The modified version of the coherent elastic generator

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The model Description

The elastic channel in a hadron-nucleon scattering takes a rather large part of a hadronic total cross-section.

At high energies the following ratio approximately is valid: $\sigma_{el} = 0.15 \div 0.25\ \sigma_{tot}$

For heavy nuclei $\sigma_{el} > 0.3\ \sigma_{tot}$

The Glauber's approach was used in the preparation of the generator. This approach works well enough at energies higher several GeV.

According to this model the scattering of the high energy hadrons from nuclei can be described using the information about interaction on free nucleons.

The main approaches of the Glauber model:

1. Adiabatic approximation of the nucleons wave function

$$F_{fi}(\vec{q}) = \frac{ik}{2\pi}\int d^2b \cdot e^{i\vec{q}\vec{b}}\Psi^*{}_f(\vec{r}_1,\ldots\vec{r}_A)\Gamma(\vec{b},\vec{r}_1,\ldots\vec{r}_A)\Psi_i(\vec{r}_1,\ldots\vec{r}_A)\prod_{j=1}^{A} d^3r_j$$

2. Independent interaction of the individual nucleons of the nucleus

=> The "additive phase" assumption:

$$\Gamma(\vec{b},\vec{r}_1,...\vec{r}_A) = 1 - \exp\{i\chi(\vec{b},\vec{r}_1,...\vec{r}_A)\} = 1 - \exp\{i\sum_{j=1}^{A}\chi_j(\vec{b}-\vec{s}_j)\}$$

3. All correlations between the nucleons positions are neglected.
Then the many-body density of the nucleus is represented as a product of one-body densities:

$$\rho_{ii}(\vec{r}_1,...\vec{r}_A) \equiv |\Psi_i|^2 = \prod_{j=1}^{A}\rho(\vec{r}_j)$$

$\rho(\vec{r}_j)$ - is an one-nucleon density.

=> $$F(\vec{q}) = \frac{ik}{2\pi}\int d^2b \cdot e^{\vec{q}\cdot\vec{b}} M(\vec{b})$$

$$M(\vec{b}) = 1 - [1 - \int d^3 r \Gamma(\vec{b} - \vec{s})\rho(\vec{r})]^A$$

$$\Gamma_j(\vec{b}) \equiv 1 - \exp[i\chi_j(\vec{b})] = \frac{\sigma_{tot}^{hN}}{2\pi i k^{hN}} \int d\vec{q} e^{-\vec{q}\cdot\vec{b}} f_j(\vec{q}),$$

$$f(q) = \frac{i k^{hN} \sigma^{hN}}{2\pi} e^{-0.5 q^2 B}$$

$$\frac{d\sigma}{d\Omega_{CM}} = | F(q) |^2 . \qquad \frac{d\sigma}{d(-t)} = \frac{d\sigma}{dq^2{}_{CM}} = \frac{\pi}{k^2{}_{CM}} | F(q) |^2 .$$

The main dependence of the differential cross-section on energy goes from the parameters of the hadron-nucleon scattering

Glauber's approach can be used not only in elastic scattering but also in production particles and excitation of nuclei.

To illustrate this possibility the calculations of the 3-π system production at P_{lab}=16 Gev/c is presented:

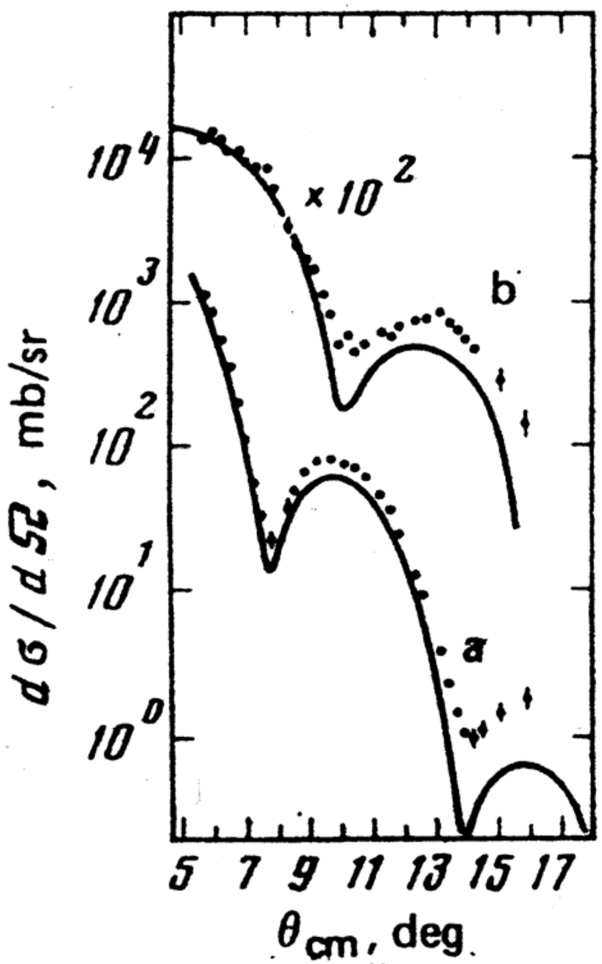
$$\pi^- + Si \to \pi^- + \pi^- + \pi^+ + Si' \text{ (exited)}$$

(Bellini et al., NIM, 107, 85,1973)

In this experiment the most part of quasielastic channels were suppressed but the first minimum was filled nevertheless. The use of Glauber's formalism allows to explain this result as the simultaneous contribution of two channels

- Si remains in ground state
- the 2^+ (1.78 Mev) level of Si is exited

1. The adjustment of nuclear parameters

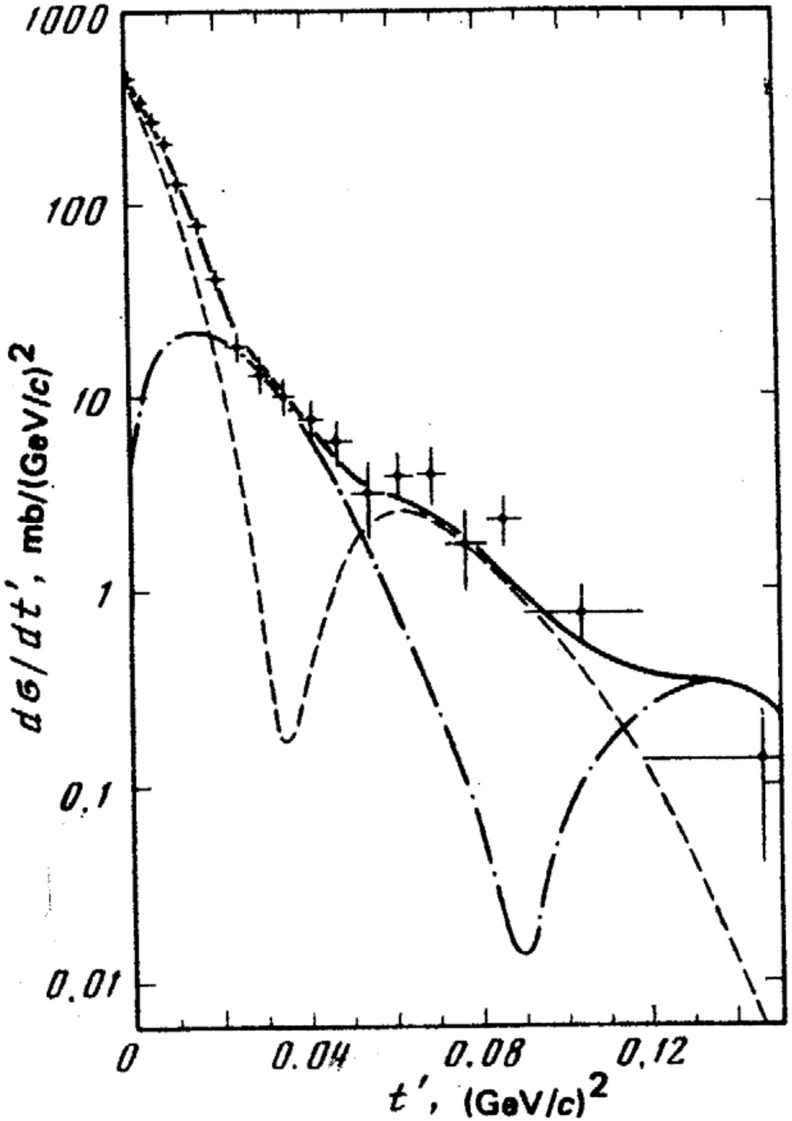


The proton-silicon elastic scattering and exitation of 2^+-level of Si by proton at T=1 GeV were used to adjust parameters of two Gauss nuclear density and transition density.

$$\rho(\vec{r}) = C[e^{-(r/R_1)^2} - pe^{-(r/R_2)^2}]$$

***a* – elastic pSi**

***b* – pSi => p Si' (2^+, 1.78 MeV)**



2. The comparison of the experimental data and calculations confirms the assumption about two-channel nature of measured Reaction:

- Si remains in ground state
- the 2+ (1.78 Mev) level of Si is exited

Legend:

dashed* – *Si in ground state

dot-dash* – *2^+ (1.78 MeV)

solid* – *combined curve

(Ermekov, Korotkikh, Starkov; Sov. Journ. Nucl. Phys., **33**, 77, 1981)

The base of the coherent-elastic generator are differential cross sections which are calculated by Glauber's formula.

Two Gauss form was chosen for the one nucleon density :

$$\rho(\vec{r}) = C[e^{-(r/R_1)^2} - pe^{-(r/R_2)^2}]$$

The experimental data of elastic scattering at T = 1 GeV were used to adjust of the nuclear density parameters.

The distribution functions are calculated by the following formula:

$$F(q^2) = \frac{\int_0^{q^2} d(q^2)\frac{d\sigma}{d(q^2)}}{\int_0^{q^2_{max}} d(q^2)\frac{d\sigma}{d(q^2)}}$$

The values of q^2_{max} are chosen near the second minimum of the hadron-nucleus differential cross section.

The structure of generator

Class: G4ElasticHadrNucleusHE

General interface functions:

1. G4HadFinalState * ApplyYourself(const G4HadProjectile& aTrack,
   G4Nucleus& G4Nucleus);

2. G4double SampleT(const G4ParticleDefinition* p, G4double plab,
   G4int Z, G4int A);

The tables of probability function are prepared by the function SampleT on fly.

Each table approximates $F(q^2)$ for fixed pairs of HADRON-NUCLEUS (hA) and describes dependence on energy T and transfer momentum q^2.

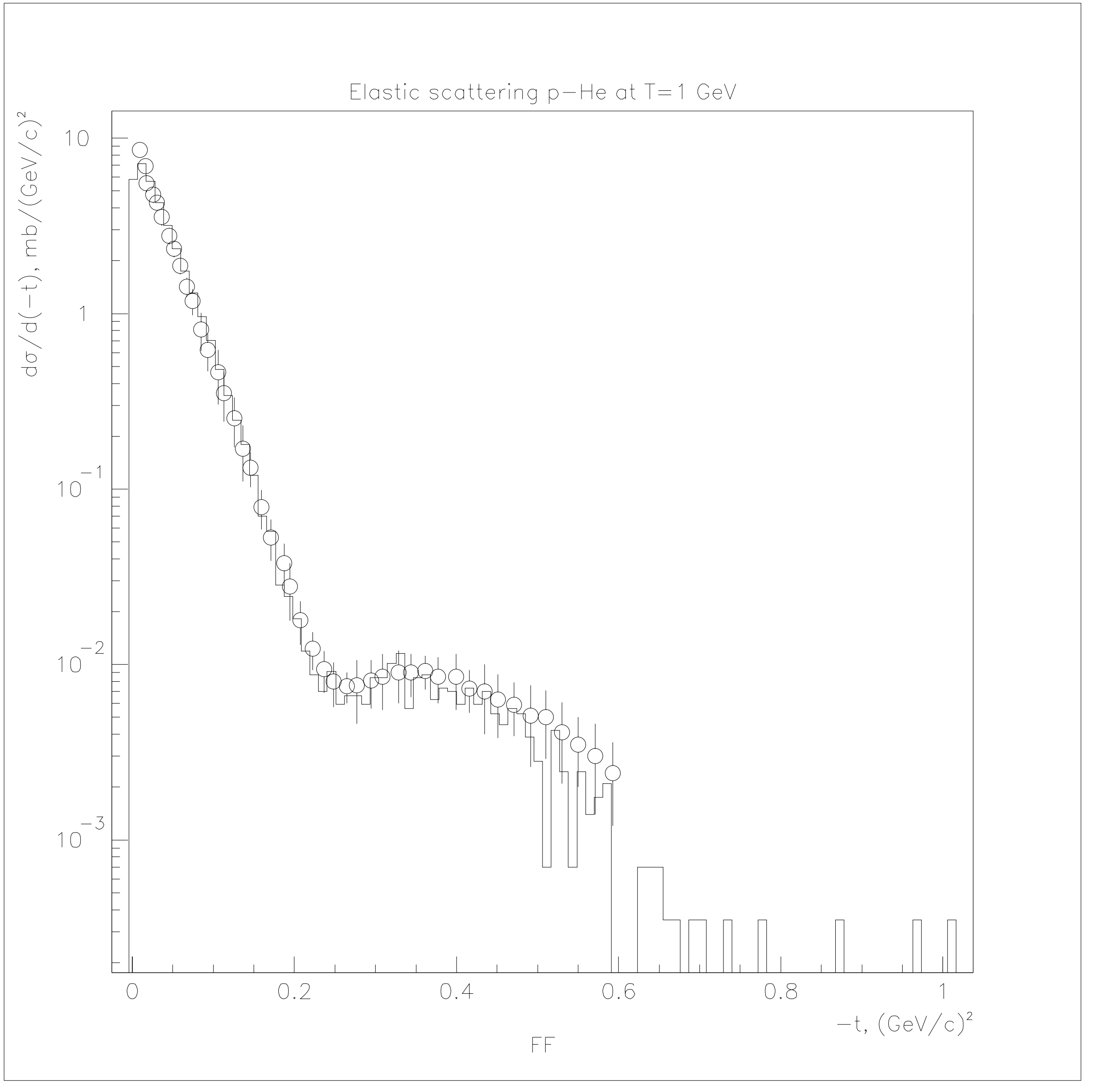
At the beginning of the generating these tables are empty and filled step-by-step as new combination (hA) and energy arises.

