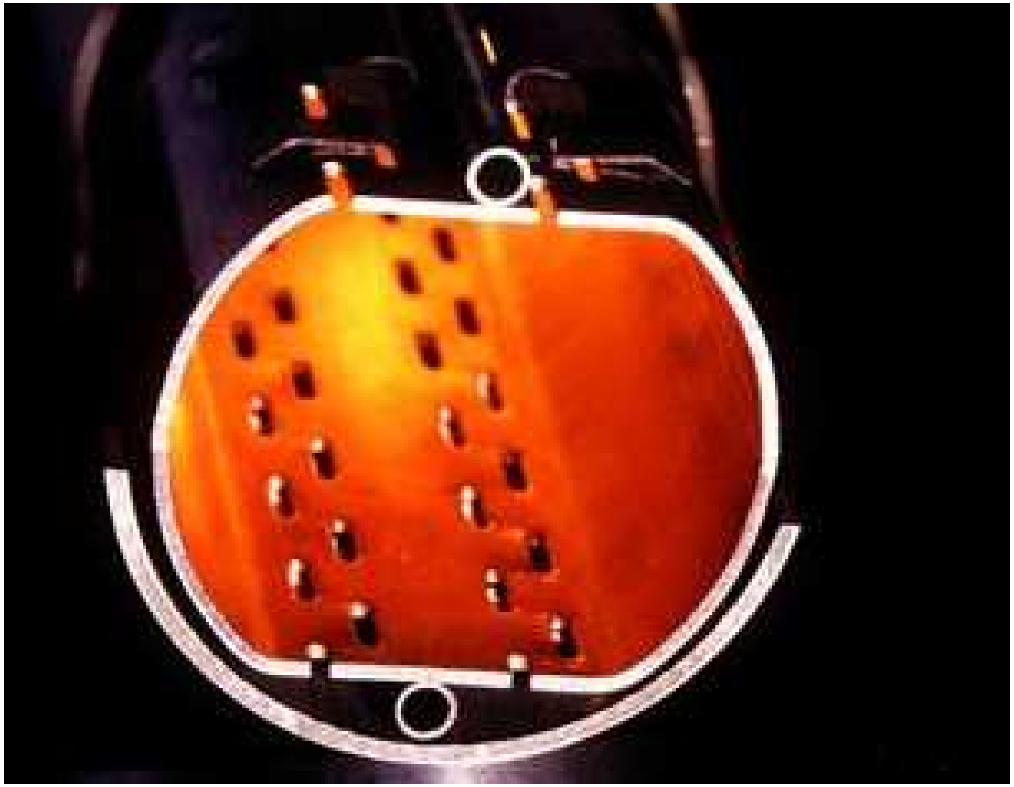
The Proton Synchrotron Booster (PSB)



The LHC Interaction Region (IR)

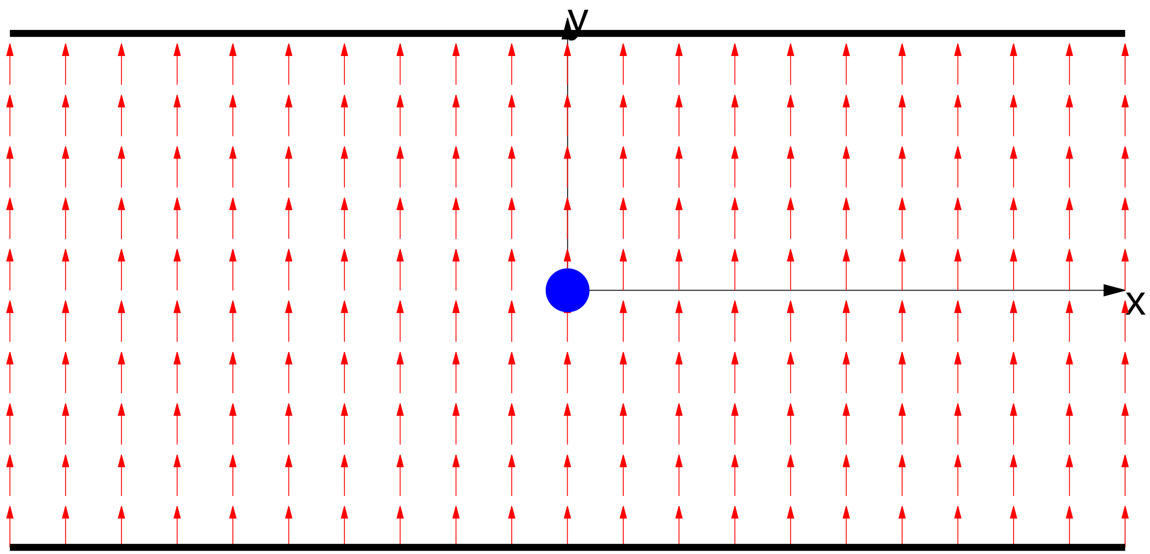


The first linear imperfection is...



...**gravity!**. The LHC vacuum chamber has a 22 mm radius. Everything takes 67 ms to fall. **Why do protons not fall?**

Dipole magnetic field

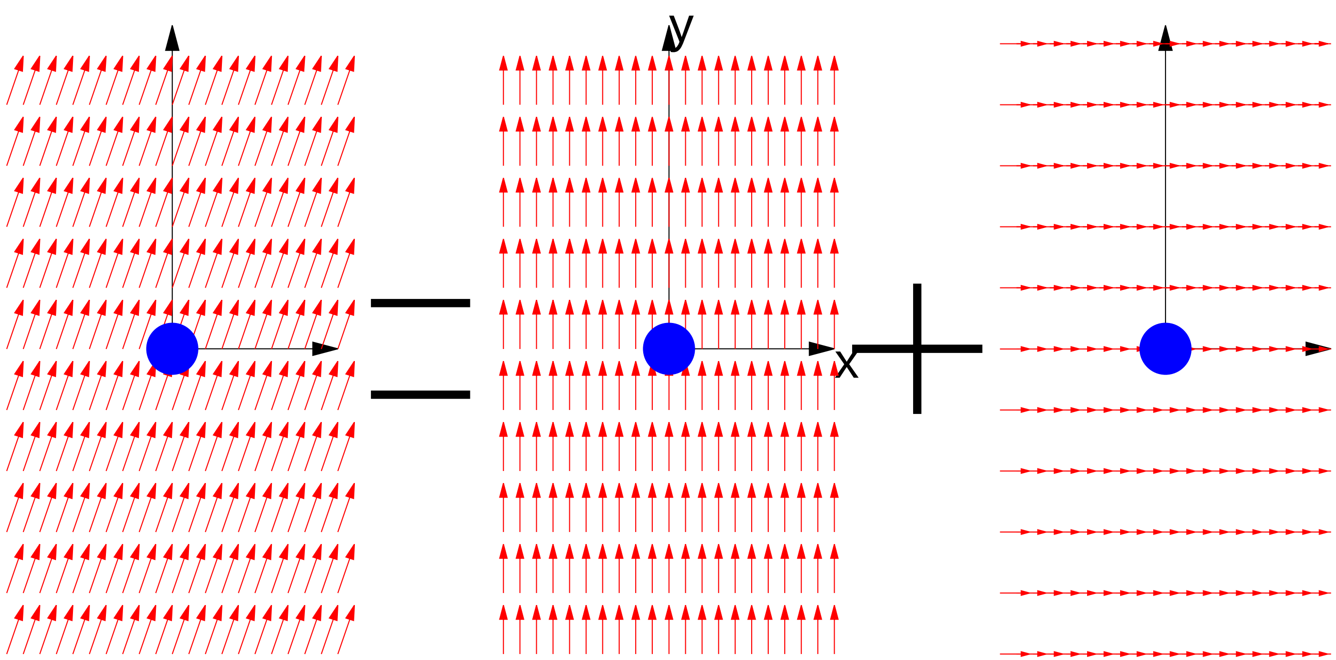


Lorentz force:

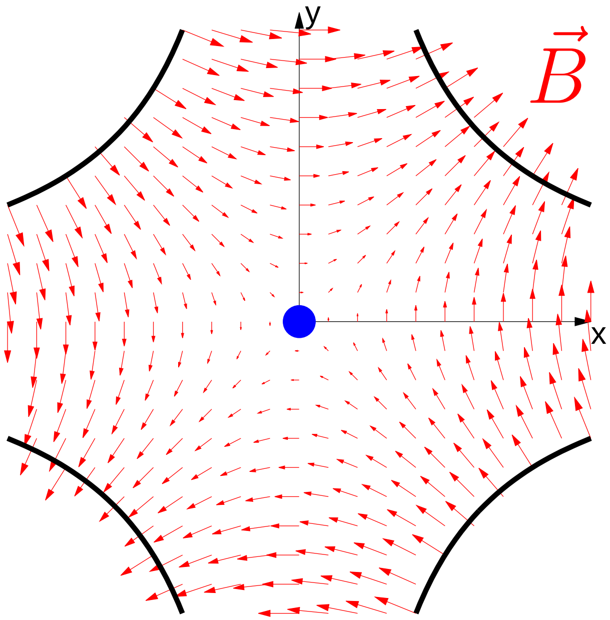
$$\vec{F} = q\vec{v} \times \vec{B}$$

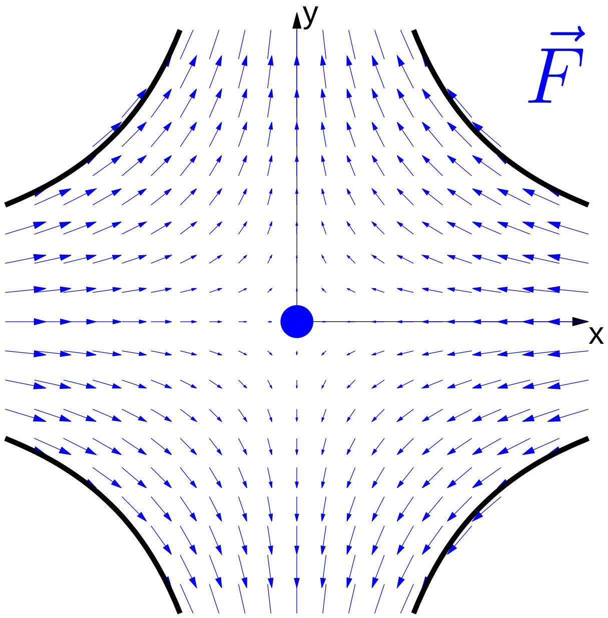
Dipole errors

- An error in the strength of a main dipole causes a perturbation on the horizontal closed orbit.
- A tilt error in a main dipole causes a perturbation on the vertical closed orbit.



