INSTITUT D'ASTROPHYSIQUE DE PARIS

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Workshop on Antimatter and Gravity (WAG 2013)

MOND PHENOMENOLOGY AND GRAVITATIONAL POLARIZATION

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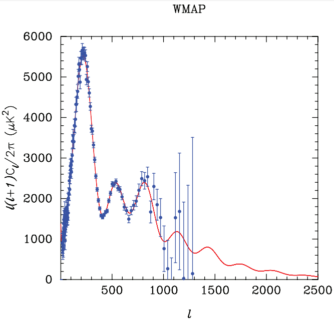
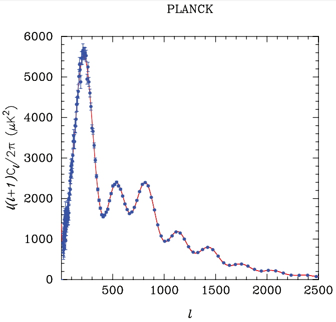
The cosmological concordance model Λ-CDM





This model brilliantly accounts for:

1. The mass discrepancy between the dynamical and luminous masses of clusters of galaxies
2. The precise measurements of the anisotropies of the cosmic microwave background (CMB)
3. The formation and growth of large scale structures as seen in deep redshift and weak lensing surveys
4. The fainting of the light curves of distant supernovae

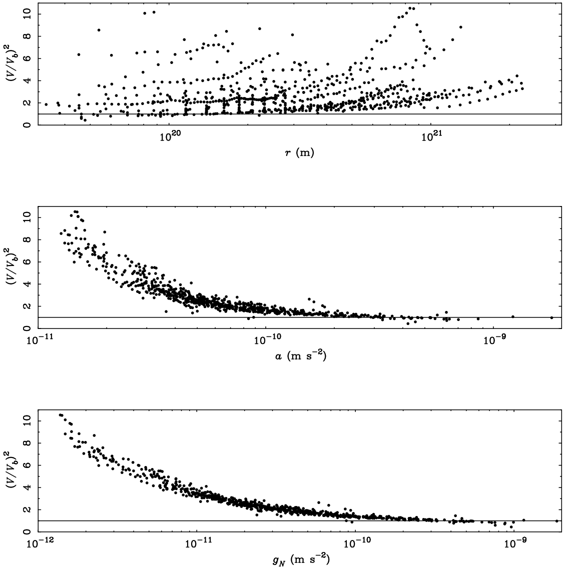
Challenges with CDM at galactic scales

The CDM paradigm faces severe challenges when compared to observations at galactic scales [McGaugh & Sanders 2004; Famaey & McGaugh 2012]

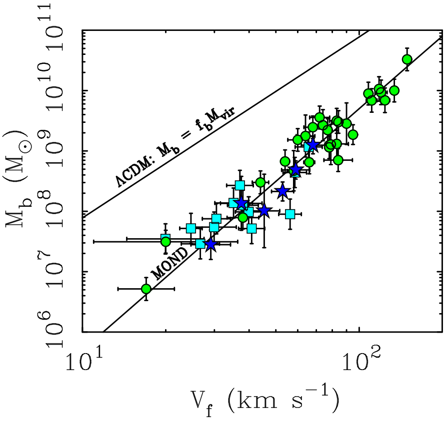
1. Unobserved predictions
   - Numerous but unseen satellites of large galaxies
   - Phase-space correlation of galaxy satellites
   - Generic formation of dark matter cusps in galaxies
   - Tidal dwarf galaxies dominated by dark matter
2. Unpredicted observations
   - Correlation between mass discrepancy and acceleration
   - Surface brightness of galaxies and the Freeman limit
   - Flat rotation curves of galaxies
   - Baryonic Tully-Fisher relation for spirals
   - Faber-Jackson relation for ellipticals

All these challenges are mysteriously solved (sometimes with incredible success) by the MOND empirical formula [Milgrom 1983]