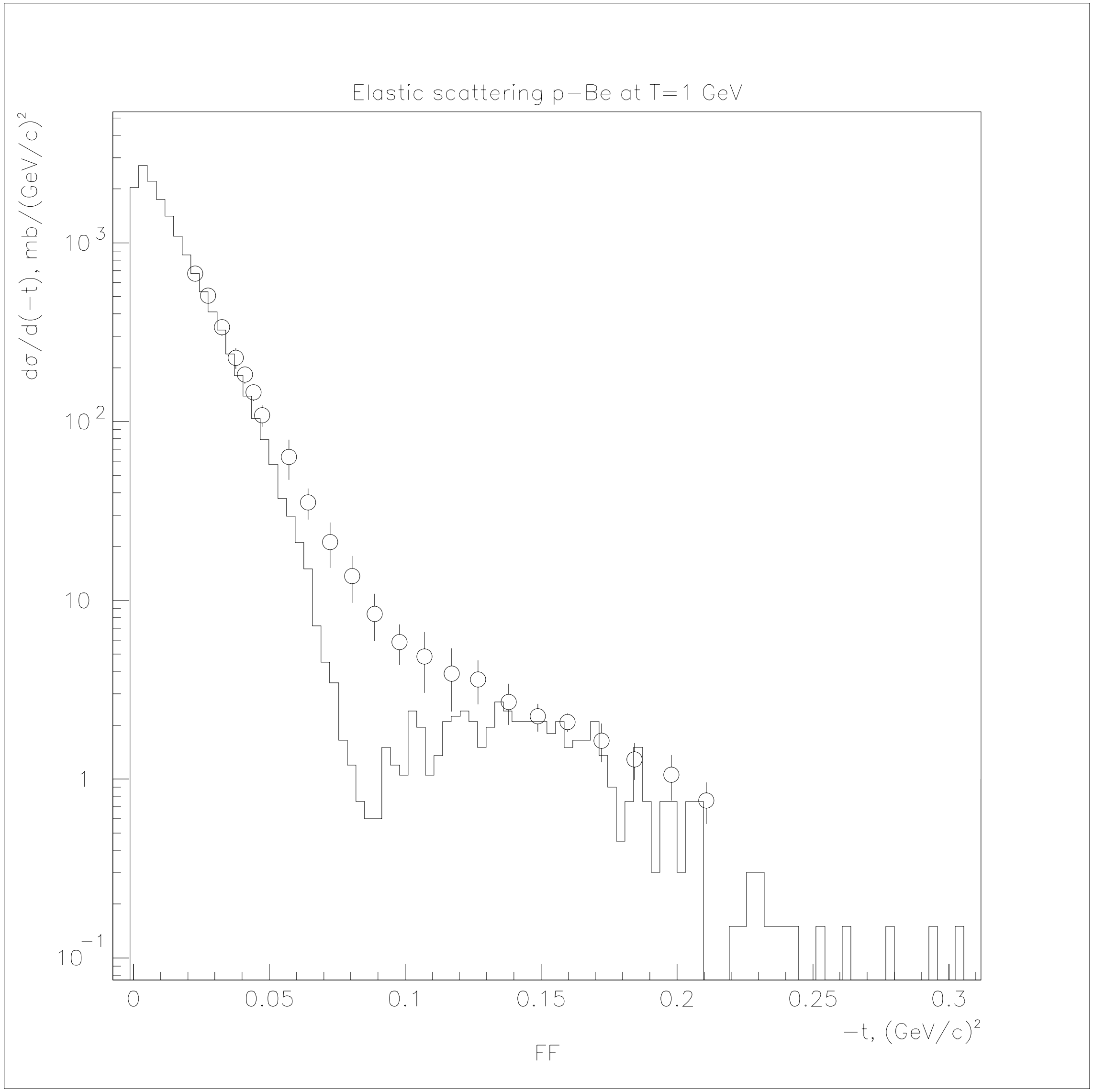
The latest upgrade

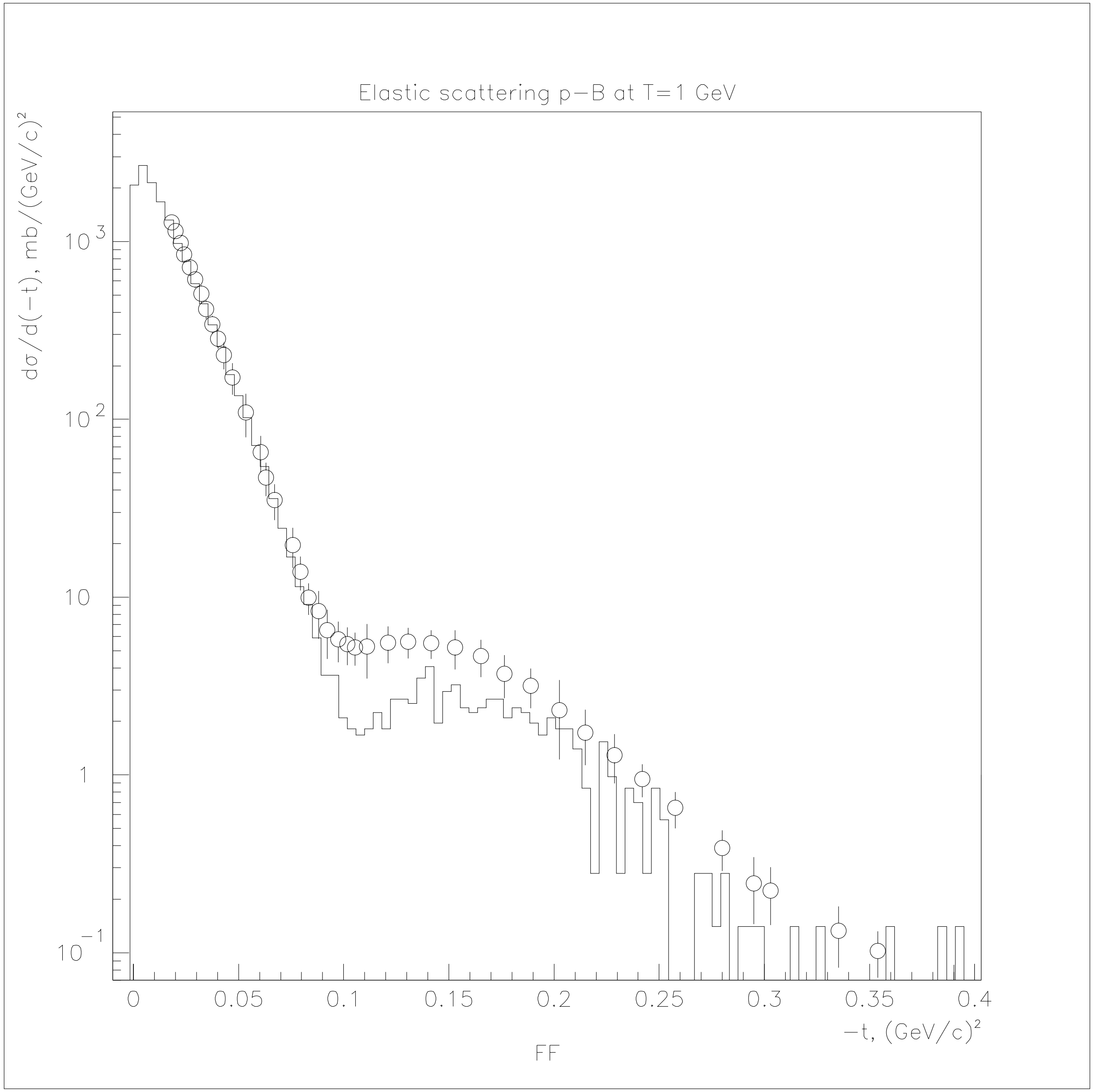
1. The more optimal algorithm of the probability function calculation is used. It allows to decrease the preparing tables time more then one order.

2. The difference between hadron-proton and hadron-neutron scattering parameters was taken into account . It leads to more exact description of cross-sections.

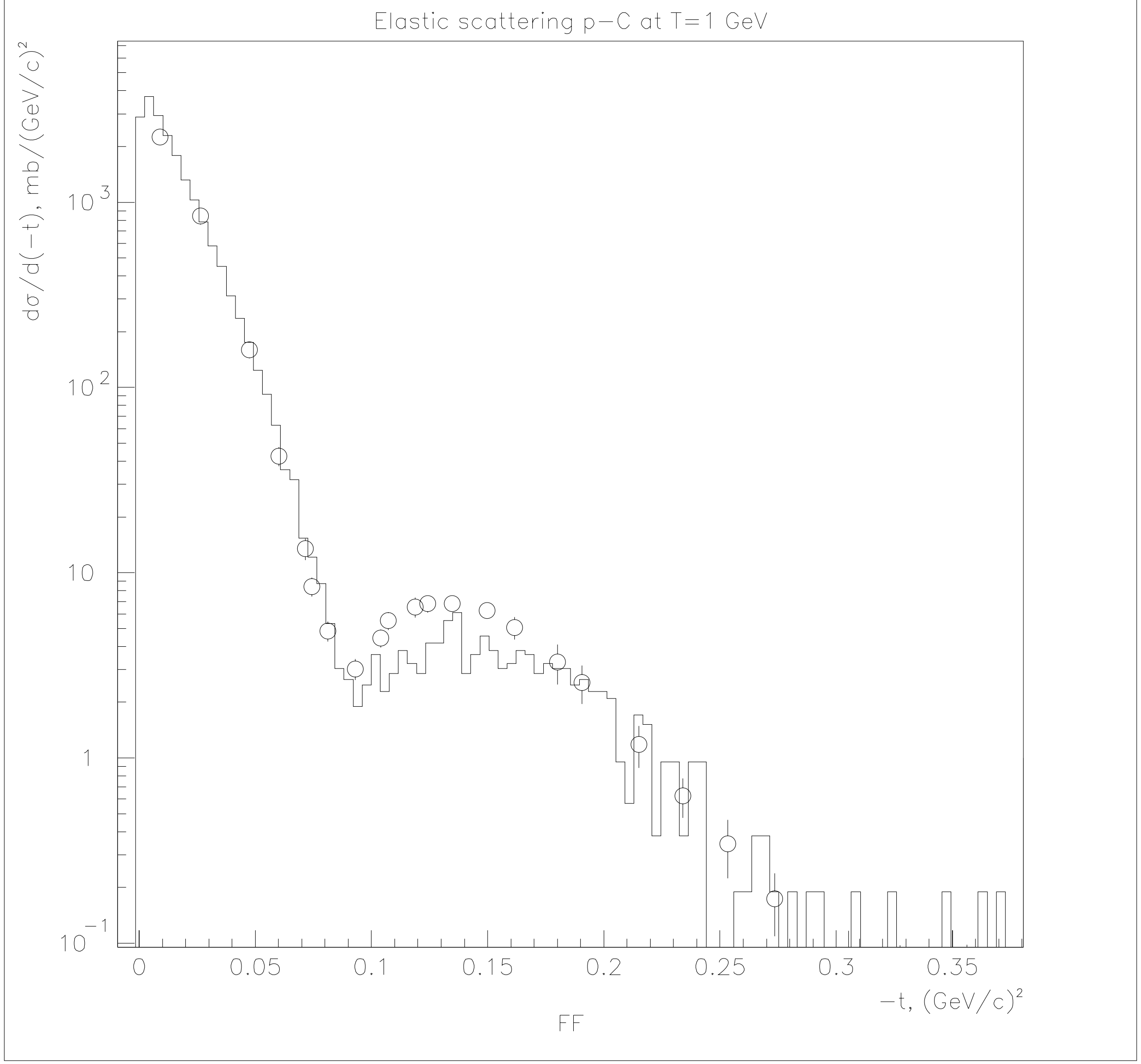
3. The nuclear parameters were adjusted so as σ_{tot}, σ_{el} and $(d\sigma/dq^2)_{el}$ are described simultaneously with enough accuracy. This allows to normalize events to absolute values.

