WAG 2013, BERN

Gravitational Quantum States of Antihydrogen

A. Voronin, V. Nesvizhevsky, P.Froelich

O.Dalkarov, E.Kupriyanova

P.N.Lebedev Physical Institute of the Russian Academy of Sciences

UPPSALA UNIVERSITET

NEUTRONS FOR SCIENCE®

Plan of the talk

- Gravitational states of antihydrogen: Is it possible?
- How can we get gravitational mass out of gravitational states?
- Properties of gravitational states
- Spectroscopy, interference and time-spatial resolution of gravitational states

Gravitational quantum states?

State of motion of a quantum particle, which is localized near reflecting surface in a gravitational field of the Earth.

$$\left[-\frac{d^2}{dx^2}+x-\lambda\right]F(x)=0,\ F(0)=0$$

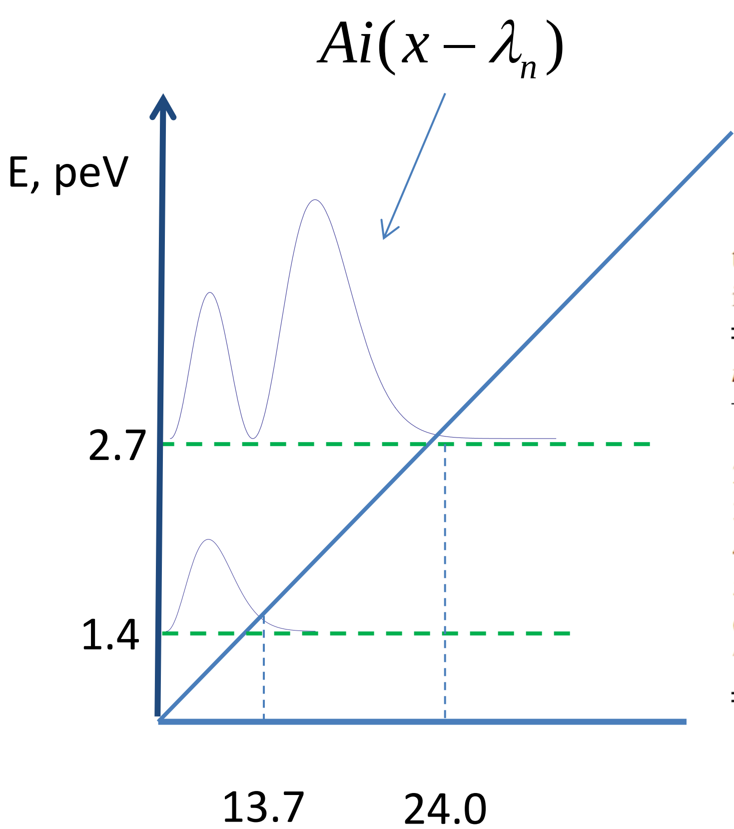
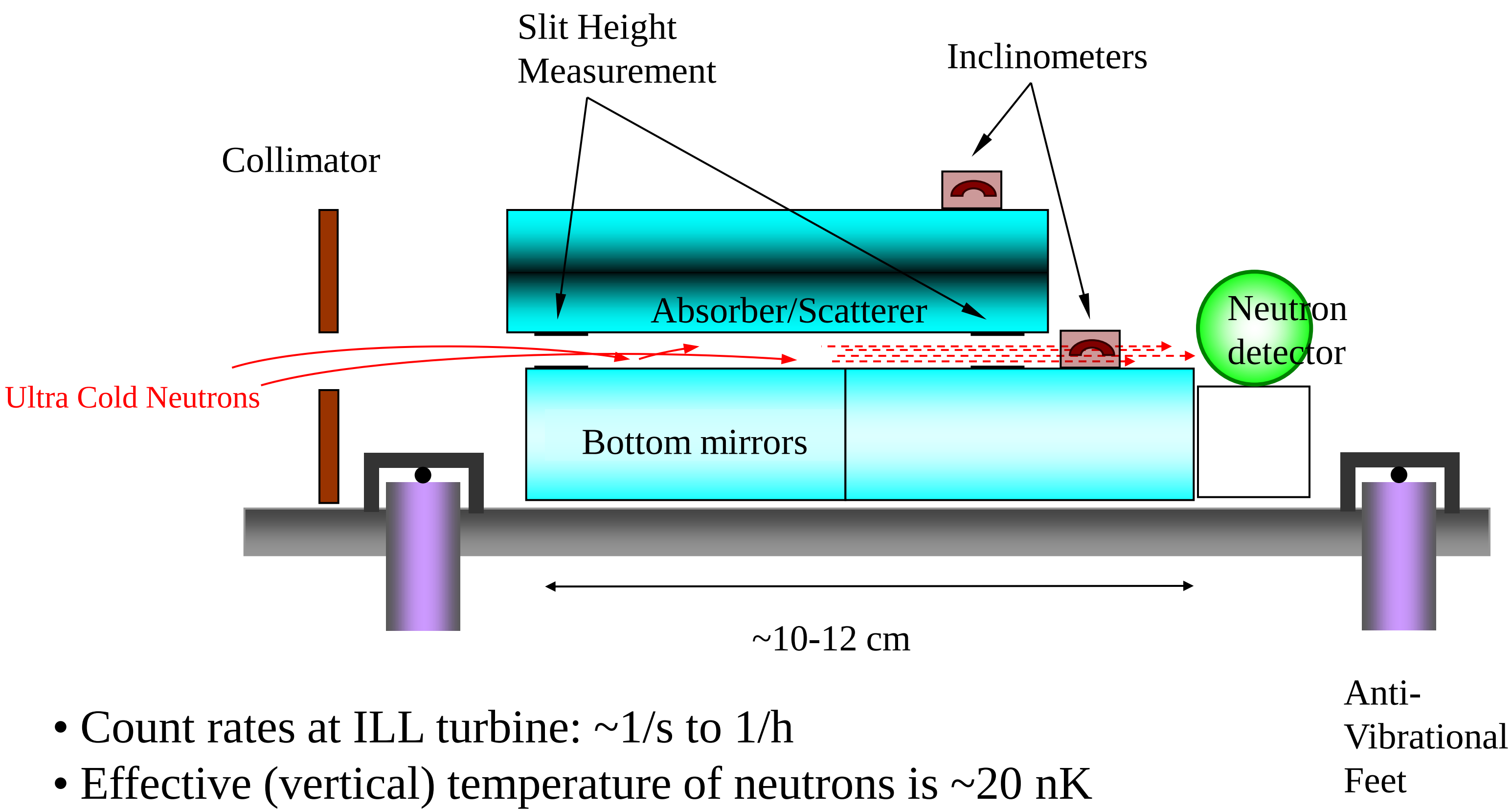


