Quantum reflection of antihydrogen on the Casimir-Polder potential above a material surface

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November 14, 2013

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- Quantum reflection of antihydrogen from the Casimir potential above matter slabs – *Phys. Rev. A 87, 012901 (Jan 2013)*
- Quantum reflection of antihydrogen from nanoporous media – *Phys. Rev. A 87, 022506 (Feb 2013)*

Outline

1. Motivation : quantum reflection in GBAR
2. Calculation of the Casimir-Polder potential
3. Quantum reflection from the Casimir-Polder potential
4. Increasing quantum reflection
5. Applications and outlook

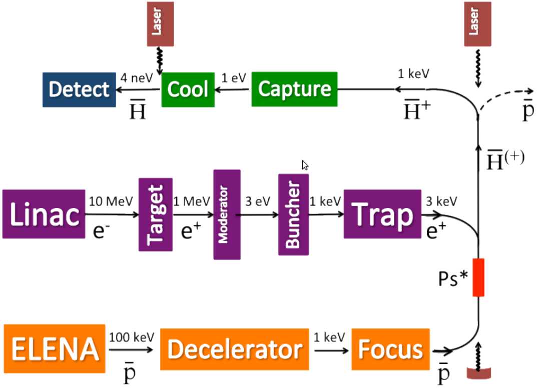
The GBAR experiment

Gravitational Behavior of Antihydrogen at Rest

Test the equivalence principle for antimatter by timing the free-fall of antihydrogen ($\overline{\mathrm{H}}$) dropped from $\sim$ 30 cm in the Earth's gravity field

The GBAR experiment

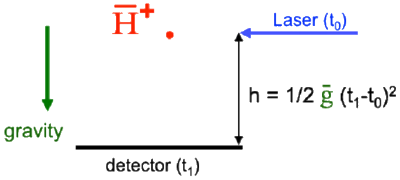
Producing, trapping and cooling $\overline{\mathrm{H}}^+$ ($\bar{p}\ e^+e^+$):



P. Perez & Y. Sacquin, *Class. Quantum Grav.* 29 (2012) 184008

The GBAR experiment

- initial state: $\overline{\mathrm{H}}^+$ in the ground state of a harmonic trap
- start: the extra e^+ is photodetached
- freefall of $\overline{\mathrm{H}}$
- stop: $\overline{\mathrm{H}}$ annihilates on the detector



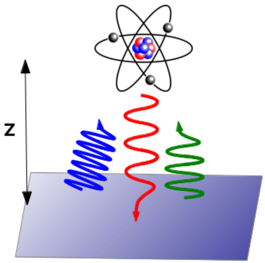
P. Perez & Y. Sacquin,
Class. Quantum Grav. 29 (2012) 184008

The free fall acceleration $\bar{g}$ of $\overline{\mathrm{H}}$ is deduced from the free fall time

Question: are there other forces than gravity acting on $\overline{\mathrm{H}}$?

The Casimir-Polder force

Electromagnetic (EM) modes are modified when the atom comes close to the detector:
$\Rightarrow$ the EM ground state (vacuum) energy changes
$\Rightarrow$ attractive Casimir-Polder force between atom and detector



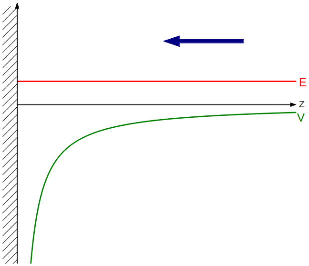
Casimir 1948 : long-range interaction energy between an atom and a perfectly conducting mirror:

$$V^*(z) = -\frac{3\hbar c}{8\pi z^4}\frac{\alpha(0)}{4\pi\epsilon_0} = -\frac{C_4^{perfect}}{z^4}$$

For H and $\overline{\mathrm{H}}$, $C_4^{perfect} \approx 73.6\ E_h a_0^4$
$V(35\text{ nm}) \approx$ - $mg \times 10$ cm

Scattering on the Casimir-Polder potential

What happens when the atom scatters on this potential ?



