5D relativistic optics for matter and antimatter interferometry.

Christian J. Bordé
LPL & SYRTE
http://christian.j.borde.free.fr

OUTLINE OF THE LECTURE

1. **Introduction to atom interferometry with labelled internal states**
2. **Application to hydrogen**
3. **5D Relativistic atom/antiatom interferometry**
4. **Theory of gravimeters and clocks and test of the EEP**

http://christian.j.borde.free.fr/

An accurate measurement of the effect of gravitation and inertia on antimatter also appears as a possibility already discussed in [65] with a transmission-grating interferometer, although we believe, for obvious reasons, that an antiatom interferometer using laser beams for the antihydrogen beam splitters (so-called Ramsey-Bordé interferometers) would be better suited for such an experiment. Such an interferometer has been recently demonstrated for hydrogen [35]. Coherent beams of antihydrogen will be produced either by Bose-Einstein condensation and/or by stimulated bosonic amplification[9] [7].

Ch. J. Bordé, Atomic clocks and inertial sensors

Metrologia, 2002, **39**, 435-463

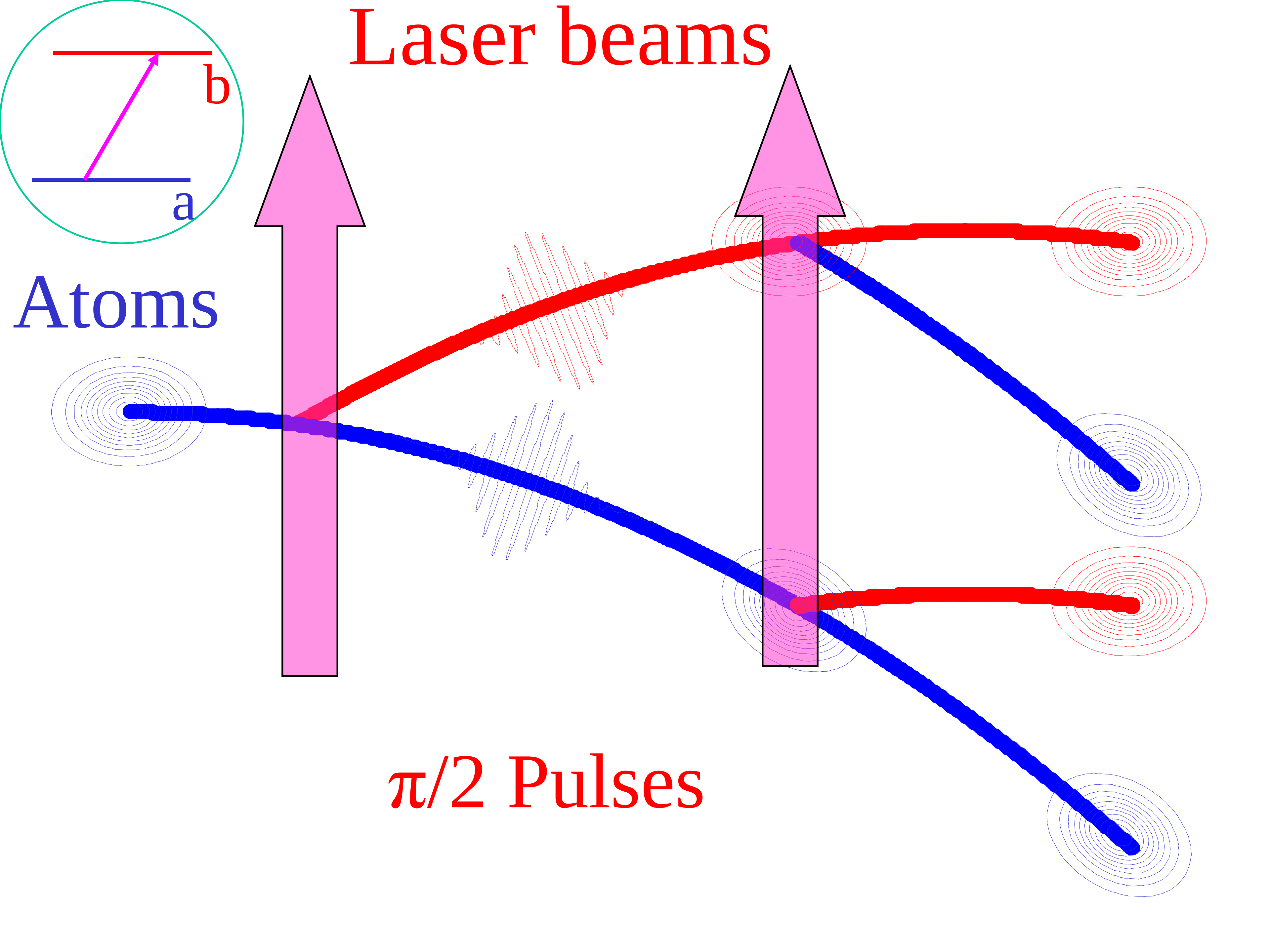
[65] T. J. Phillips (1997)

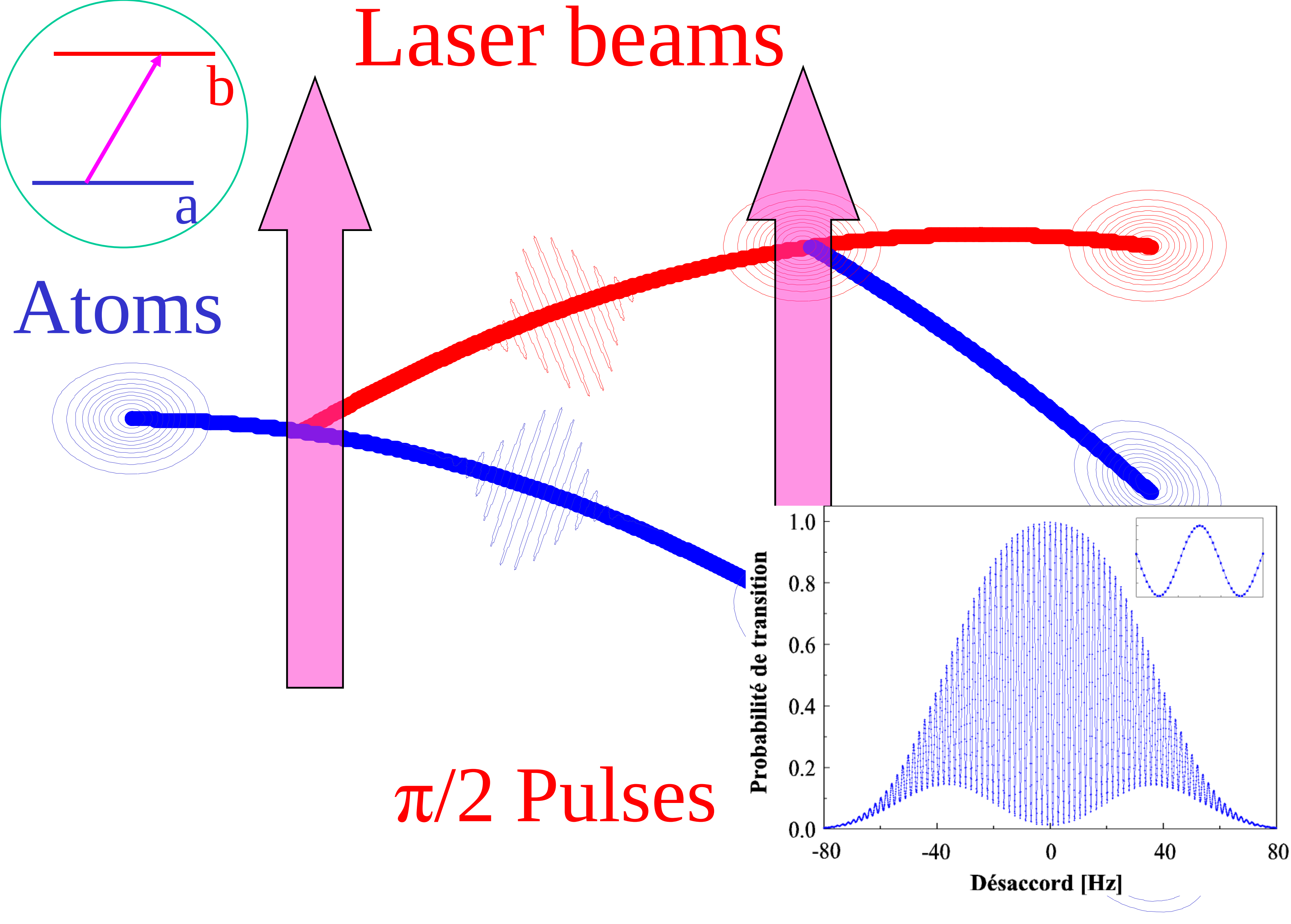
Volume 140, number 1,2 PHYSICS LETTERS A 4 September 1989

ATOMIC INTERFEROMETRY WITH INTERNAL STATE LABELLING

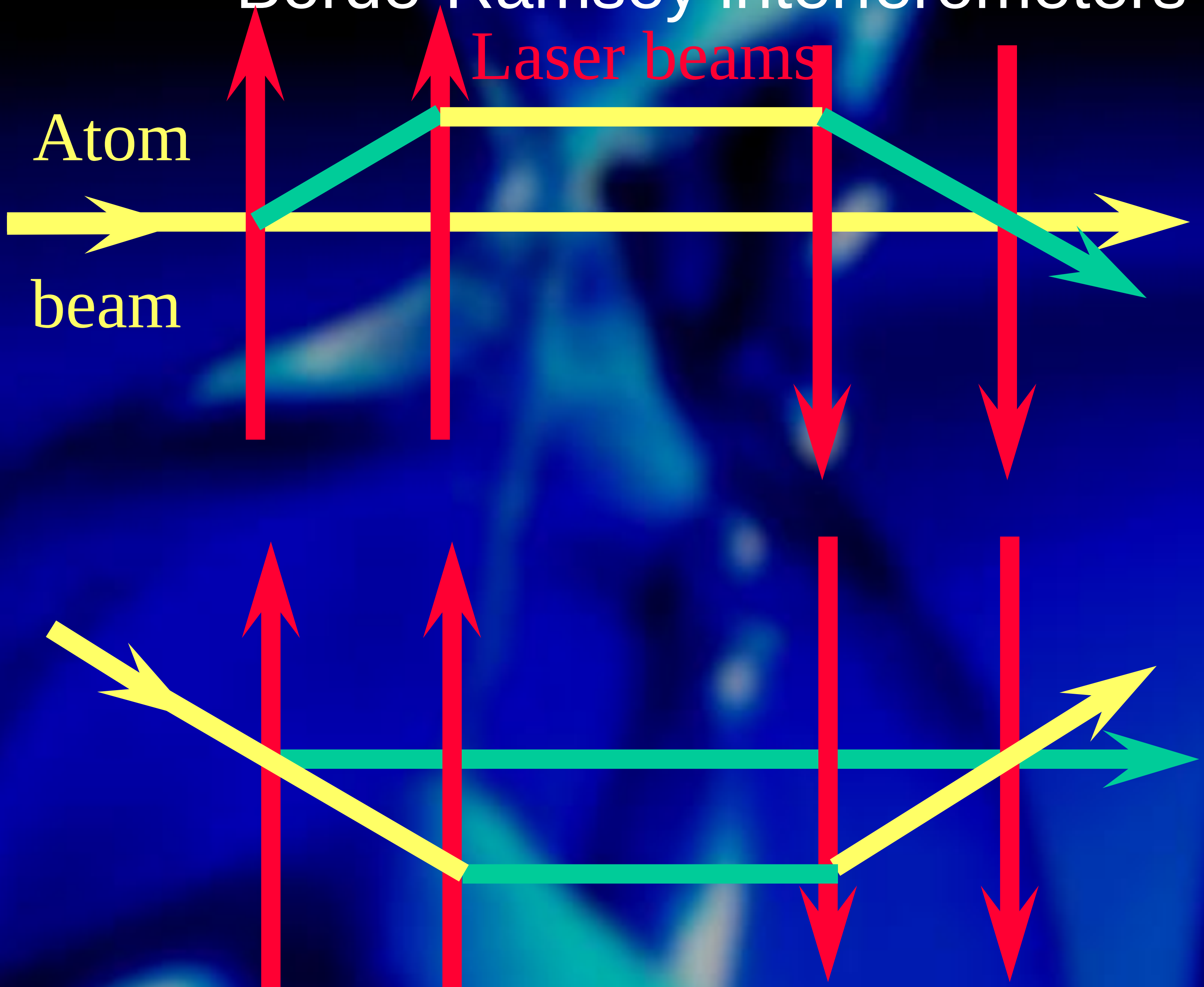
Ch.J. BORDÉ

It is shown that the interaction geometry comprising four travelling laser waves which is used to obtain optical Ramsey fringes in atomic spectroscopy, is also well suited to build an atom interferometer based on the atomic recoil. Since two different internal states are associated with the two arms of the interferometer, the de Broglie phase, induced by rotation or acceleration, manifests itself as a frequency shift of the Ramsey fringes.