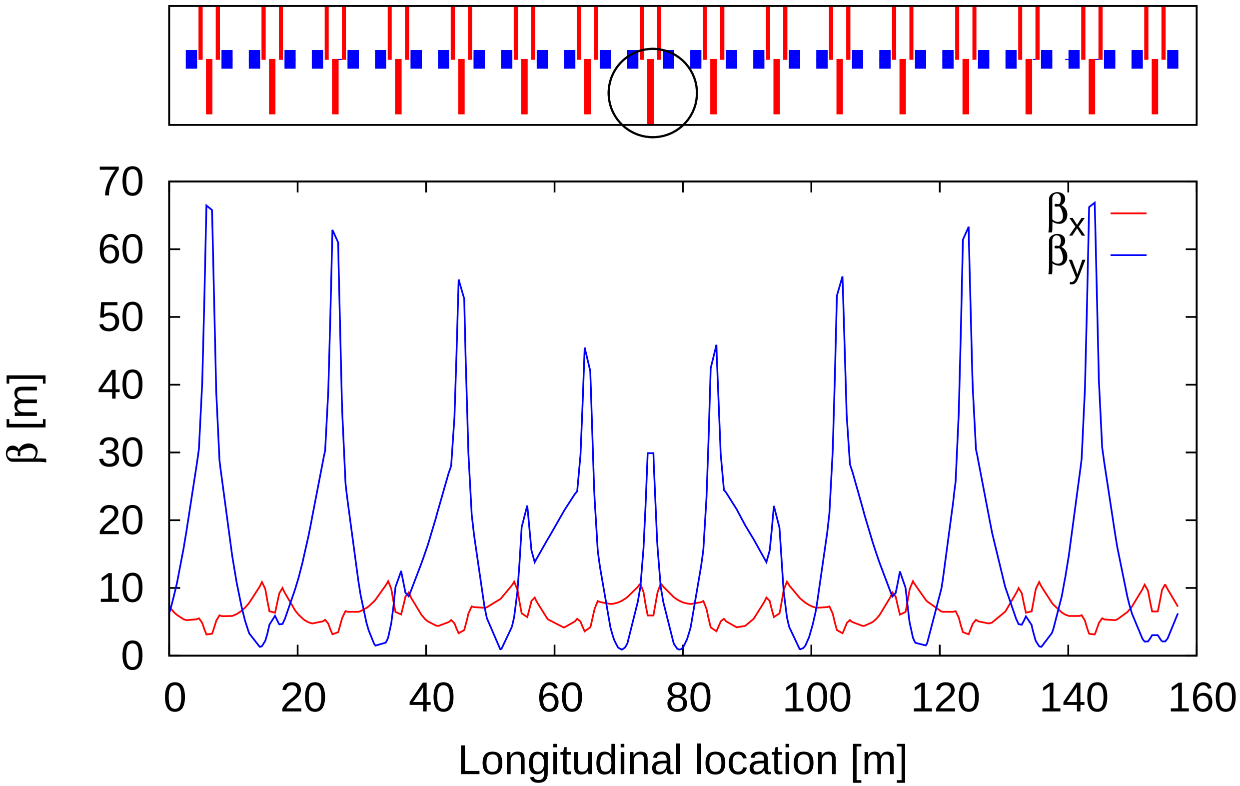
Quadrupole field, and force on the beam





Note that $F_x = -kx$ and $F_y = ky$ making horizontal dynamics totally decoupled from vertical.

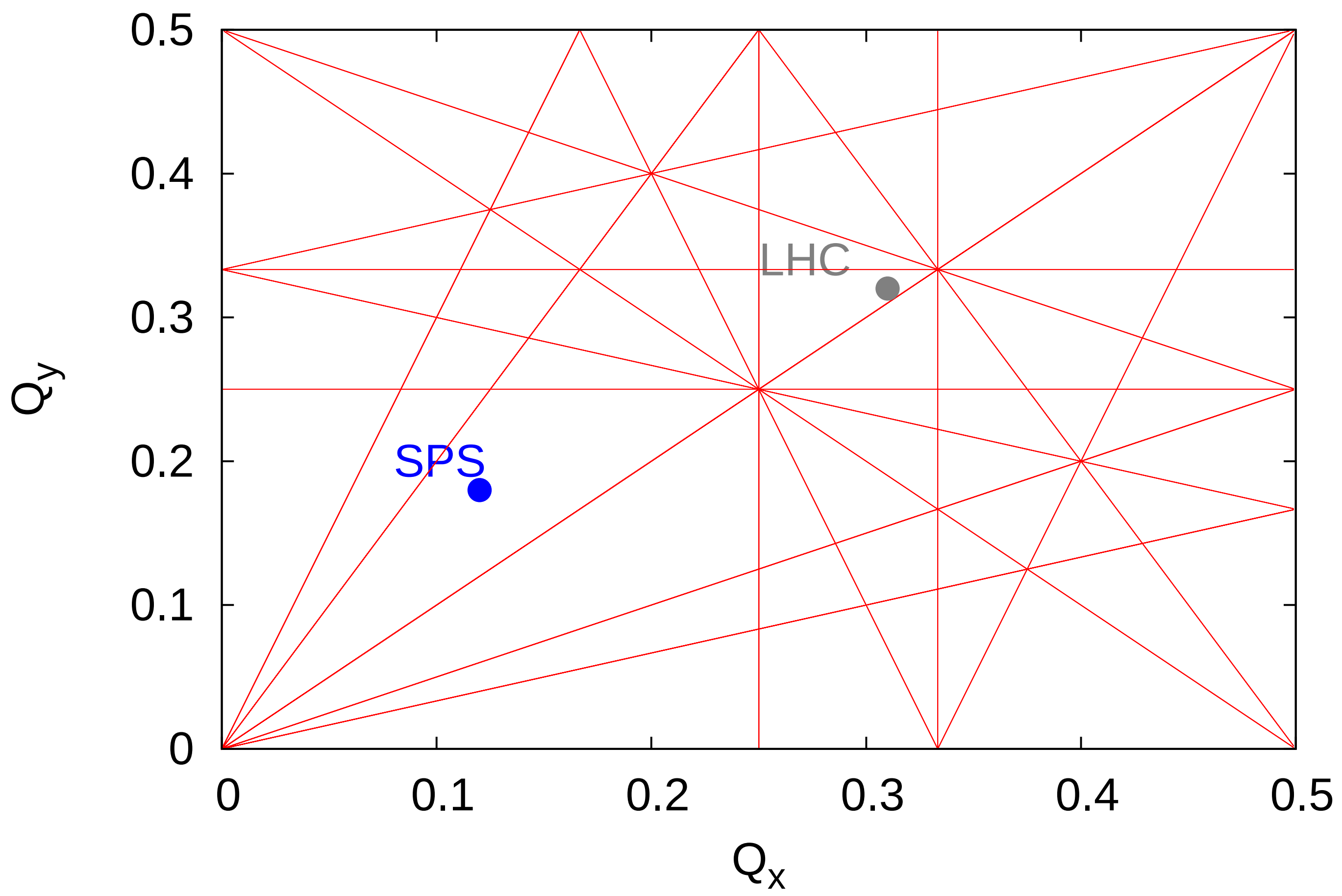
Quadrupole strength error



The β functions change, β-beating: $\frac{\Delta\beta}{\beta} = \frac{\beta_{pert}-\beta_0}{\beta_0}$.

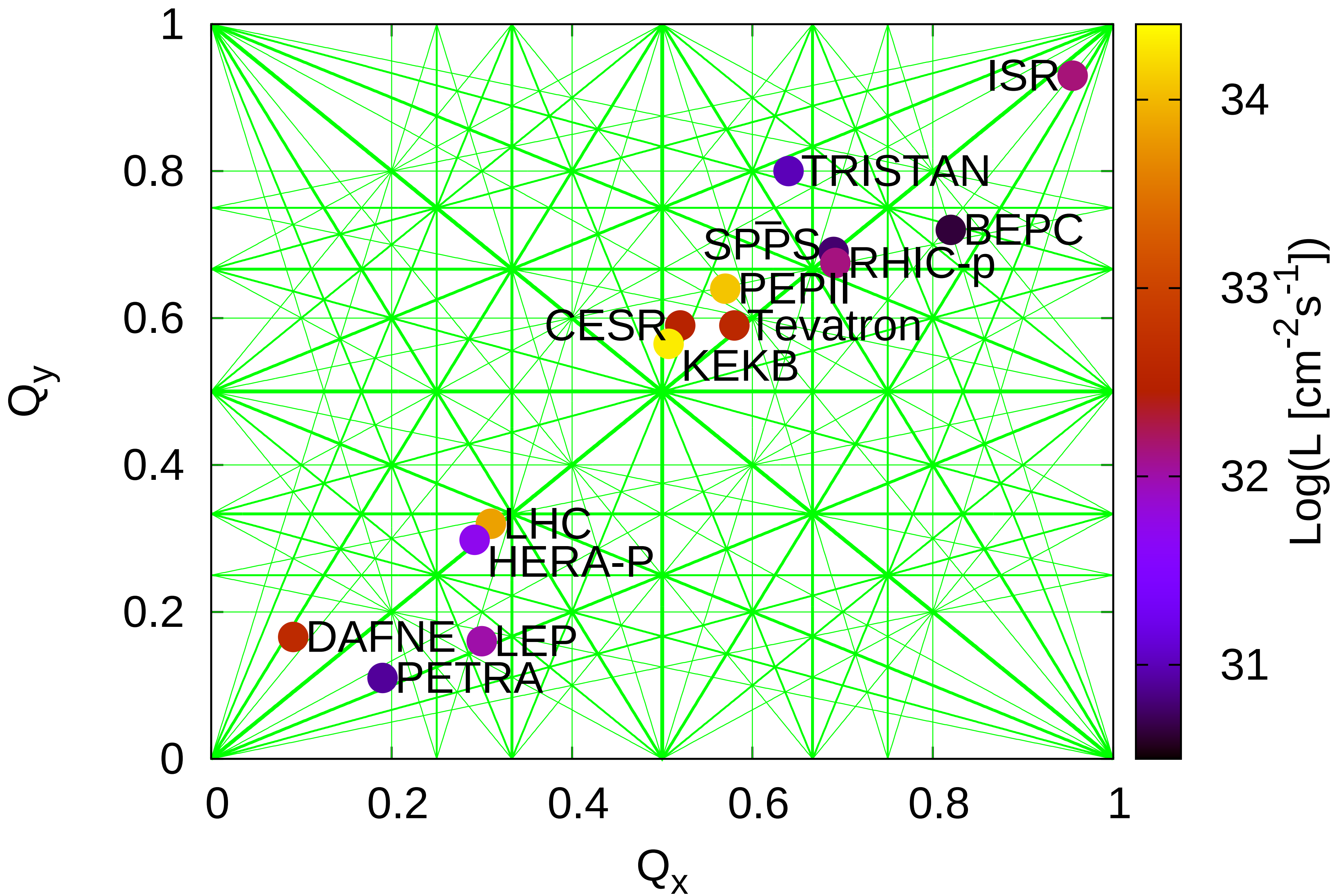
$$\Delta Q_x \approx \frac{1}{4\pi}\beta_x \Delta k, \quad \Delta Q_y \approx -\frac{1}{4\pi}\beta_y \Delta k$$