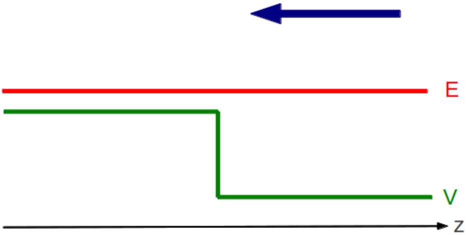
Length scales :

- free fall height : $h \approx 30$ cm
- quantum gravitational scale : $l_{grav} = \left(\hbar^2/2m^2 g\right)^{1/3} \approx 6\ \mu$m
- Casimir-Polder scale : $l_{CP} = \sqrt{2mC_4}/\hbar \approx 30$ nm

We can decouple the free-fall and the scattering on the potential

Quantum reflection on a step

Schrödinger plane wave incident on a potential step:



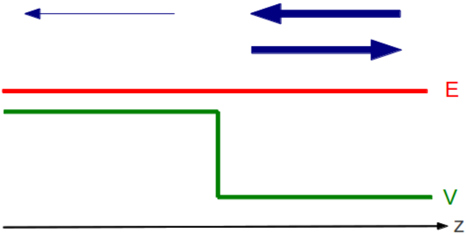
$$\psi_{in}(z) \propto \exp(-ikz)$$

with a wavevector

$$k = \sqrt{2m(E-V)}/\hbar$$

Quantum reflection on a step

The wavefunction is partly reflected, partly transmitted



Reflection:

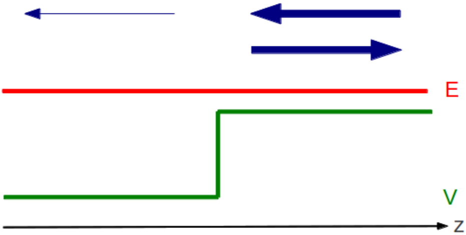
$$r_{12} = \frac{k_2 - k_1}{k_1 + k_2}$$

Transmission:

$$t_{12} = \frac{2\sqrt{k_1 k_2}}{k_1 + k_2}$$

Quantum reflection on a step

Reflection from an attractive potential: “quantum reflection”

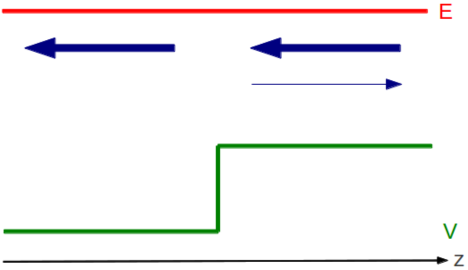


Reflection probability unchanged when $1 \leftrightarrow 2$:

$$|r_{12}|^2 = |r_{21}|^2$$

Quantum reflection on a step

No quantum reflection at large energies: classical regime



Reflection:

$$\frac{k_2 - k_1}{k_1 + k_2} \underset{E \to \infty}{\longrightarrow} 0$$

Transmission:

$$\frac{2\sqrt{k_1 k_2}}{k_1 + k_2} \underset{E \to \infty}{\longrightarrow} 1$$

Observation of quantum reflection

Shimizu 2001: Ne* on Silicon and BK7 glass, grazing incidence

VOLUME 86, NUMBER 6 PHYSICAL REVIEW LETTERS 5 FEBRUARY 2001

Specular Reflection of Very Slow Metastable Neon Atoms from a Solid Surface

Fujio Shimizu
Institute for Laser Science and CREST, University of Electro-Communications, Chofu-shi, Tokyo 182-8585, Japan
(Received 7 July 2000)

An ultracold narrow atomic beam of metastable neon in the $1s_3[(2s)^5 3p\,{:}^1P_0]$ state is used to study specular reflection of atoms from a solid surface at extremely slow incident velocity. The reflectivity on a silicon (1,0,0) surface and a BK7 glass surface is measured at the normal incident velocity between 1 mm/s and 3 cm/s. The reflectivity above 30% is observed at about 1 mm/s. The observed velocity dependence is explained semiquantitatively by the quantum reflection that is caused by the attractive Casimir–van der Waals potential of the atom-surface interaction.