TABLE I. The eigenvalues, gravitational energies, and classical turning points of a quantum bouncer with the mass of (anti)hydrogen in the Earth's gravitational field.

n	λ_n^0	E_n^0 (peV)	z_n^0 (μm)
1	2.338	1.407	13.726
2	4.088	2.461	24.001
3	5.521	3.324	32.414
4	6.787	4.086	39.846
5	7.944	4.782	46.639
6	9.023	5.431	52.974
7	10.040	6.044	58.945

First Observation: Gravitational States of Neutrons
Nesvizhevsky et al. Nature 415, 297 (2002)



- Count rates at ILL turbine: ~1/s to 1/h
- Effective (vertical) temperature of neutrons is ~20 nK
- Background suppression is a factor of $\sim 10^8$-10^9
- Parallelism of the bottom mirror and the absorber/scatterer is $\sim 10^{-6}$

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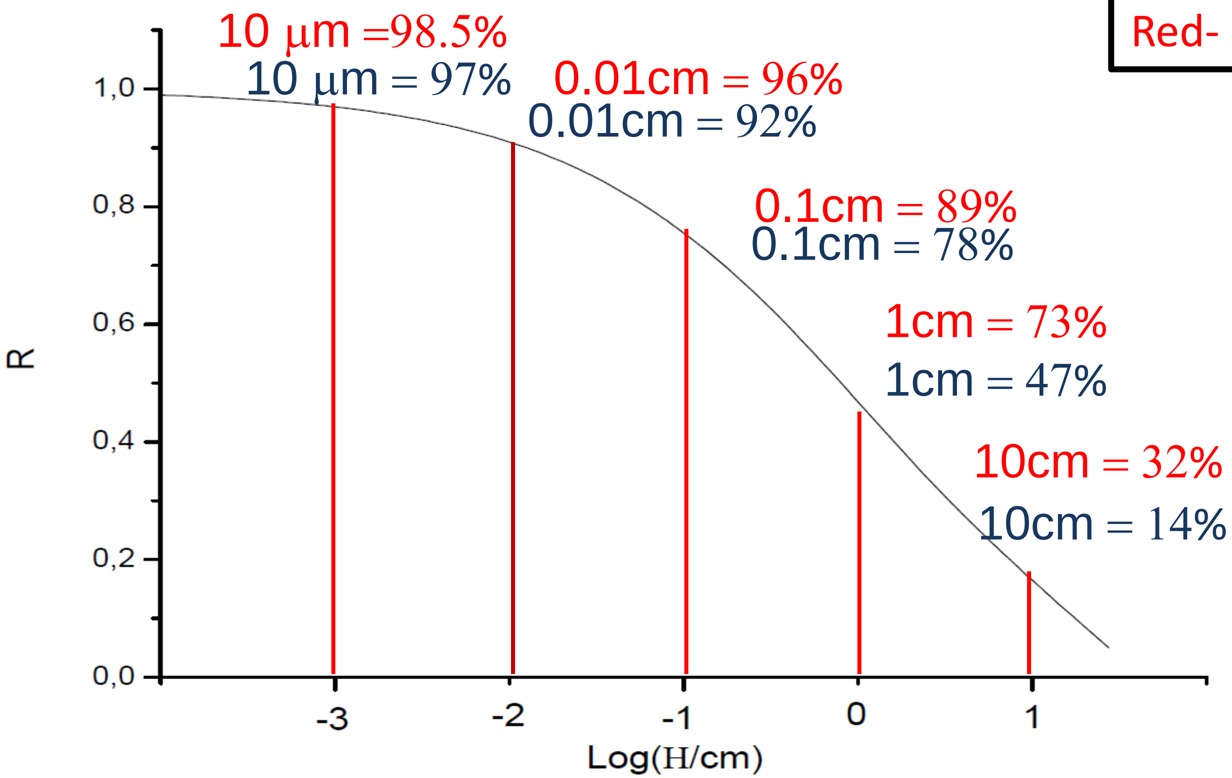
GRANIT SPECTROMETER

Gravitational states of Antihydrogen: Seems Impossible? Quantum Reflection!

Red- silica, black- gold



A. Yu. Voronin, P. Froelich, and B. Zygelman, Phys. Rev. A **72**, 062903 (2005).

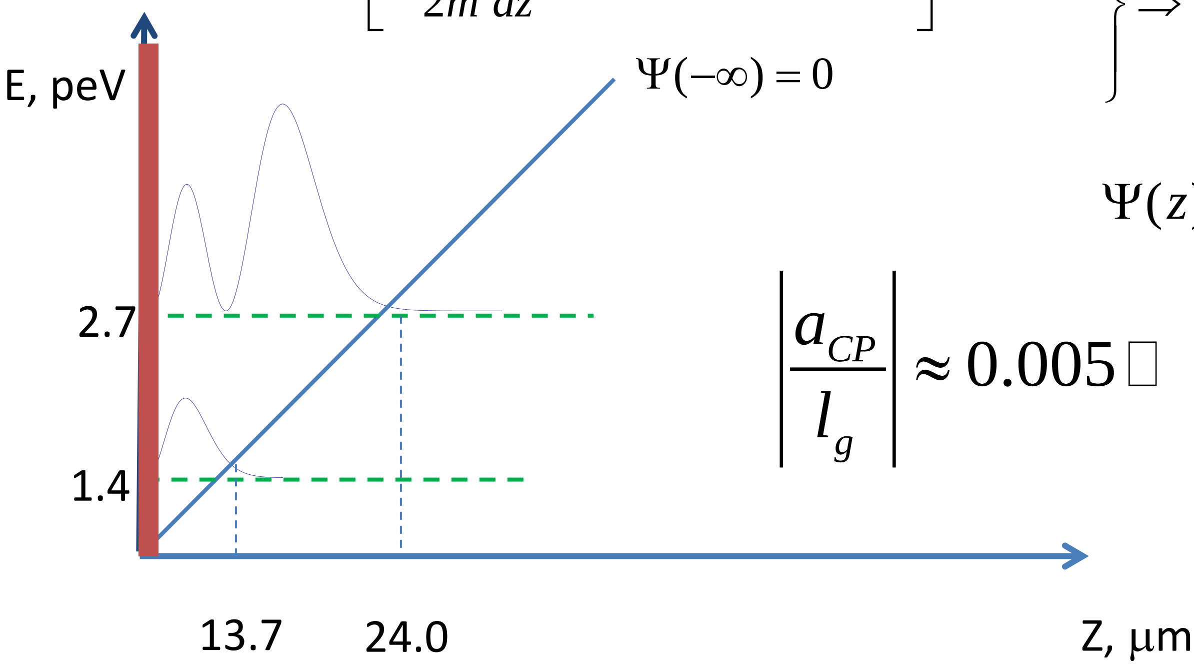
G. Dufour, A. Gérardin, R. Guérout, A. Lambrecht, V. V. Nesvizhevsky, S. Reynaud, A. Yu. Voronin Phys. Rev. A 87, 012901 (2013)