Quadrupole strength error - Tune change



Quadrupole errors can push tunes into resonances (dangerous) $nQ_x + mQ_y = N$

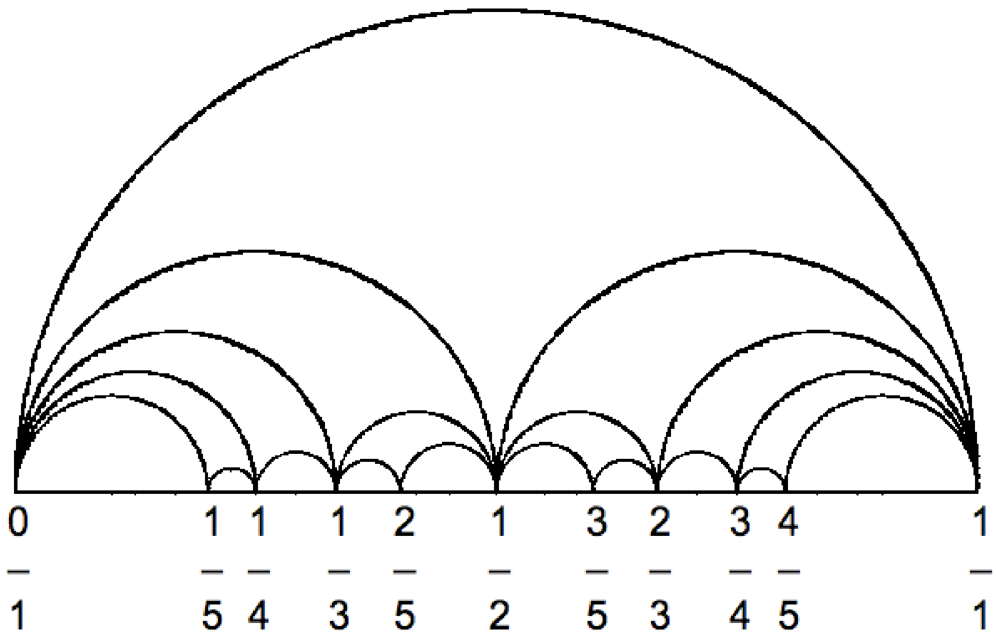
Colliders in the resonance world



Interlude: Farey sequences (1802)

The Farey sequence of order n is the sequence of completely reduced fractions between 0 and 1 which, when in lowest terms, have denominators less than or equal to $N \rightarrow$ **Resonances of order N or lower** (in one plane)

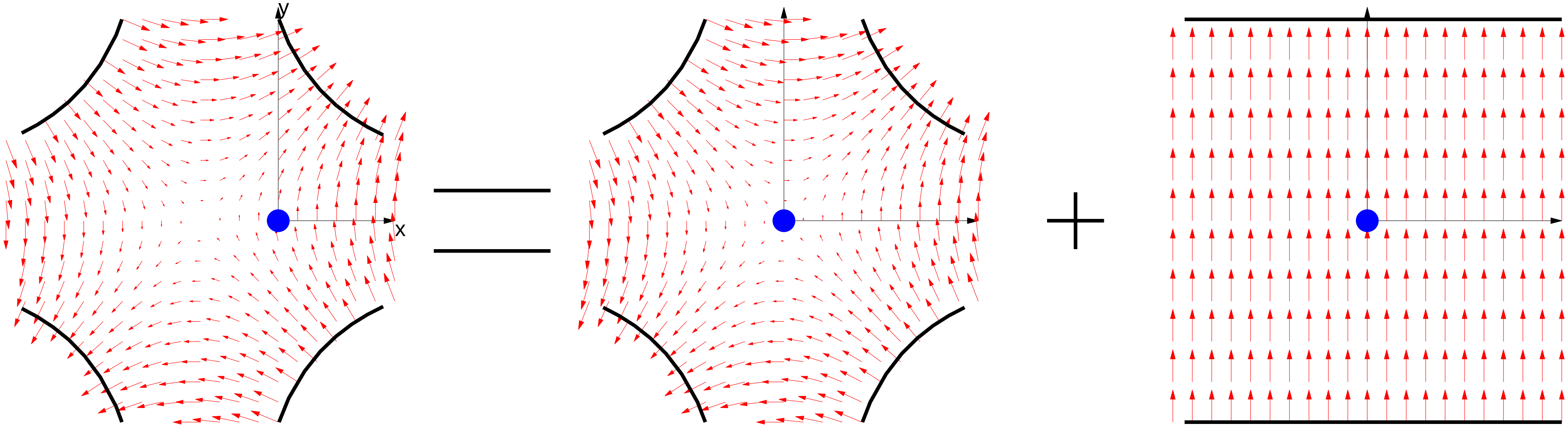
Farey diagram of order 5 (F_5)



Some properties of Farey sequences

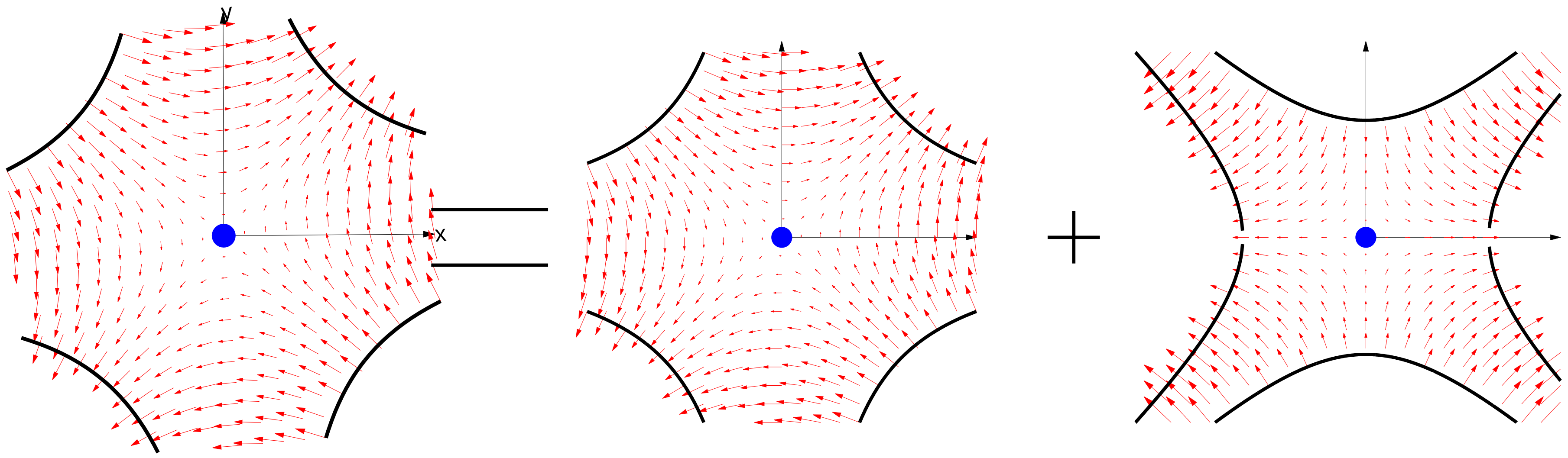
- ⋆ The distance between neighbors in a Farey sequence (aka two consecutive resonances) a/b and c/d is equal to $1/(bd)$
- ⋆ The next leading resonances in between two consecutive resonances a/b and c/d is $(a+c)/(b+d)$.
- ⋆ The number of 1D resonances of order N or lower tends asymptotically to $3N^2/\pi^2$

Offset quadrupole - Feed-down



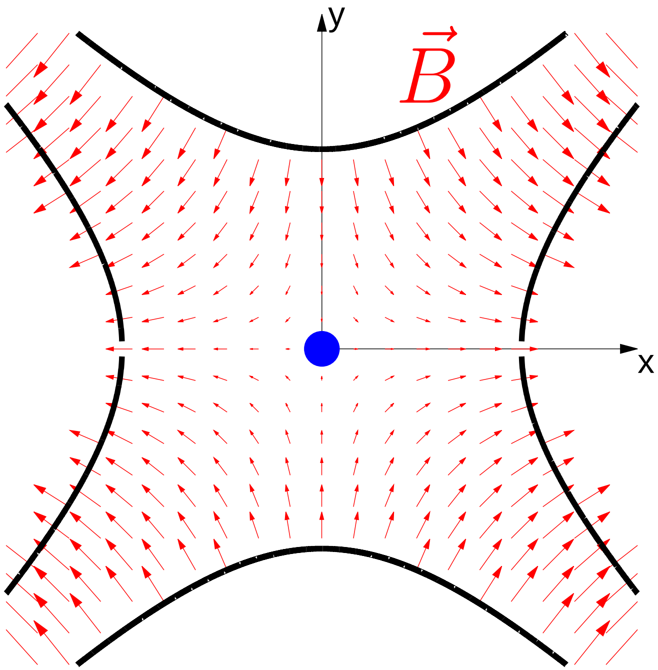
An offset quadrupole is seen as a centered quadrupole plus a dipole. This is called feed-down.

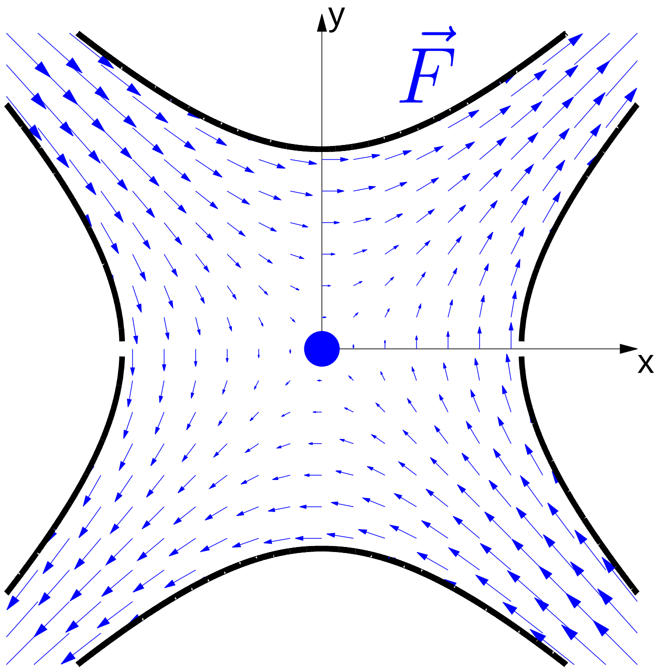
Tilted quadrupole



A tilted quadrupole is seen as a normal quadrupole plus another quadrupole tilted by 45° (this is called a skew quadrupole).