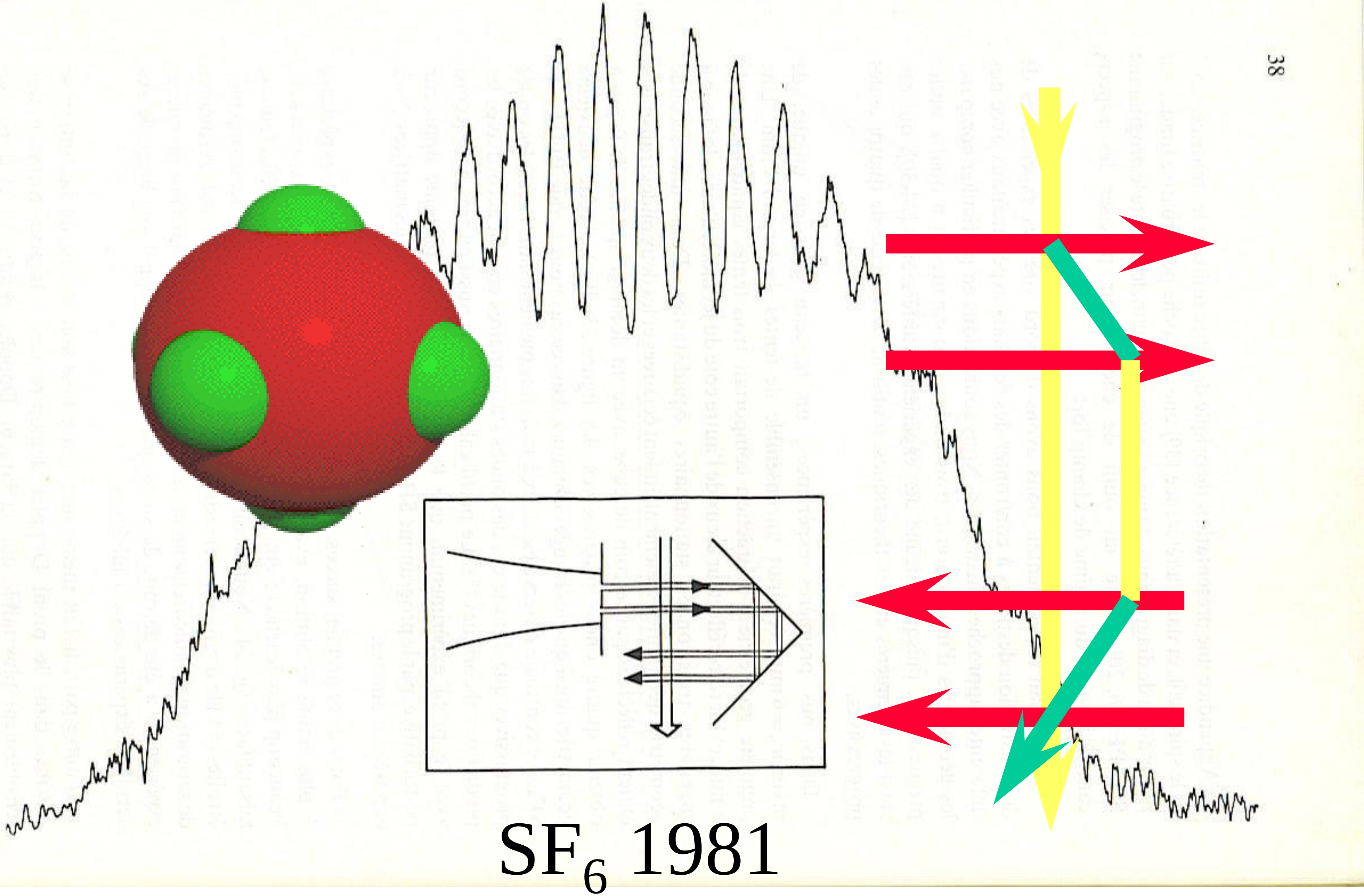

Laser beams
b
a
Atoms
π/2 Pulses


Laser beams
b
a
Atoms
π/2 Pulses
1.0
0.8
0.6
0.4
0.2
0.0
-80
-40
0
40
80
Probabilité de transition
Désaccord [Hz]