Correlation between mass discrepancy and acceleration



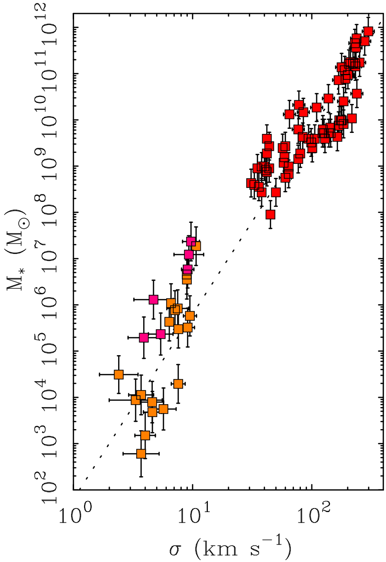
Baryonic Tully-Fisher relation [Tully & Fisher 1977, McGaugh 2011]



The relation between the asymptotic flat velocity and the mass of galaxies is

$$V \approx \left(G\, M_{\rm b}\, a_0\right)^{1/4} \quad \text{where} \quad a_0 \approx 1.2 \times 10^{-10} \text{m/s}^2$$

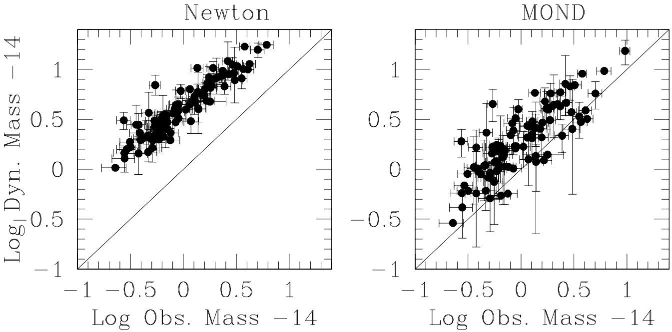
Mass velocity dispersion relation [Faber & Jackson 1976]



We have approximately

$$\sigma \approx \left(G\, M_\mathrm{b}\, a_0\right)^{1/4}$$

Problem with galaxy clusters [Gerbal, Durret *et al.* 1992, Sanders 1999]



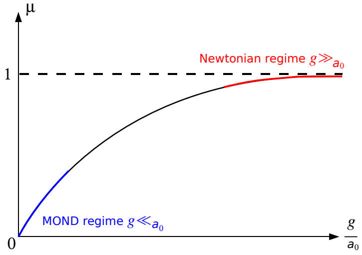
- The mass discrepancy is $\approx 4-5$ with Newton and ≈ 2 with MOND
- The bullet cluster and more generally X-ray emitting galaxy clusters can be fitted with MOND only with a component of baryonic dark matter and hot/warm neutrinos [Angus, Famaey & Buote 2008]

The MOND equation [Milgrom 1983; Bekenstein & Milgrom 1984]

The MOND equation can be written as the modified Poisson equation

$$\nabla \cdot \Big[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \boldsymbol{g} \Big] = -4\pi\, G\, \rho_{\mathsf{b}}$$

where $\boldsymbol{g} = \nabla U$ is the gravitational field and ρ_{b} the density of ordinary matter



In the MOND regime $g \ll a_0$ we have $\mu = g/a_0 + \mathcal{O}(g^2)$

Modified gravity ?

Traditional approach of MOND

There is a fundamental modification of the law of gravity in a regime of weak gravity. This has led to many relativistic theories modifying GR:

- Tensor-vector-scalar theory [Bekenstein 2004, Sanders 2005]
- Non-canonical Einstein-Æther theories [Zlosnik *et al.* 2007, Halle *et al.* 2008]
- Bimetric theory (BIMOND) [Milgrom 2012]
- Khronon field [Blanchet & Marsat 2012, Sanders 2012]
- ···

Modified dark matter ?

Alternative approach to MOND

The law of gravity is not modified but dark matter is endowed with special properties which makes it:

- Reproducing the phenomenology of MOND (flat rotation curves of galaxies and the baryonic Tully-Fisher relation) at the scales of galaxies
- Appearing to be made of CDM particles (*i.e.* a perfect fluid without pressure) at cosmological scales

The dielectric analogy [Blanchet 2007]

- In electrostratics the Gauss equation is modified by the polarization of the dielectric (dipolar) material

$$\nabla\cdot\Big[\underbrace{(1+\chi_e)\boldsymbol{E}}_{\boldsymbol{D}\text{ field}}\Big]=\frac{\rho_e}{\varepsilon_0}\qquad\Longleftrightarrow\qquad\nabla\cdot\boldsymbol{E}=\frac{\rho_e+\rho_e^{\text{polar}}}{\varepsilon_0}$$