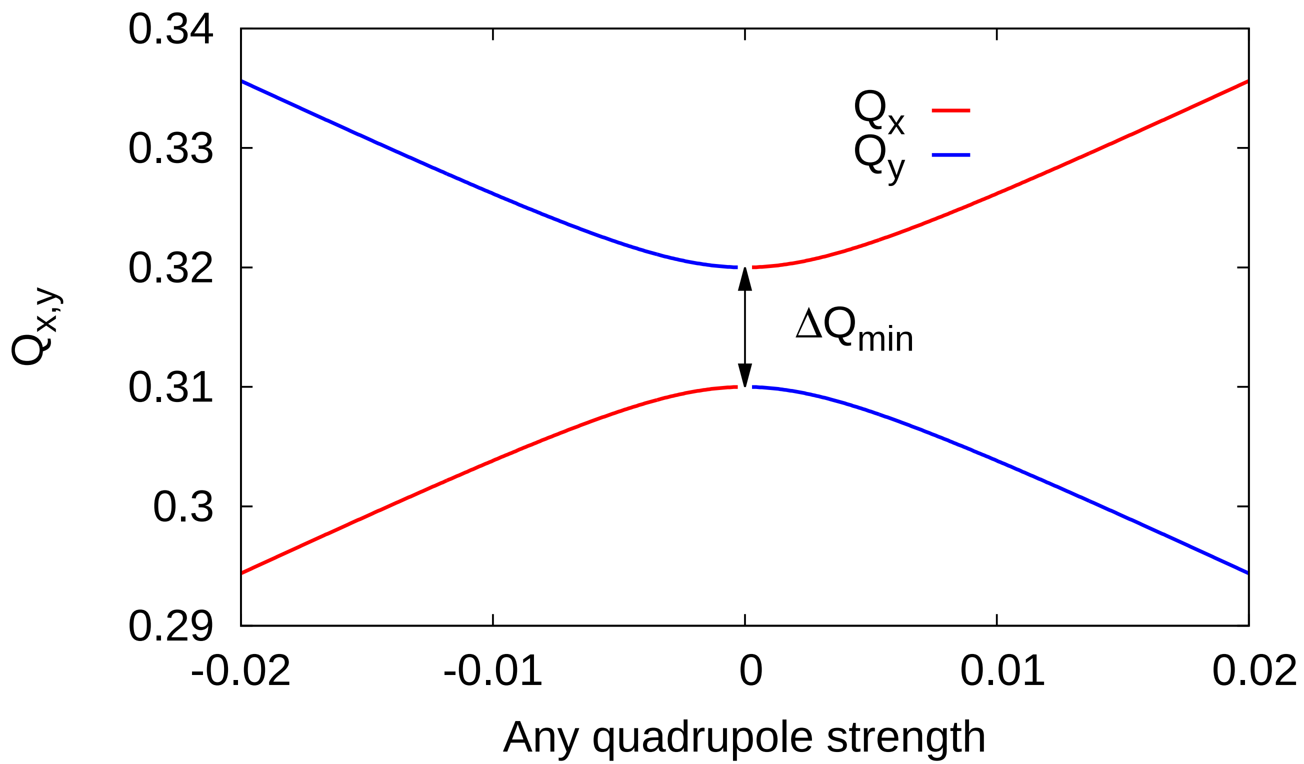
Skew quadrupole $\rightarrow$ x-y Coupling





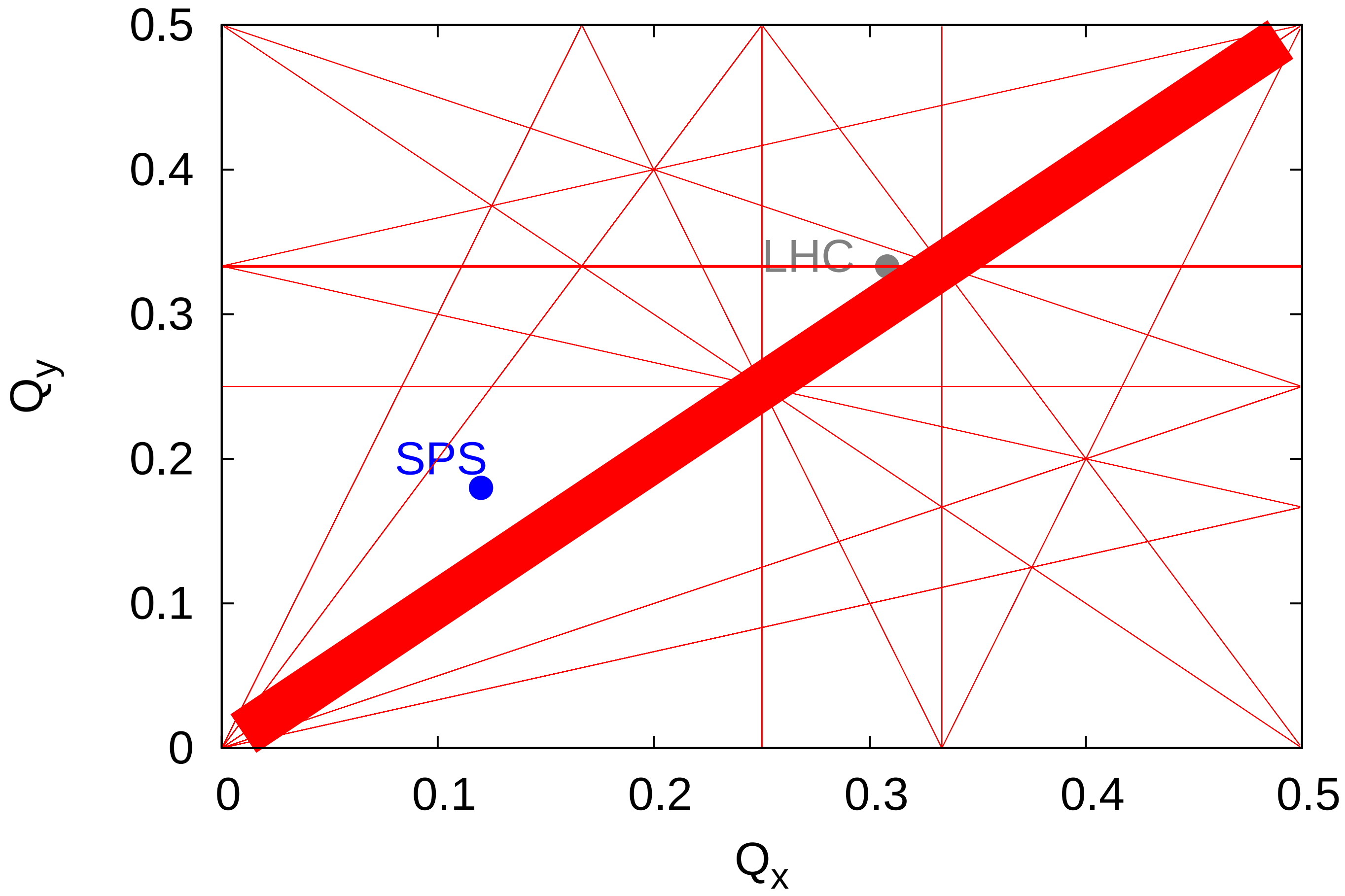
Note that $F_x = ky$ and $F_y = kx$ making horizontal and vertical dynamics to couple.

Skew quadrupole → x-y Coupling



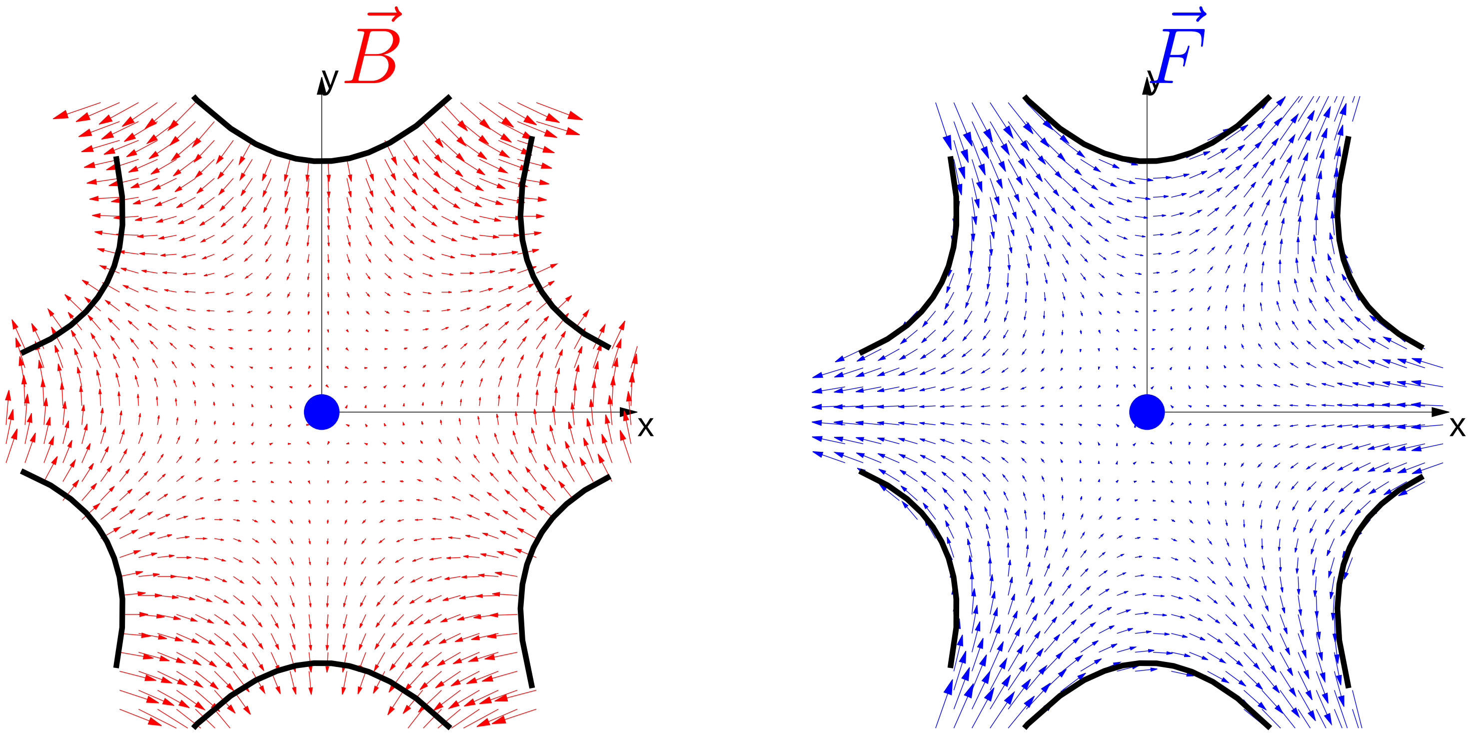
Coupling makes it impossible to approach tunes below $\Delta Q_{min} = |C^-|$, where C^- is a complex number characterizing the difference resonance $Q_x - Q_y = N$.

Coupling



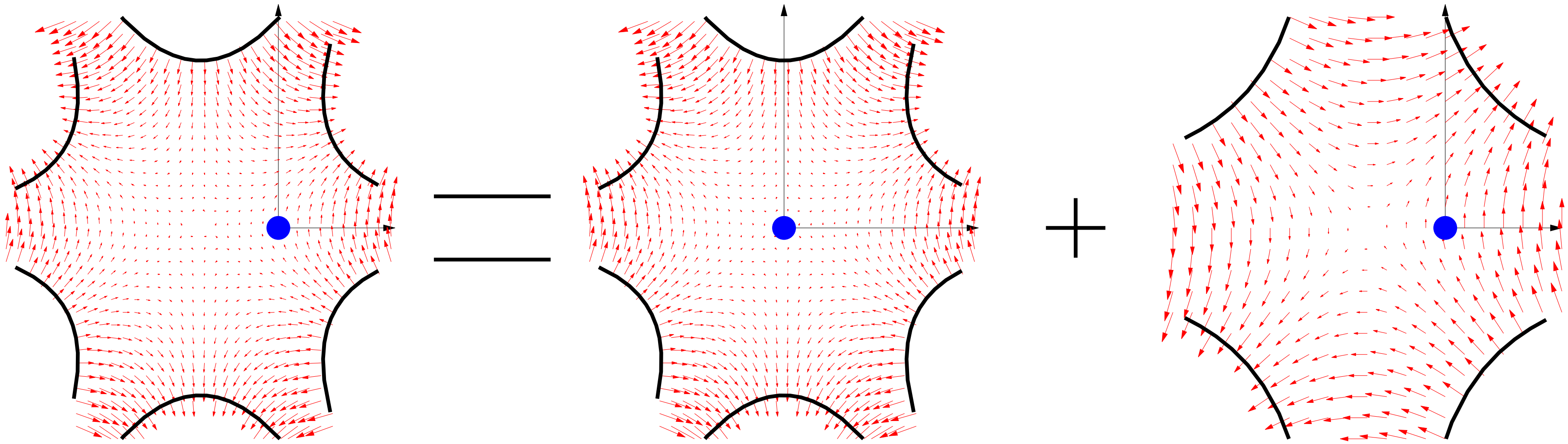
Coupling can push tunes into resonances.