DOI: 10.1103/PhysRevLett.86.987 PACS numbers: 34.50.Dy, 03.75.–b, 34.20.Cf

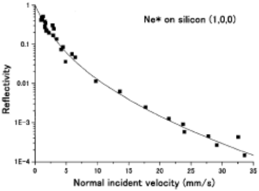


FIG. 3. The reflectivity vs the normal incident velocity on the Si(1,0,0) surface. The solid curve is the reflectivity calculated by using the potential Eq. (1) with $\lambda = 0.4\ \mu$m and $C_4 = 6.8 \times 10^{-56}$ J m^4, which corresponds to $\alpha = 2.0 \times 10^{-39}$ F m^2 of Casimir's theory.

Observation of quantum reflection

Pasquini et al. 2004: dilute BEC of Na on silicon, normal incidence

PRL **93,** 223201 (2004) PHYSICAL REVIEW LETTERS week ending 26 NOVEMBER 2004

Quantum Reflection from a Solid Surface at Normal Incidence

T. A. Pasquini, Y. Shin, C. Sanner, M. Saba, A. Schirotzek, D. E. Pritchard, and W. Ketterle*
Department of Physics, MIT-Harvard Center for Ultracold Atoms, and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts, 02139, USA
(Received 15 June 2004; published 24 November 2004)

We observed quantum reflection of ultracold atoms from the attractive potential of a solid surface. Extremely dilute Bose-Einstein condensates of ^{23}Na, with peak density 10^{11}–10^{12} atoms/cm^3, confined in a weak gravitomagnetic trap were normally incident on a silicon surface. Reflection probabilities of up to 20% were observed for incident velocities of 1–8 mm/s. The velocity dependence agrees qualitatively with the prediction for quantum reflection from the attractive Casimir-Polder potential. Atoms confined in a harmonic trap divided in half by a solid surface exhibited extended lifetime due to quantum reflection from the surface, implying a reflection probability above 50%.

DOI: 10.1103/PhysRevLett.93.223201 PACS numbers: 34.50.Dy, 03.75.Be

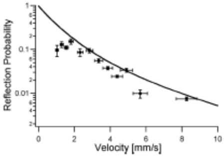


FIG. 3. Reflection probability vs incident velocity. Data were collected in a magnetic trap with trap frequencies $2\pi \times (3.3, 2.5, 6.5)$ Hz. Incident and reflected atom numbers were averaged over several shots. Vertical error bars show the standard deviation of the mean of six measurements. Horizontal error bars reflect the uncertainty in deducing $v_\perp$ from the applied magnetic field $B_\perp$. The solid curve is a numerical calculation for individual atoms incident on a conducting surface as described in the text.

Effect of the atom-detector interaction

Attractive Casimir-Polder interaction between atom and detector :

- no noticeable change in time of fall
- BUT part of the atomic wavepacket is reflected

Quantum reflection : classically forbiden reflection of a matter wave from a rapidly changing attractive potential

Need to estimate and master this bias :

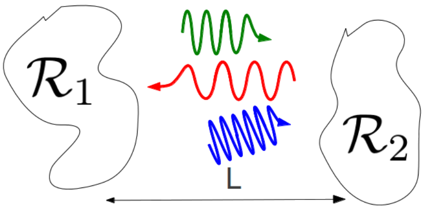
- How much quantum reflection can we expect?
- How does it depend on the atom's velocity?
- How is this affected by the materials used?

