9th DITANET Topical Workshop on Non-Invasive Beam Size Measurement for High Brightness Proton and Heavy Ion Accelerators

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CERN

Theory of optical Reconstruction for Synchrotron Light Sources

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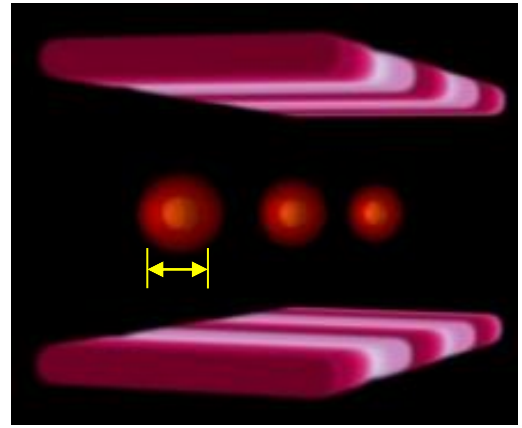
- Introduction
- Resolution in Frame of Geometrical Optics
- Analytical Approach
- Numerical Near Field Calculations

Size Measurements



courtesy:

J. Amundson (FNAL)

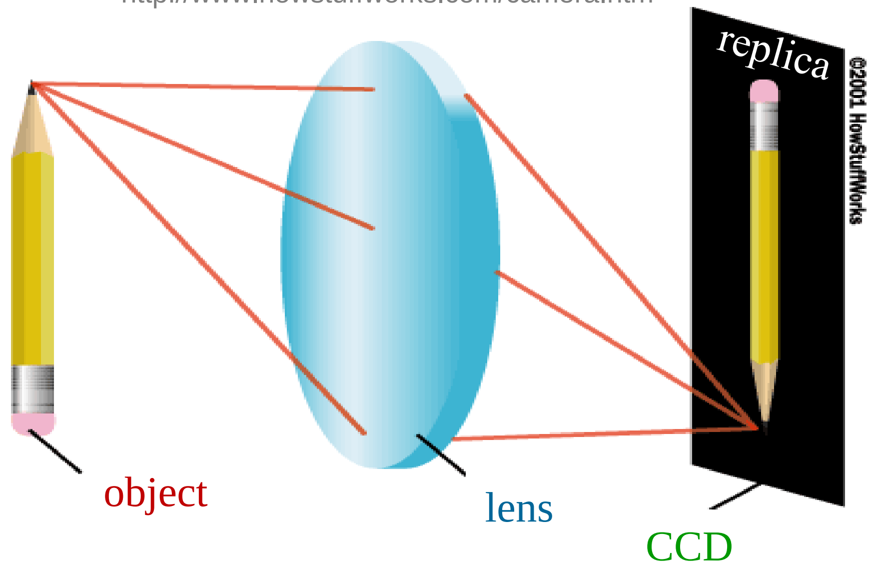
- **task**
  - determination of beam profile
    - → measurement of characteristical size (rms, …)
- **conventional size measurement**
  - take object and measure

- **difficulties**
  - object extremely small
  - object not directly accessible
    - → inside vacuum beam pipe, accelerator environment, …
- **optical imaging**
  - generate replica in comfortable environment
  - adjust replica size (image) to size of measuring device (CCD)

Optical Imaging

- principle

http://www.howstuffworks.com/camera.htm



- resolution

extended object (depth of field)
moving object

→ σ_{dof}

fundamental resolution limit (diffraction)
aberrations, …

→ σ_{dfr}

finite sampling width (pixel size)
cross talk, …

→ σ_{ccd}

› spatial resolution:

$$\boxed{\sigma = \sqrt{\sigma_{dof}^2 + \sigma_{dfr}^2 + \cancel{\sigma_{ccd}^2}}}$$