Sextupole field and force



Ooops, We are entering the non-linear regime, however...

Offset sextupole

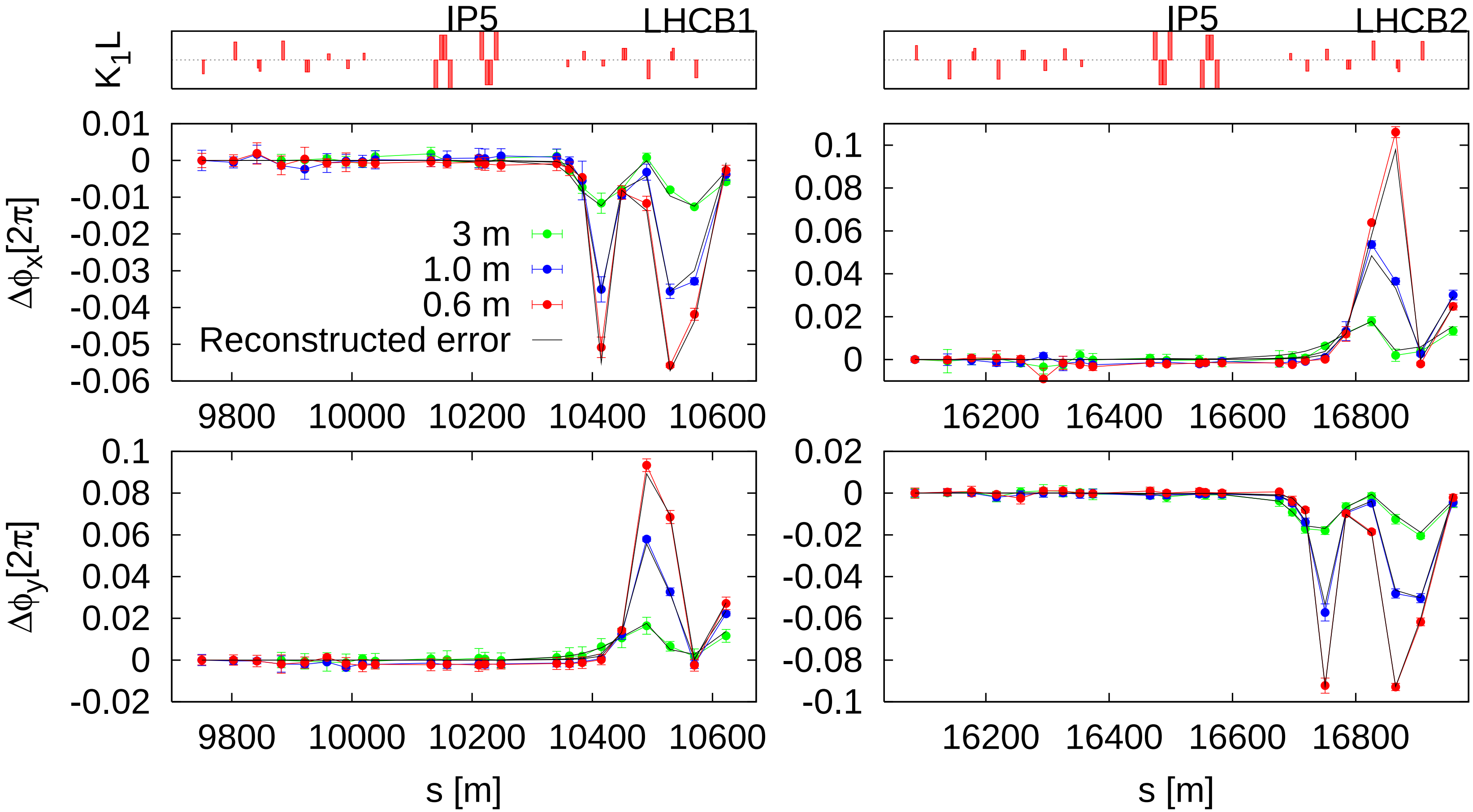


A sextupole horizontally (vertically) displaced is seen as a centered sextupole plus an offset quadrupole (skew quadrupole). Offset sextupoles are also sources of quadrupole and skew quadrupole errors.

Correction

- Local corrections
  - Ideal correction: Error source identification and repair.
  - Effective local error correction.
  - MICADO (ISR-MA/73-17): Best few correctors (no guarantee of locality).
- Global corrections
  - Pre-designed knobs for varying particular observables in the least invasive way (like tunes, coupling, β^*, etc.)
  - MICADO: Best N correctors
  - Response matrix approach

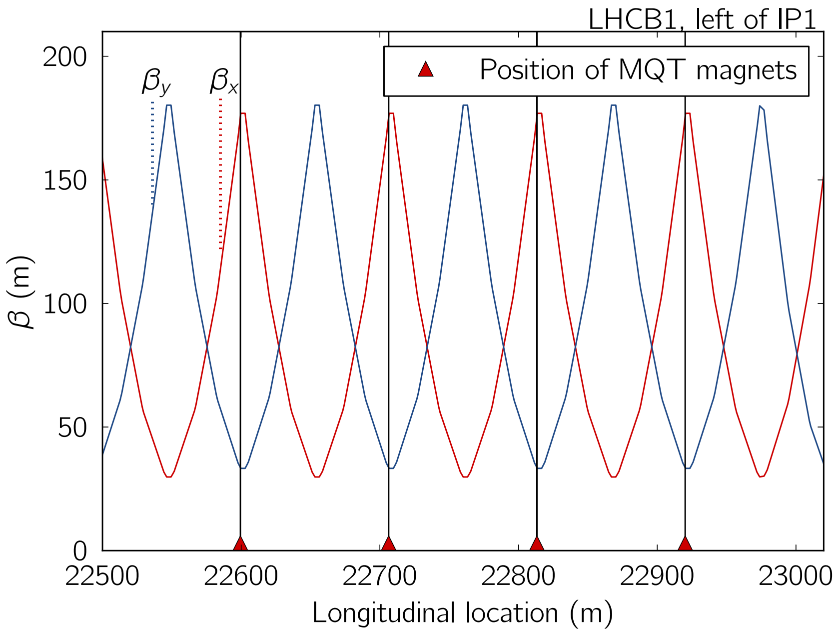
Local correction: segment-by-segment



Key point: Isolate a segment of the machine by imposing boundary conditions from measurements and find corrections.

Pre-designed knobs - Tunes

- In most machines it is OK to use all focusing quads to change Q_x and all defocusing quads for Q_y: PSB, PS, SPS
- In the LHC dedicated tune correctors (MQT) are properly placed to minimize impact on other quantities:



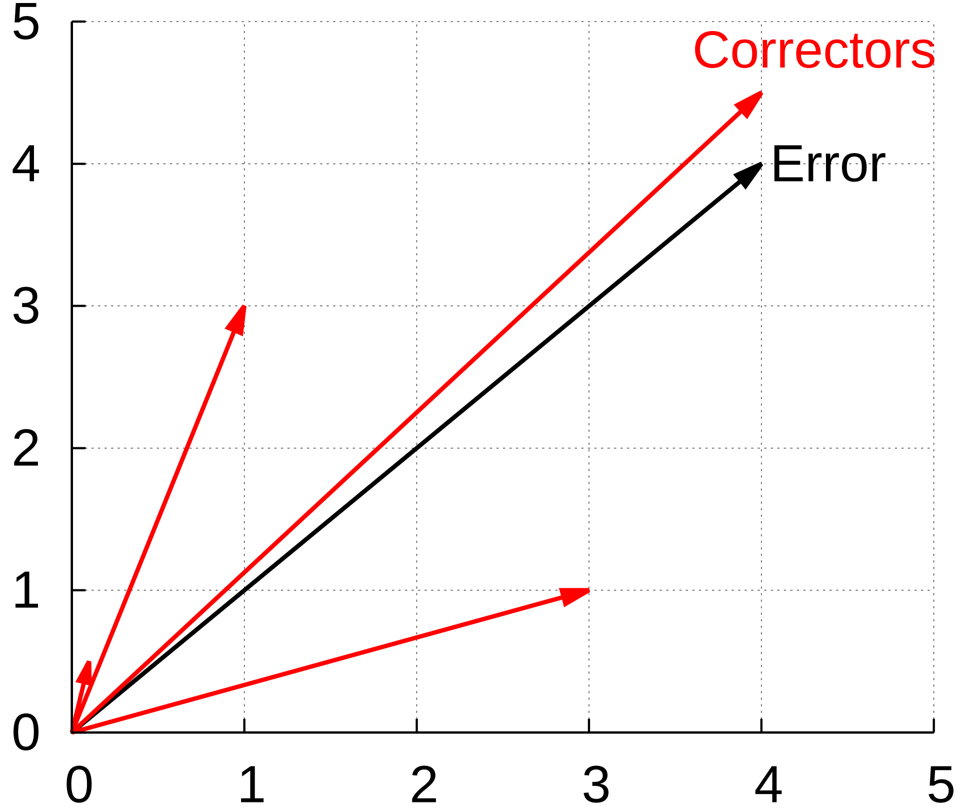
A.S. Langner

Pre-designed knobs - Coupling

- ⋆ The full control of the difference resonance (C^-) needs two independent families of skew quadrupoles.
- ⋆ PSB, PS and SPS can survive only with one family since $\mathrm{int}(Q_x) = \mathrm{int}(Q_y)$, making errors in phase with correctors.
- ⋆ In LHC there are two families to vary the real and imaginary parts of C^- independently.

Best corrector concepts from MICADO

Correctors $\pm\vec{v}_i$ are to be added linearly.



Which is the **best** corrector?
Which is the best **second** corrector? (using the 1^{st})
Which are the **two best correctors**?

MICADO challenge

- ⋆ LHC has about 500 orbit correctors per plane and per beam.
- ⋆ Imagine you want to find the best 20 correctors
- ⋆ How many combinations of these 500 correctors taking 20 at a time exist?
- ⋆ ...

Response matrix approach

- ⋆ Available correctors: $\vec{c}$
- ⋆ Available observables: $\vec{a}$
- ⋆ Assume for small changes of correctors linear approximation is good: $R\Delta\vec{c} = \Delta\vec{a}$
- ⋆ Use, e.g., MADX to compute R
- ⋆ Invert or pseudo-invert R to compute an effective global correction based on measured $\Delta\vec{a}$:

$$\Delta\vec{c} = R^{-1}\Delta\vec{a}$$

- ⋆ This works for orbit, $\Delta\beta/\beta$, coupling, etc.