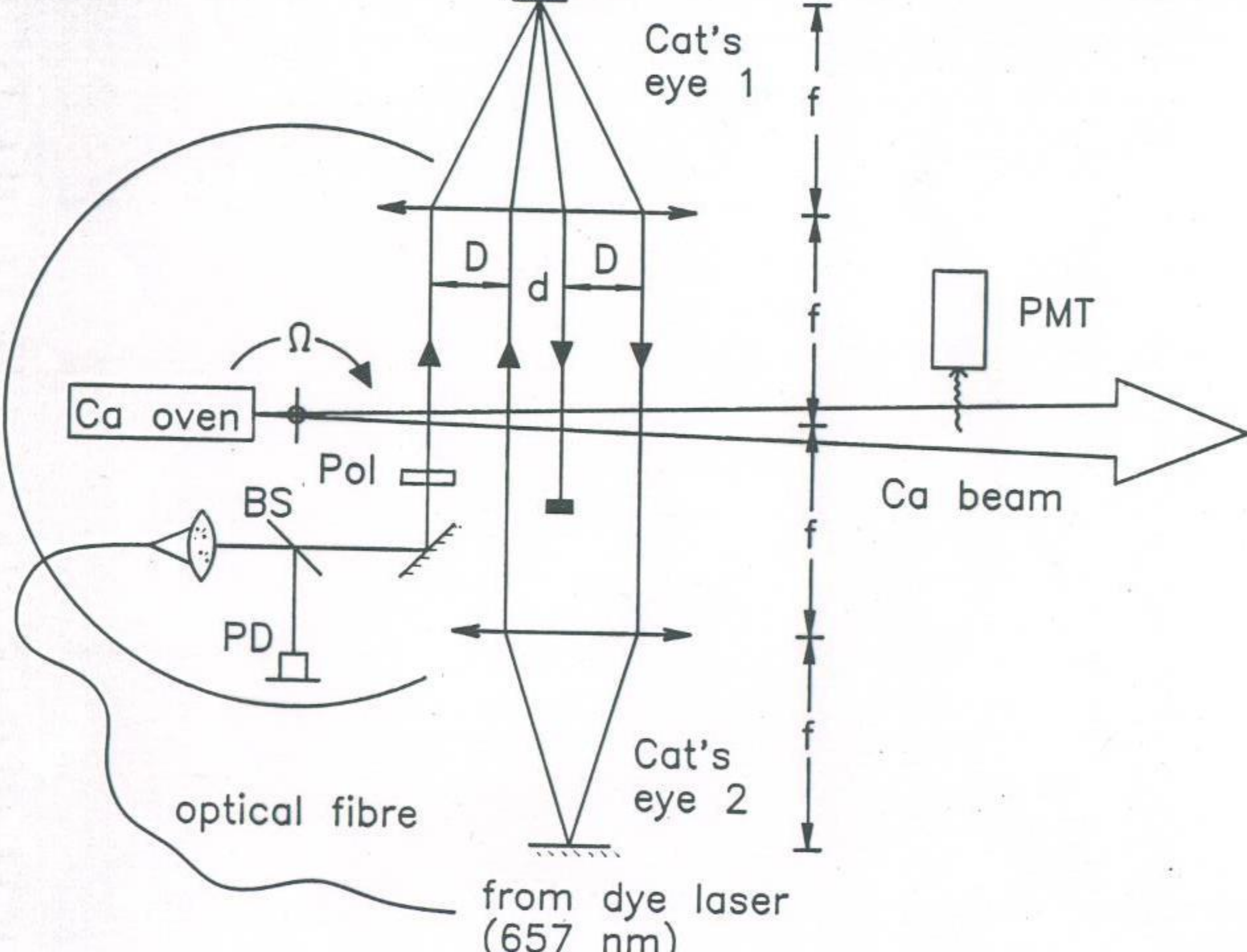
Bordé-Ramsey interferometers

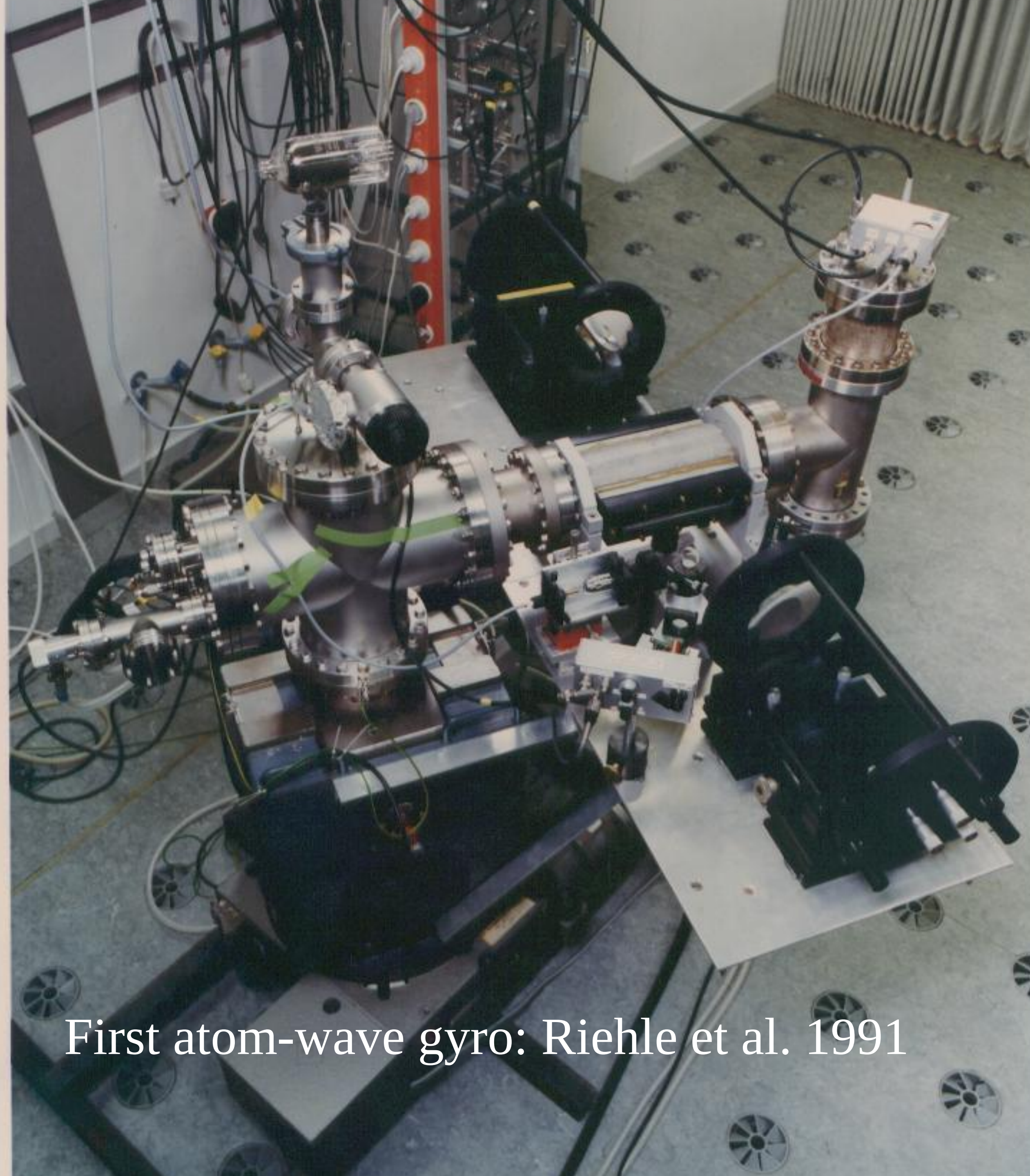


MOLECULAR INTERFEROMETRY

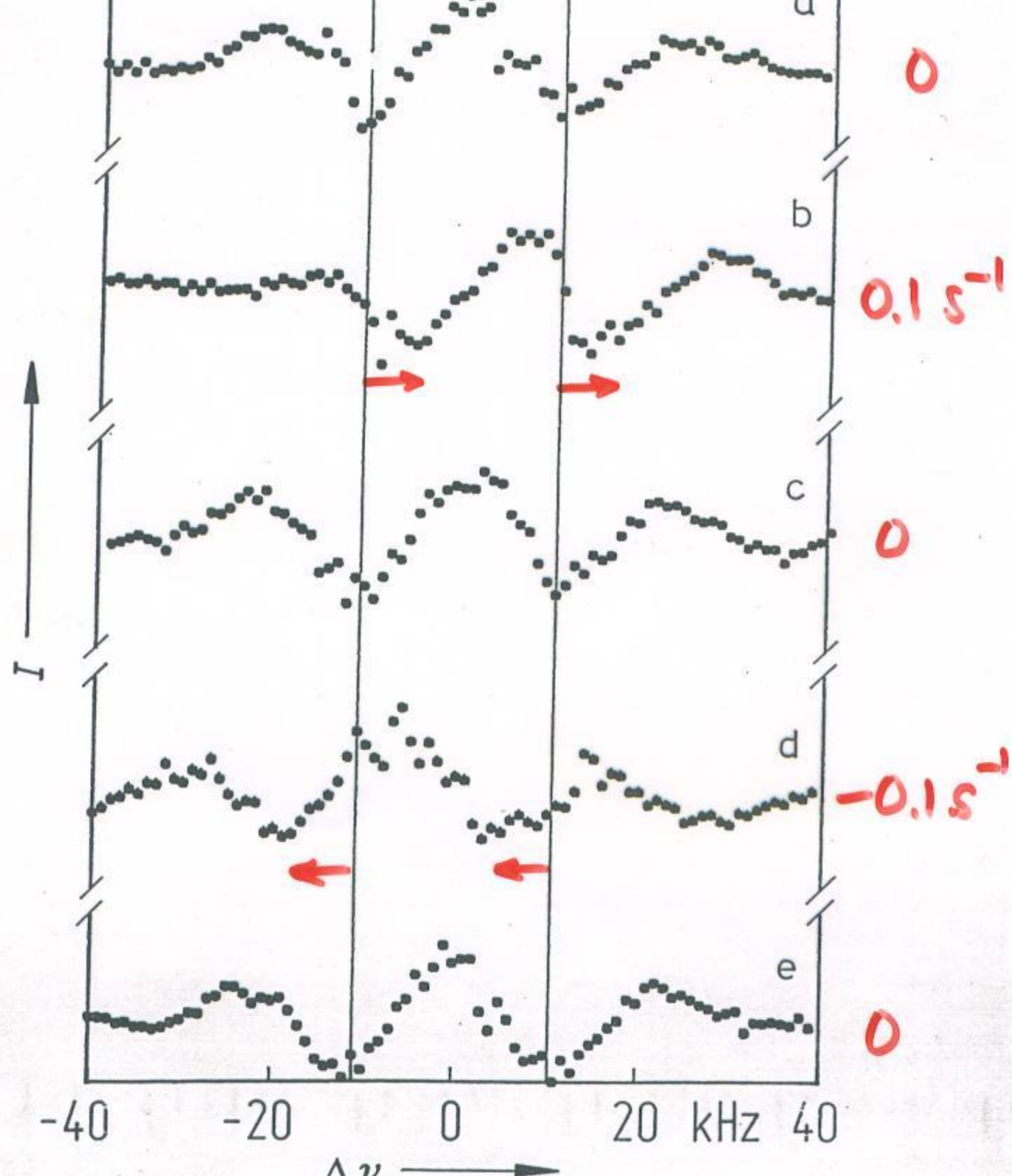


SF_6 1981


Cat's eye 1
f
D
D
d
f
PMT
Ω
Ca oven
Pol
BS
Ca beam
f
PD
f
Cat's eye 2
optical fibre
from dye laser (657 nm)



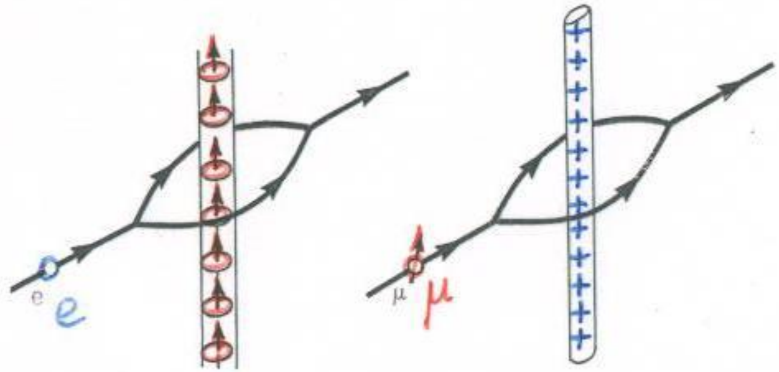
First atom-wave gyro: Riehle et al. 1991


a
0
b
0.1 s⁻¹
c
0
d
−0.1 s⁻¹
e
0
I
-40
-20
0
20
kHz
40
Δν

NEUTRONS CANNOT ENCIRCLE LINES OF ELECTRIC CHARGE UNPHASED

Glasses were raised at a conference at the University of South Carolina in December to toast the 30-year-old prediction that gave reality to the magnetic vector potential. According to that prediction, made by Yakir Aharonov (University of South Carolina and the University of Tel Aviv) and David Bohm (Birkbeck College of London University), an electron will experience a phase shift as it encircles an infinitely long solenoid, even though there is no magnetic field—only a vector potential—in the region where it travels. Almost all those who originally doubted this conclusion have long since been persuaded by several elegant experimental confirmations.

Now there may be a possible analog to the Aharonov–Bohm effect: At the recent anniversary conference on the fundamental aspects of quantum theory, researchers described their mea-



Switch the charges (blue) and the dipoles (red) in the Aharonov–Bohm effect (left) and you have its "dual" (right). In the former, the different phases acquired along the two paths shift the electron interference pattern. A similar interference occurs for neutral particles with magnetic dipoles (such as neutrons), as shown by a recent experiment done by researchers from the University of Melbourne and the University of Missouri at Columbia. (Adapted from ref. 1.)

Aharonov-Casher phase shift

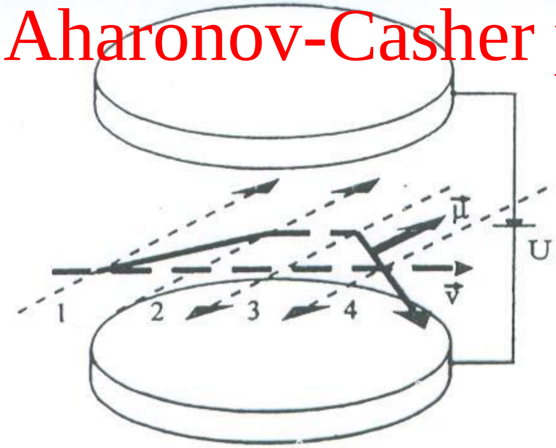


Fig. 5. Bordé interferometer in an electric field (only the high-frequency recoil component is shown). Between the first and second and also between the third and fourth laser beam, one partial wave is in the ground state (*dashed line*) and the other in the excited state (*full line*). Since the different states have different polarizabilities α_i and different magnetic moments μ_i, an applied electric field will generate different potentials for the respective partial wave. Consequently, a phase shift occurs due to the interaction with the electric moment (Stark effect) and the magnetic moment (AC effect)

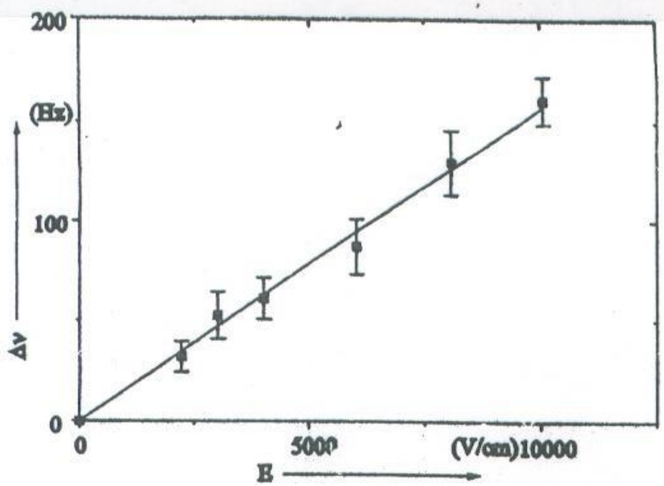


Fig. 6. The measured shift $\Delta\nu_{AC}$ vs the applied electric field strength (*squares*). The *line* corresponds to a least-squares fit which is used for the comparison with the expected value of the Aharonov-Casher phase shift

From Laser Spectroscopy X, 1991

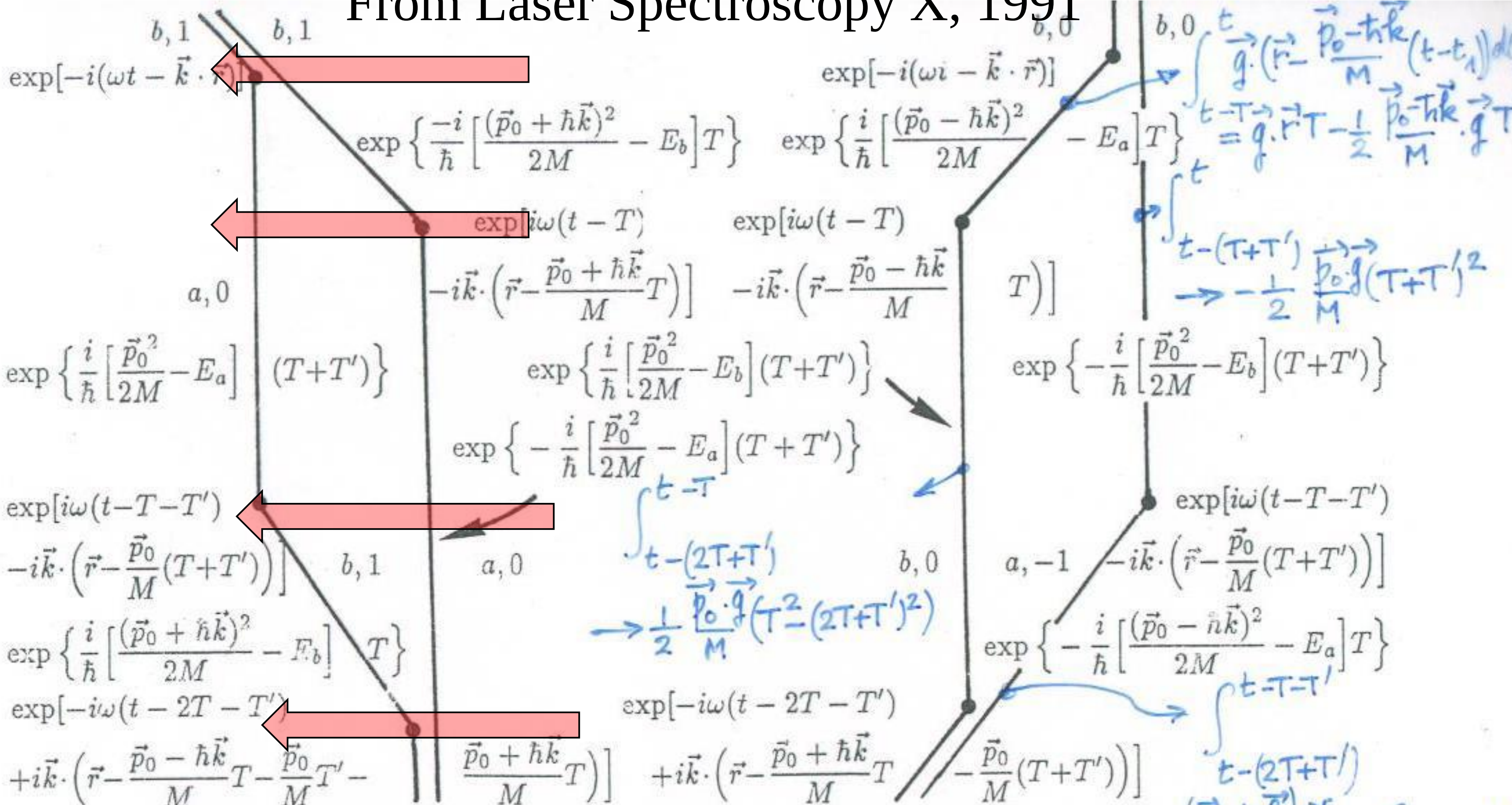


Figure 2

2nd vertex: $\vec{r}-\frac{\vec{p}_0+\hbar\vec{k}}{M}T \longrightarrow \vec{r}-\frac{\vec{p}_0+\hbar\vec{k}}{M}T-\vec{g}\frac{T^2}{2}$

$\vec{v} \longrightarrow \vec{v}-\vec{g}\,T$

$$\Delta\phi = -\vec{k}\cdot\vec{g}\,(T+T')\,T \quad \propto \frac{1}{v^2}$$

Mass independent (equivalence principle)