Gravitational states of antihydrogen

Quantum reflection is about 97% - it works like a reflecting wall

$$\left.\begin{array}{l}\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}+V_{CP}(z)+Mgz-E\right]\Psi(z)=0\\ \Psi(-\infty)=0\end{array}\right\}\Rightarrow\begin{array}{l}\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}+Mgz-E\right]\Psi(z)=0\\ \frac{\Psi(0)}{\Psi'(0)}=-\frac{a_{CP}}{l_g}\approx i0.005\end{array}$$

$$\Psi(z)=\mathrm{Ai}(z/l_g-\lambda_n)/|\mathrm{Ai}'(-\lambda_n)|$$



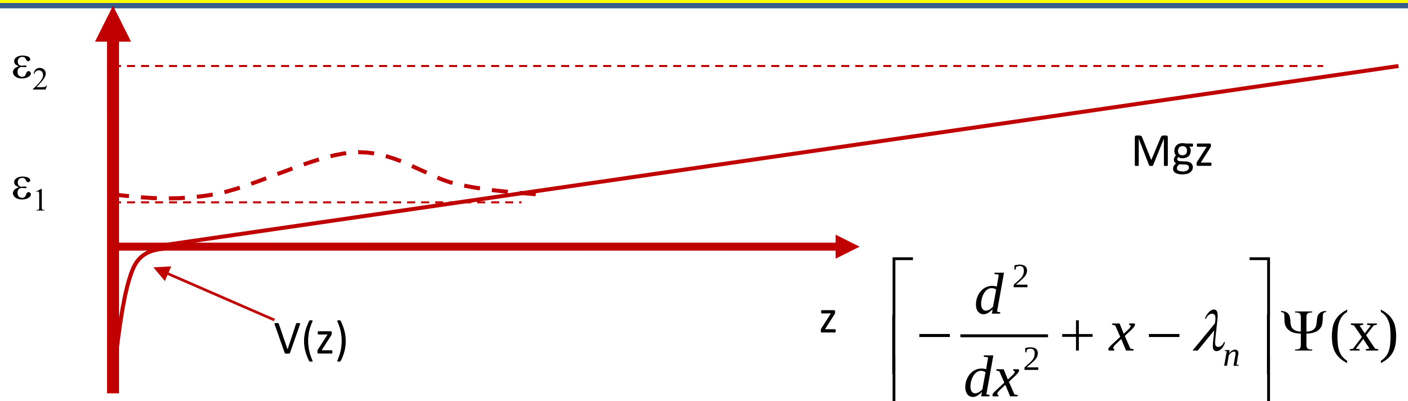
$$\left|\frac{a_{CP}}{l_g}\right|\approx 0.005 \ll 1$$

$$\lambda_n\approx\left(\frac{3\pi}{4}(2n-\frac{1}{2})\right)^{2/3}$$

Effects of surface

- Hierarchy of scales $l_g \gg |a_{CP}|$: gravity and surface-atom interaction are factorized
- Annihilation in the bulk of the wall: short –range atom-wall interactions are washed out
- Small annihilation width of gravitational states: compromise between long life-time and observation

Correction by Casimir-Polder potential + annihilation



$$\left[-\frac{d^2}{dx^2}+x-\lambda_n\right]\Psi(x)=0$$

$$|a|/l_0 \approx 0.005$$

$$\frac{\Psi(0)}{\Psi'(0)}=-\frac{a_{CP}}{l_g}\approx i0.005$$

Correction by Casimir-Polder and annihilation:

$$\tilde{\lambda}_n=\lambda_n+a/l_g$$

$$\varepsilon_n=\varepsilon_0(\lambda_n+\operatorname{Re}a/l_g)\quad \Gamma=2\varepsilon_0\left|\operatorname{Im}a\right|/l_g$$

$$\tau=\frac{l_g}{\varepsilon_g}\frac{\hbar}{2\left|\operatorname{Im}a\right|}=\frac{\hbar}{2Mg\left|\operatorname{Im}a\right|}\approx 0.1s$$

All states have equal shift and lifetime~0.1s $\Rightarrow$

No surface effects in transition frequiencies

Gravitational states and Gravitational mass

Classical: $m\ddot{z} = Mg \rightarrow \ddot{z} = g \rightarrow T = \sqrt{2H/g}$

Quantum: $$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + Mgz - E\right]\Psi(z) = 0 \Rightarrow \left[-\frac{d^2}{dx^2} + x - \lambda_n\right]F(x) = 0$$

$$\varepsilon_g = \sqrt[3]{\frac{\hbar^2 M^2 g^2}{2m}} = 0.61\ 10^{-12}\text{eV};\quad l_g = \sqrt[3]{\frac{\hbar^2}{2Mmg}} = 5.87\ 10^{-6} m$$

$$E_n = \varepsilon_g \lambda_n \qquad z = l_g x \qquad \Psi_n(z) = \frac{1}{\sqrt{l_g}} F(z/l_g - E_n/\varepsilon_g)$$

$$m = \frac{\hbar^2}{2\varepsilon_g l_g^2};\quad M = \frac{\varepsilon_g}{g l_g}$$

$$M = m \Rightarrow \frac{\hbar}{\varepsilon_g} = \sqrt{\frac{2l_g}{g}} \text{ or } T = \sqrt{\frac{2H}{g}}$$