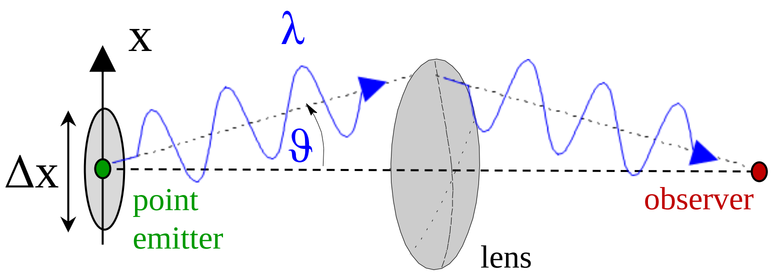
- outline

› this talk: image formation (generate replica onto CCD)

› talk E. Bravin: image registration

Resolution Consideration (1)

- geometrical imaging
  - first order image is a „perfect“ (but scaled) replica → no blurring
- real image blurred
  - by aberrations → e.g. spherical aberrations, astigmatism, coma, … **lens imperfections**
  - by diffraction → wave nature of light **fundamental**
- neglect aberrations
  - **diffraction limited** systems → high quality, aberration-free systems
- fundamental resolution limit
  - point observer detecting photons from point emitter → location of emission point ?



Δx

$$\Delta p_x = 2\hbar k \cdot \sin\vartheta \approx 2 \cdot \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda} \cdot \sin\vartheta$$

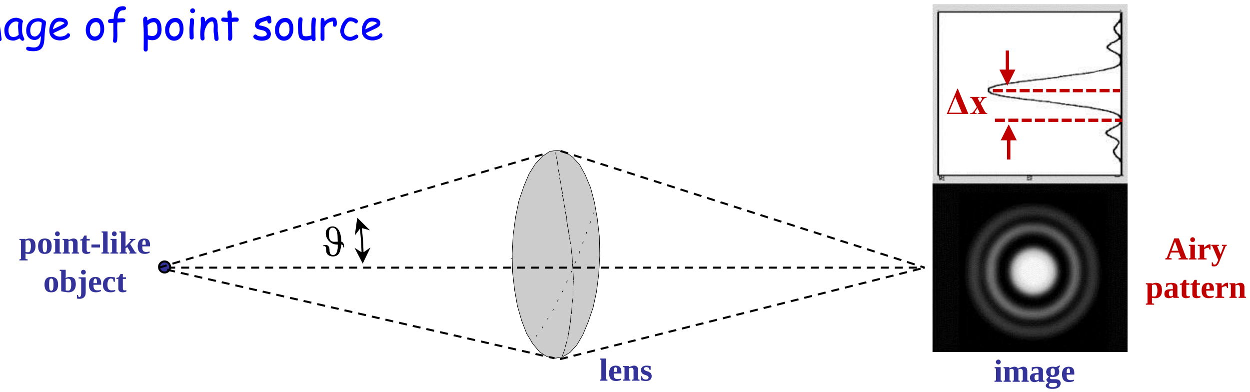
NA = sinϑ: numerical aperture

uncertainty principle: $\Delta x \cdot \Delta p_x \approx h \quad \Rightarrow \quad \boxed{\Delta x \approx \frac{\lambda}{2\sin\vartheta}}$

high resolution: (i) **small λ** (ii) **high NA**

Resolution Consideration (2)

- image of point source



http://www.astro.ljmu.ac.uk

$$\Delta x = 0.61 \frac{M\lambda}{\sin\vartheta}$$

magnification M

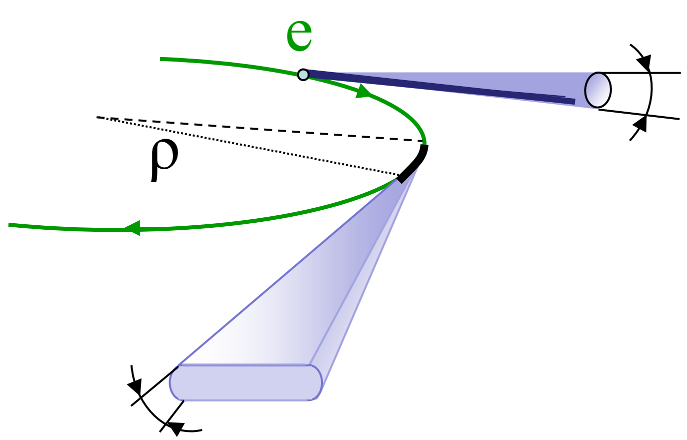
- synchrotron radiation (long bending magnet)
  - strongly collimated emission