- Similarly MOND can be viewed as a modification of the Poisson equation by the polarization of some dipolar medium

$$\nabla\cdot\left[\mu\left(\frac{g}{a_0}\right)\boldsymbol{g}\right]=-4\pi\,G\,\rho_\text{b}\qquad\Longleftrightarrow\qquad\nabla\cdot\boldsymbol{g}=-4\pi\,G\left(\rho_\text{b}+\underbrace{\rho^{\text{polar}}}_{\text{dark matter}}\right)$$

The MOND function can be written $\mu=1+\chi$ where χ appears as a susceptibility coefficient characterizing some dipolar DM medium

Dark matter made of a polarizable medium ?

1. The density of DM medium is modelled by particles $(m_\mathrm{i}, m_\mathrm{g}) = (m, \pm m)$ and described by individual dipole moments and a polarization field

$$\boldsymbol{P} = n\,\boldsymbol{p} \qquad \text{with} \qquad \boldsymbol{p} = m\,\boldsymbol{\xi}$$

2. The polarization of the DM medium is induced by the gravitational field of ordinary masses

$$\boldsymbol{P} = -\frac{\chi}{4\pi\,G}\,\boldsymbol{g} \qquad \text{with} \qquad \chi = \mu - 1$$

Because like masses attract and unlike ones repel we have

$$\chi < 0$$

- This corresponds to anti-screening of ordinary masses by polarization masses
- This is in agreement with MOND phenomenology

Need of a new non-gravitational internal force

1. The constituents of the dipole will repel each other so we need to invoke a non-gravitational force (i.e. a fifth force) to stabilize the dipolar medium

$$m\frac{\mathrm{d}^2\boldsymbol{x}_\pm}{\mathrm{d}t^2} = \pm m\big(\boldsymbol{f} + \boldsymbol{g}\big)$$

2. Like in ordinary plasma physics the internal force is generated by the gravitational charge which is the mass in this case

$$\boldsymbol{\nabla}\cdot\boldsymbol{f} = -\frac{4\pi G m}{\chi}\big(n_+ - n_-\big)$$

The DM medium appears as a polarizable plasma of particles $(m, \pm m)$ oscillating at the natural plasma frequency

$$\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} + \omega^2\boldsymbol{\xi} = 2\boldsymbol{g} \qquad \text{with} \qquad \omega = \sqrt{-\frac{8\pi\, G\, m\, n}{\chi}}$$

Could DM be due to vacuum quantum fluctuations ?

Suppose that the DM medium $(m, \pm m)$ is made of virtual particle-antiparticle pairs. The classical separation ξ between pairs should be

$$\xi \sim \frac{\hbar}{m\,c}$$

In the MOND regime the polarization is $P \sim \frac{a_0}{4\pi G}$

- Since $P = nm\,\xi$ we have

$$n \sim \frac{a_0\,c}{4\pi G\,\hbar} \sim 4.3 \times 10^{35}\,\text{cm}^{-3}$$

- This corresponds to a typical separation

$$\xi \sim n^{-1/3} \sim 1.3 \times 10^{-12}\,\text{cm}$$

- And a typical mass $\qquad m \sim 14\,\text{Mev}$

Such a medium looks very much like the QCD vacuum [Chardin 2008, Hajdukovic 2011]

Could DM be due to vacuum quantum fluctuations ?

Such an interpretation poses a lot of problems

1. Assuming that anti-particles have mass $(m_\mathrm{i}, m_\mathrm{g}) = (m, -m)$ is at odds with all theoretical expectations [Morrison & Gold 1957]
2. This is ruled out by the K^0-$\bar{K}^0$ system which shows that the gravitational mass of $\bar{K}^0$ is the same as that of K^0 [Good 1960]
3. This is severely constrained by equivalence principle Eötvos-type experiments using the virtual e^+-e^- and q-$\bar{q}$ pairs in ordinary materials [Schiff 1959]
4. The description of vacuum fluctuations based on Compton's separation is very loose and merely semi-classical
5. The model is non-relativistic so cannot address crucial questions related to
   - light deflection (DM seen in galaxy clusters)
   - cosmology (DM seen by the CMB anisotropies)