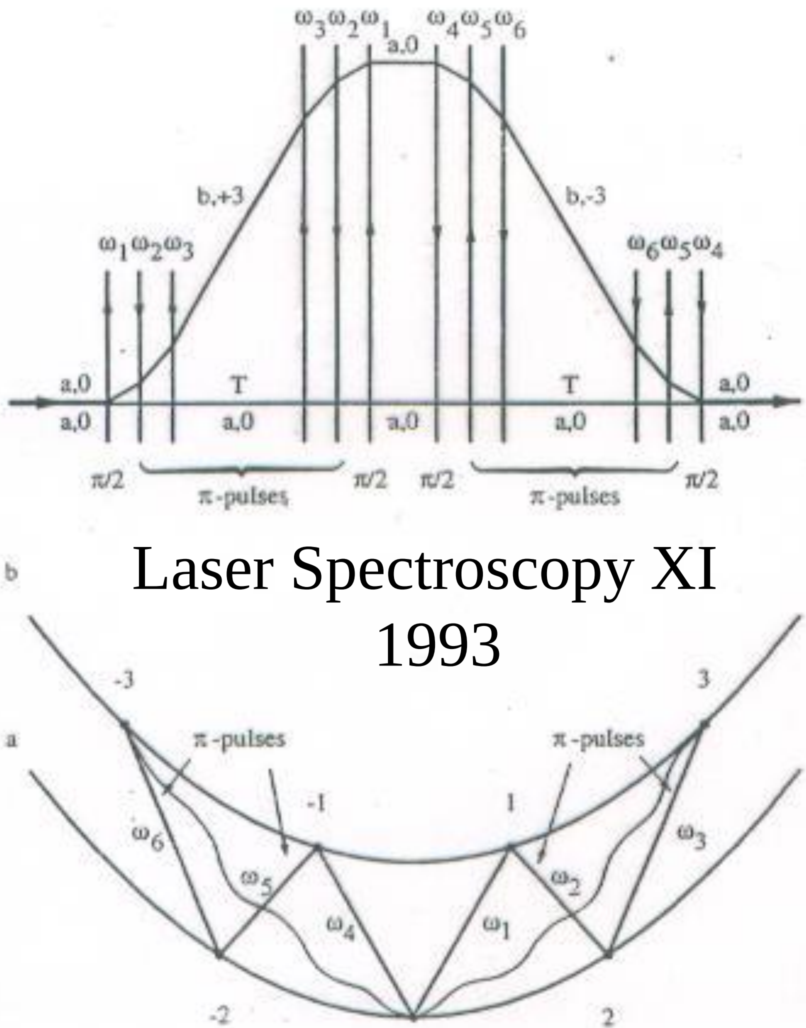
NEW OPTICAL ATOMIC INTERFEROMETERS FOR PRECISE MEASUREMENTS OF RECOIL SHIFTS. APPLICATION TO ATOMIC HYDROGEN

Christian J. Bordé

Laboratoire de Physique des Lasers, UPN, URA 282, Villetaneuse and Laboratoire de Gravitation et Cosmologie Relativistes, UPMC, URA 769, Paris, France

M. Weitz * and T. W. Hänsch,

Max-Planck-Institut für Quantenoptik, D-8046 Garching, Germany



Laser Spectroscopy XI

1993

Fig. 1. Space-time and energy-momentum diagrams for an interferometer with $|m_1| = 3$ exchanged momentum quanta per interaction sequence. The two-level system interacts with effective multiphoton fields of opposite directions either perpendicular or collinear to the atomic motion. The space-time diagram displays the deflection along the optical axis versus the proper time in the "atomic frame" at the velocity $\vec{p}_0/M$. A coherent superposition of the two states $|a, 0>$ and $|b, m_1>$ is created (wiggly line) and travels freely during the time T leading to a phase shift $\varphi = (\omega_1 - \omega_2 + \omega_3 + \omega_4 - \omega_5 + \omega_6 - 2\omega_0 - 18\delta)T$. A second interferometer with opposite recoil shift is obtained by exchanging the roles of states a and

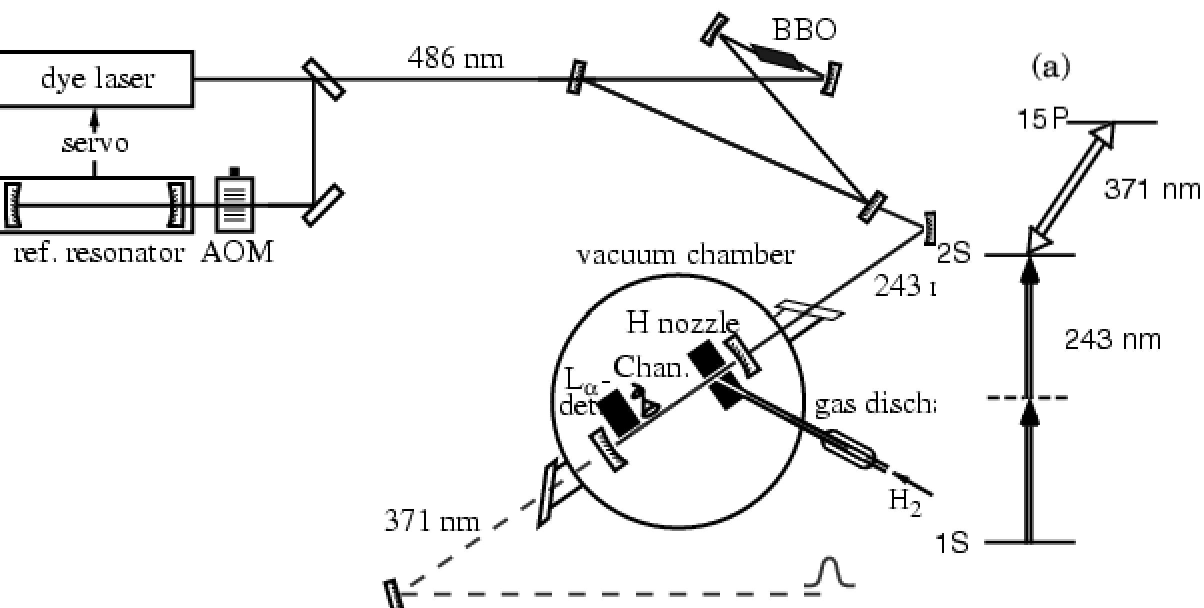
EUROPHYSICS LETTERS 15 January 2002

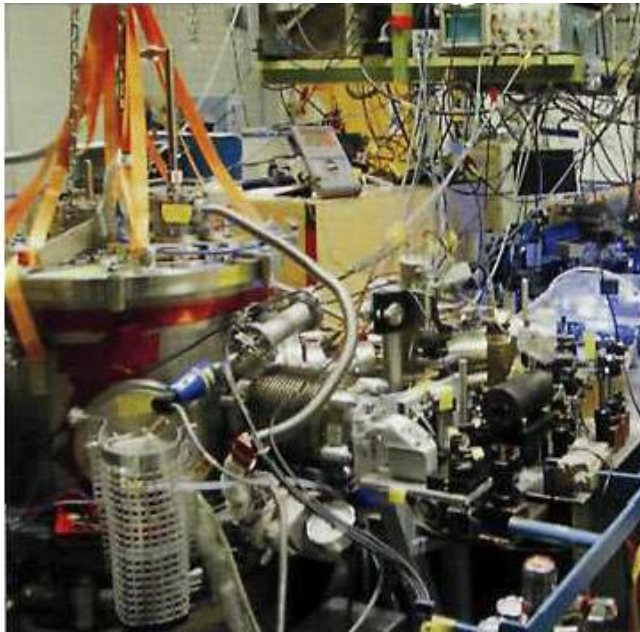
Europhys. Lett., **57** (2), pp. 158–163 (2002)

Hydrogen atom interferometer with short light pulses

T. Heupel[1], M. Mei[1], M. Niering[1], B. Gross[1], M. Weitz[1],
T. W. Hänsch[1] and Ch. J. Bordé[2]

Abstract. – We report the realization of a hydrogen atom interferometer experiment using light as the atomic beam splitter. The wave packets of hydrogen atoms excited to the metastable $2S$ state are coherently split up and later recombined with the help of intense nanosecond light pulses. The pulses are generated by a novel phase-coherent source. These experiments can be seen as a step towards a precision measurement of the recoil energy of a hydrogen atom when absorbing a photon and thus of $\hbar/m_{\text{hydrogen}}$.





Hydrogen atom interferometer with short light pulses

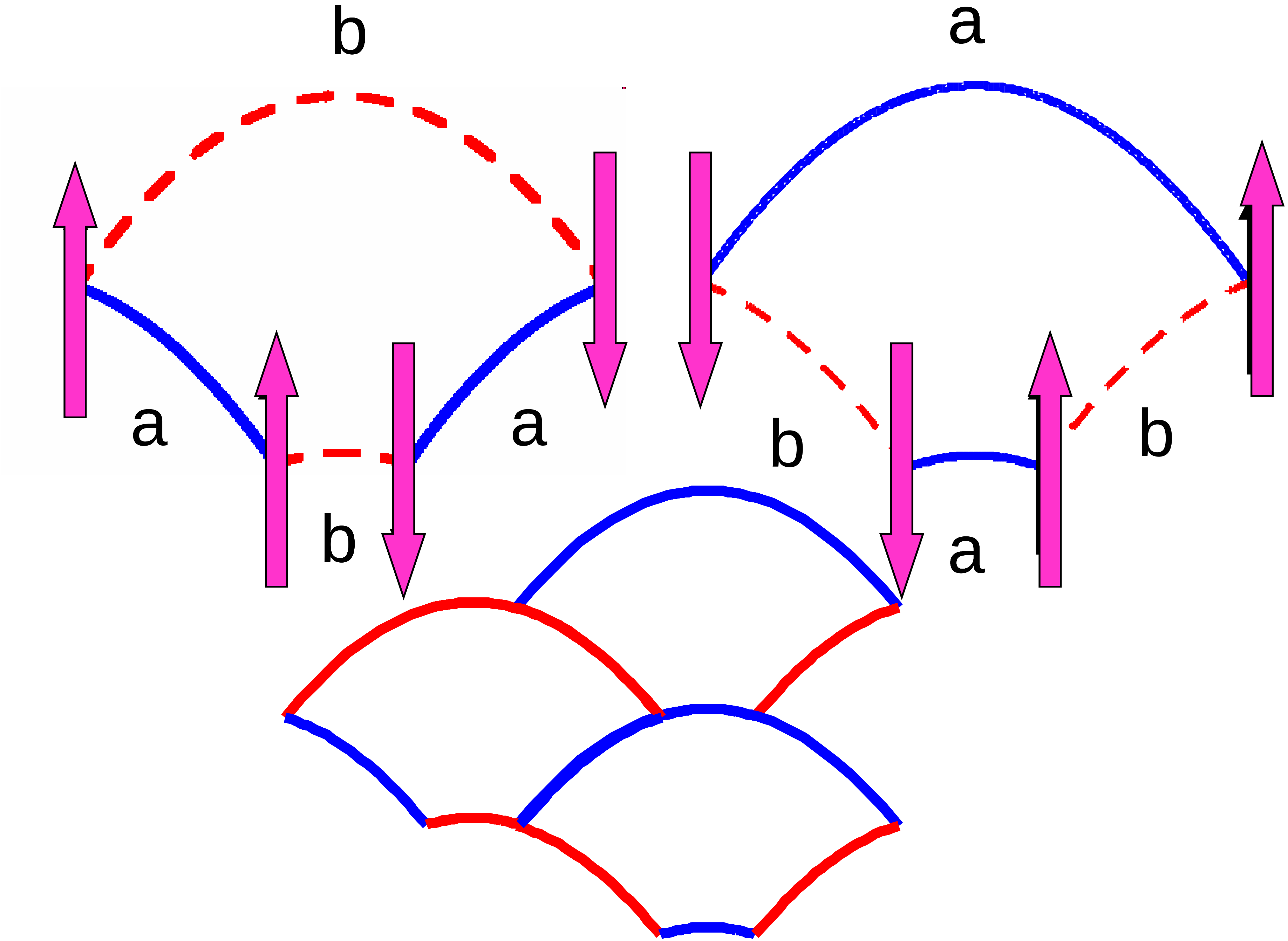
T. Heupel[1], M. Mei[1], M. Niering[1], B. Gross[1], M. Weitz[1],
T. W. Hänsch[1] and Ch. J. Bordé[2]

Europhys. Lett., **57** (2), pp. 158–163 (2002)