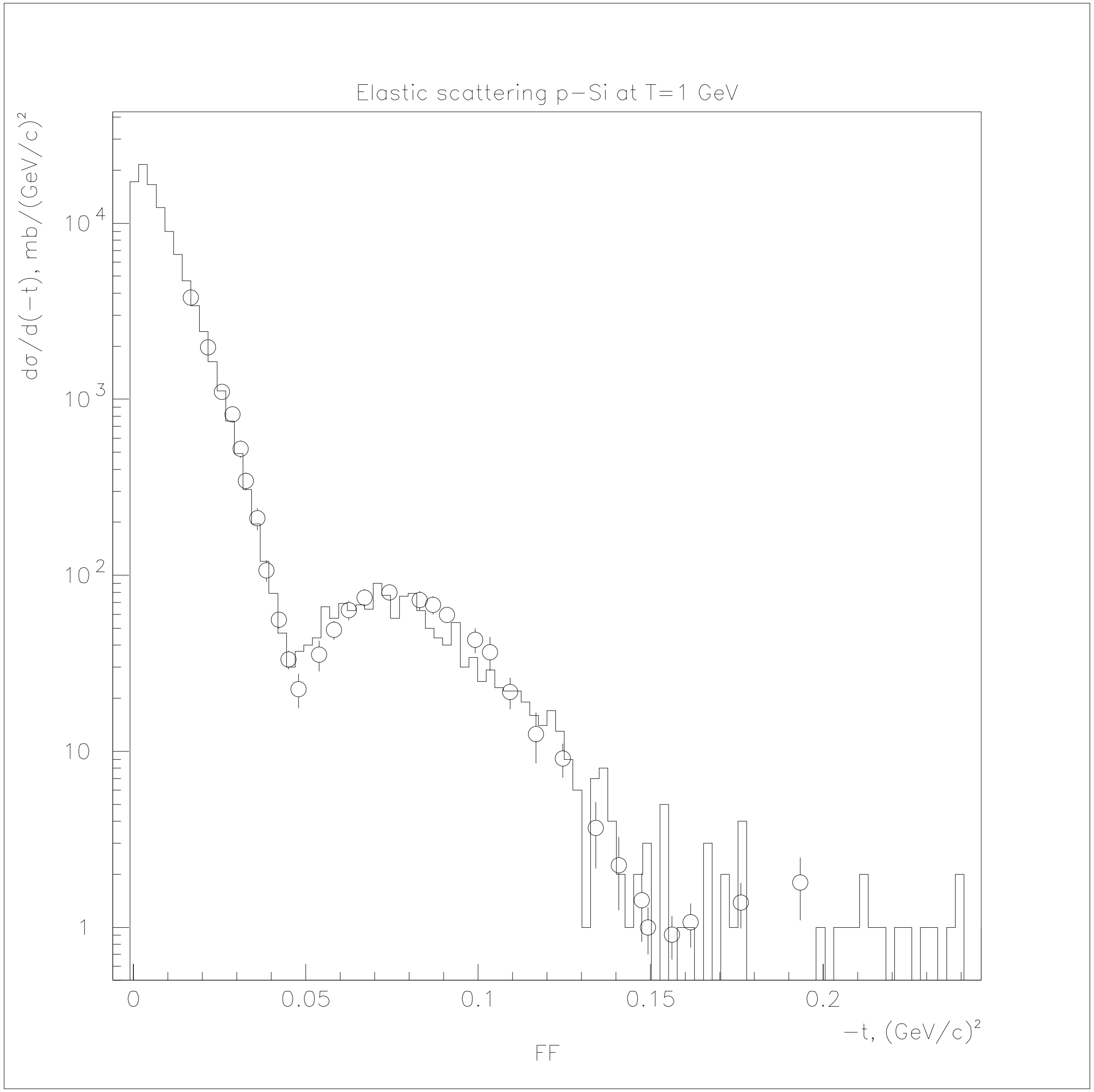

Elastic scattering p−He at T=1 GeV
dσ/d(−t), mb/(GeV/c)²
10
1
10⁻¹
10⁻²
10⁻³
0
0.2
0.4
0.6
0.8
1
−t, (GeV/c)²
FF

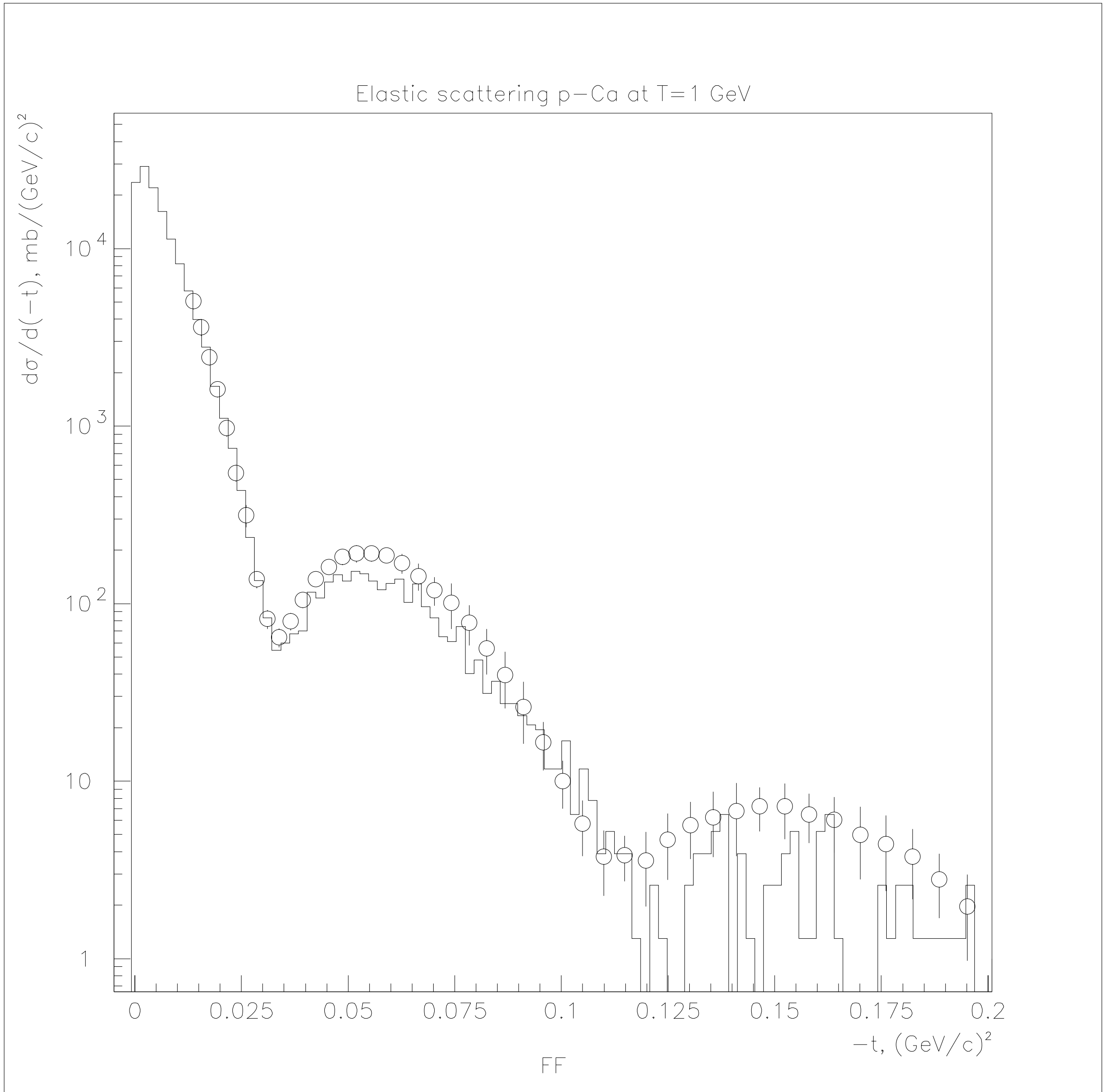

Elastic scattering p−Be at T=1 GeV
dσ/d(−t), mb/(GeV/c)²
10³
10²
10
1
10⁻¹
0
0.05
0.1
0.15
0.2
0.25
0.3
−t, (GeV/c)²
FF