Scattering approach to Casimir forces

Scattering formula for Casimir energy (here at $T = 0$)

$$V = \hbar \int_0^\infty \frac{\mathrm{d}\xi}{2\pi} \mathrm{Tr} \log \left(1 - \mathcal{R}_1 e^{-\kappa L} \mathcal{R}_2 e^{-\kappa L}\right)$$

Objects described by EM reflection matrices $\mathcal{R}_1, \mathcal{R}_2$

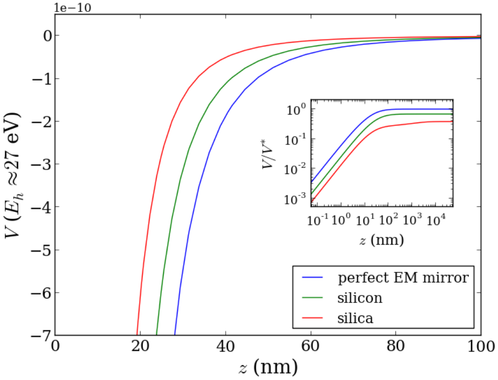


- $\mathcal{R}_{atom}$ function of the dynamic polarizability of the atom
- $\mathcal{R}_{plane}$ function of Fresnel reflection coefficients on the surface, which depend on the permittivity of the material

R. Messina, D.A.R. Dalvit, P.A. Maia Neto, A. Lambrecht, S. Reynaud, *Phys. Rev. A 80 (2009) 022119*

Calculation of the Casimir-Polder potential

Casimir-Polder potential above various semi-infinite media, numerical results (inset : normalized potential V/V^*):



- long distance (retarded regime): $V(z) \simeq -C_4/z^4$
- short distance (van der Waals regime): $V(z) \simeq -C_3/z^3$
- weaker potential for materials weakly coupled to the EM field

Phys. Rev. A 87 (Jan 2013) 012901

Reflection equations and boundary conditions

Exact wavefunction written as a sum of up- and downward semiclassical (WKB) waves with non-constant coefficients :

$$\psi(z) = b_+(z)\frac{\exp(+i\phi(z))}{\sqrt{p(z)}} + b_-(z)\frac{\exp(-i\phi(z))}{\sqrt{p(z)}}$$

$$p(z) = \sqrt{2m(E - V(z))}, \qquad \hbar\phi(z) = \int^z p(z')\mathrm{d}z'$$

Schrödinger's equation $\Rightarrow$ coupled equations for $b_\pm(z)$

M.V. Berry and K.E. Mount, *Rep. Prog. Phys. 35 (1972) 315*

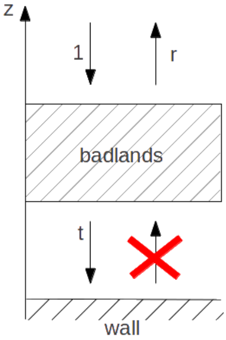
Annihilation of $\overline{\mathrm{H}}$ on the surface: no reflected wave $b_+(z = 0) = 0$
$\Rightarrow$ different from matter atoms & less sensitive to surface physics

Reflection and transmission probabilities

The WKB approximation becomes exact

- as $z \to \infty$ since the potential goes to 0
- as $z \to 0$ because the momentum becomes very large

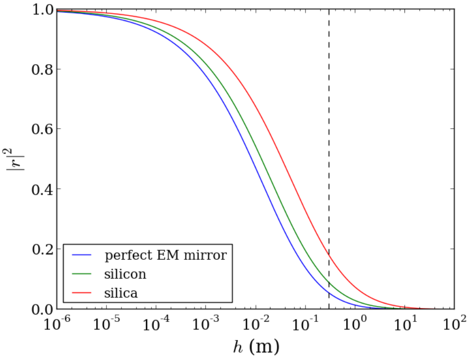
$\Rightarrow$ reflection only occurs in an intermediate region, the “badlands”



reflection probability:
$|r|^2 = |b_+(\infty)/b_-(\infty)|^2$

transmission/annihilation probability:
$|t|^2 = |b_-(0)/b_-(\infty)|^2$

Reflection probability versus energy



Atomic reflection probabilities for a free fall height $h = 30$ cm

surface	$\|r\|^2$
perfect EM mirror	5%
silicon	9%
silica	18%

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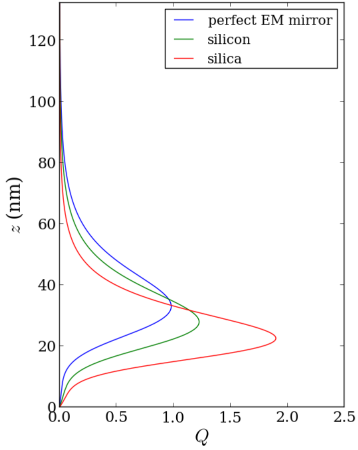
Atomic reflection probability:

- significant for GBAR fall heights
- bias : high energy atoms more likely to be detected
- weaker reflectors of EM field are better reflectors of atoms !