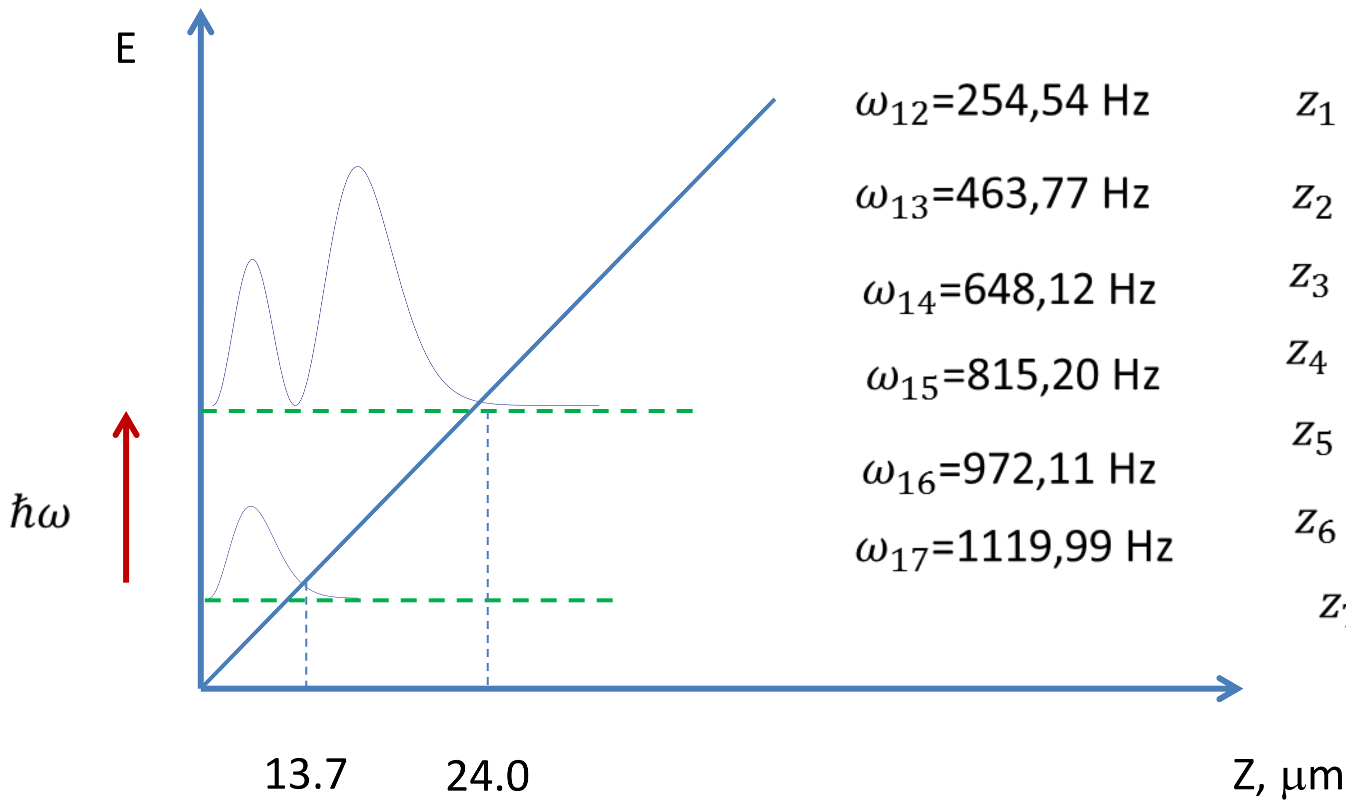
Gravitational states are all about energy and spatial scales

Methods of observation

- Spectroscopy: induced transition between gravitational states
- Interference: temporal and spatial oscillations of annihilation signal of superposition of gravitational states
- Time and spatial resolution of free-fall events: mapping of momentum distribution of gravitational state into time-of-fall or spatial distribution

Spectroscopy- to induce transitions between gravitational states with alternating magnetic field

Developed for neutrons by V. Nesvizhevsky, S. Baessler, G. Pignol, K. Protassov, A.Voronin



ω_{12}=254,54 Hz

ω_{13}=463,77 Hz

ω_{14}=648,12 Hz

ω_{15}=815,20 Hz

ω_{16}=972,11 Hz

ω_{17}=1119,99 Hz

$z_1 = 13.7\ \mu m$

$z_2 = 24.0\ \mu m$

$z_3 = 32.4 \mu m$

$z_4 = 39.8\ \mu m$

$z_5 = 46.6\ \mu m$

$z_6 = 52.9\ \mu m$

$z_7 = 58.9\ \mu m$

Antihydrogen in Magnetic Field

$$\hat{T}_z + Mgz + \hat{H}_c + \frac{\alpha_{HF}}{2}\left(\boldsymbol{F}^2 - \frac{3}{2}\right) + \boldsymbol{B}(z,t)\left(\mu_B \hat{\boldsymbol{s}}_{\bar{e}} + \mu_{\bar{p}} \hat{\boldsymbol{s}}_{\bar{p}}\right) + \hat{H}_{st}$$