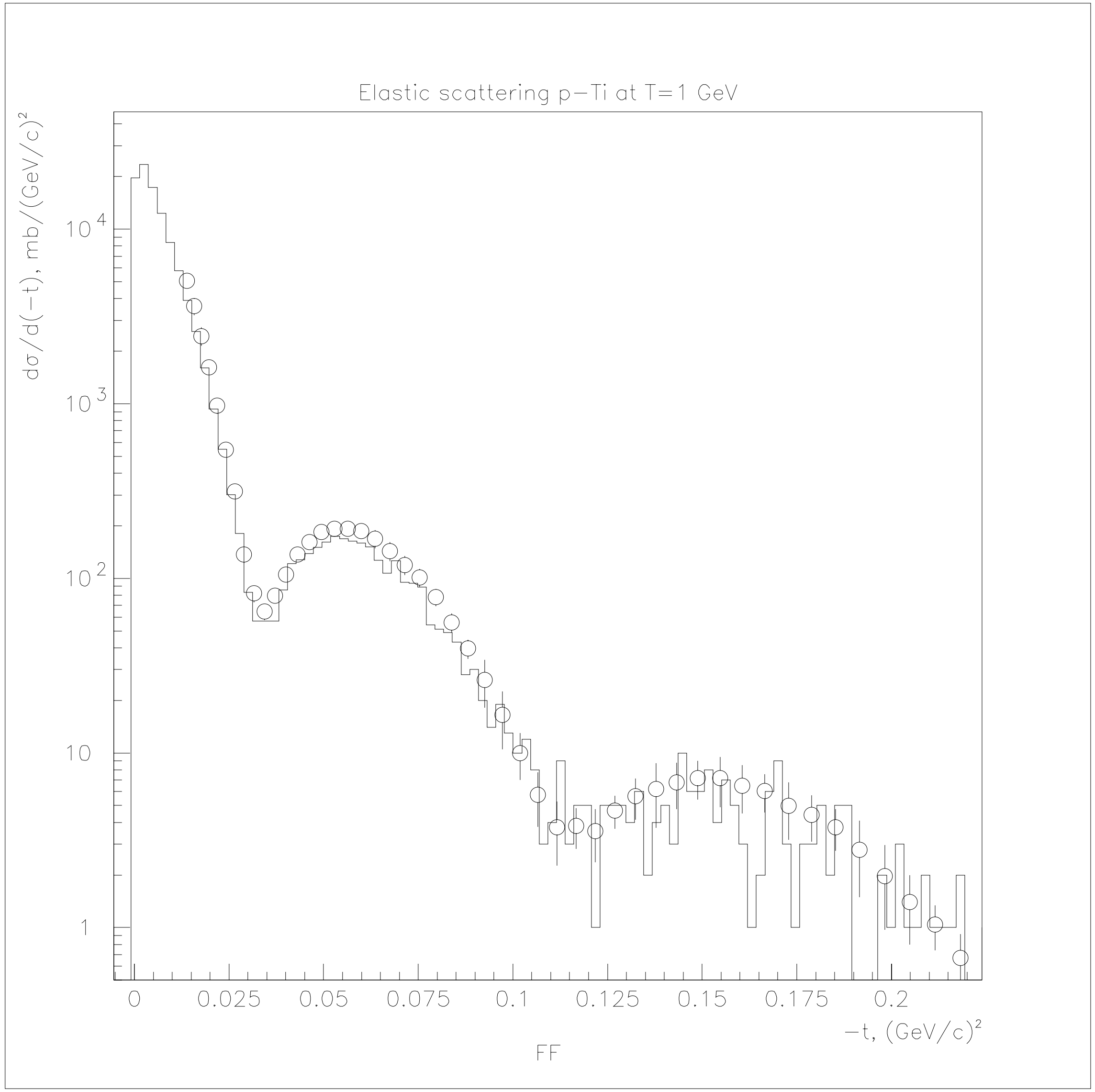


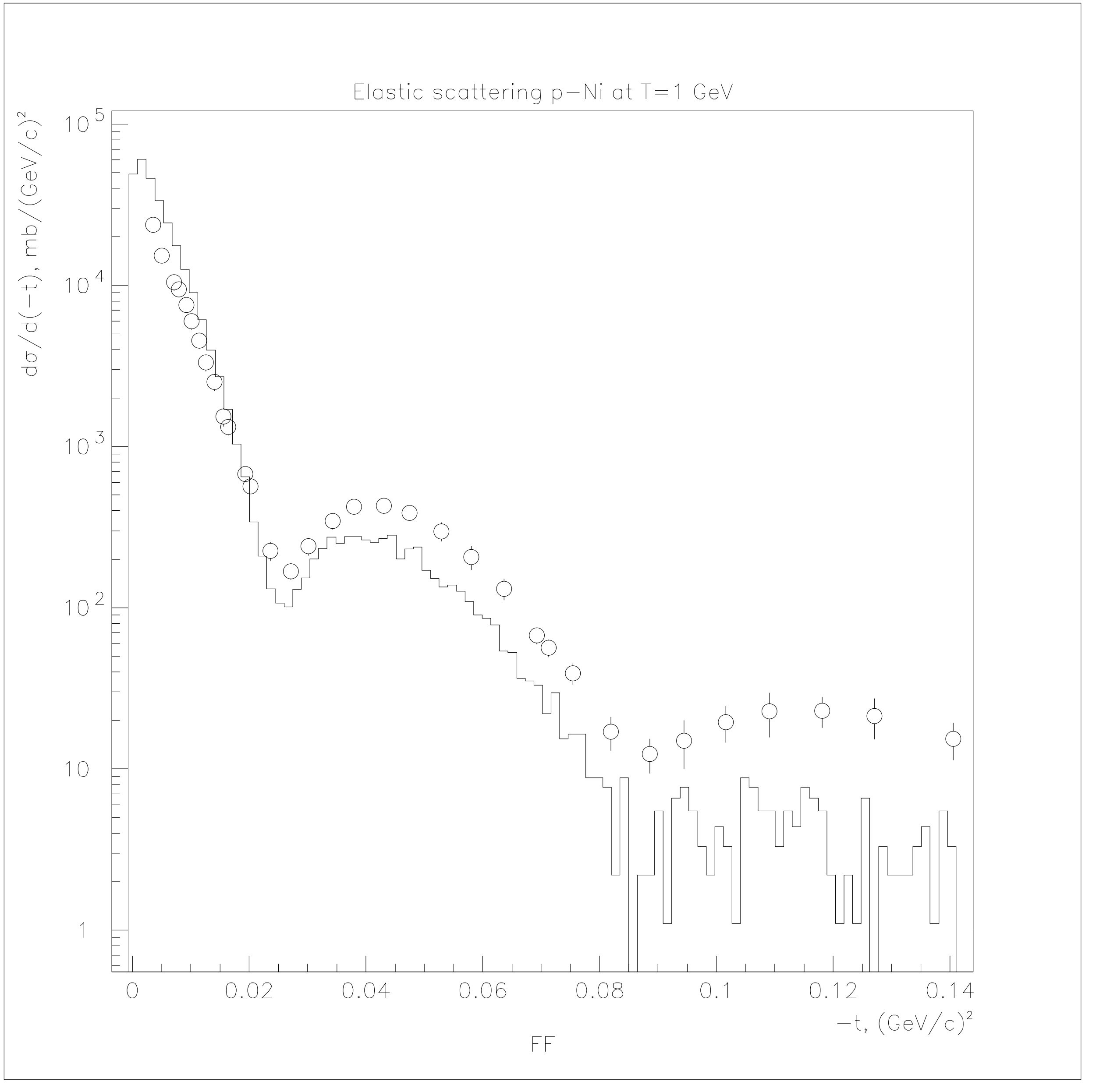
Elastic scattering p−B at T=1 GeV
dσ/d(−t), mb/(GeV/c)²
10³
10²
10
1
10⁻¹
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
−t, (GeV/c)²
FF

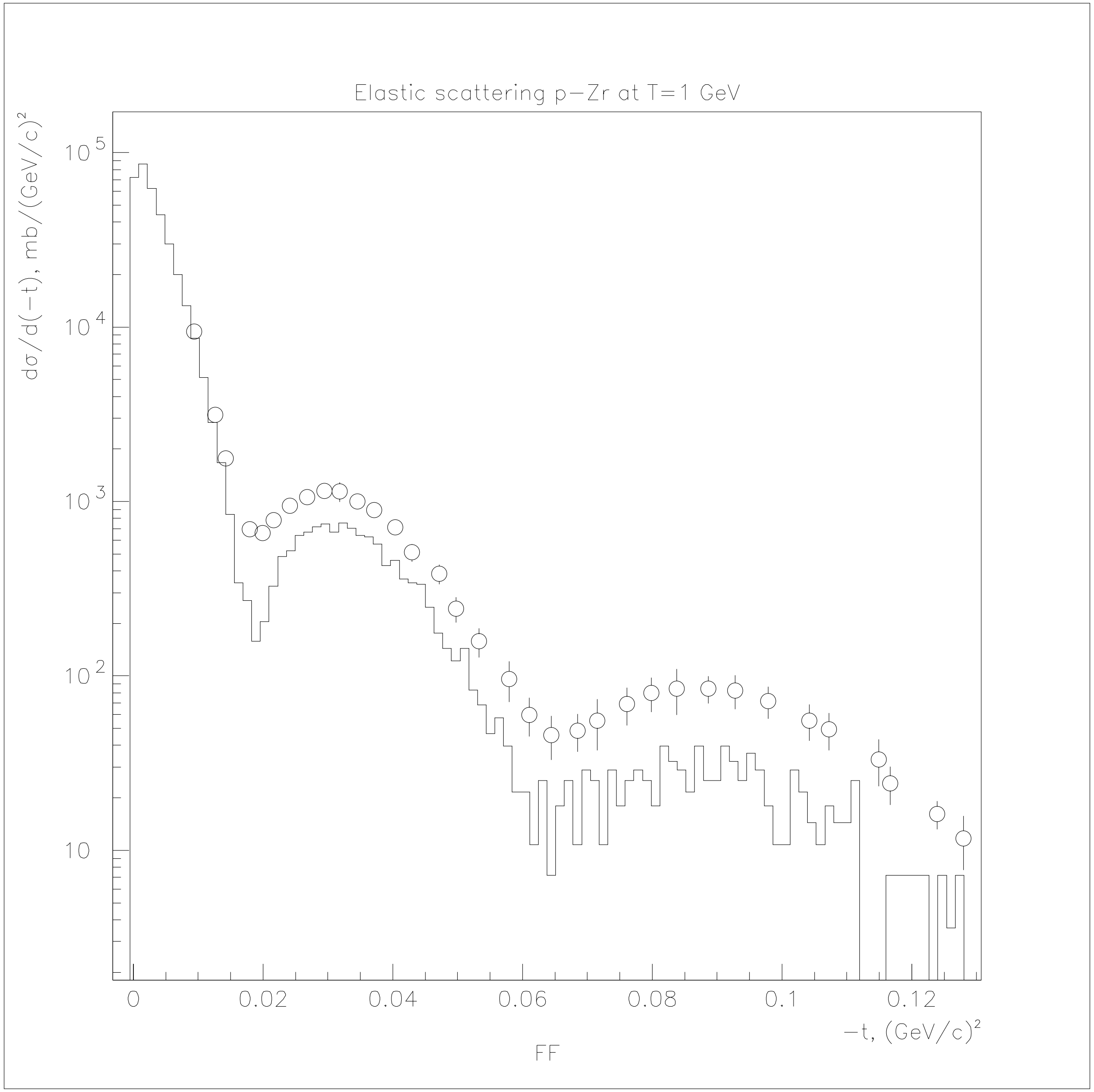

Elastic scattering p–C at T=1 GeV
dσ/d(–t), mb/(GeV/c)²
10³
10²
10
1
10⁻¹
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
–t, (GeV/c)²
FF


Elastic scattering p−Si at T=1 GeV
dσ/d(−t), mb/(GeV/c)²
10⁴
10³
10²
10
1
0
0.05
0.1
0.15
0.2
−t, (GeV/c)²
FF

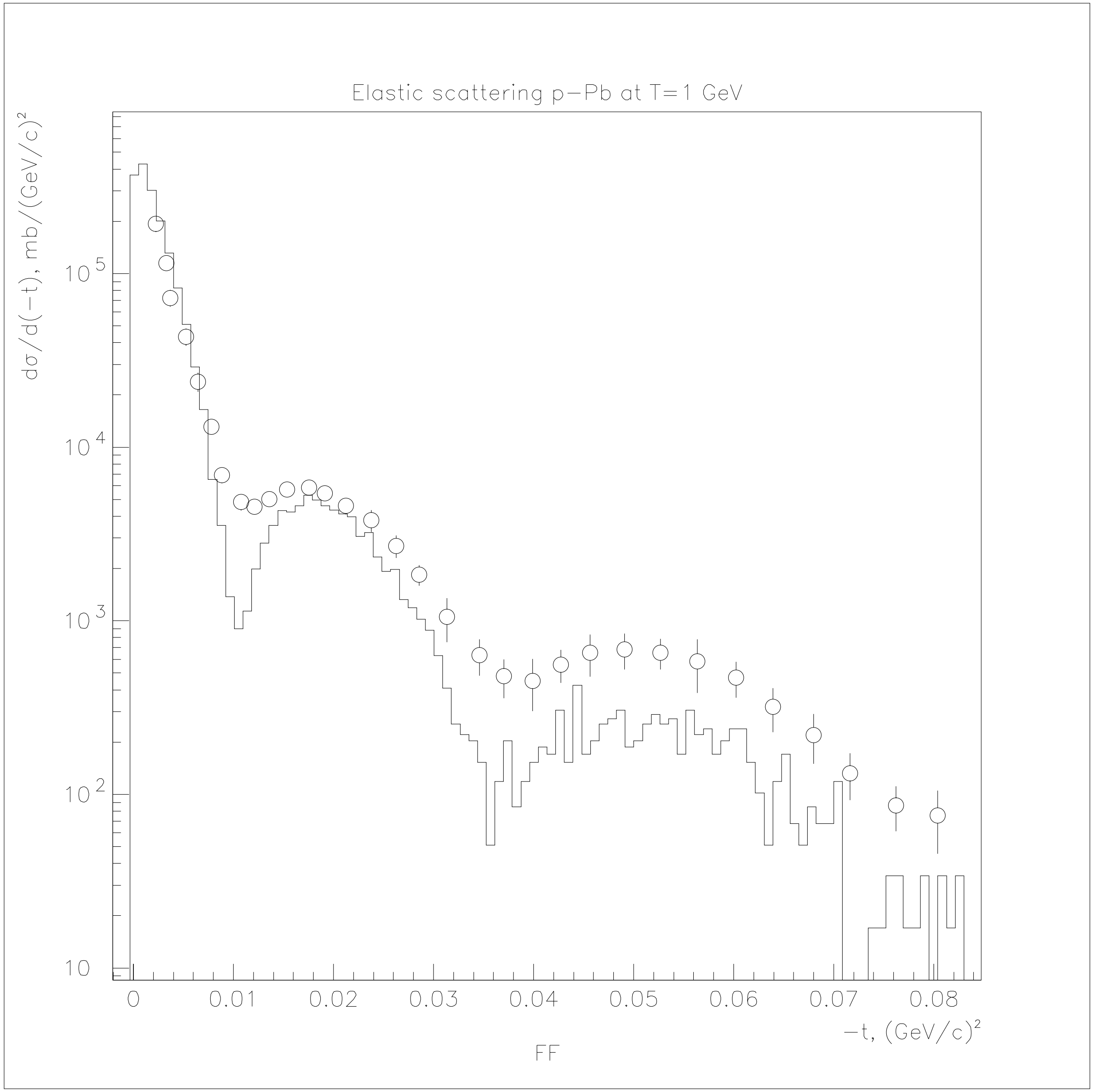

Elastic scattering p−Ca at T=1 GeV
dσ/d(−t), mb/(GeV/c)²
10⁴
10³
10²
10
1
0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
−t, (GeV/c)²
FF