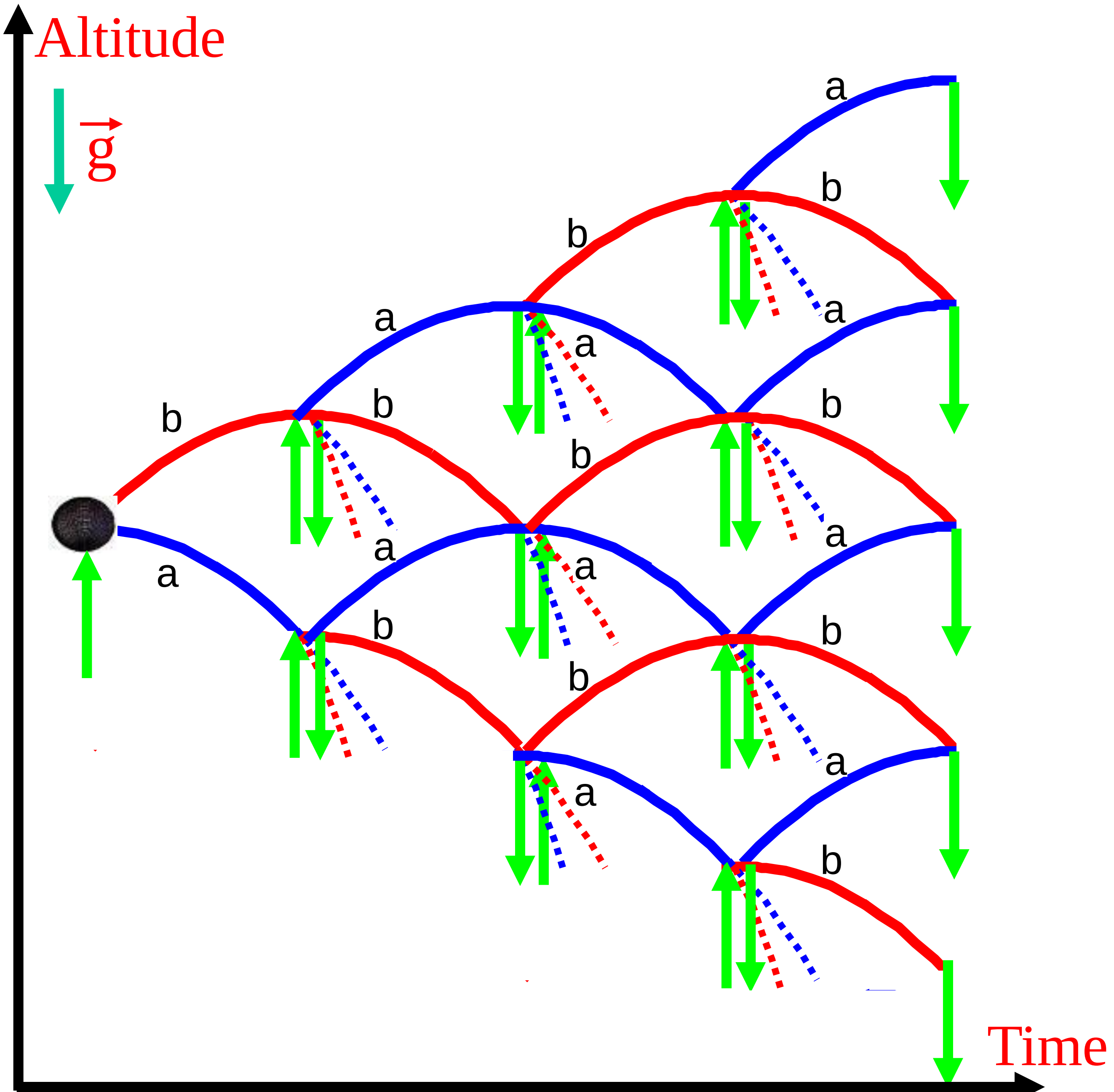
in magnetically trapped atomic hydrogen, which can be evaporatively cooled down to the microkelvin regime [18], makes this light atom a very promising candidate for precision measurements of the photon recoil. Finally, the presently used "photon-echo" atom interferometer has a geometry sensitive to the Earth's acceleration g and could be used when testing for a possible difference in g for hydrogen and antihydrogen. We wish to point out that in contrast to atom interferometric schemes relying on scattering from material gratings, the use of light as a beamsplitter is feasible both for matter and antimatter wave packets.

In summary, we have realized the first hydrogen atom interferometer making use of light pulse interferometer techniques to split and later redirect the atomic paths. This is a first step towards the realization of a hydrogen atom interferometer experiment for measuring the recoil energy shift of the hydrogen atom when absorbing a photon. With the geometry of fig. 3, a precise determination of $\hbar/m_{\text{hyd}}$ and of the fine-structure constant α seems possible.

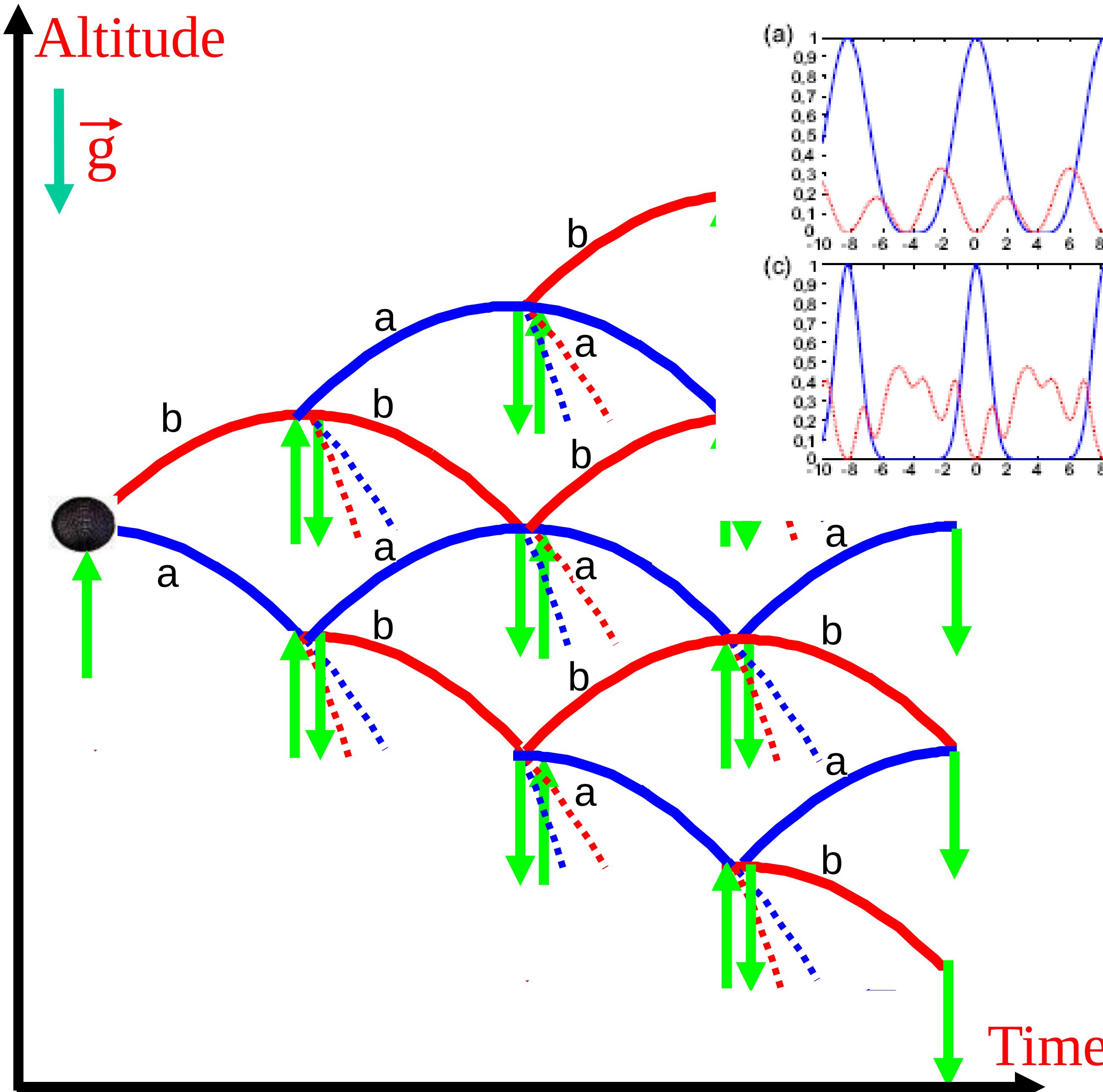
BORDÉ-RAMSEY INTERFEROMETERS

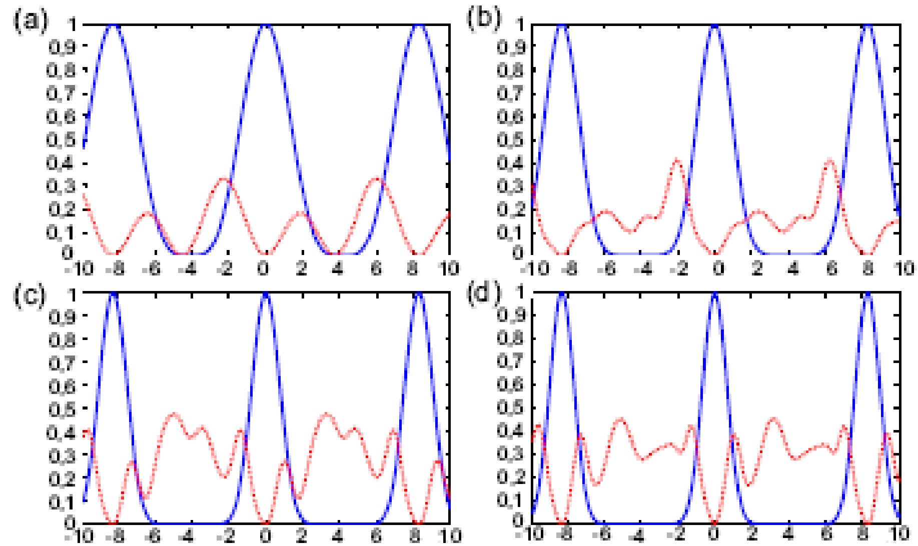


Multiple wave interferometer



Multiple wave interferometer





Theoretical tools for atom optics and interferometry

Christian J. BORDÉ [a,b]

[a] Laboratoire de physique des lasers, UMR 7538 CNRS, université Paris-Nord, 99 avenue J.-B. Clément, Villetaneuse, 93430 France

[b] Institut für Quantenoptik, Universität Hannover, Welfengarten 1, 30167 Hannover, Germany
E-mail: borde@lpl.univ-paris13.fr

(Reçu le 12 janvier 2001, accepté le 13 mars 2001)

Abstract. The development of high sensitivity and high accuracy atom interferometers requires [illegible] in the presence of a weak gravitational field treated as a spin two external field in a flat Minkowski background. This theory is used to calculate and discuss relativistic phase shifts in the context of matter-wave interferometry (especially atom or antiatom interferometry). In this way, many effects are introduced in a unified relativistic framework, including spin-gravitation terms: gravitational red shift, Thomas precession, Sagnac effect, spin-rotation effect, orbital and spin Lense–Thirring effects, de Sitter geodetic precession and finally the effect of gravitational waves.

ENERGY

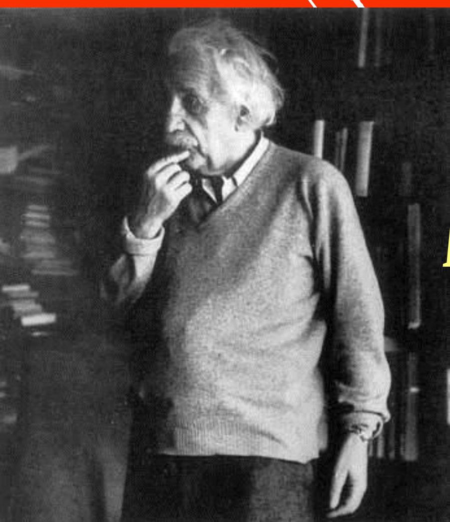
E(p)