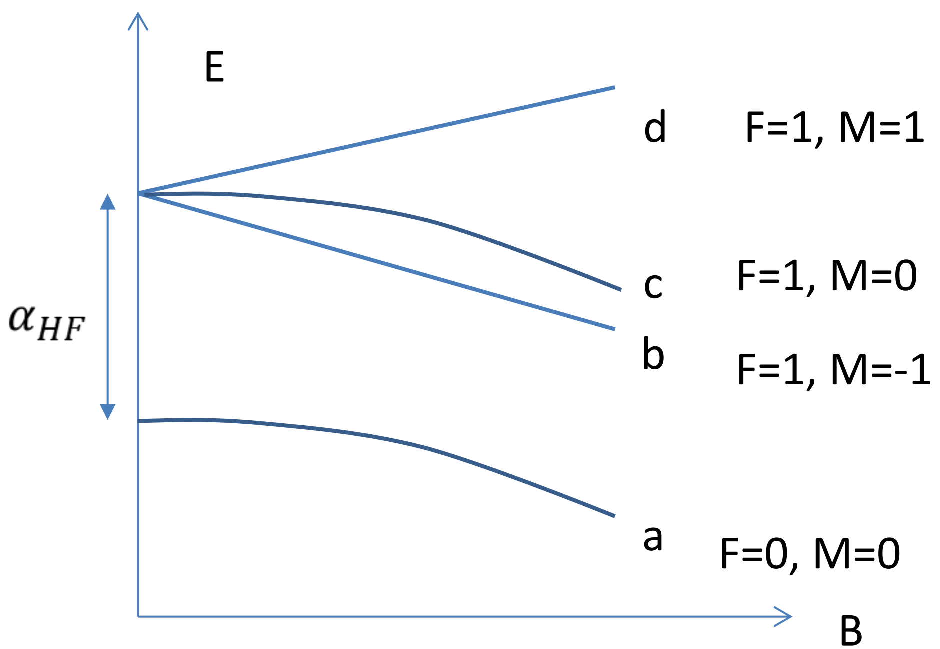
Center-of-mass fall

Coulomb

HF splitting

Field-magnetic moment interaction

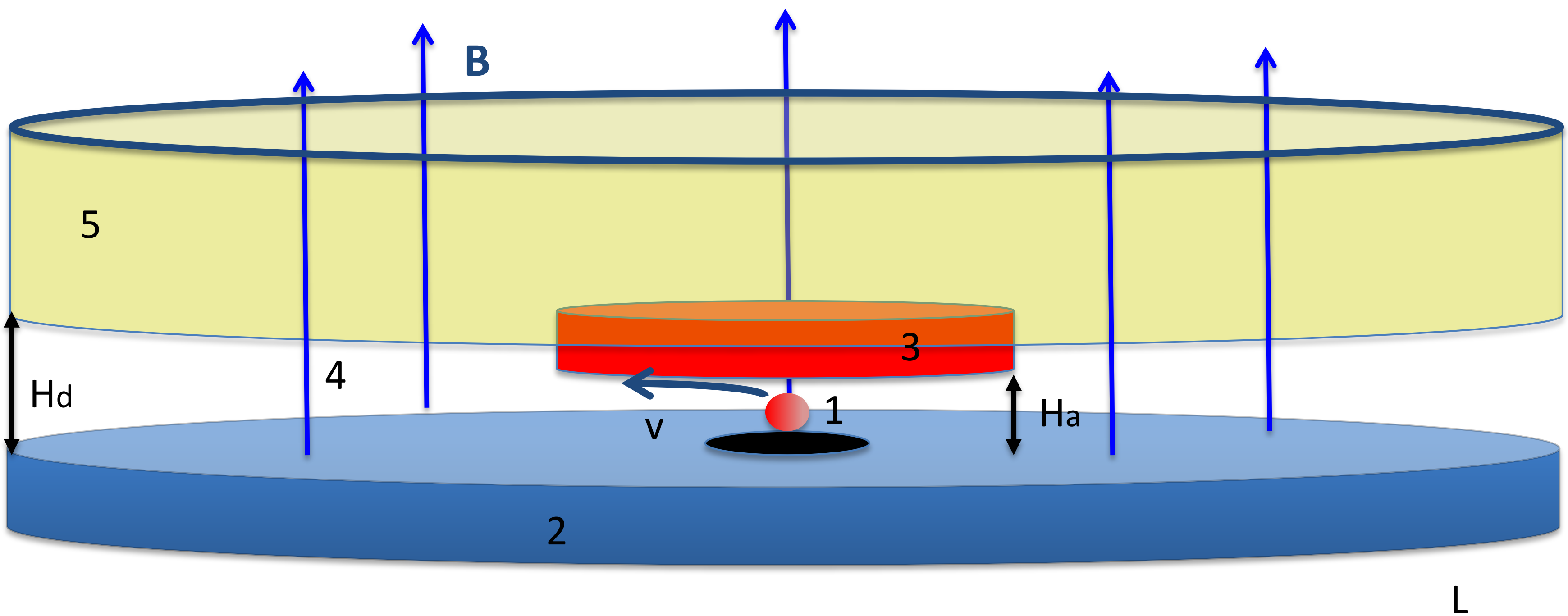
Effective Stark-effect



$$\frac{\alpha_{HF}}{\hbar} \cong 1420\ Mhz$$

$$\varepsilon_{grav} \ll \mu B \ll \alpha_{HF}$$

Possible scheme of flow-throw experiment



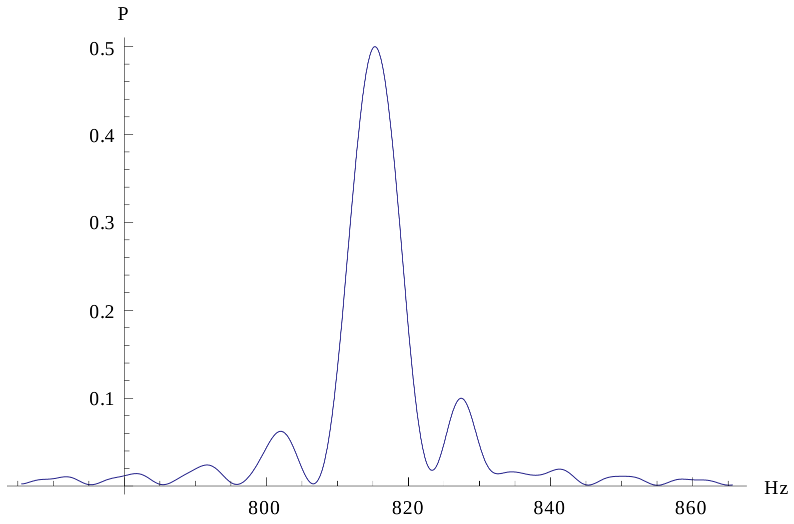
1-source of ultracold antihydrogen, 2-mirror, 3- absorber, 4- magnetic field, 5- detector

$$v \approx 1m/s, H_a = 15\mu m, H_d = 25\mu m, B_0 \approx 10Gs, \beta \approx 10Gs/m, L = 30cm$$

Transition probability

$$P_{ik} = \frac{1}{2}\frac{\Omega_{ik}^2}{\Omega_{ik}^2 + \hbar^2\Delta^2}\sin^2\left(\frac{t}{\hbar}\sqrt{\Omega_{ik}^2 + \hbar^2\Delta^2}\right)\exp(-\Gamma\, t)$$

$$\Omega_{ik} = \frac{(\mu_B + \mu_{\bar{p}})\beta l_g^3}{\hbar(z_k - z_i)^2}$$



Transition probability as a function of frequency. Transition 1->5

Time of observation t=0.1 s

EP and gravitational mass

$$\varepsilon_g = \sqrt[3]{\frac{\hbar^2 M^2 g^2}{2m}} = 0.61\ 10^{-12}\text{eV}$$

$$\hbar\omega_{ik} = \sqrt[3]{\frac{\hbar^2 M^2 g^2}{2m}}\left(\lambda_k - \lambda_i\right) \Rightarrow M = \sqrt{\frac{2m\hbar\omega_{ik}^3}{g^2\left(\lambda_k - \lambda_i\right)^3}}$$

$$M = m \Rightarrow M = \frac{2\omega_{ik}^3}{(\lambda_k - \lambda_i)^3}\frac{\hbar}{g^2}$$