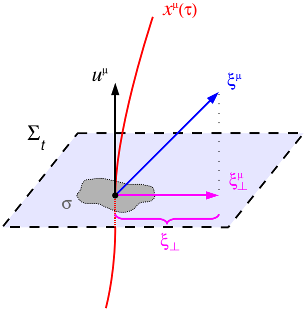
Dipolar dark matter [Blanchet & Le Tiec 2008; 2009]

Tries to implement in a relativistic way the dielectric analogy of MOND

The DDM action in standard general relativity is

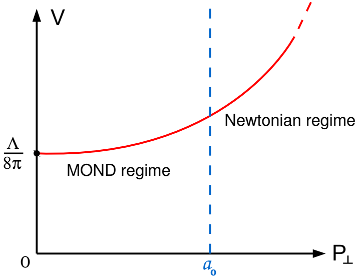
$$S_{\mathrm{DDM}} = \int \mathrm{d}^4x \sqrt{-g}\Big[-\rho + J^\mu \dot{\xi}_\mu - V(P_\perp)\Big]$$

where the current density J^μ and the dipole moment ξ^μ are two independent dynamical variables



- Mass term ρ in an ordinary sense like for CDM
- Interaction term between the fluid's current $J^\mu = \rho u^\mu$ and the covariant time derivative of the dipole moment ξ_μ
- Potential term V depending on the norm of the polarization field $P_\perp = \rho\, \xi_\perp$

The internal potential



The potential V is phenomenologically determined through third order

$$V = \frac{\Lambda}{8\pi} + 2\pi\, P_\perp^2 + \frac{16\pi^2}{3a_0} P_\perp^3 + \mathcal{O}\left(P_\perp^4\right)$$

- The minimum of that potential is the cosmological constant Λ and the third-order deviation from the minimum contains the MOND scale a_0
- The natural order of magnitude of the cosmological constant is comparable with a_0 namely $\Lambda \sim a_0^2$ in agreement with observations

Agreement with Λ-CDM at cosmological scales

In a cosmological perturbation around a FLRW background, the dipole moment, which is space-like, must belong to the first-order perturbation

$$\xi_\perp^\mu = \mathcal{O}(1)$$

The stress-energy tensor reduces to $T^{\mu\nu} = T^{\mu\nu}_{\text{DE}} + T^{\mu\nu}_{\text{DDM}}$ where

- the DE is given by the cosmological constant Λ
- the DDM takes the form of a perfect fluid with zero pressure

$$T^{\mu\nu}_{\text{DDM}} = \varepsilon\, u^\mu u^\nu + \mathcal{O}(2)$$

where $\varepsilon = \rho - \nabla_\mu P^\mu_\perp$ is a dipolar energy density

- The dipolar fluid is undistinguishable from standard Λ-CDM at the level of first-order cosmological perturbations
- At second-order cosmological perturbation it induces some non-Gaussianity in the bispectrum of curvature perturbation [Blanchet, Langlois, Le Tiec & Marsat 2012]

Weak clustering of dipolar DM

- Baryonic matter follows the geodesic equation $\dot{u}^\mu = 0$, therefore collapses in regions of overdensity
- Dipolar dark matter obeys $\dot{u}^\mu = -\mathcal{F}^\mu$, with the internal force $\boldsymbol{\mathcal{F}}$ balancing the gravitational field $\boldsymbol{g}$ created by an overdensity

The mass density of dipolar dark matter in a galaxy at low redshift should be smaller than the baryonic density and maybe close to its mean cosmological value

$$\sigma \approx \overline{\sigma} \ll \rho_\mathsf{b} \quad \text{and} \quad \boldsymbol{v} \approx \mathbf{0}$$

- Using this hypothesis one can show that the phenomenology of MOND is recovered at galactic scales
- However the model involves an instability and does not recover the polarizable DM medium of masses $(m, \pm m)$ in the Newtonian limit

Bimetric approach to gravitational polarization

[Bernard & Blanchet, in preparation]