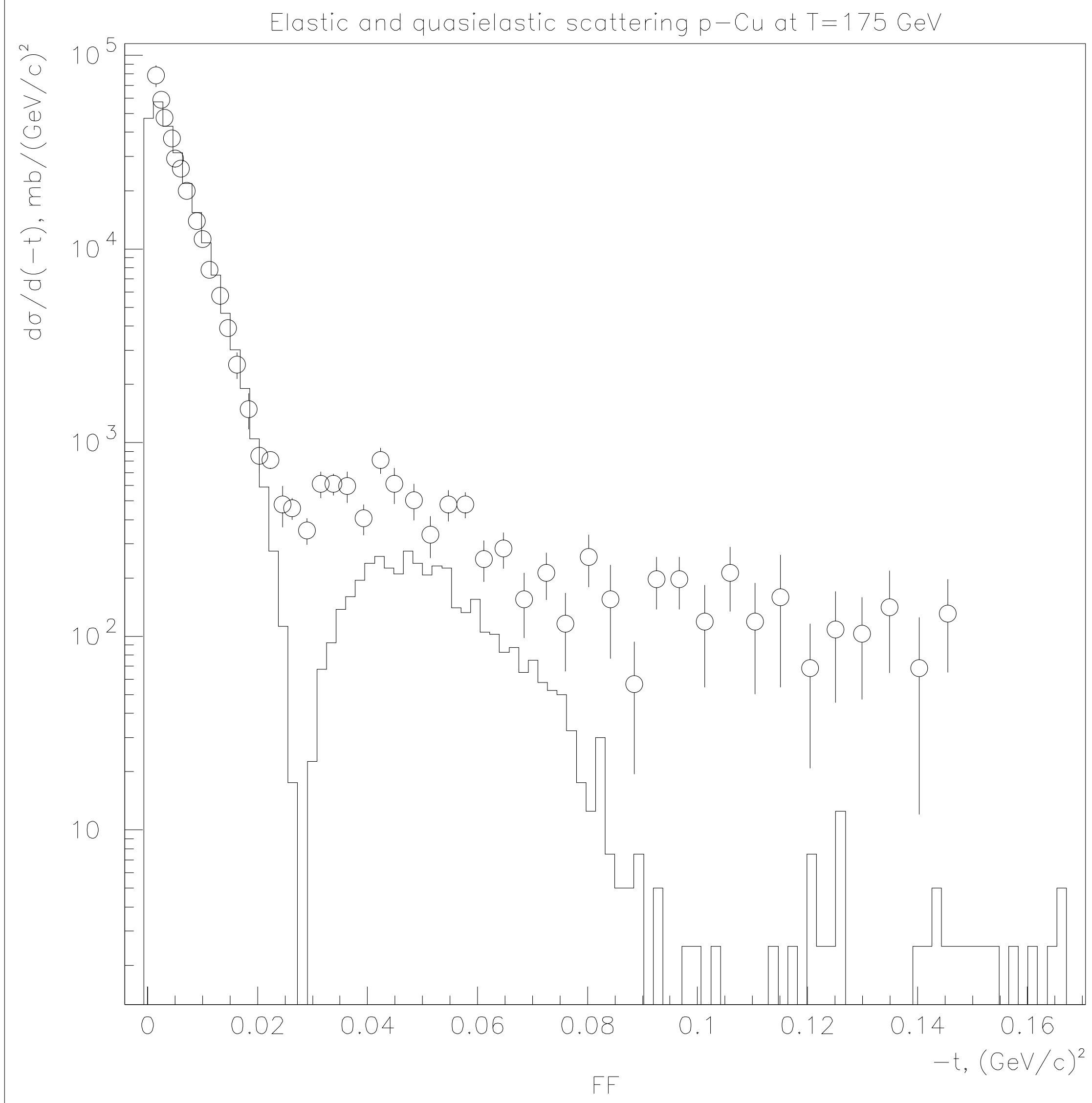

Elastic scattering p−Ti at T=1 GeV
dσ/d(−t), mb/(GeV/c)²
10⁴
10³
10²
10
1
0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
−t, (GeV/c)²
FF


Elastic scattering p−Ni at T=1 GeV
dσ/d(−t), mb/(GeV/c)²
10⁵
10⁴
10³
10²
10
1
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
−t, (GeV/c)²
FF

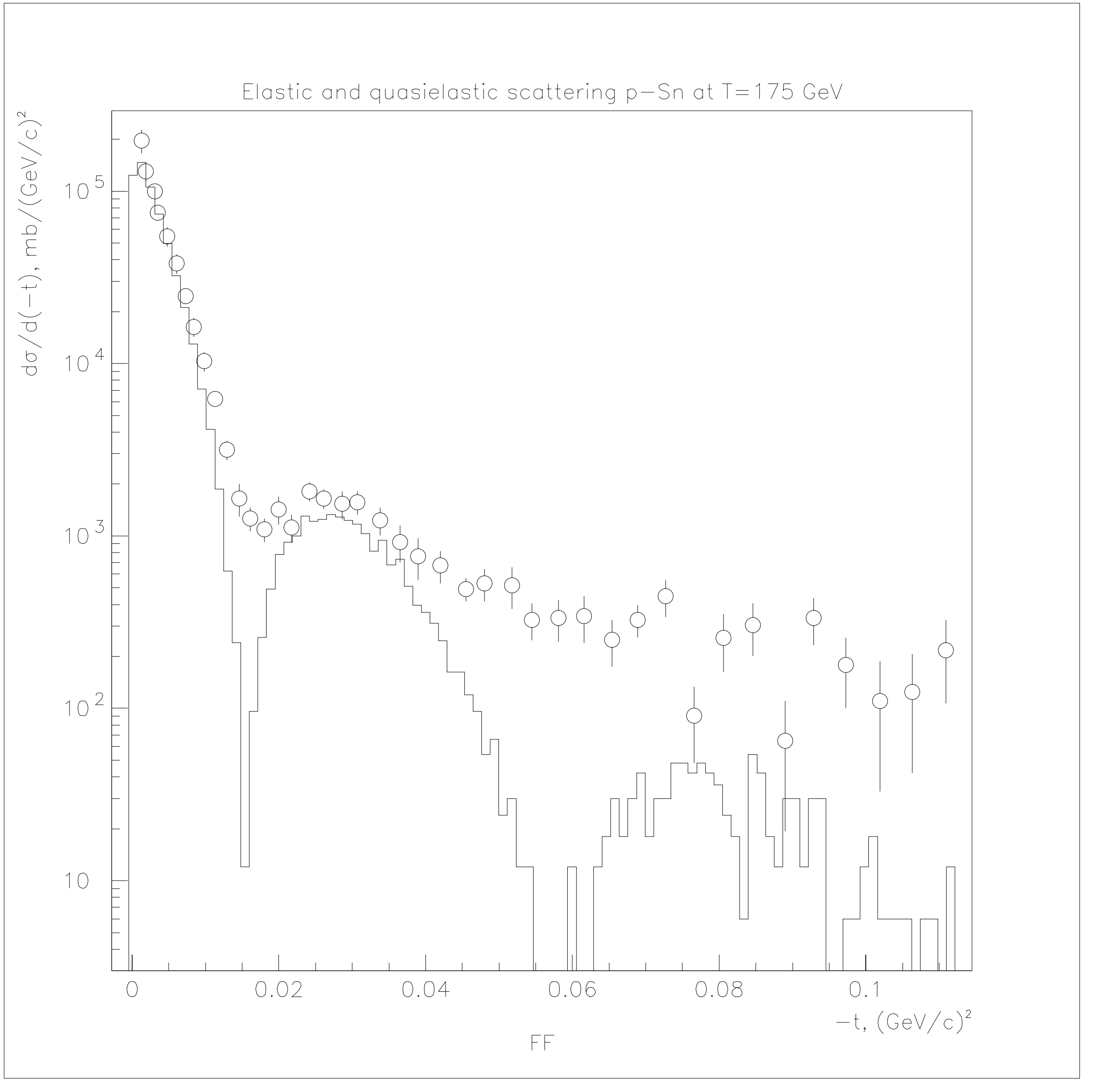


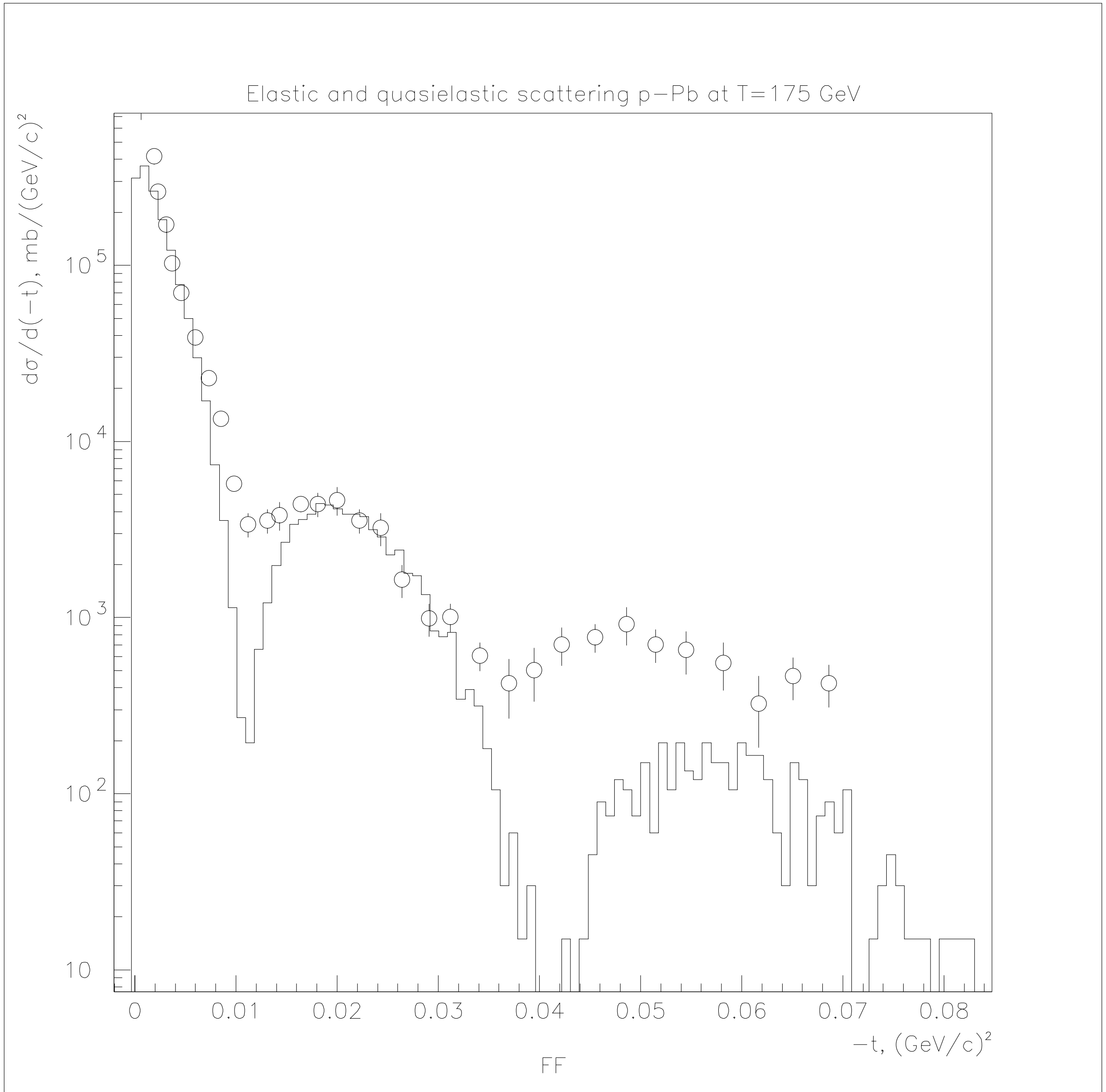
Elastic scattering p−Zr at T=1 GeV
dσ/d(−t), mb/(GeV/c)²
10⁵
10⁴
10³
10²
10
0
0.02
0.04
0.06
0.08
0.1
0.12
−t, (GeV/c)²
FF

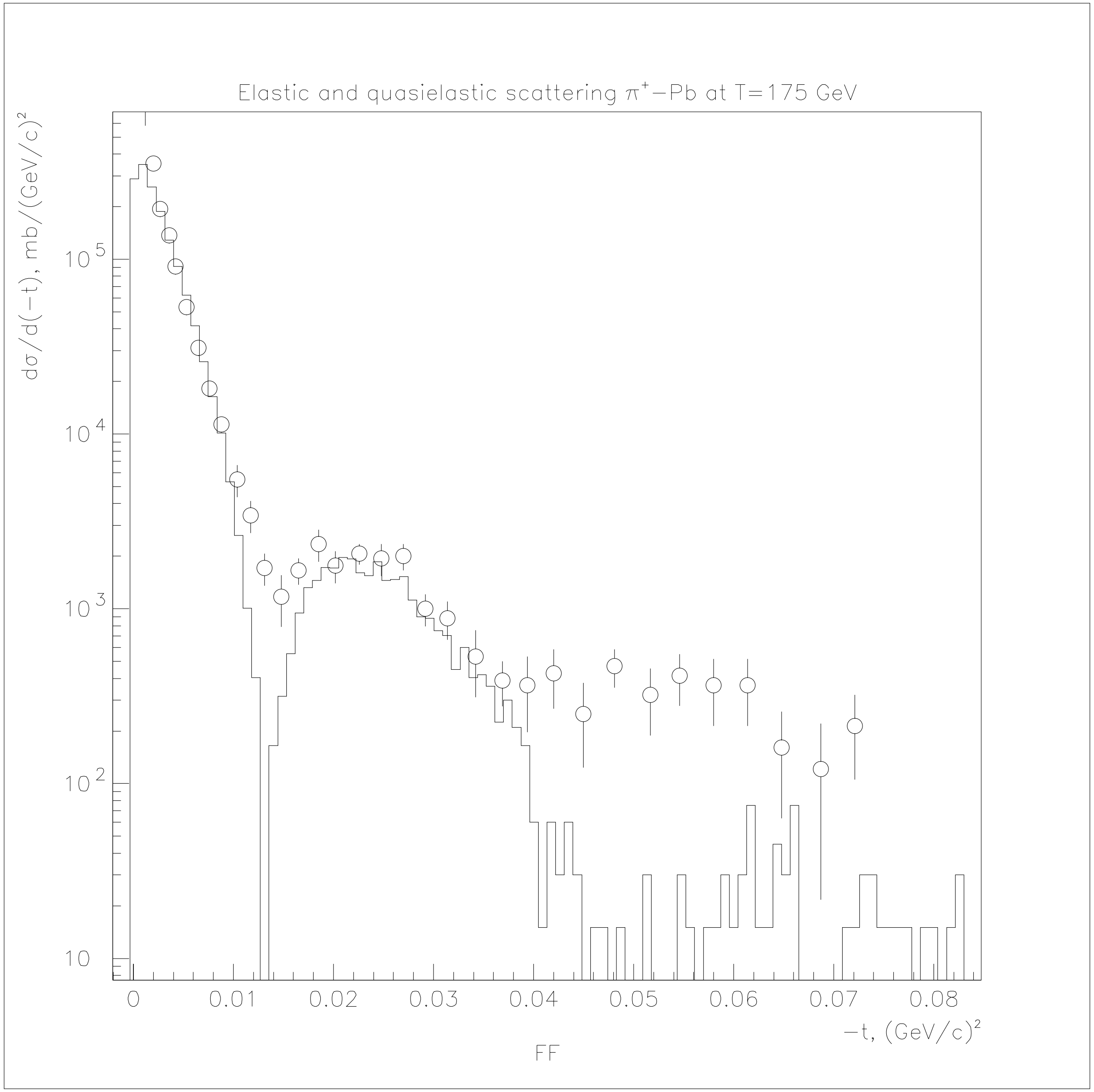

Elastic scattering p−Pb at T=1 GeV
dσ/d(−t), mb/(GeV/c)²
10⁵
10⁴
10³
10²
10
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
−t, (GeV/c)²
FF