$$p^{\mu} = (E/c, p_x, p_y, p_z)$$

$$p_{\mu} = (E/c, -p_x, -p_y, -p_z) = g_{\mu\nu} p^{\nu}$$

$$p^{\mu} p_{\mu} = E^2/c^2 - p^2 = m^2 c^2$$

$$E(\vec{p}) = \sqrt{m^2 c^4 + p^2 c^2}$$



1905

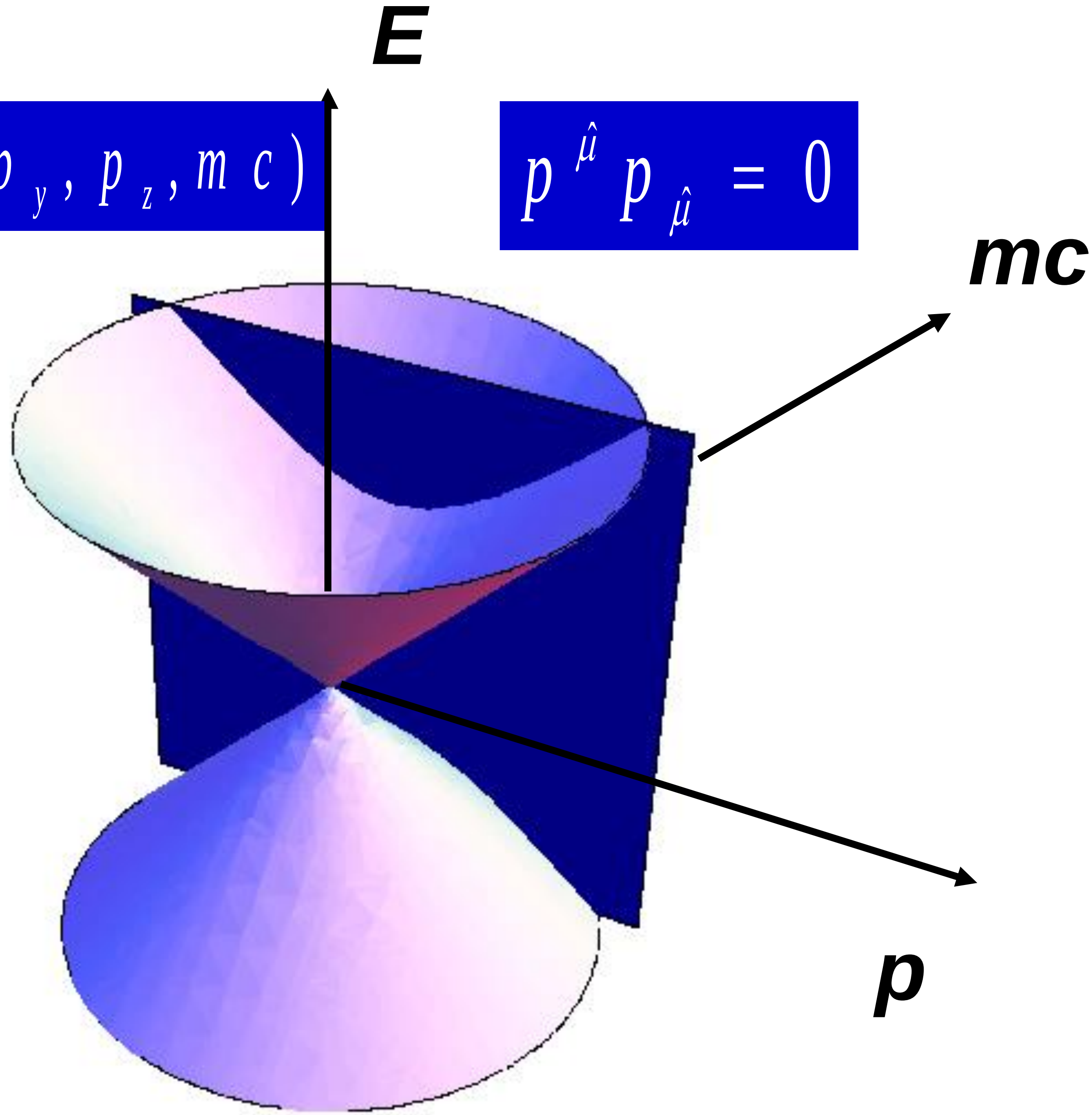
Mass

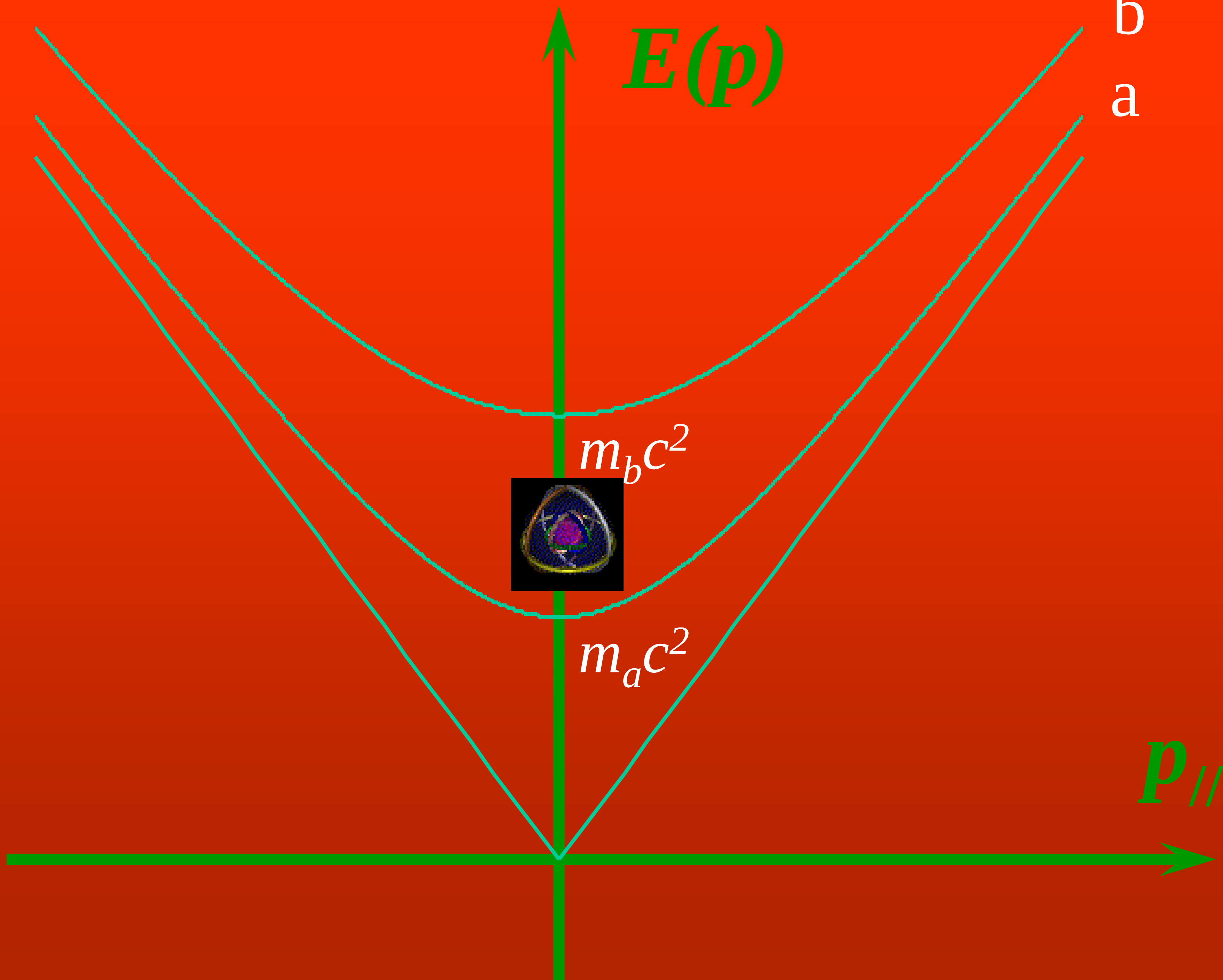
MOMENTUM

$$p^{\hat{\mu}} = (E/c, p_x, p_y, p_z, mc)$$

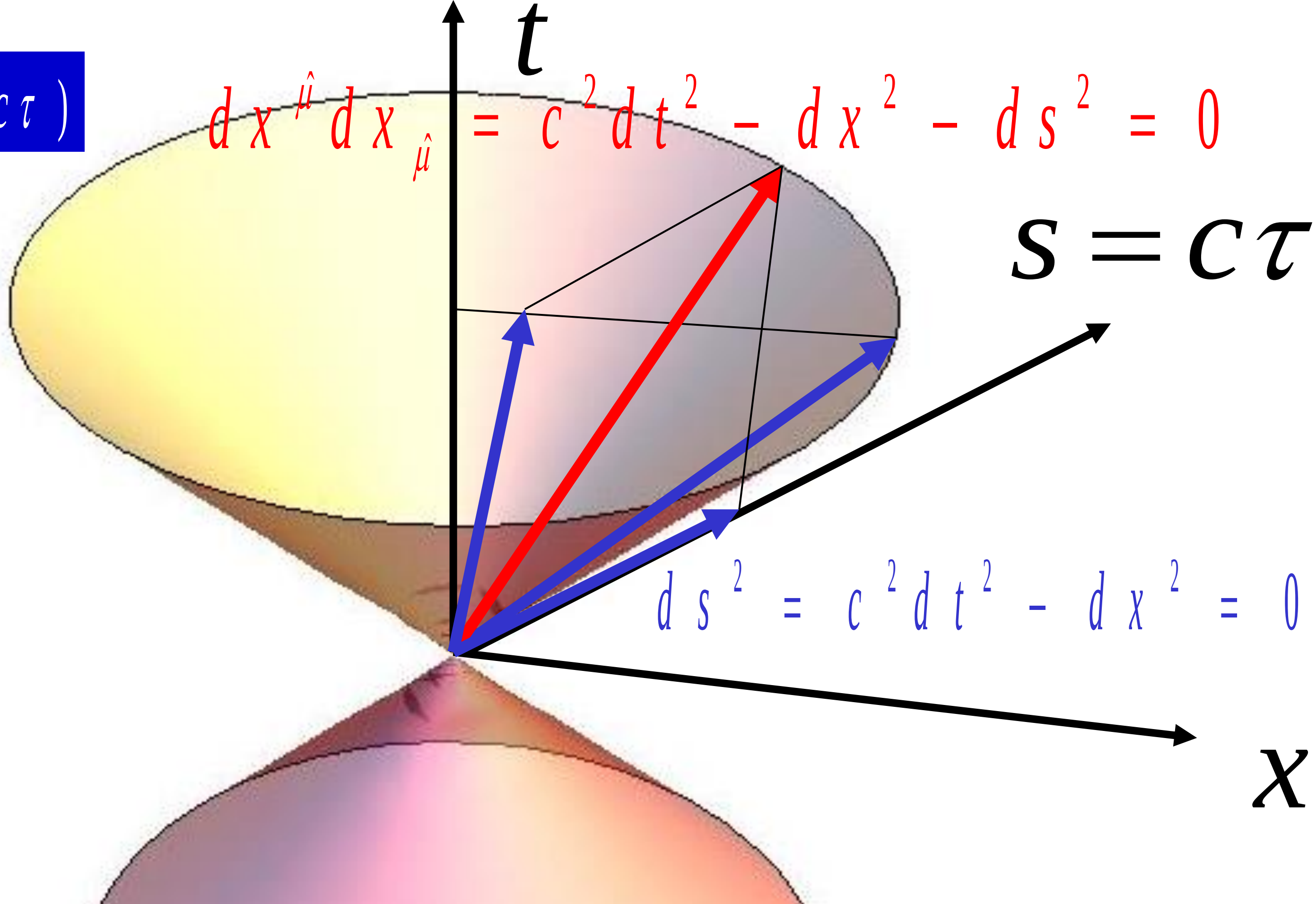
$$\hat{\mu} = 0,1,2,3,4$$

$$p^{\hat{\mu}} p_{\hat{\mu}} = 0$$




b
a
E(p)
$m_b c^2$
$m_a c^2$
$p_{//}$

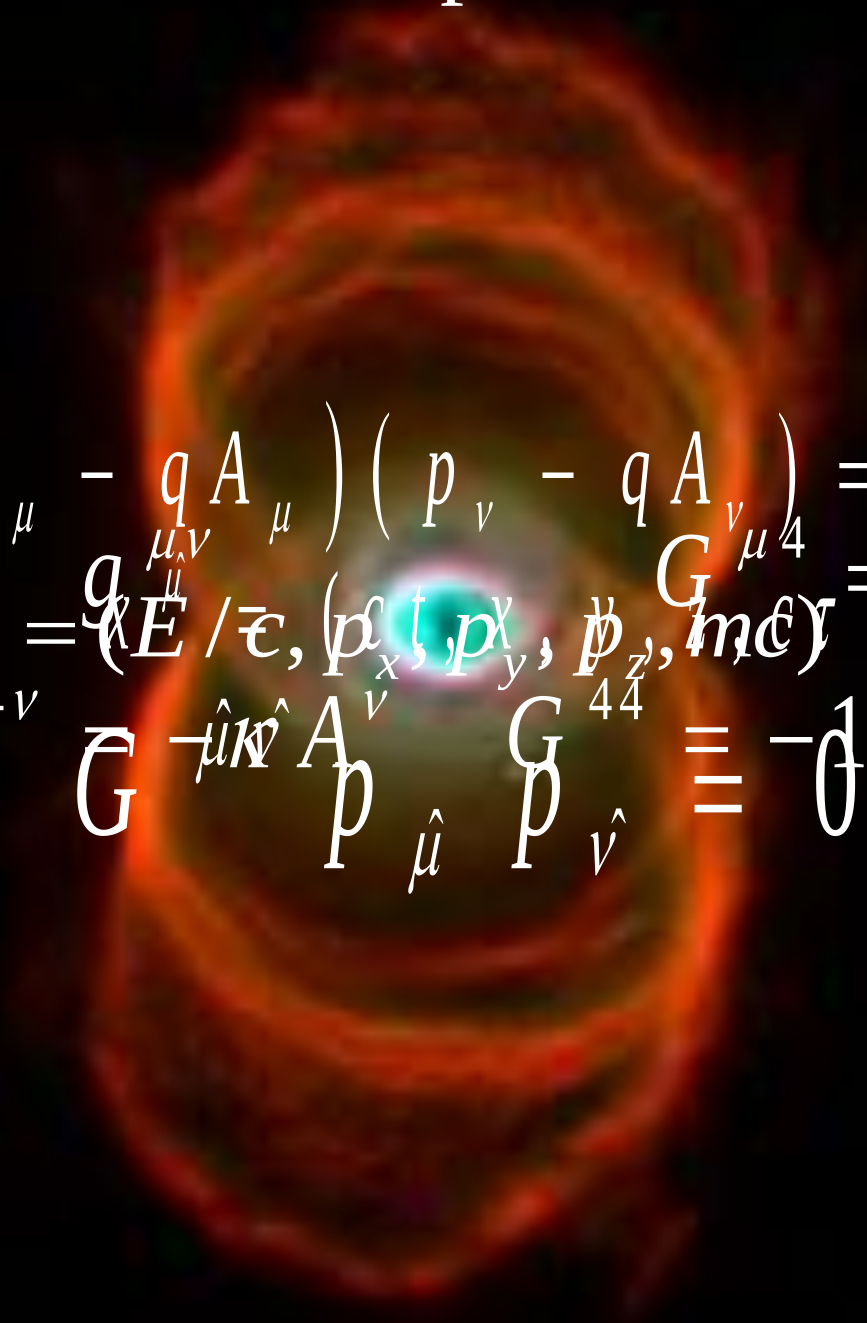
$$x^{\hat{\mu}} = (ct, x, y, z, c\tau)$$



$$d\sigma^2 = G_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}} = 0$$

$$G_{\hat{\mu}\hat{\nu}} = (1, -1, -1, -1, -1)$$

Generalization in the presence of interactions



$$g^{\mu\nu}\left(p_\mu - qA_\mu\right)\left(p_\nu - qA_\nu\right) = m^2c^2$$

$$x^{\hat{\mu}} = (ct, x, y, z, c\tau)$$

$$p^{\hat{\mu}} = (E/c, p_x, p_y, p_z, mc)$$

$$G^{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g^{\mu\nu} & G^{\mu 4} = -\kappa A^\mu \\ G^{4\nu} = -\kappa A^\nu & G^{44} = -1 + \kappa^2 A^\lambda A_\lambda \end{pmatrix}$$