1. To describe a relativistic DM medium made of particles $(m, \pm m)$ one needs in principle two metrics
$$g^{\pm}_{\mu\nu} = f_{\mu\nu} \pm h_{\mu\nu}$$
2. We look for an action depending on $f_{\mu\nu}$ (considered to be the metric) and a second-rank tensor field $h_{\mu\nu}$ propagating on that metric
$$S = \int \mathrm{d}^4x\, \mathcal{L}\Big[f_{\mu\nu}, h_{\mu\nu}, \text{matter}\Big]$$
3. We are interested in the weak field MOND limit hence we assume that $h_{\mu\nu}$ is a small perturbation of the metric $f_{\mu\nu}$
$$G^{\mu\nu}_{\pm} = \frac{\sqrt{-f}}{\sqrt{-g_{\pm}}}\Bigg[G^{\mu\nu}[f] \mp \underbrace{\frac{1}{2}\mathcal{D}h^{\mu\nu}}_{\text{linear perturbation}}\Bigg] + \mathcal{O}(h^2)$$

This is a mixed modified gravity/modified DM approach

Matter fields are

1. Ordinary baryonic matter ρ_b coupled to $g^+_{\mu\nu}$
2. DM particles $\rho_\pm$ coupled to $g^\pm_{\mu\nu}$
3. A $U(1)$ gauge field A_μ for the internal force of the DM medium

$$
\begin{aligned}
S \;=\; \int \mathrm{d}^4x \Bigg\{ & \sqrt{-g_+}\left(\frac{R_+}{2\kappa} - \rho_\mathrm{b} - \rho_+\right) + \sqrt{-g_-}\left(\frac{R_-}{2\kappa} - \rho_-\right) \\
& + \sqrt{-f}\left[\frac{R[f]}{\epsilon} + \left(J^\mu_+ - J^\mu_-\right) A_\mu + a_0^2\, f\left(\frac{F^{\mu\nu}F_{\mu\nu}}{a_0^2}\right)\right]\Bigg\}
\end{aligned}
$$

- The interesting case is $\epsilon \ll \kappa$ for which the two metric deviations $g^\pm_{\mu\nu} = f_{\mu\nu} \pm h_{\mu\nu}$ are essentially inverse to each other
- The adjustable function f in the action is fine-tuned to reproduce MOND

Agreement with Λ-CDM and MOND

Introducing the stress-energy tensor $T_0^{\mu\nu}$ at equilibrium and a small displacement ξ^λ of order $h^{\mu\nu}$ between $\pm$ particles

$$\left.\begin{aligned} T_+^{\mu\nu} + T_-^{\mu\nu} &= 2T_0^{\mu\nu} \\ T_+^{\mu\nu} - T_-^{\mu\nu} &= -\nabla_\lambda\left(\xi^\lambda T_0^{\mu\nu}\right) \end{aligned}\right\} + \mathcal{O}(h^2)$$

$$G_\pm^{\mu\nu} = \pm\frac{\kappa}{2}\Big[T_{\mathsf{b}}^{\mu\nu} \underbrace{-\nabla_\lambda\left(\xi^\lambda T_0^{\mu\nu}\right)}_{\text{DM in galaxies (MOND)}}\Big] + \underbrace{\epsilon\, T_0^{\mu\nu}}_{\text{DM in cosmology (CMB)}} + \mathcal{O}(h^2)$$

- The models reproduces MOND and the polarizable DM medium in the non-relativistic approximation when $\epsilon \ll \kappa$
- It agrees with the cosmological model Λ-CDM at first order cosmological perturbation with $\rho_{\text{DM}} = \frac{\epsilon}{\kappa}\rho_0 \ll \rho_0$

Conclusions

1. Λ-CDM is an extremely successful model in cosmology but
   - poses the problem of the fundamental constituents of the Universe
   - faces severe challenges when compared to observations at galactic scale
2. MOND is a successful alternative for interpreting the galactic rotation curves and the baryonic Tully-Fisher relation but
   - is based on an empirical formula not explained in terms of fundamental physics
   - does not work at galaxy cluster scale and in cosmolgy
3. Reconciling Λ-CDM at cosmological scale and MOND at galactic scale into a single relativistic theory is a great challenge
4. A non-standard form of dark matter might exist, possibly with some non standard coupling to gravity, and might explain the two antinomic aspects of DM that we see at cosmological and galactic scales