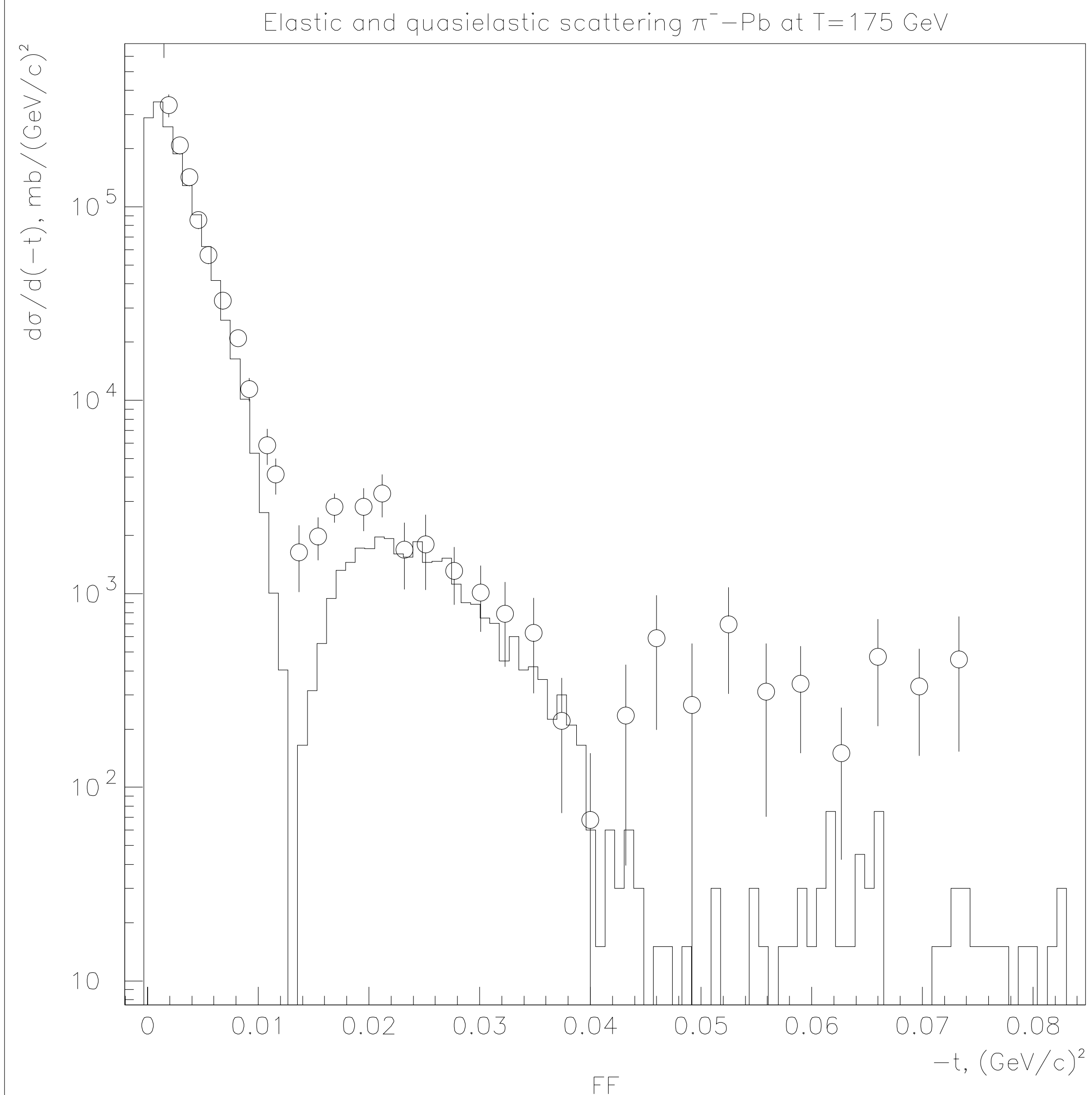


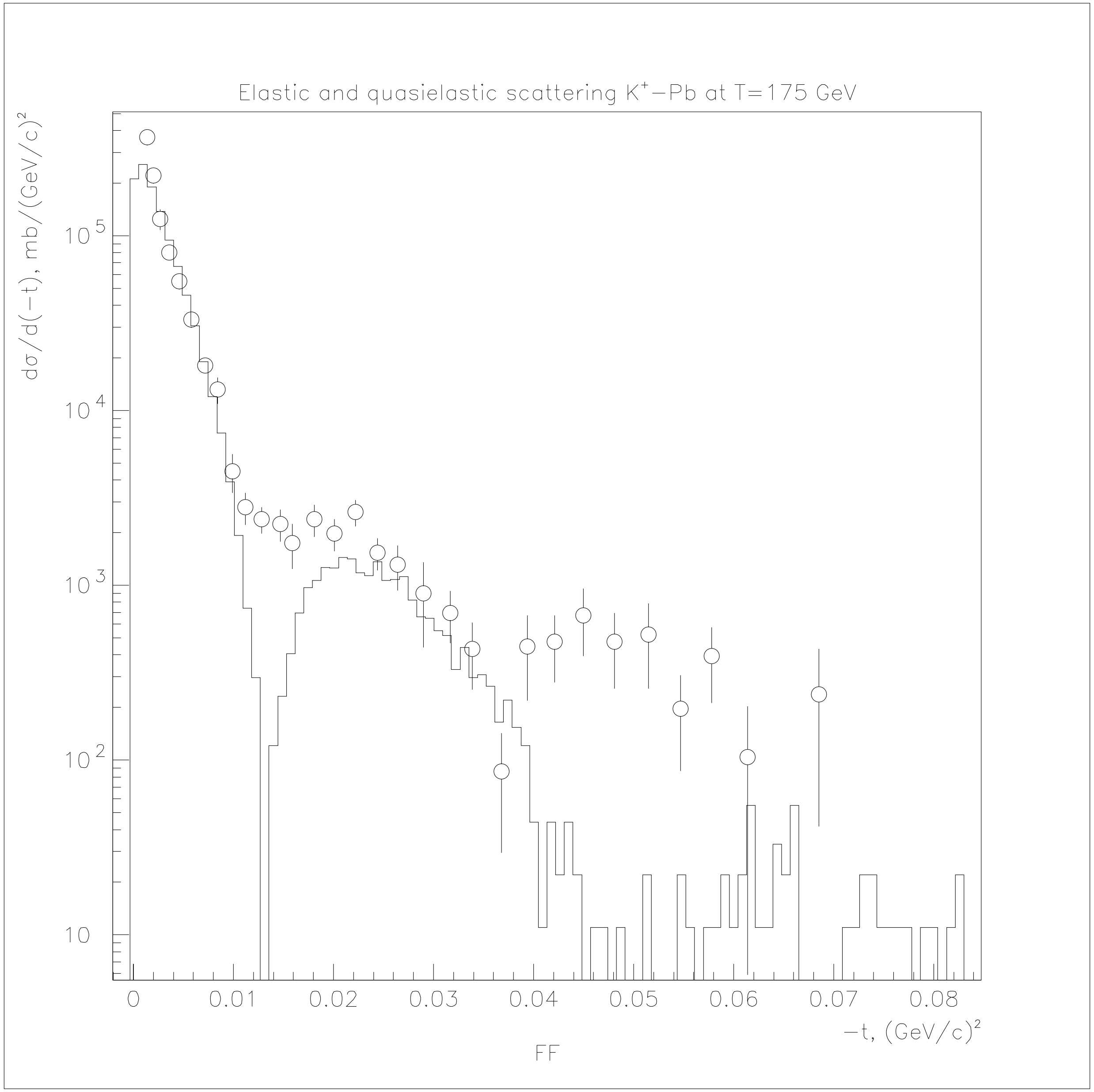
Elastic and quasielastic scattering p−Cu at T=175 GeV
dσ/d(−t), mb/(GeV/c)²
10⁵
10⁴
10³
10²
10
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
−t, (GeV/c)²
FF

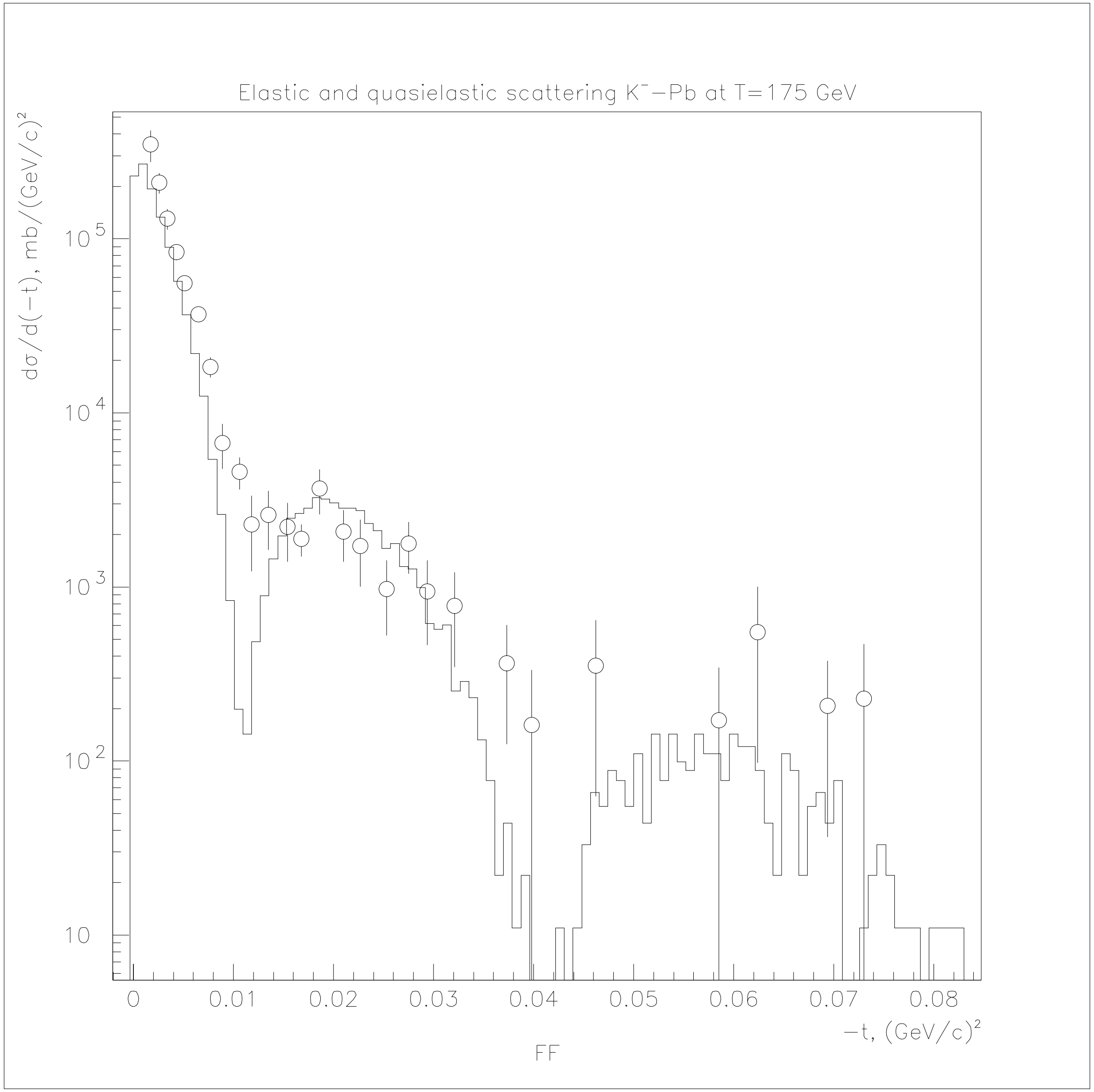

Elastic and quasielastic scattering p−Sn at T=175 GeV
dσ/d(−t), mb/(GeV/c)²
10⁵
10⁴
10³
10²
10
0
0.02
0.04
0.06
0.08
0.1
−t, (GeV/c)²
FF