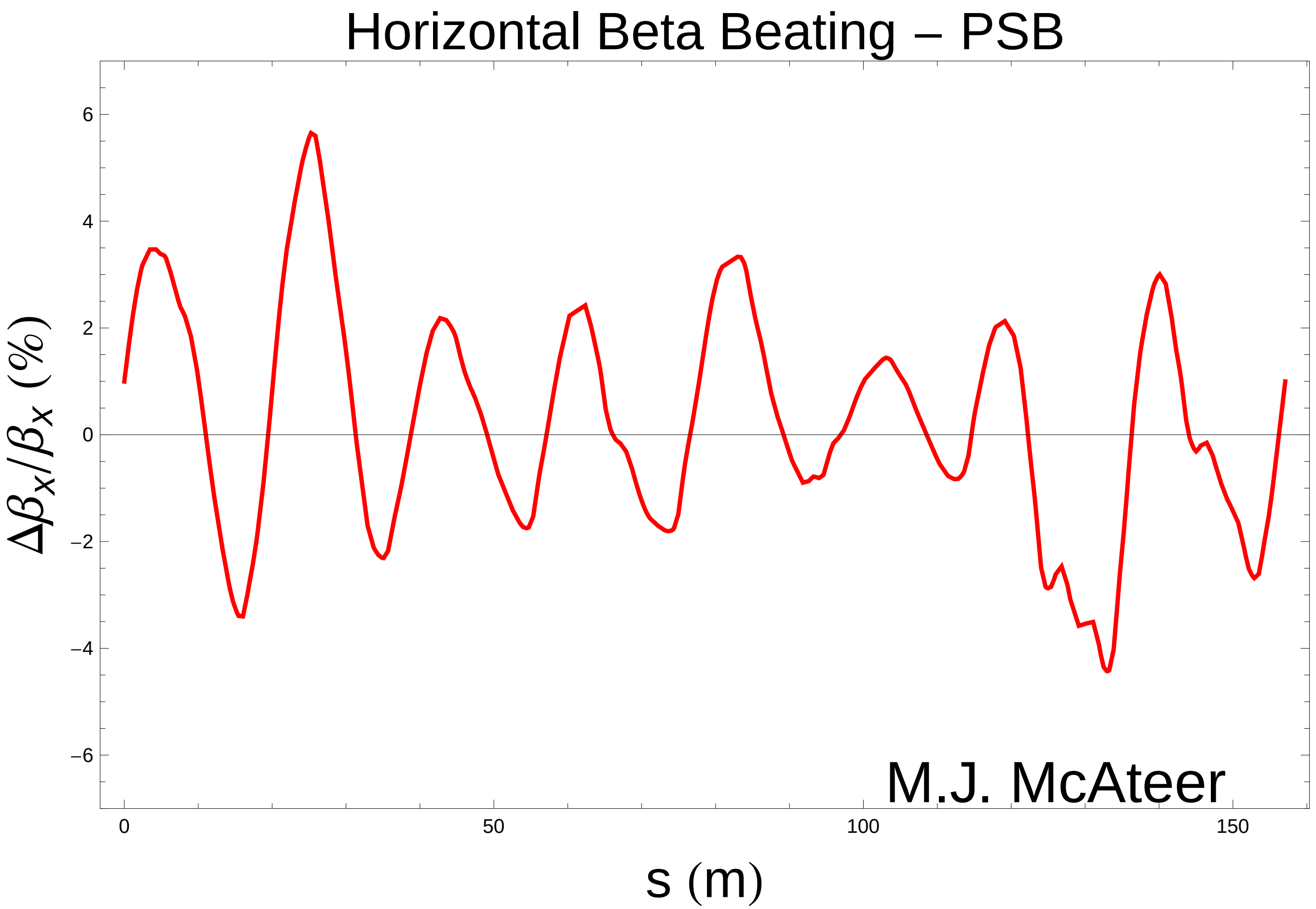
Pseudo-inverse via SVD

$$R = U \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \\ 0 & 0 & 0 \end{pmatrix} V'$$

Imagine $\sigma_3 << \sigma_2$, then just neglect it:

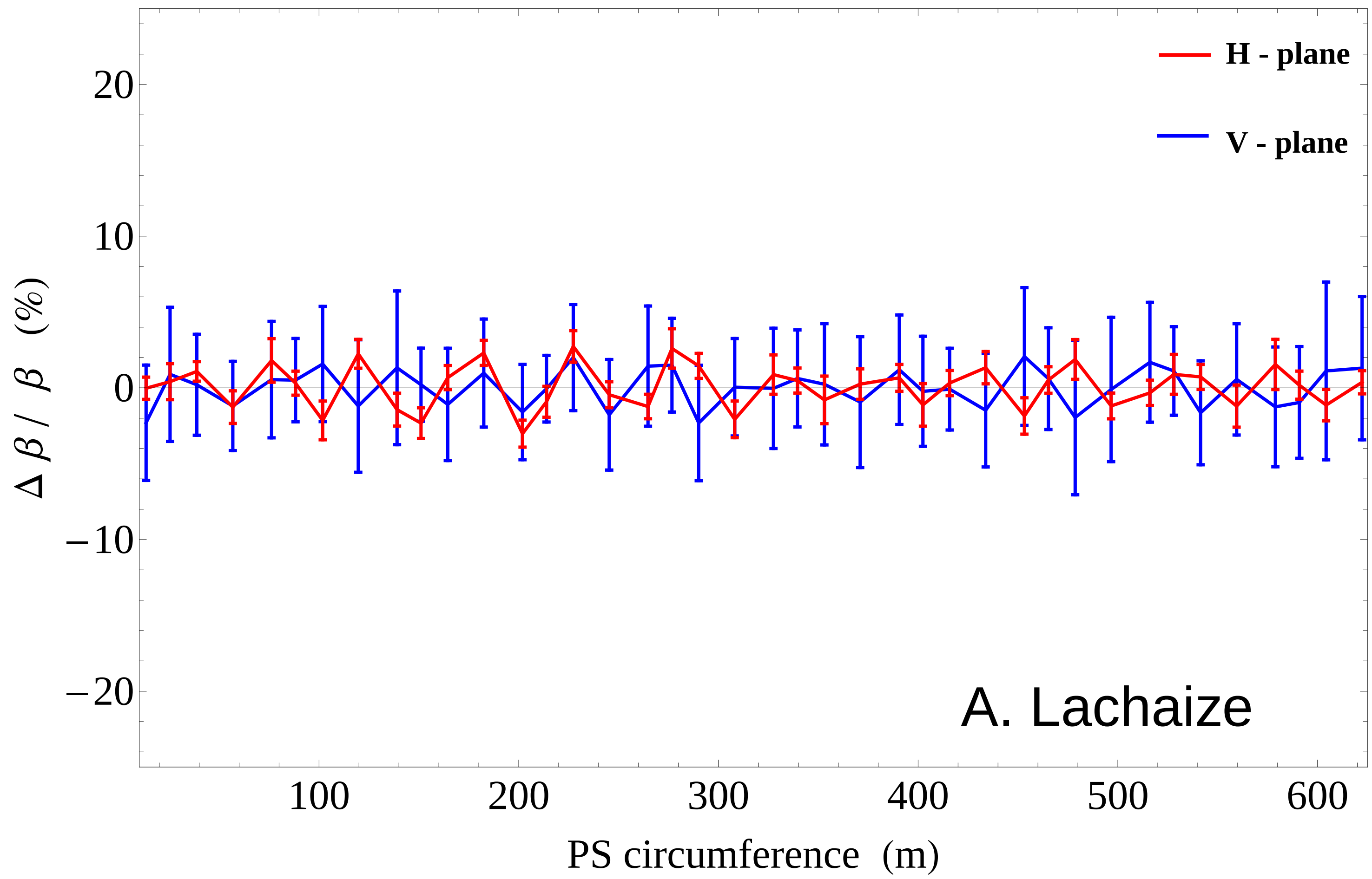
$$R^{-1} = V \begin{pmatrix} \frac{1}{\sigma_1} & 0 & 0 \\ 0 & \frac{1}{\sigma_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} U'$$