    → typical emission angle (out of orbit plane):

    $$\Psi_{SR} \approx 1/\gamma$$

  - emission angle << acceptance of optical system

    → resolution fully dominated by SR emission characteristics



$$\Psi_{SR} \propto 1/\gamma \text{ rad}$$

- trends in modern light sources
  - X-ray imaging

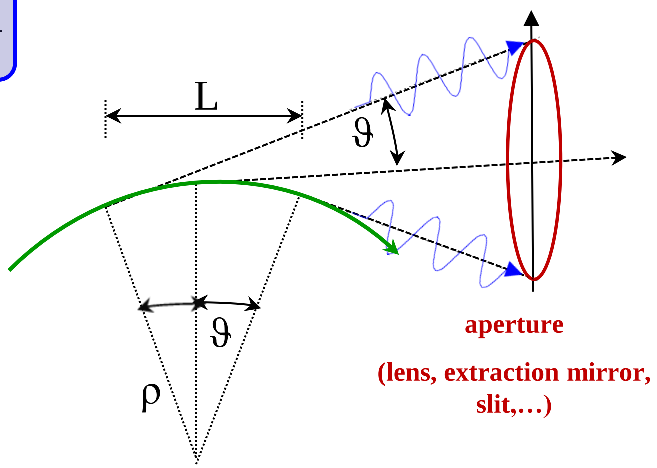
    → PETRA3 @ DESY with E = 6 GeV, $\sigma_v \approx 10$ μm : Δx (500nm) = 145μm

    Δx (20keV) = 0.4μm

Resolution: 1st Level

- diffraction error
  - assumption: plane wave diffraction broadening

$$\boxed{\Delta x_{dfr} \approx 0.61 \frac{\lambda}{\vartheta}}$$



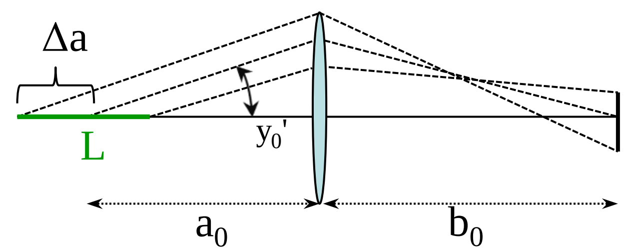
different acceptance angles ϑ for horizontal/vertical plane:

→ horizontal plane: ϑ defined by acceptance of aperture

→ vertical plane: ϑ defined by SR natural opening angle Ψ_{SR}

$$\Psi_{SR} \approx \left(\frac{3\lambda}{4\pi\rho}\right)^{1/3} \quad \text{for} \quad \lambda > \lambda_c \quad \text{(optical SR)}$$

- depth of field
  - radiation from finite part of trajectory: **L**



→ imaging with misalignment Δa:

$a = a_0 + \Delta a$ and $1/f = 1/a_0 + 1/b_0$

→ calculation in frame of paraxial optics:

$$\begin{pmatrix} y_1 \\ y_1' \end{pmatrix} = \begin{pmatrix} 1 & b_0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y_0' \end{pmatrix} \Rightarrow y_1 = -\left(\frac{b_0}{a_0}\right) \cdot \left(y_0 + y_0' \cdot \Delta a\right)$$