PRECISION

$$\varepsilon \approx \frac{3\gamma}{2\sqrt{N_{\bar{H}}}\,\omega};\ N_{\bar{H}} = 10^2$$

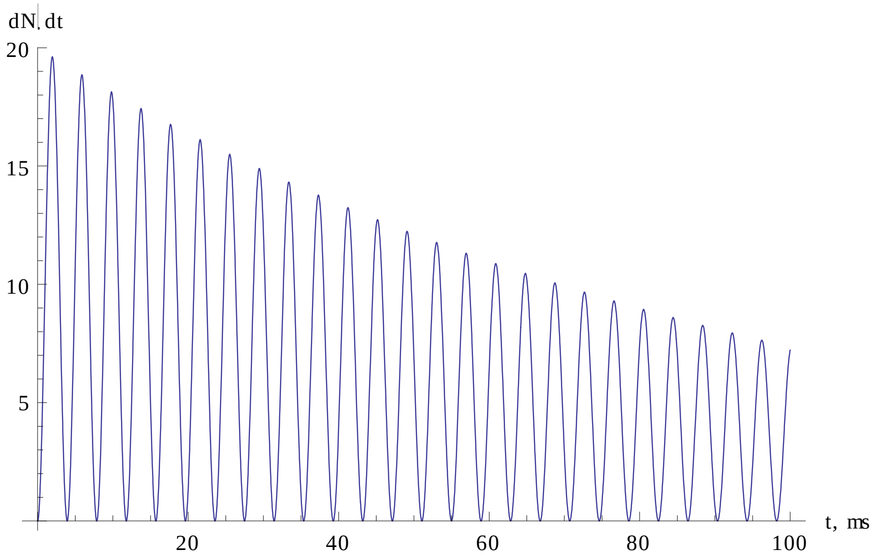
$$\varepsilon_{1-5} \approx\ 10^{-3}$$

Interference of gravitational states

$$\Psi = \sum_{i=1}^{N} C_i \Psi_i$$

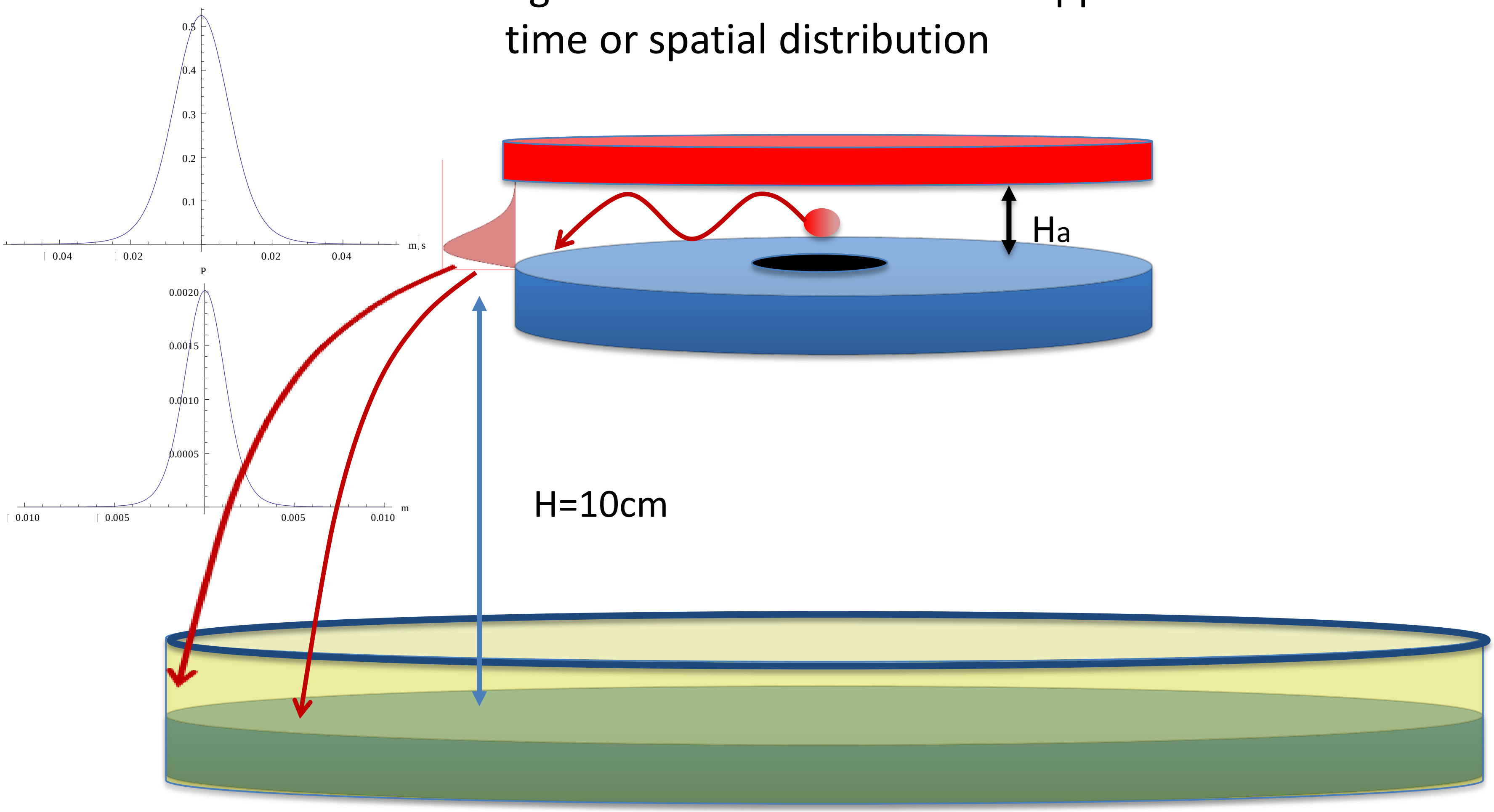
$$j(z,t) = \frac{i\hbar}{2m}\left[\Psi(z,t)\frac{d\Psi^*(z,t)}{dz} - \Psi^*(z,t)\frac{d\Psi(z,t)}{dz}\right]$$

$$\frac{dF_{12}}{dz} = -\frac{\Gamma}{\hbar}\exp(-\frac{\Gamma}{\hbar}t)\left(1-\cos(\omega_{12}t)\right) \qquad \omega_{12} = \frac{E_2 - E_1}{\hbar} \qquad \hbar\Gamma^{-1} \approx 0.1s$$