PSB β-beating



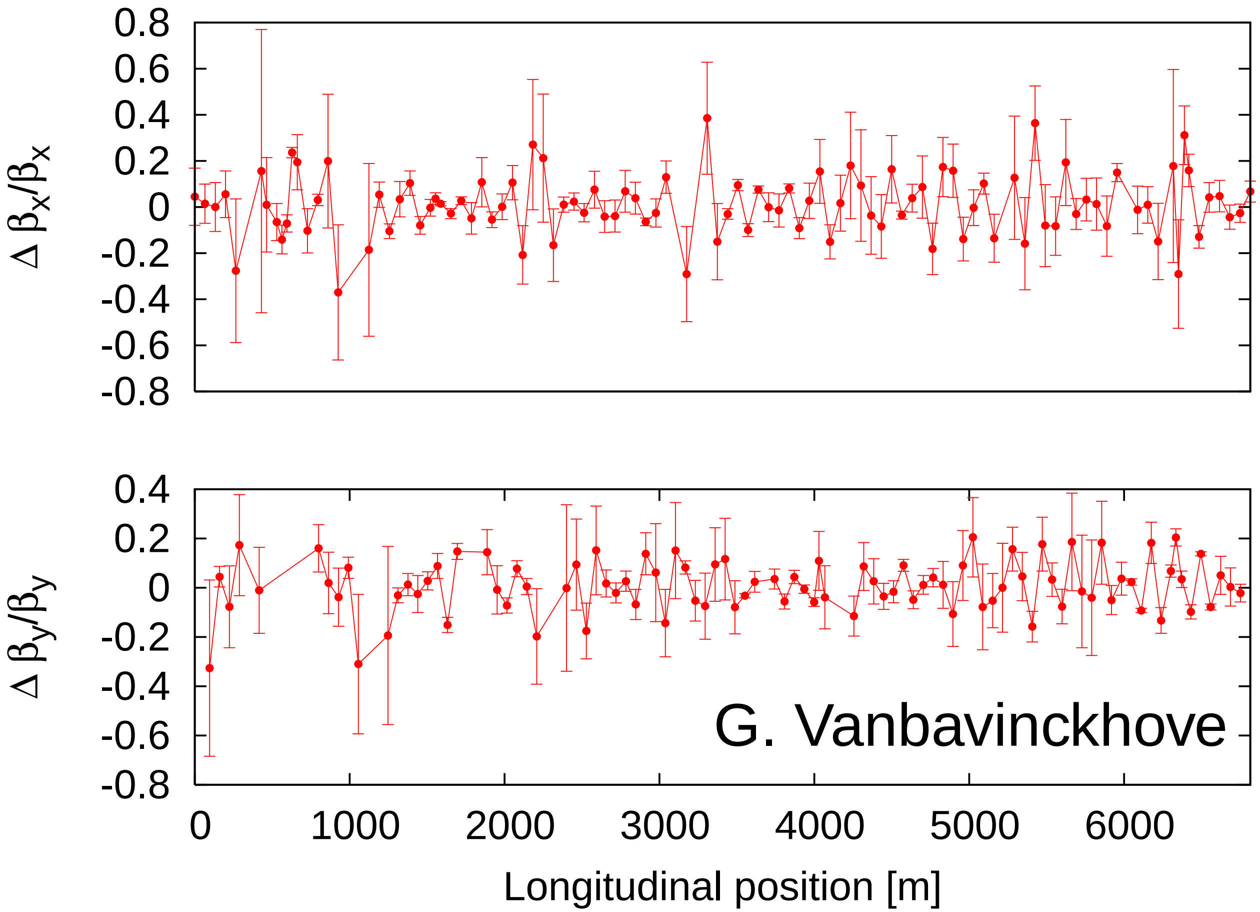
Peak β-beating of $\approx$5%

PS β-beating



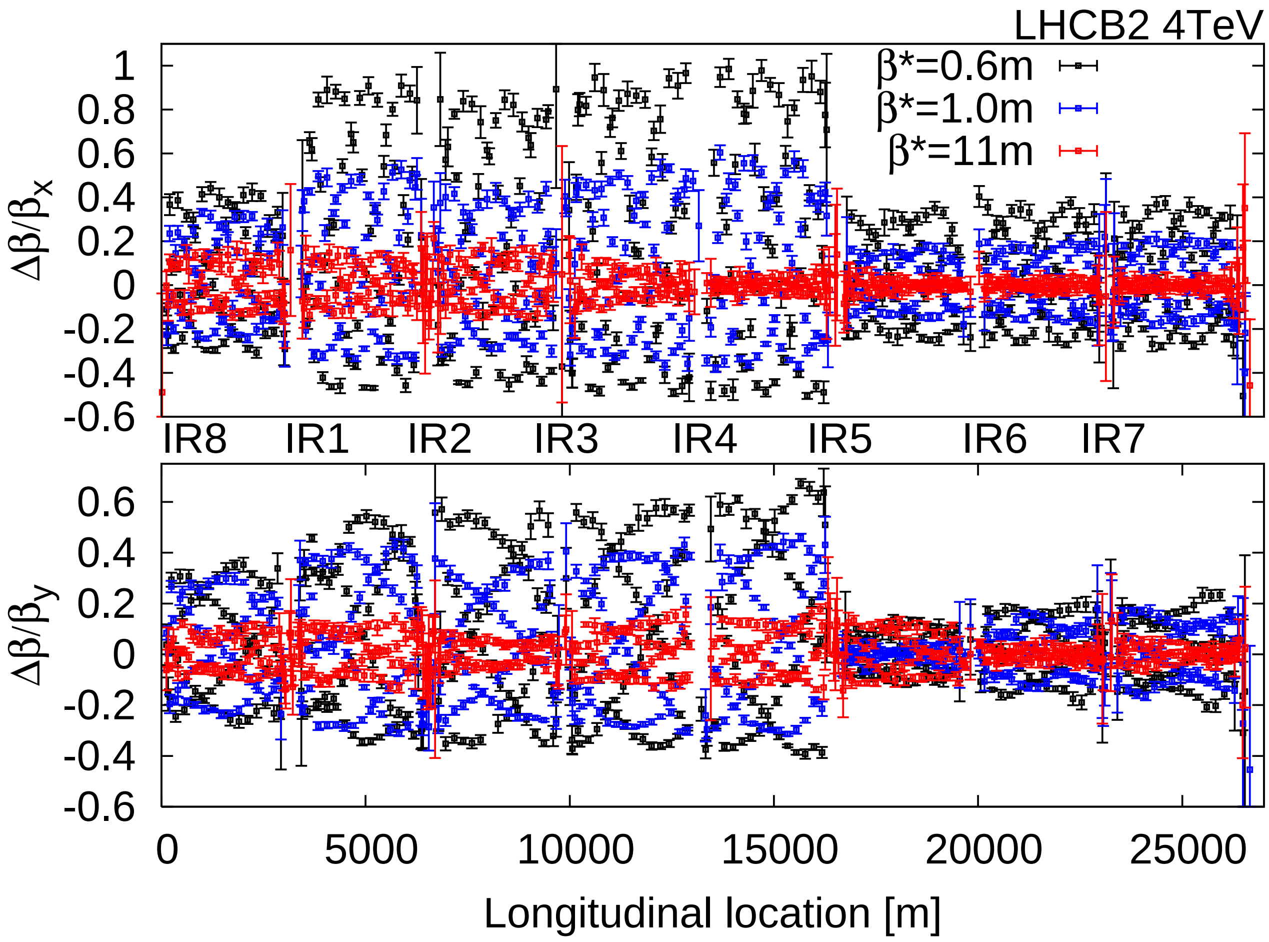
Peak β-beating of $\approx$ 4%

SPS β-beating (Q20)



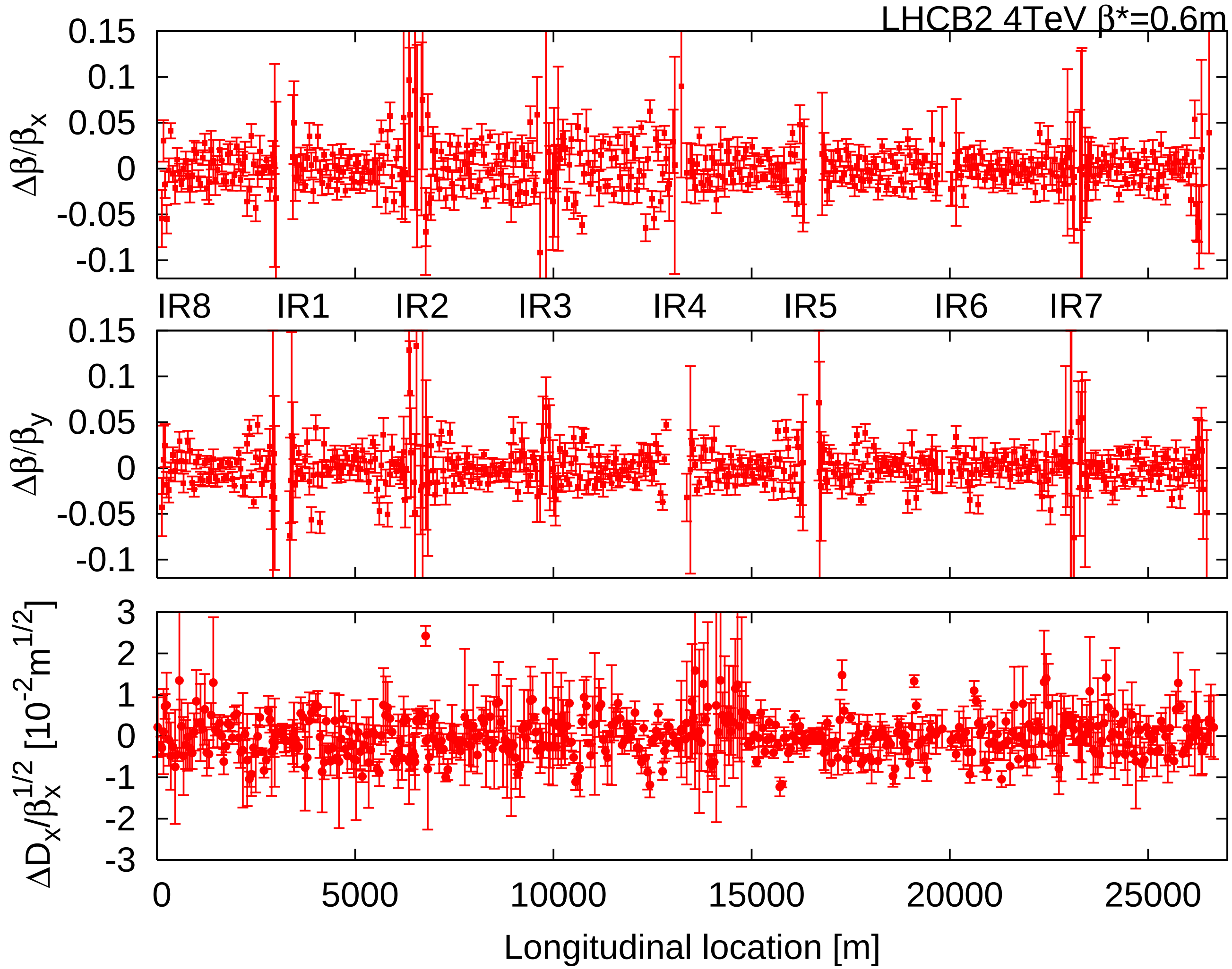
Peak β-beating of $\approx$ 25%

LHC β-beating, before correction



Peak β-beating of $\approx$ 100% !!!

LHC β-beating, after correction



Correction brings peak β-beating to $\approx$ 7%

Dynamic linear imperfections

- Ground motion and vibrations in quadrupoles produce sinusoidal dipolar fields
- Electrical noise can cause currents in quadrupoles and dipole to oscillate in time
- Electromagnetic pollution can act directly on the beam.
- Slow variations ($f << Q_{x,y} \times f_{rev}$) just cause a time varying orbit and optics
- Fast variations ($f \approx Q_{x,y} \times f_{rev}$) can cause resonances and emittance growth

An oscillating dipolar field

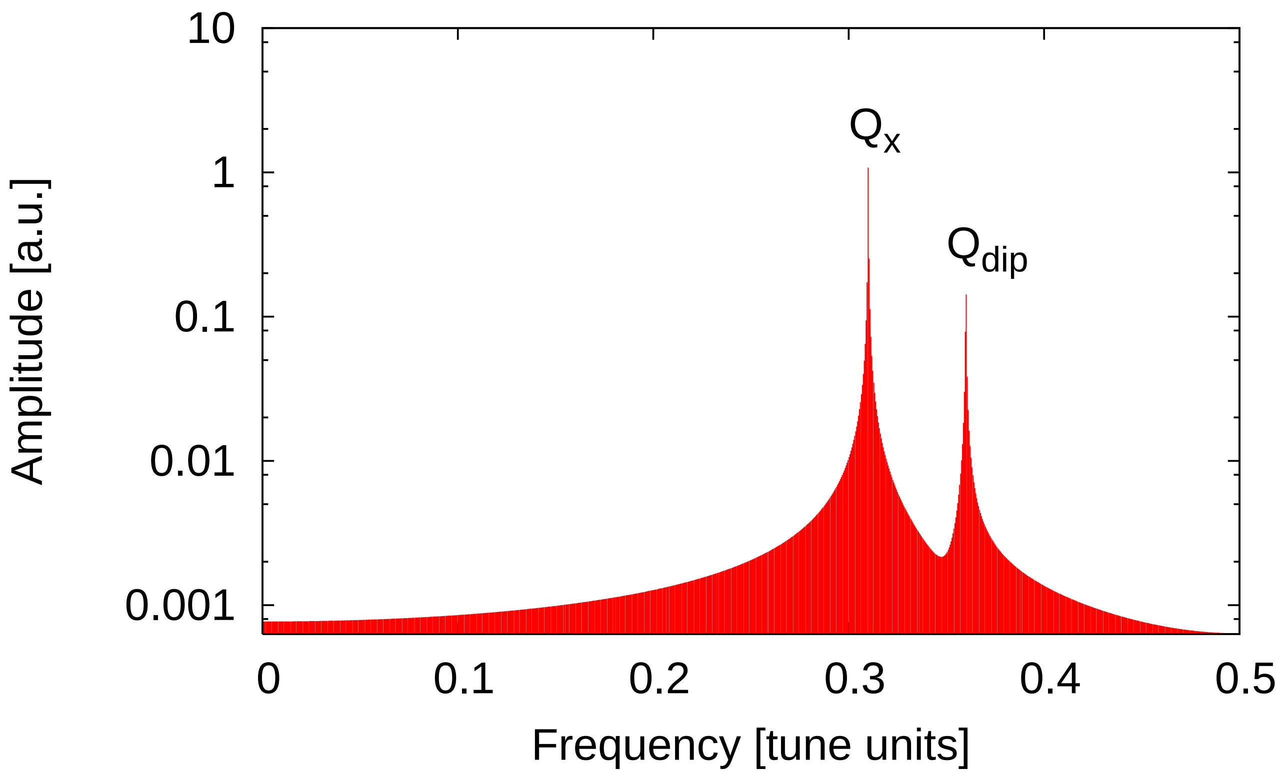
- Let $Q_{dip} = f_{dip}/f_{rev}$ be the tune of the dipolar field oscillation.
- This causes the appearance of new resonances
- Linear resonances: $Q_x \pm Q_{dip} = N$
- Non-linear resonances of sextupolar order:

$$Q_x \pm 2Q_{dip} = N$$

$$2Q_x \pm Q_{dip} = N$$

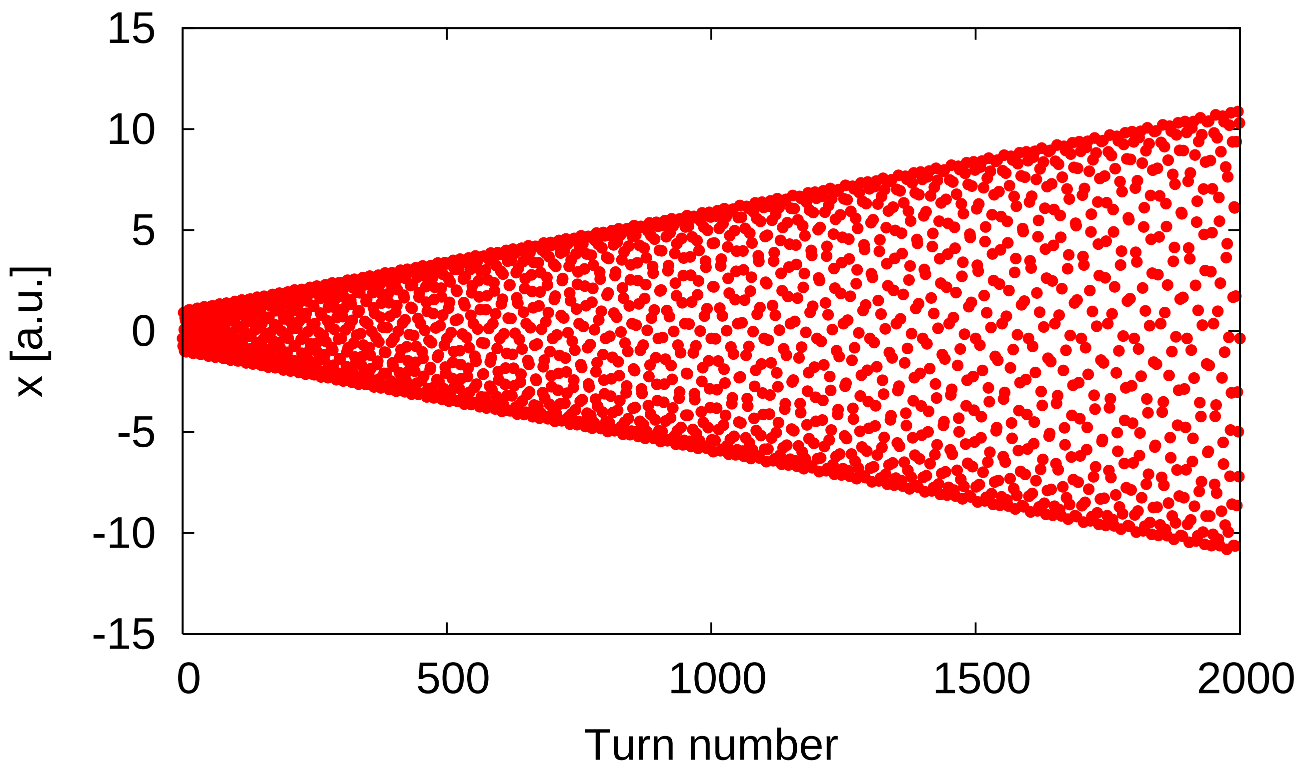
- Note that $mQ_{dip} = N$ is not a problem

Oscillating dipolar field, $Q_x \neq Q_{dip}$



Orbit oscillates with Q_{dip} but there is no emittance growth far from resonances.

Oscillating dipolar field, $Q_x = Q_{dip}$

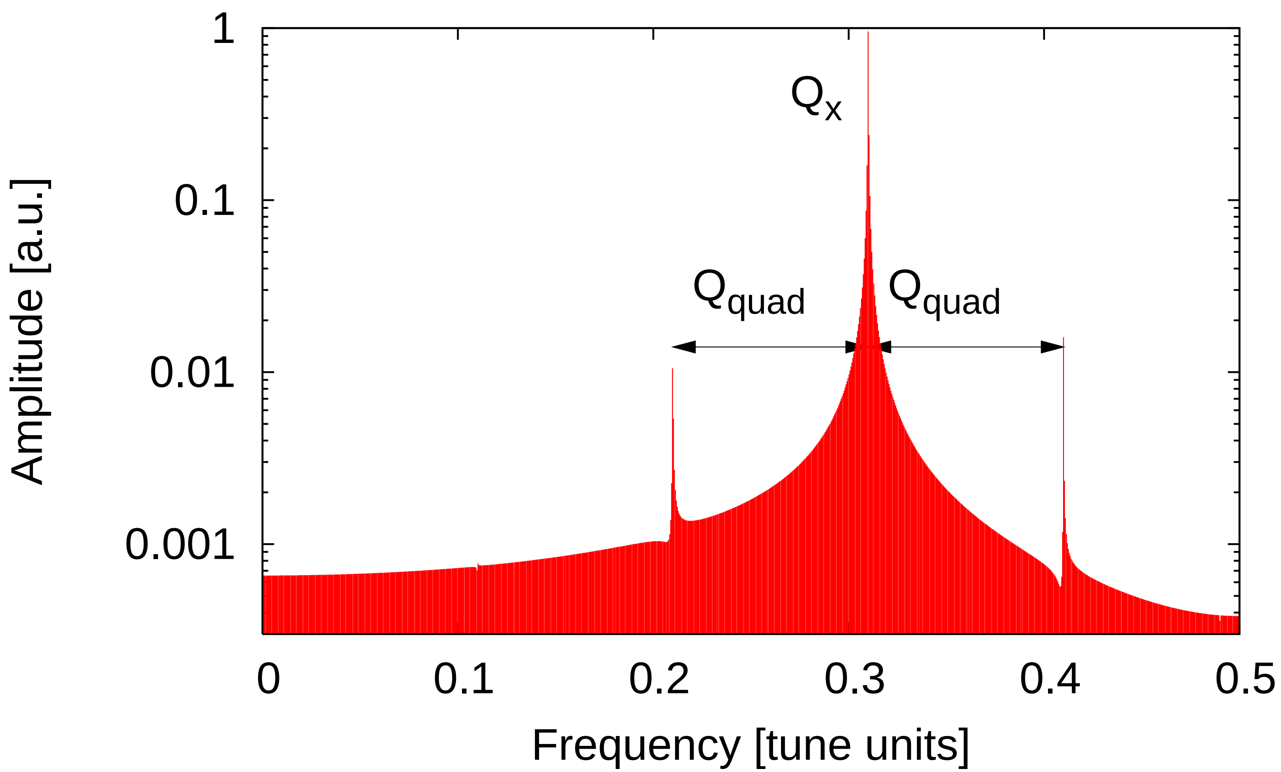


Linear growth in time $\longrightarrow$ Emittance growth.

An oscillating quadrupolar field

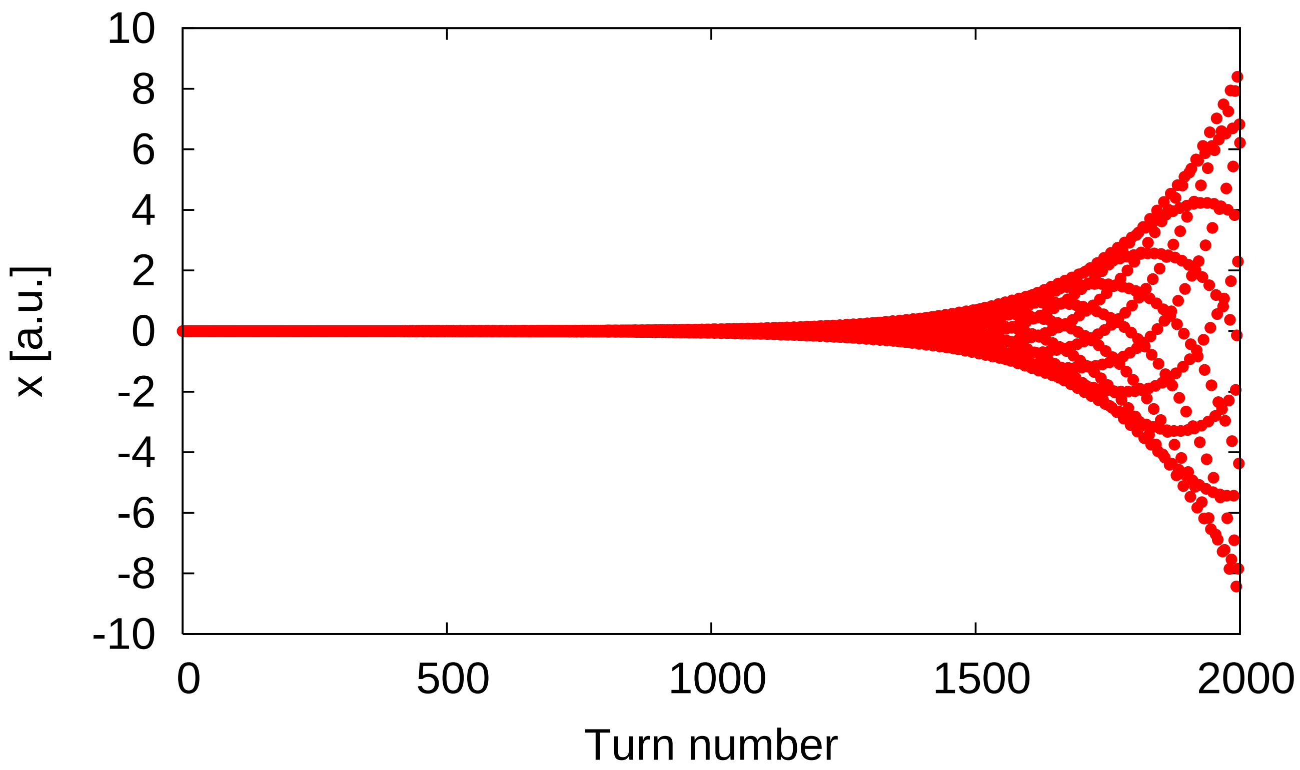
- Let $Q_{quad} = f_{quad}/f_{rev}$ be the tune of the quadrupolar field oscillation.
- This causes the appearance of new resonances
- Linear resonances: $2Q_x \pm Q_{quad} = N$

Oscillating quadrupolar field, $2Q_x \neq Q_{quad}$



Tune is modulated with Q_{quad}, displaying side-bands at $Q_x \pm Q_{quad}$ but there is no emittance growth far from resonances.

Oscillating quadrupolar field, $2Q_x = Q_{quad}$



Exponential growth, clear signatures depending on the oscillating field type.

Mismatched injections

- ⋆ Beam must be injected on the closed orbit to avoid emittance growth from decoherence
- ⋆ Decoherence from amplitude detuning: movie
- ⋆ Similarly, β-functions at the end of injection line must be equal to those at the injection point.