Elastic and quasielastic scattering p−Pb at T=175 GeV
dσ/d(−t), mb/(GeV/c)²
10⁵
10⁴
10³
10²
10
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
−t, (GeV/c)²
FF

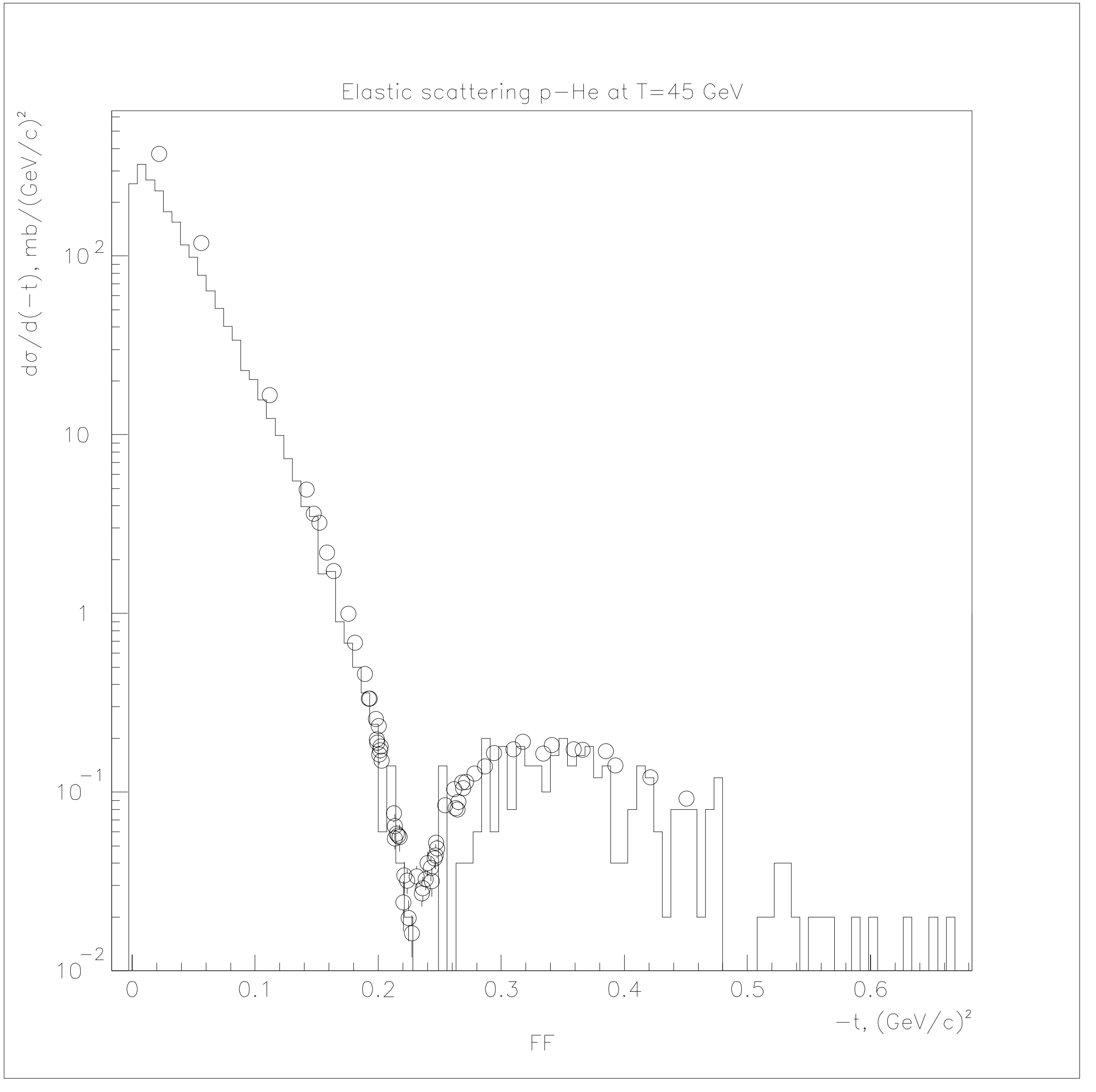

Elastic and quasielastic scattering π+−Pb at T=175 GeV
dσ/d(−t), mb/(GeV/c)²
10⁵
10⁴
10³
10²
10
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
−t, (GeV/c)²
FF

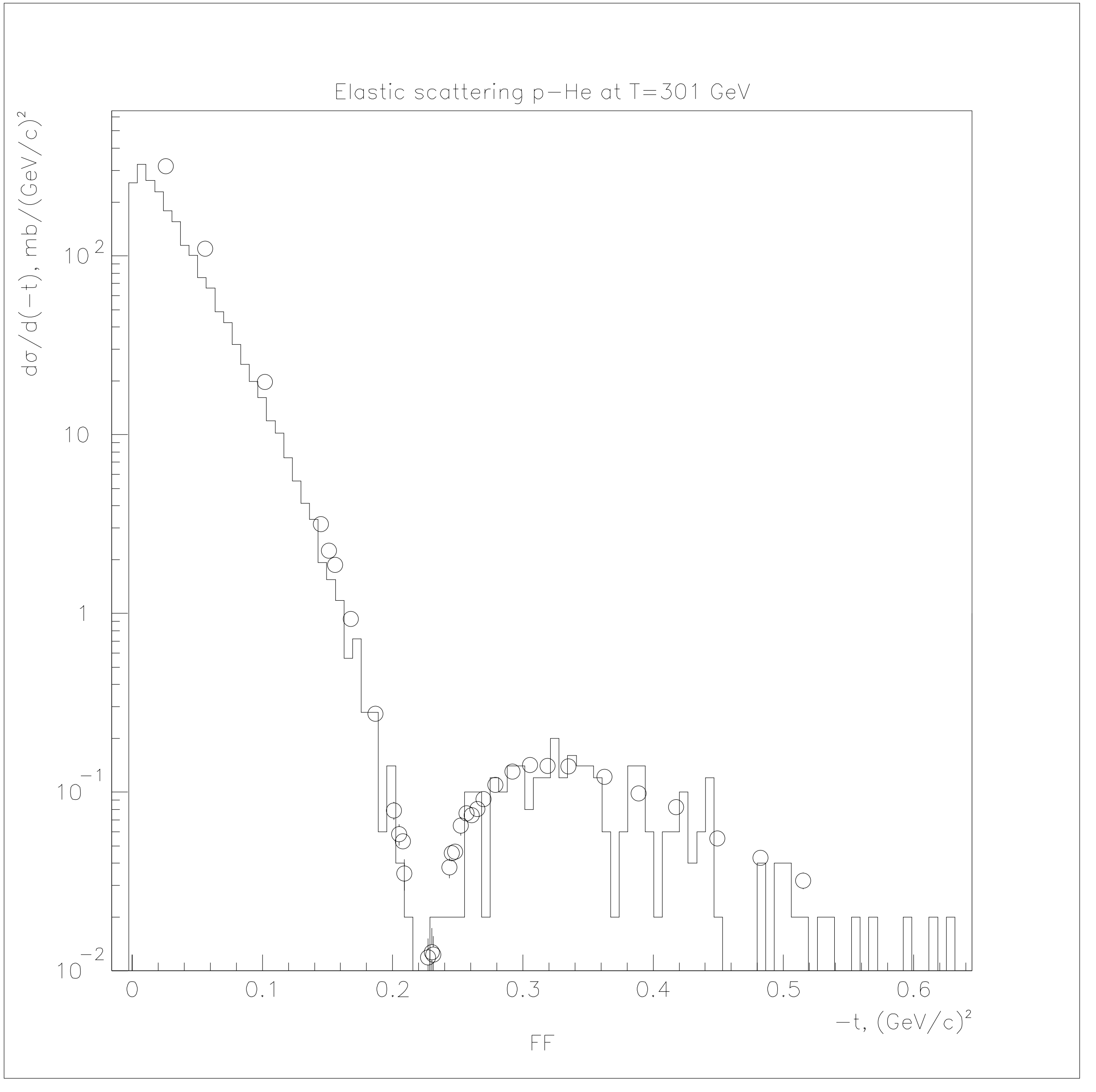

Elastic and quasielastic scattering π^-–Pb at T=175 GeV
dσ/d(−t), mb/(GeV/c)²
10⁵
10⁴
10³
10²
10
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
−t, (GeV/c)²
FF


Elastic and quasielastic scattering K+−Pb at T=175 GeV
dσ/d(−t), mb/(GeV/c)2
10
10 2
10 3
10 4
10 5
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
−t, (GeV/c)2
FF



Elastic and quasielastic scattering K⁻–Pb at T=175 GeV
dσ/d(–t), mb/(GeV/c)²
10⁵
10⁴
10³
10²
10
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
–t, (GeV/c)²
FF


Elastic scattering p−He at T=45 GeV
dσ/d(−t), mb/(GeV/c)²
10²
10
1
10⁻¹
10⁻²
0
0.1
0.2
0.3
0.4
0.5
0.6
−t, (GeV/c)²
FF


Elastic scattering p−He at T=301 GeV
dσ/d(−t), mb/(GeV/c)²
10²
10
1
10⁻¹
10⁻²
0
0.1
0.2
0.3
0.4
0.5
0.6
−t, (GeV/c)²
FF

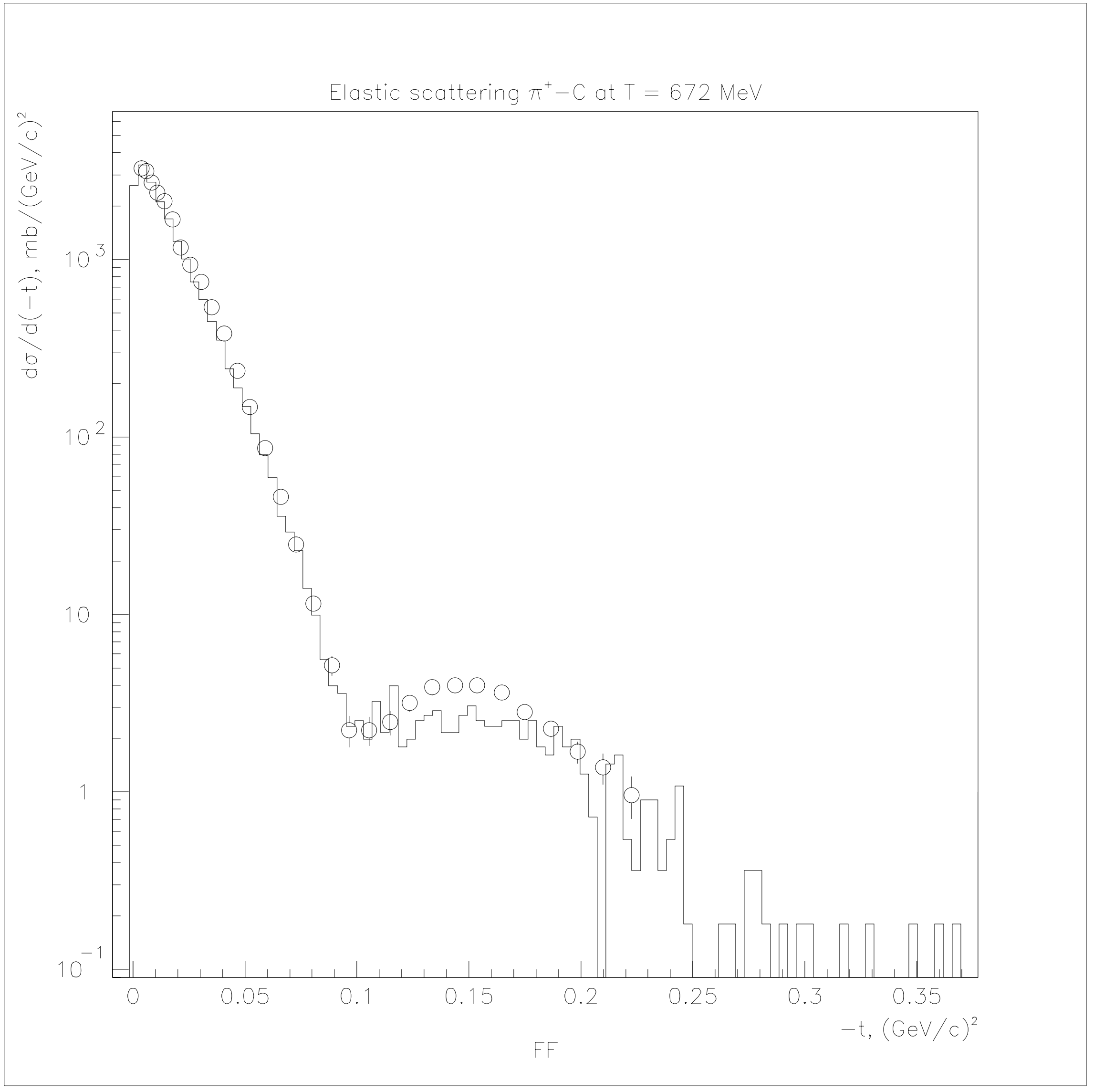
The energies lower 1 GeV.