magnification & inversion | original size | depth of field

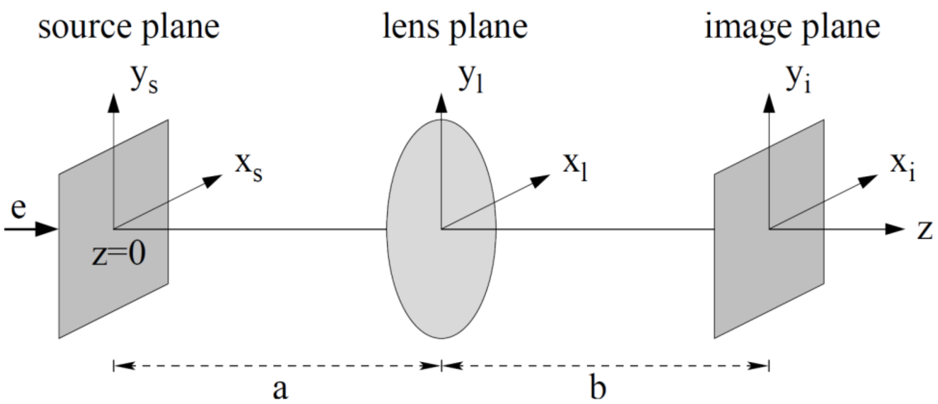
→ misalignment is length of trajectory: $\Delta a = L/2 = \rho\vartheta$

$$\boxed{\Delta x_{dof} = y_0' \cdot \rho\vartheta}$$

horizontal plane: $y_0' = \vartheta$ **vertical plane:** $y_0' = \Psi_{SR}$

Fundamentals of Image Formation

- detailed resolution information
  - requires basic knowledge of image formation
- simple imaging setup



- procedure
  - calculate image of point source (single particle radiation) → **Point Spread Function (PSF)**
  - image of extended object → 2-dim. convolution of source distribution and PSF
  - resolution → difference beween source distribution and image (resp. PSF)
- PSF calculation
  - el. field in source plane (radiation field) → discussed at a later stage
  - field propagation from element to element → in frame of scalar diffraction theory

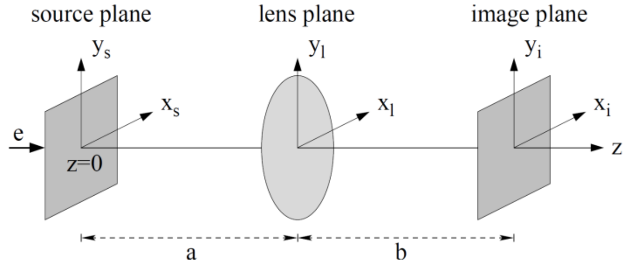
    (i) source plane – lens input (ii) lens input – lens output (iii) lens output – image plane
  - intensity distribution in the image plane

Fundamentals of Image Formation

- source field
  - synchrotron radiation field: **two** different representations

    → form basis of **two different resolution treatments**

    (discussed later)



- propagation
  - scalar diffraction theory

    (here: from source to lense plane)

$$E^{l}_{x_l,y_l}(\vec{r}_l,\omega) = -i\,\frac{e^{ika}}{\lambda a}\cdot e^{i\frac{k}{2a}\left(x_l^{2}+y_l^{2}\right)} \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} dx_s\,dy_s\,E^{s}_{x_s,y_s}(\vec{r}_s,\omega)\cdot e^{i\frac{k}{2a}\left(x_s^{2}+y_s^{2}\right)}\cdot e^{-ik\frac{x_s x_l+y_s y_l}{a}}$$

aperture boundaries

  - far field (Fraunhofer) approximation: $\frac{k}{2}\left(x_s^2+y_s^2\right)_{\max} << a$

$$\Rightarrow\quad E^{m}_{x_l,y_l}(\vec{r}_l,\omega) = -i\,\frac{e^{ika}}{\lambda a}\cdot e^{i\frac{k}{2a}\left(x_l^{2}+y_l^{2}\right)} \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} dx_s\,dy_s\,E^{s}_{x_s,y_s}(\vec{r}_s,\omega)\cdot e^{-i\left(k_x x_s+k_y y_s\right)} \propto \mathcal{F}\left(E^{s}_{x_s,y_s}\right) \quad \left(k_{x,y} = k\,\frac{x_l,y_l}{a}\right)$$

→ basis of **Fourier Optics**

- thin lens approximation
  - quadratic phase shift:

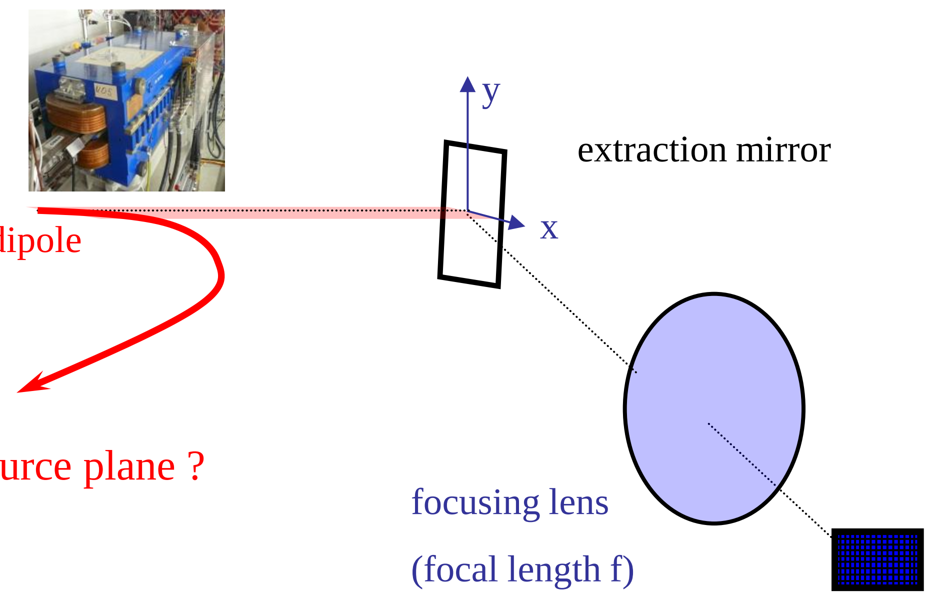
$$E^{l_{out}}_{x_l,y_l}(\vec{r}_l,\omega) = E^{l_{in}}_{x_l,y_l}(\vec{r}_l,\omega)\cdot e^{-i\frac{k}{2f}\left(x_l^{2}+y_l^{2}\right)} \qquad \text{with} \qquad \frac{1}{f}=\frac{1}{a}+\frac{1}{b}$$

- intensity

$$\frac{d^2W}{d\omega\, d\Omega} = \frac{c}{4\pi^2}\left(\left|\vec{E}^{i}_{x_i}(\vec{r}_i,\omega)\right|^2 + \left|\vec{E}^{i}_{y_i}(\vec{r}_i,\omega)\right|^2\right)$$

Profile Monitor Considerations

- point of interest: monitor resolution
  - consider single particle
  - calculate „point spread function“ (PSF) in image plane
  - convolute beam profile with PSF



- tasks
  - radiation source
    - ⟹ SR dipole radiation field ⟹ source plane ?
  - extraction out of vacuum system
    - ⟹ horizontal aperture limitation
  - focusing optics (lens)
    - ⟹ thin lens approximation
  - measurement of spatial intensity distribution
    - ⟹ PSF on CCD detector

Radiation Propagation

- ...in frame of scalar diffraction theory

  - source plane to extraction mirror

$$E^m_{x_m,y_m}(\vec{r}_m,\omega) = -i\frac{e^{ika_1}}{\lambda a_1}\cdot e^{i\frac{k}{2a_1}\left(x_m^2+y_m^2\right)}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx_s dy_s\, E^s_{x_s,y_s}(\vec{r}_s,\omega)\cdot e^{i\frac{k}{2a_1}\left(x_s^2+y_s^2\right)}\cdot e^{-ik\frac{x_s x_m + y_s y_m}{a_1}}$$