Time and Spatial Resolving of Gravitational States

Momentum distribution of gravitational state can be mapped into measurable time or spatial distribution



Mapping of momentum distribution

$$\Psi(z,t)=\frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{ipz/\hbar}G(p,t,p')F_0(p')dpdp'$$

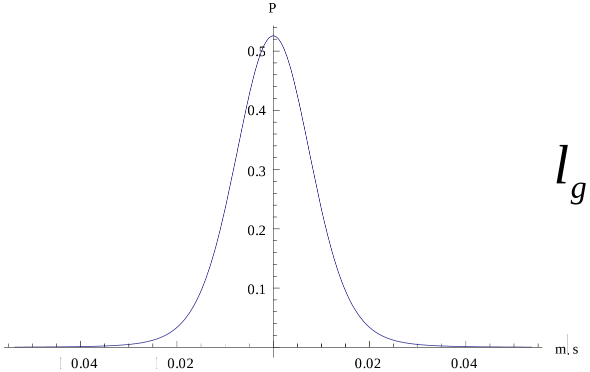
$$G(p,t,p')=\exp\left[-\frac{it}{2m\hbar}(p^2-Mgpt+M^2g^2t^2/3)\right]\delta(p-Mgt-p')$$

$$\Psi(z,t)\approx\sqrt{\frac{m}{t}}e^{\frac{imz^2}{2t\hbar}+-\frac{it^3M^2g^2}{2m\hbar}}F_0(p_0-Mgt);\ p_0=(z+\frac{gt^2}{2})\frac{m}{t}$$

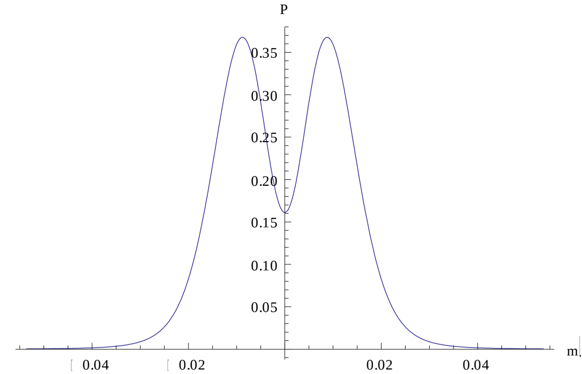
$$|\Psi(z,t)|^2\approx\frac{m}{t}|F_0(k)|^2$$

$$1)\ \mathrm{z}=\mathrm{z}_0:k=mg(t-t_0),\ t_0=\sqrt{2g/\mathrm{z}_0}$$

$$2)\ t=t_0=L/v:k=\frac{m(\mathrm{z}-\mathrm{z}_0)}{t_0}$$



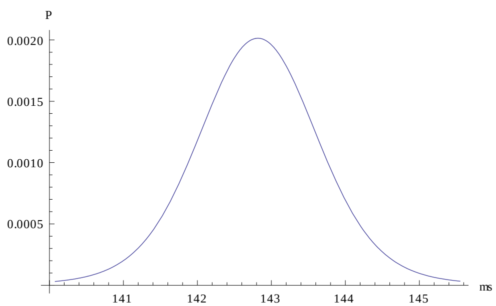
$$l_g = \frac{\delta(n)\hbar}{\Delta k_n}$$

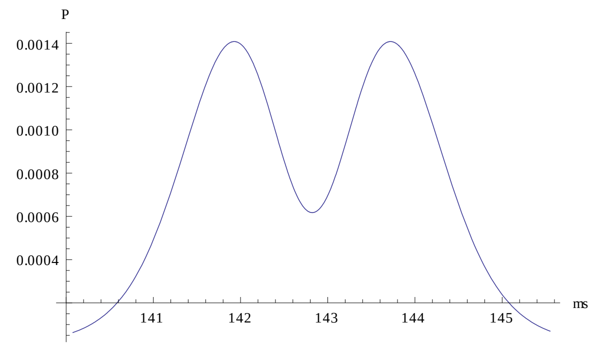


1 state

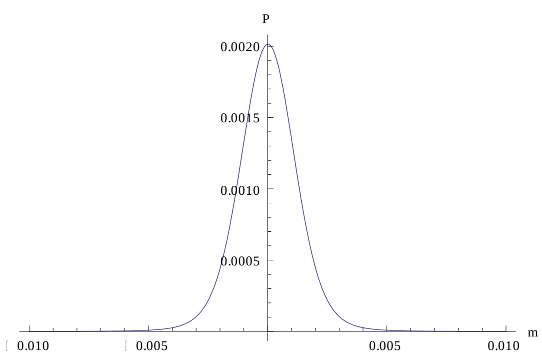
velocity distribution

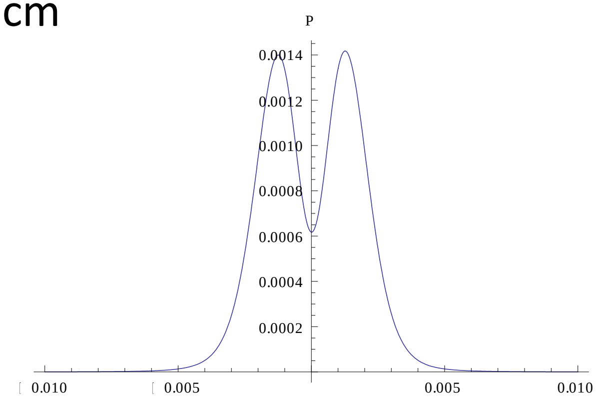
2 state





Time-of –fall distribution H= 10 cm

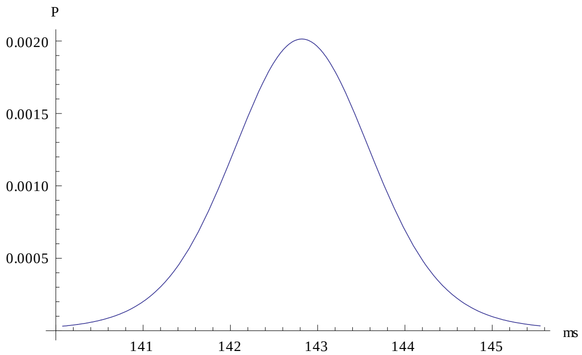




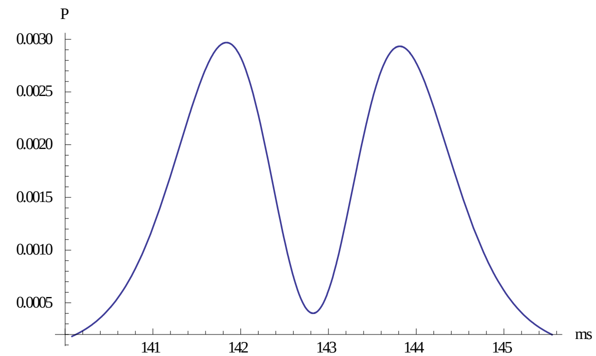
Spatial distribution time-of-flight T=0.1s