Explanation of the "paradox"



Badlands function Q for h=10cm

"Badlands" function:

$$Q(z) = \hbar^2 \left| \frac{p''(z)}{2p(z)^3} - \frac{3p'(z)^2}{4p(z)^4} \right|,$$

indicates regions where the semiclassical approximation breaks down

For the Casimir-Polder potential, reflection is localized in the region where $|V(z)| \simeq E$

$\Rightarrow$ For weaker Casimir potentials, the atom comes closer to the surface, where V(z) varies rapidly, so that reflection is enhanced

Interest and general idea

Increasing quantum reflection opens many possibilities:

⇒ storing antimatter: antimatter bottles, pipes, ...

⇒ levitation of anti-atoms above a surface

A.Yu. Voronin, P. Froelich, *J. Phys. B 38 (2005) L301*
A.Yu. Voronin, P. Froelich, B. Zygelman, *Phys. Rev. A 72 (2005) 062903*
A.Yu. Voronin, P. Froelich, V.V. Nesvizhevsky, *Phys. Rev. A 83 (2011) 032903*

We can use our understanding of quantum reflection to enhance it :
to increase reflection, weaken the Casimir-Polder interaction

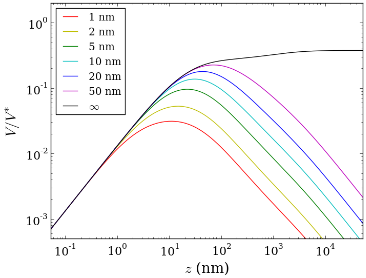
- thin slabs
- graphene
- nanoporous materials

Thin slabs

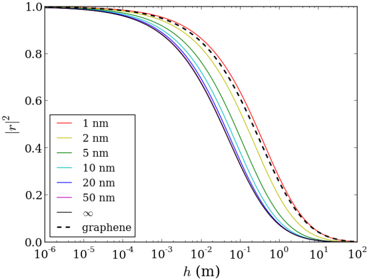
Thin slabs invisible to large EM wavelengths : reduction of Casimir-Polder potential

Naive argument: $V_{slab}(z) \underset{z\to\infty}{\sim} -\frac{C_4}{z^4} + \frac{C_4}{(z+d)^4} \simeq -4dC_4/z^5$

Exact calculation of the potential above thin silica slabs



Reflection on silica slabs and graphene (Dirac model)

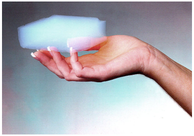


Phys. Rev. A 87 (Jan 2013) 012901

Nanoporous materials

Materials that incorporate a significant fraction of gas or vacuum

Examples : aerogels (dried silica gels $\sim$ 98% porosity), powders of nanodiamonds, porous silicon



NASA

Lifetime of 1st quantum state

surface	Lifetime (s)
perfect EM mirror	0.11
bulk silicon	0.14
bulk silica	0.22
5 nm silica slab	0.33
graphene	0.55
silica aerogel (90% porosity)	1.1
silica aerogel (98% porosity)	4.6

Pore size in the 10-100 nm range allows use of an effective permittivity model if atoms are slow enough

Phys. Rev. A 87 (Feb 2013) 022506

Velocity selector for GBAR

Uncertainty on measured arrival time :

$$\frac{\Delta t}{t} = \sqrt{\left(\frac{\Delta z}{2h}\right)^2 + \left(\frac{\Delta p}{m\sqrt{2gh}}\right)^2} = \frac{1}{2}\frac{\Delta \bar{g}}{\bar{g}}$$