$$G^{\hat{\mu}\hat{\nu}} p_{\hat{\mu}} p_{\hat{\nu}} = 0$$

Generalization in the presence of interactions

$$x^{\hat{\mu}} = (ct, x, y, z, c\tau) \qquad p^{\hat{\mu}} = (E/c, p_x, p_y, p_z, mc)$$

$$G^{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g^{\mu\nu} & G^{\mu 4} = -\kappa A^{\mu} \\ G^{4\nu} = -\kappa A^{\nu} & G^{44} = -1 + \kappa^2 A^{\lambda} A_{\lambda} \end{pmatrix}$$

$$g^{\mu\nu}(p_{\mu} - qA_{\mu})(p_{\nu} - qA_{\nu}) = m^2 c^2$$

$$G^{\hat{\mu}\hat{\nu}} p_{\hat{\mu}} p_{\hat{\nu}} = 0$$

$$d\sigma^2 = G_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}} = 0$$

$$\hat{m}^2 c^2 = G^{\hat{\mu}\hat{\nu}} p_{\hat{\mu}} p_{\hat{\nu}} = 0$$

$$\delta d\sigma^2 = 0$$

$\rightarrow$ 5D equations of motion

$$\hat{L} = -\frac{m^*}{2} G_{\hat{\mu}\hat{\nu}} \dot{\hat{x}}^{\hat{\mu}} \dot{\hat{x}}^{\hat{\nu}}$$

$$\hat{p}_{\hat{\mu}} = -\frac{\partial \hat{L}}{\partial \dot{\hat{x}}^{\hat{\mu}}} = m^* \dot{\hat{x}}_{\hat{\mu}}$$

$$\Rightarrow p^0 = m^* \dot{\hat{x}}^0 = m^* c$$

$$\hat{p}_4 = m^* \dot{\hat{x}}_4 \Rightarrow \dot{\tau} = \frac{m}{m^*}$$

$$\dot{\hat{p}}_i = \frac{m^* c^2}{2} \partial_i g_{00} \Rightarrow m^* g_i$$

5D OPTICAL PATH

$$\Box \varphi = 0$$

5D eikonal equation $(\hat{\mu}, \hat{\nu} = 0,1,2,3,4)$:

$$G^{\hat{\mu}\hat{\nu}} \partial_{\hat{\mu}} \phi \, \partial_{\hat{\nu}} \phi = 0$$

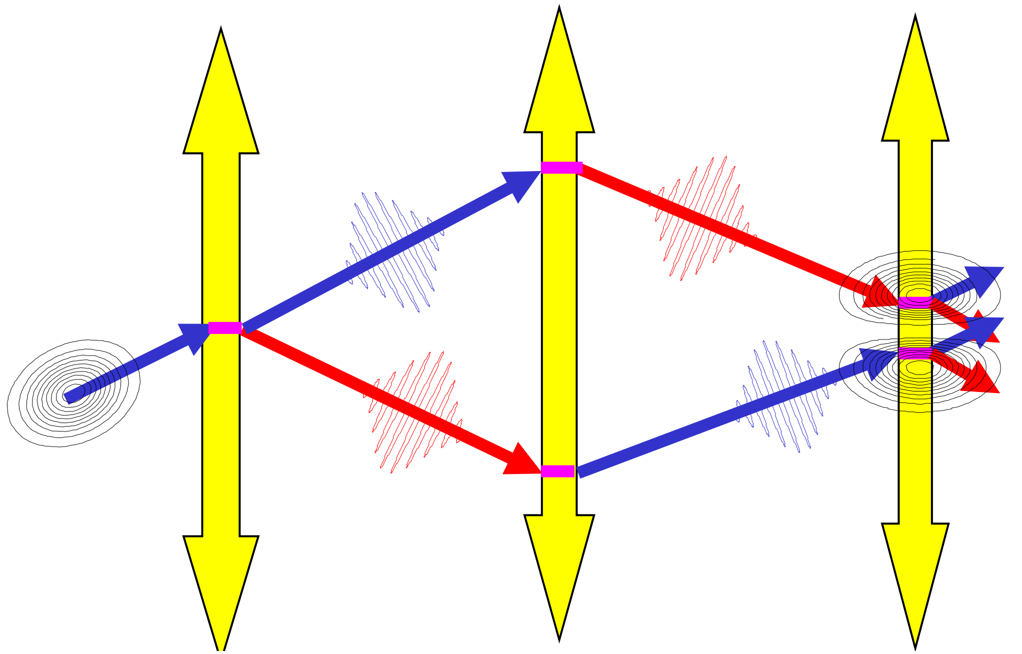
$$h\phi = -\int p_{\hat{\mu}} dx^{\hat{\mu}} = -\frac{E}{c}\left(\int c\,dt - \int \frac{dl^{(4)}}{\sqrt{G_{00}}} + \int \frac{G_{\hat{j}0}}{G_{00}} dx^{\hat{j}} \right)$$

$$dl^{(4)} = \sqrt{-\hat{f}_{\hat{i}\hat{j}} dx^{\hat{i}} dx^{\hat{j}}}$$

$$\hat{f}_{\hat{i}\hat{j}} = G_{\hat{i}\hat{j}} - \frac{G_{0\hat{i}} G_{0\hat{j}}}{G_{00}}$$

$$\lambda^{(4)} = \frac{h}{E} c \sqrt{G_{00}}$$

GENERAL FORMULA FOR THE PHASE SHIFT OF AN ATOM/PHOTON INTERFEROMETER

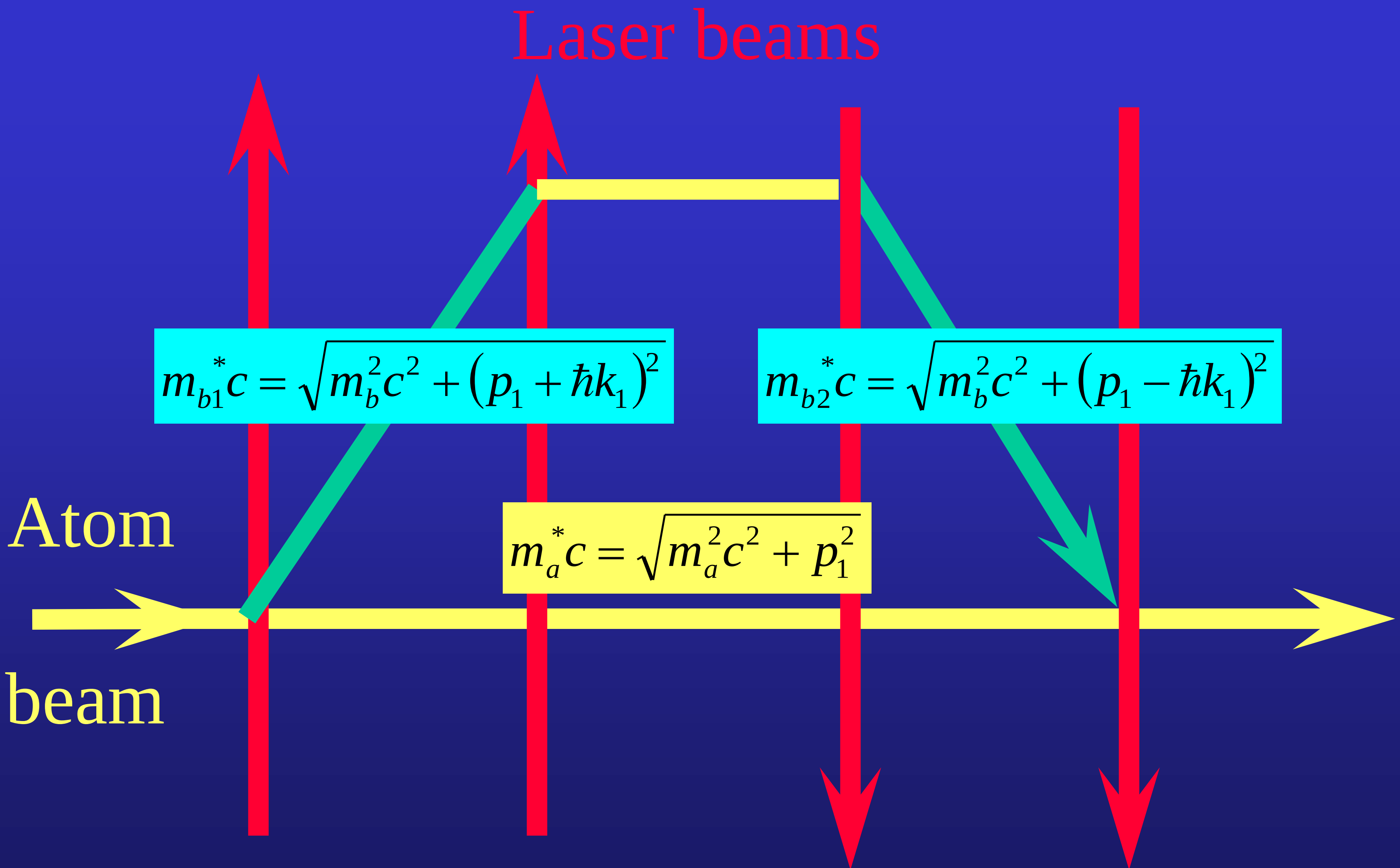


$$\delta\varphi = \sum_{j=1}^{N} (\tilde{k}^{(5)}_{\beta j} \cdot q^{(5)}_{\beta j} + \varphi_{\beta j}) - (\beta \leftrightarrow \alpha)$$

$$+\left(\tilde{p}_{\beta,D} + \tilde{p}_{\alpha,D}\right)\left(q_{\alpha,D} - q_{\beta,D}\right)/2\hbar$$

$$\tilde{k}^{(5)} = \left[(k_x, k_y, k_z, \frac{\omega^{(0)}}{c}), \frac{\omega}{c}\right]; \quad \tilde{q}^{(5)} = \left[(x, y, z, c\tau), ct\right]$$

Bordé-Ramsey interferometers



$$m_{b1}^{*}c = \sqrt{m_b^2 c^2 + \left(p_1 + \hbar k_1\right)^2}$$

$$m_{b2}^{*}c = \sqrt{m_b^2 c^2 + \left(p_1 - \hbar k_1\right)^2}$$

$$m_a^{*}c = \sqrt{m_a^2 c^2 + p_1^2}$$

$$\delta\varphi = \left(2\omega - (m_{b1}^{*} + m_{b2}^{*} - 2m_a^{*})c^2 / \hbar\right)T$$

Bordé-Ramsey interferometers

$$\vec{q}_{\alpha 2} = \vec{q}_1 + \frac{\vec{p}_1 T}{m_a^*}; \quad \tau_{\alpha 2} = \tau_1 + \frac{m_a}{m_a^*}T; \quad \vec{p}_{\alpha 2} = \vec{p}_1$$

$$\vec{q}_{\alpha 3} = \vec{q}_1 + \frac{\vec{p}_1 (T + T')}{m_a^*}; \quad \tau_{\alpha 3} = \tau_1 + \frac{m_a}{m_a^*}(T + T'); \quad \vec{p}_{\alpha 3} = \vec{p}_1$$

$$\vec{q}_{\alpha 4} = \vec{q}_1 + \frac{\vec{p}_1 (2T + T')}{m_a^*}; \quad \tau_{\alpha 4} = \tau_1 + \frac{m_a}{m_a^*}(2T + T'); \quad \vec{p}_{\alpha 4} = \vec{p}_1$$

$$m_{b1}^{*}c = \sqrt{m_b^2c^2 + \left(p_1 + \hbar k_1\right)^2}$$

$$m_{b2}^{*}c = \sqrt{m_b^2c^2 + \left(p_1 - \hbar k_1\right)^2}$$

Atom

$$m^{*}c = \sqrt{m^2c^2 + p^2}$$

$$t_2 = t_1 + T; \quad \vec{q}_{\beta 2} = \vec{q}_1 + \frac{\left(\vec{p}_1 + \hbar\vec{k}\right)T}{m_{b1}^*}; \quad \tau_{\beta 2} = \tau_1 + \frac{m_b}{m_{b1}^*}T; \quad \vec{p}_{\beta 2} = \vec{p}_1 + \hbar\vec{k}$$

$$t_3 = t_2 + T'; \quad \vec{q}_{\beta 3} = \vec{q}_1 + \frac{\left(\vec{p}_1 + \hbar\vec{k}\right)T}{m_{b1}^*} + \frac{\vec{p}_1 T'}{m_a^*}; \quad \tau_{\beta 3} = \tau_1 + \frac{m_b}{m_{b1}^*}T + \frac{m_a}{m_a^*}T'; \quad \vec{p}_{\beta b3} = \vec{p}_1$$