Phase monitoring

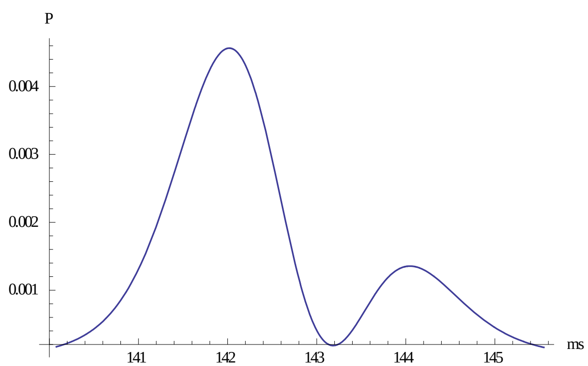
$$\Psi = \Psi_1 + Exp(-i\omega_{21}t)\Psi_2$$



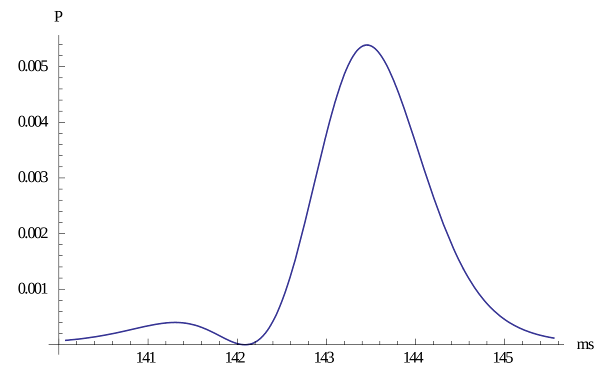
$t = 0$



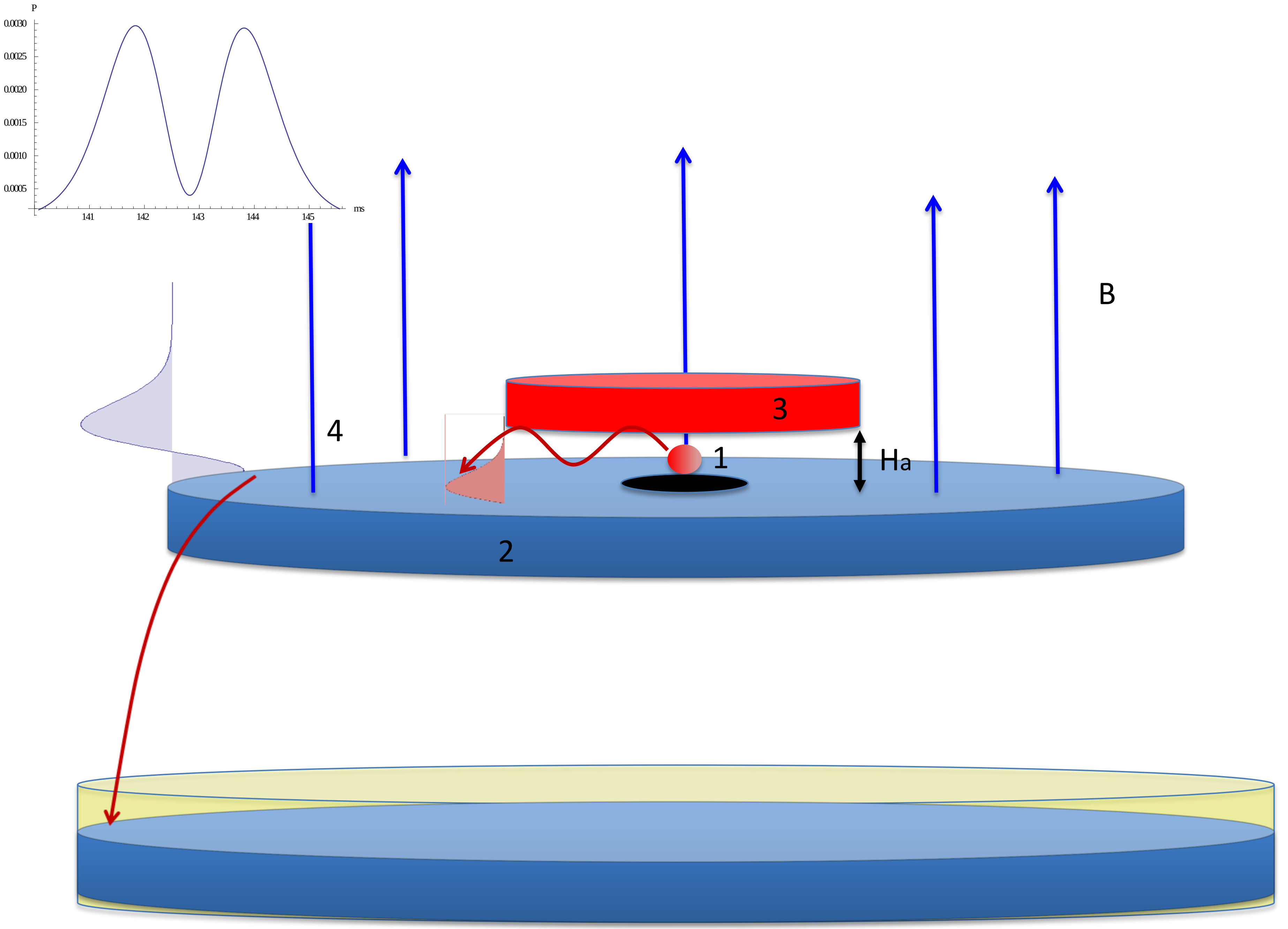
$t = 0.0019s$



$t = 0.0014s$



$t = 0.0029s$


P
0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
141
142
143
144
145
ms
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1
Ha
2

Conclusions

- Gravitational states of Antihydrogen: simplest bound antimatter quantum system, determined by gravity. Effects of surface are canceled out.
- Gravitational states of Antihydrogen- metastable and long-living, easy to study due to annihilation signal
- Gravitational states- a way to precision measurement of the gravitational mass M