An optical models are usually used at energy lower then 1 GeV.
They are based on Schredinger, Dirak and Kleine-Gordon equations.

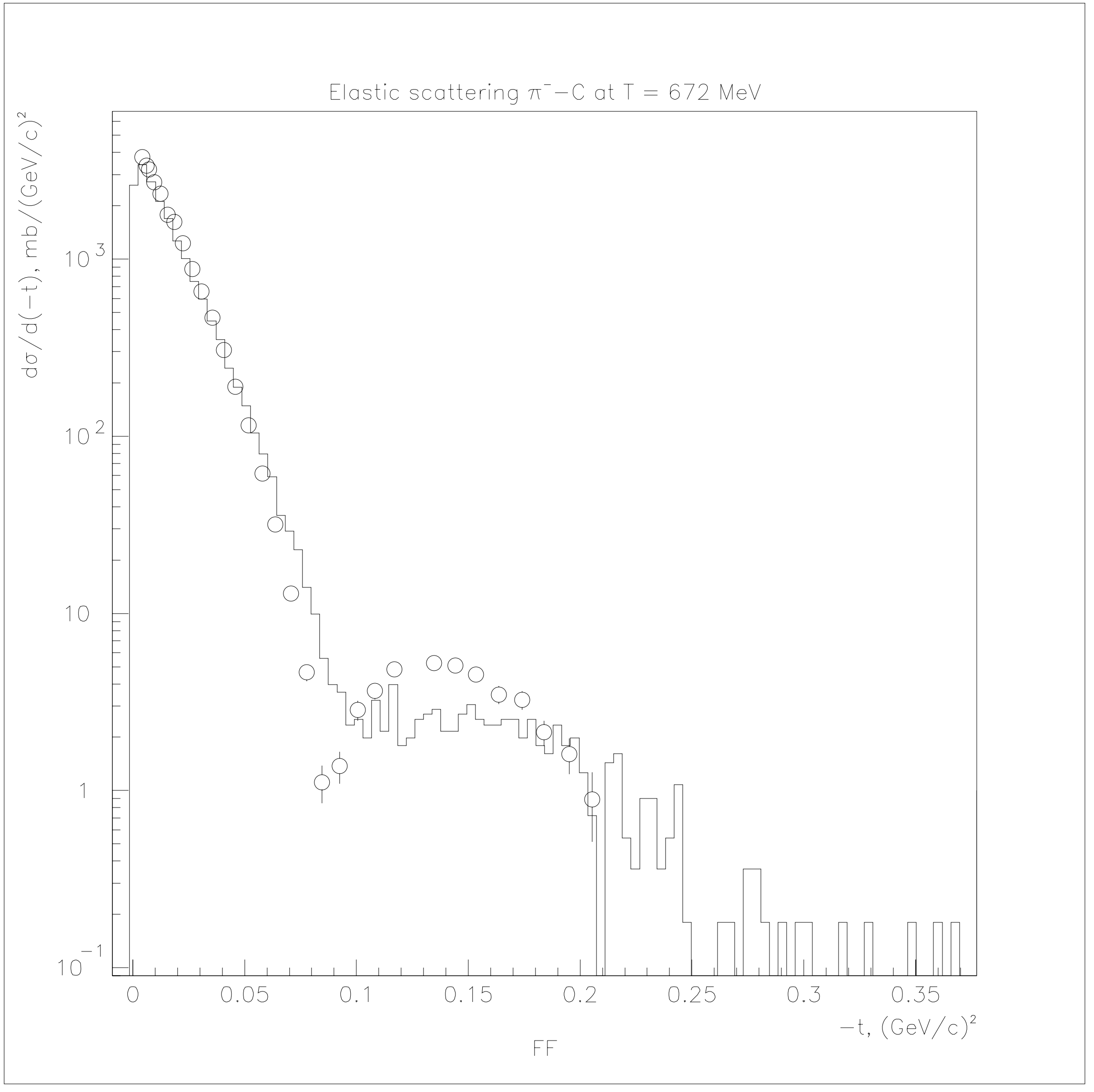
In our works we used the eiconal approximation for solutions of Dirak equations.
(Balashov, Starkov, Sheinov; "Vestnik of Moscow State University", **29**, No 3, 35, 1988)

Nevertheless the Glauber's approach gives satisfactory results when is used at low energy. This accuracy is enough for our purpose I think.

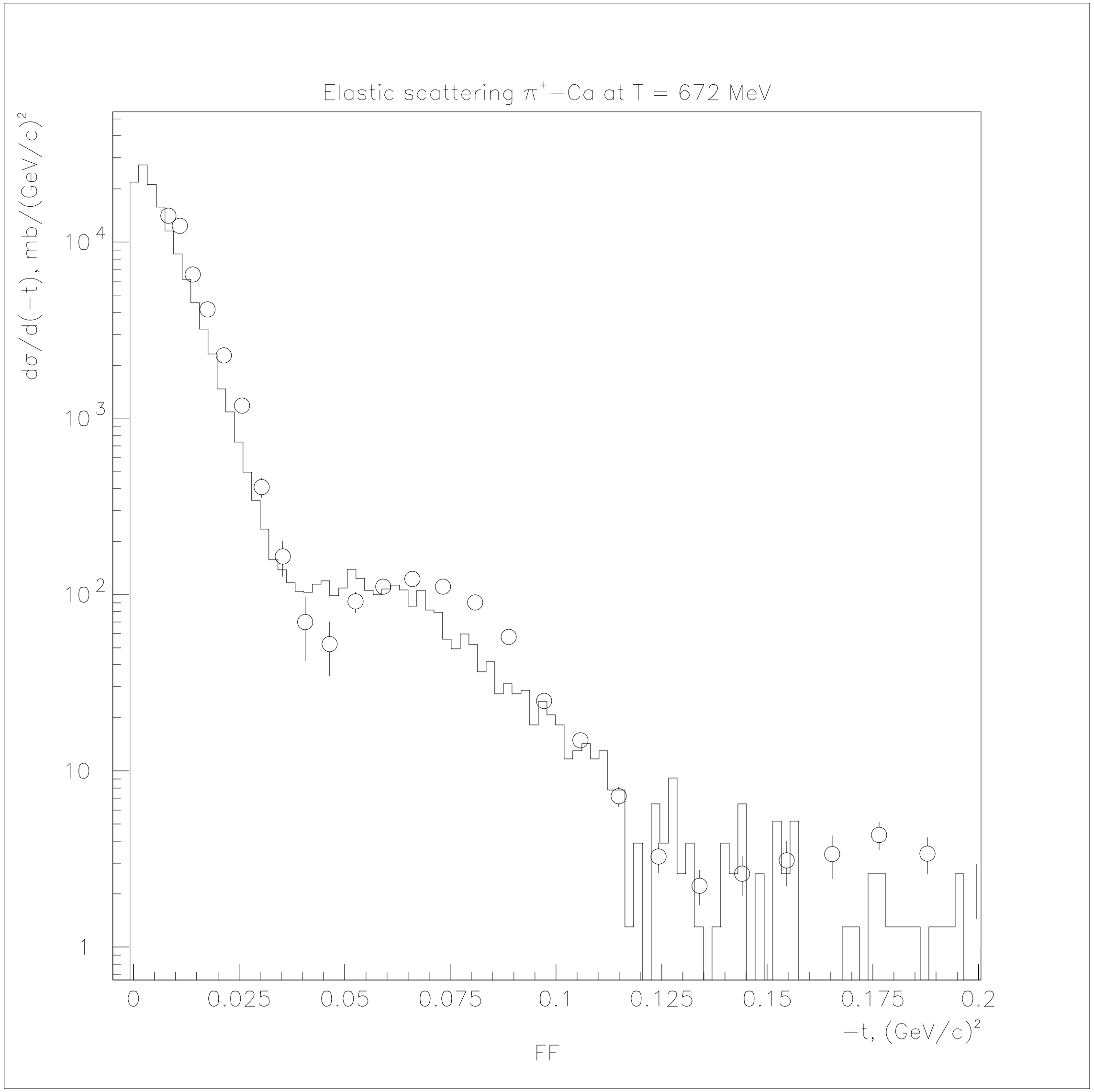
Therefore the low energy boundary of the use of the coherent elastic generator is T = 400 MeV.



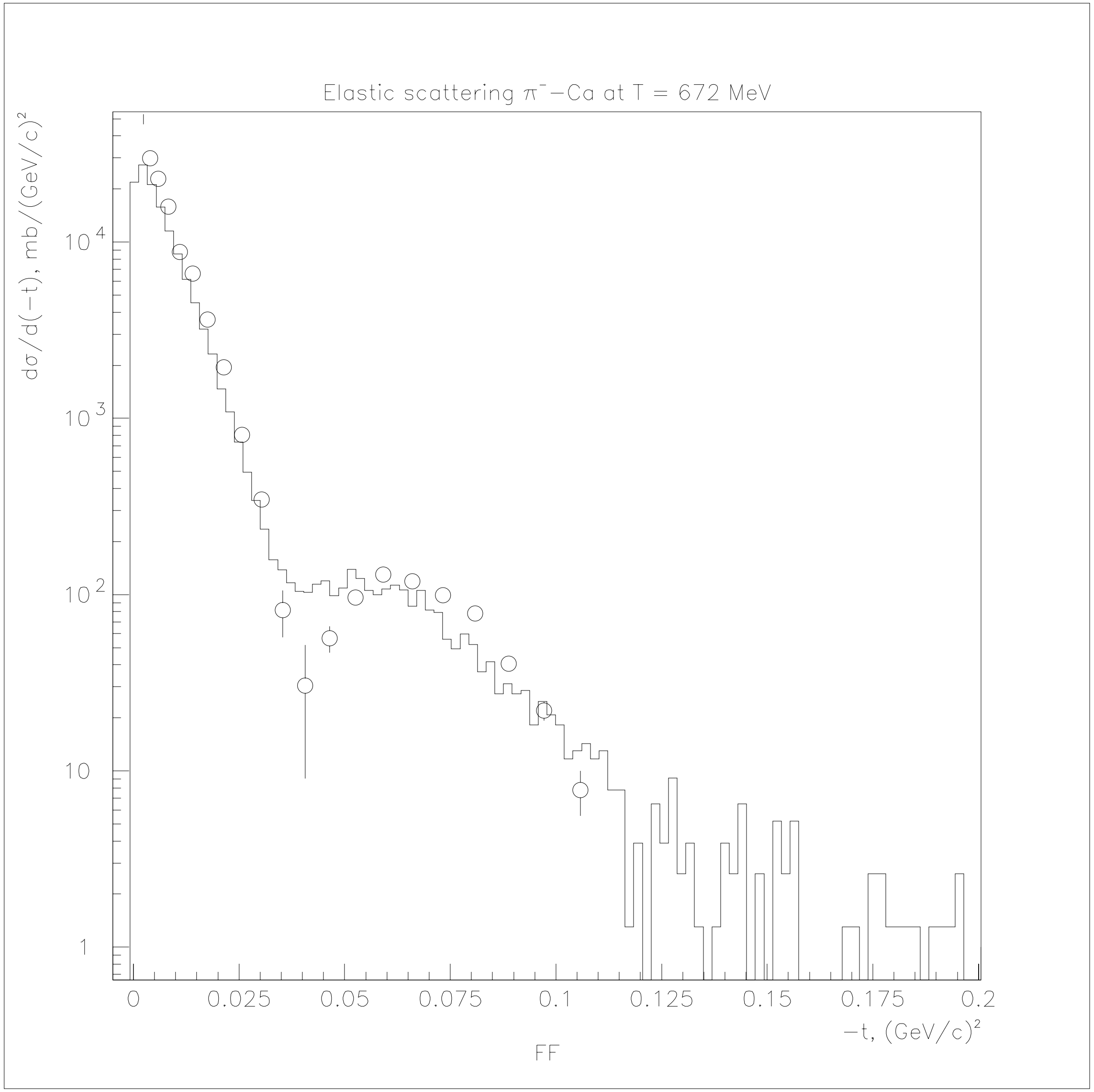
Elastic scattering π^+–C at T = 672 MeV
$d\sigma/d(-t)$, mb/(GeV/c)^2
10^3
10^2
10
1
10^{-1}
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
$-t$, (GeV/c)^2
FF



Elastic scattering π^-–C at T = 672 MeV
$d\sigma/d(-t)$, mb/(GeV/c)^2
10^3
10^2
10
1
10^{-1}
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
$-t$, (GeV/c)^2
FF



Elastic scattering π⁺−Ca at T = 672 MeV
dσ/d(−t), mb/(GeV/c)²
10⁴
10³
10²
10
1
0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
−t, (GeV/c)²
FF


Elastic scattering π⁻−Ca at T = 672 MeV
dσ/d(−t), mb/(GeV/c)²
10⁴
10³
10²
10
1
0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
−t, (GeV/c)²
FF

Conclusions

- The Glauber model shows good description of experimental data
- The model can be extended to low energies
- Additional validation is needed to improve the model performance