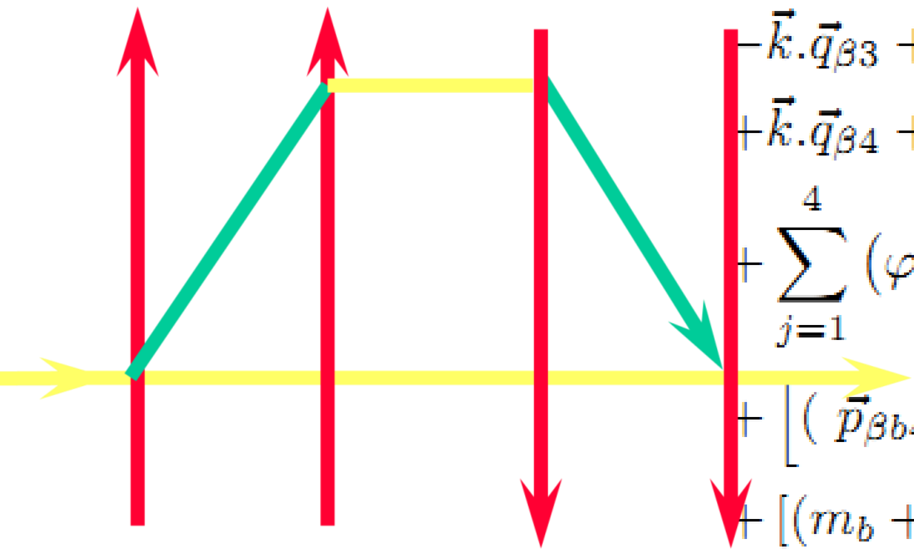
$$t_4 = t_3 + T; \quad \vec{q}_{\beta 4} = \vec{q}_1 + \frac{\left(\vec{p}_1 + \hbar\vec{k}\right)T}{m_{b1}^*} + \frac{\left(\vec{p}_1 - \hbar\vec{k}\right)T}{m_{b2}^*} + \frac{\vec{p}_1 T'}{m_a^*};$$

$$\tau_{\beta 4} = \tau_1 + \frac{m_b}{m_{b1}^*}T + \frac{m_b}{m_{b2}^*}T + \frac{m_a}{m_a^*}T'; \quad \vec{p}_{\beta b4} = \vec{p}_1 - \hbar\vec{k} \qquad (93)$$

$$\delta\phi((q_{\beta,D}+q_{\alpha,D})/2) = \sum_{j=1}^{N}\left(\tilde{k}_{\beta j}q_{\beta j}-\tilde{k}_{\alpha j}q_{\alpha j}\right)-(\omega_{\beta j}-\omega_{\alpha j})t_j+(\varphi_{\beta j}-\varphi_{\alpha j}) + \left[(\tilde{p}_{\beta,D}+\tilde{p}_{\alpha,D})(q_{\alpha,D}-q_{\beta,D})/2\right]/\hbar \qquad (88)$$

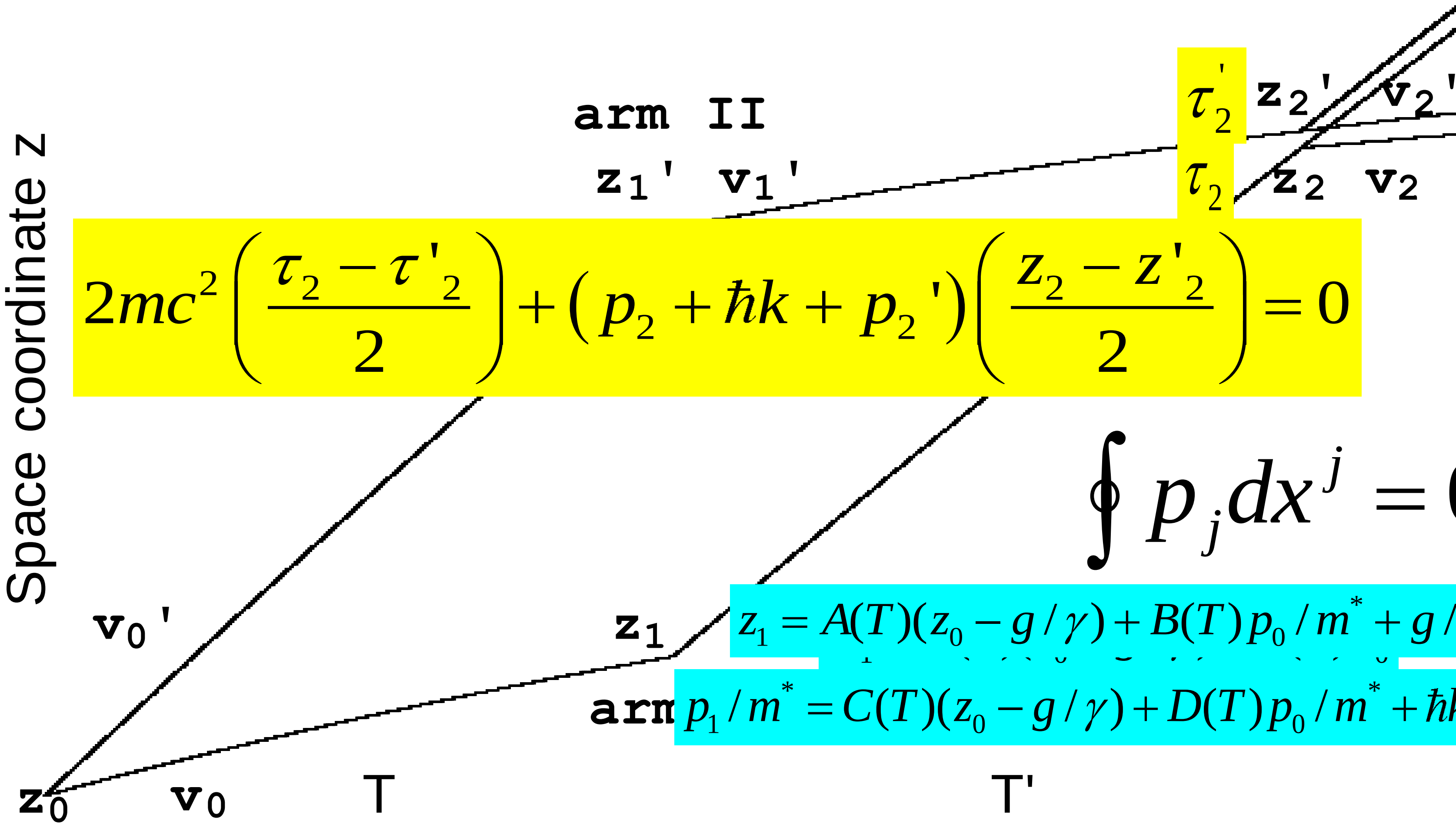
Formula (88) gives for the mid-point 5D phase:

$$\begin{aligned}\delta\phi((\widehat{q}_{\beta 4}+\widehat{q}_{\alpha 4})/2) = {} & \vec{k}.\vec{q}_1+(m_b-m_a)c^2\tau_1/\hbar-\omega t_1 \\ & -\vec{k}.\vec{q}_{\beta 2}+(-m_b+m_a)c^2\tau_{\beta 2}/\hbar+\omega t_2 \\ & -\vec{k}.\vec{q}_{\beta 3}+(m_b-m_a)c^2\tau_{\beta 3}/\hbar-\omega t_3 \\ & +\vec{k}.\vec{q}_{\beta 4}+(-m_b+m_a)c^2\tau_{\beta 4}/\hbar+\omega t_4 \\ & +\sum_{j=1}^{4}(\varphi_{\beta j}-\varphi_{\alpha j}) \\ & +\left[(\vec{p}_{\beta b4}+\vec{p}_{\alpha a4}+\hbar\vec{k}).(\vec{q}_{\alpha 4}-\vec{q}_{\beta 4})/2\right]/\hbar \\ & +\left[(m_b+m_a+m_a-m_b)(\tau_{\alpha 4}-\tau_{\beta 4})/2\right]c^2/\hbar \qquad (92)\end{aligned}$$



$$\delta\varphi = \left(2\omega-(m_{b1}^{*}+m_{b2}^{*}-2m_a^{*})c^2/\hbar\right)T$$

Atomic Gravimeter



$$\tau'_2 \qquad \tau_2$$

$$2mc^2\left(\frac{\tau_2-\tau'_2}{2}\right)+\left(p_2+\hbar k+p_2'\right)\left(\frac{z_2-z'_2}{2}\right)=0$$

$$\oint p_j dx^j = 0$$

$$z_1 = A(T)(z_0 - g/\gamma) + B(T)p_0/m^* + g/\gamma$$

$$p_1/m^* = C(T)(z_0 - g/\gamma) + D(T)p_0/m^* + \hbar k/m^*$$

$$\delta\varphi = -kz_0 + 2k(z_1 + z'_1)/2 - k(z_2 + z'_2)/2$$

Exact phase shift for the atom gravimeter

$$\delta\varphi = -kz_0 + 2k(z_1 + z'_1)/2 - k(z_2 + z'_2)/2$$

$$= \frac{k}{\sqrt{\gamma}}\left\{\left[\sinh\left(\sqrt{\gamma}(T+T')\right) - 2\sinh\left(\sqrt{\gamma}T\right)\right]\left(\mathrm{v}_0 + \frac{\hbar k}{2m^*}\right)\right.$$

$$\left. + \sqrt{\gamma}\left[1 + \cosh\left(\sqrt{\gamma}(T+T')\right) - 2\cosh\left(\sqrt{\gamma}T\right)\right]\left(z_0 - \frac{g}{\gamma}\right)\right\}$$

which can be written to first-order in γ, with T=T':

$$\delta\varphi = kgT^2 + k\gamma T^2\left[\frac{7}{12}gT^2 - \left(\mathrm{v}_0 + \frac{\hbar k}{2m^*}\right)T - z_0\right]$$

Reference: Ch. J. B., Theoretical tools for atom optics and interferometry, C.R. Acad. Sci. Paris, 2, Série IV, p. 509-530, 2001

Atominterferometrie und Gravitation

Ch. J. Bordé und C. Lämmerzahl

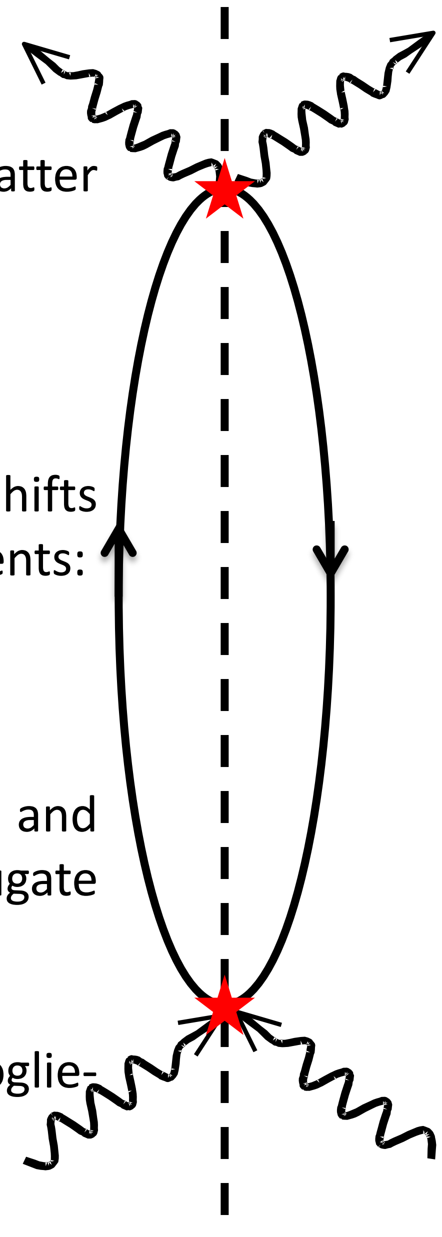
Phys. Bl. 52 (1996) Nr. 3

die träge Masse m_t und die schwere Masse m_s im kinetischen bzw. gravitativen Teil der Schrödinger-Gleichung ein, kann man dieses Experiment auch zum Test des schwachen Äquivalenzprinzips im Quantenbereich heranziehen: man erhält in der Phasenverschiebung (1) einen zusätzlichen Faktor m_s/m_t. Somit kann man durch Verwendung verschiedener Atomsorten direkt das Äquivalenzprinzip testen. Mit der Genauigkeit des Kasevich-Chu-Interferometers würde ein Null-… …zip im Sinne … $m_s/m_t)^{(a)}$ $-(m_s/$… 10^{-8} bestätige… …nzeich-nen zu… Dieses Experiment wäre ein …

$$\delta\varphi = \frac{m_g}{m_I} k g T^2$$

$$+\left(2\omega - (m_{b1}^* + m_{b2}^* - 2m_a^*)c^2/\hbar\right)T$$

Conclusions



5D theoretical framework well-adapted to matter/antimatter interferometry

Unified treatment of clocks and gravito-inertial sensors

Single formula for the various contributions to the phase shifts including effects involving the electric charge and dipole moments: scalar and vector AB effect, AC and HMW shifts
leading to a complete test of charge conjugation

This framework has required an explicit introduction of mass and proper time as associated dynamical variables and conjugate quantum mechanical observables

Related controversial matters: gravitational red shift of de Broglie-Compton clocks, clocks versus anticlocks …