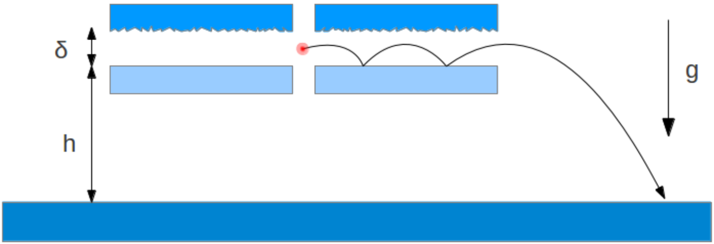
If $\Delta z \Delta p = \hbar/2$, uncertainty minimal for $\Delta z = \left(\frac{\hbar^2 h}{2m^2 g}\right)^{1/4} \sim 90\ \mu\text{m}$

In current GBAR proposal, the ion trap imposes $\Delta z \sim 0.1\ \mu\text{m}$:
precision is limited by ~ 1 m/s uncertainty on initial velocity

Idea : keep only atoms with a small initial vertical velocity,
the gain in precision outweighs loss in statistics

Velocity selector for GBAR

Atoms bounce on (without touching!) a mirror, above which an absorber is placed (similar to GRANIT experiment with neutrons)



Output : $\Delta z \sim \delta$ and $\Delta p \sim m\sqrt{2g\delta}$

Precision of GBAR experiment taken from 1% to 1‰

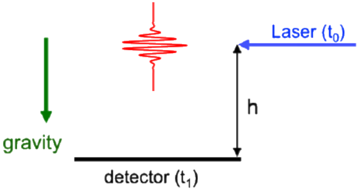
Quantum effects appear for $\delta \lesssim 50\ \mu$m

Submitted to EPJC

Quantum reflection of a free-falling wavepacket

Until now the treatment of atomic motion was classical BUT:

- initial state is a gaussian wavepacket
- close to the surface the energy spectrum is discrete



Need for a fully quantum, time-dependent treatment with

- proper treatment of wavepacket falling in gravity field
- decoherence from annihilation

⇒ Quantum effects in GBAR ?

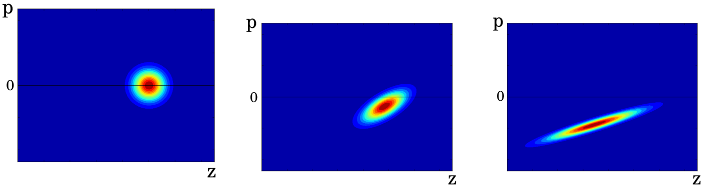
→

Wavepacket falling high above the surface

If $V(z) = mgz$ over the extent of the wavepacket, the Wigner phase-space function obeys the classical equations of motion:

$$\frac{\partial}{\partial t} W(z,p,t) = -\frac{p}{m}\frac{\partial}{\partial z} W(z,p,t) + mg\frac{\partial}{\partial p} W(z,p,t)$$

whereas in general the potential term is nonlocal

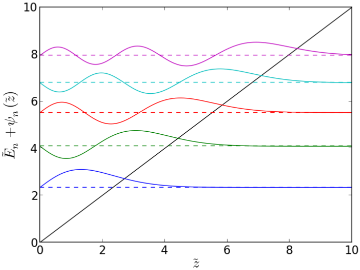


Can we describe reflection in this picture ?

Gravitational quantum states near the surface

Eigenstates in a linear potential: $\psi(z) \propto \mathrm{Ai}(\tilde{z} - \tilde{E})$

Full reflection : $\psi_n(0) = 0$:
$-\tilde{E}_n$ zero of the Airy function



Interaction with the surface is described by a complex scattering length $\tilde{a}$

$\tilde{E}_n \to \tilde{E}_n + \tilde{a}$

$\mathrm{Re}(\tilde{a})$: energy level shift

$\mathrm{Im}(\tilde{a})$: decay rate

perfect conductor : $a = -2.8 - i28.7$ nm

$\Rightarrow$ quasi-stationary states

Description of reflection and dissipation in this framework, beyond the scattering length approximation?

The end

Thank you for your attention

Any questions?