  - extraction mirror to lens input

$$E^{l_{in}}_{x_l,y_l}(\vec{r}_l,\omega) = -i\frac{e^{ika_2}}{\lambda a_2}\cdot e^{i\frac{k}{2a_2}\left(x_l^2+y_l^2\right)}\int_{-x_m/2}^{x_m/2} dx_m \int_{-\infty}^{+\infty} dy_m\, E^m_{x_m,y_m}(\vec{r}_m,\omega)\cdot e^{i\frac{k}{2a_2}\left(x_m^2+y_m^2\right)}\cdot e^{-ik\frac{x_m x_l + y_m y_l}{a_2}}$$

  - lens input to lens output (thin lens approximation)

$$E^{l_{out}}_{x_l,y_l}(\vec{r}_l,\omega) = E^{l_{in}}_{x_l,y_l}(\vec{r}_l,\omega)\cdot e^{-i\frac{k}{2f}\left(x_l^2+y_l^2\right)} \qquad \text{with} \qquad \frac{1}{f}=\frac{1}{a}+\frac{1}{b}, \qquad a = a_1 + a_2$$

  - lens output to image plane (CCD detector)

$$E^i_{x_i,y_i}(\vec{r}_i,\omega) = -i\frac{e^{ikb}}{\lambda b}\cdot e^{i\frac{k}{2b}\left(x_i^2+y_i^2\right)}\int_{-x_l/2}^{x_l/2} dx_l \int_{-y_l/2}^{y_l/2} dy_l\, E^{l_{out}}_{x_l,y_l}(\vec{r}_l,\omega)\cdot e^{i\frac{k}{2b}\left(x_l^2+y_l^2\right)}\cdot e^{-ik\frac{x_l x_i + y_l y_i}{b}}$$

- measured quantity

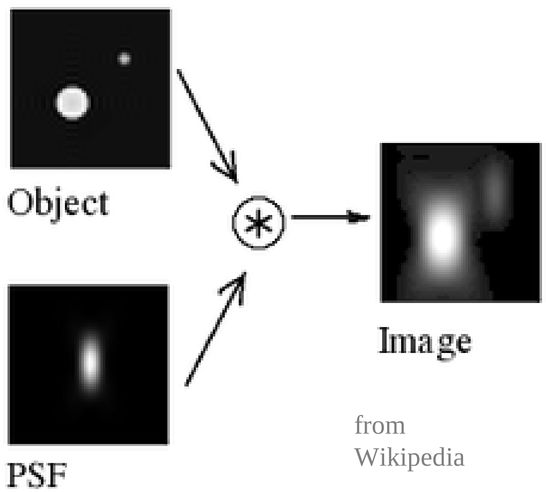
  - spatial intensity distribution

$$\frac{\mathrm{d}^2W}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \frac{\mathrm{c}}{4\pi^2}\left(\left|\vec{E}^i_{x_i}(\vec{r}_i,\omega)\right|^2 + \left|\vec{E}^i_{y_i}(\vec{r}_i,\omega)\right|^2\right)$$

Image Formation: Systems Approach

- image formation

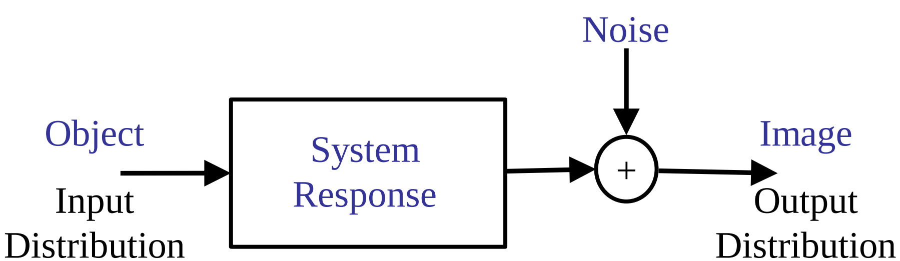
$$\boxed{\text{Image} = \text{PSF} \otimes \text{Object} + \text{Noise}}$$



from Wikipedia

- Point Spread Function (PSF)
  - → image of a point source (single particle)
  - → characteristic of the imaging instrument
  - → deterministic function
- noise
  - → nondeterministic function
  - → described in terms of statistical distributions

- systems approach to imaging (Fourier Optics)

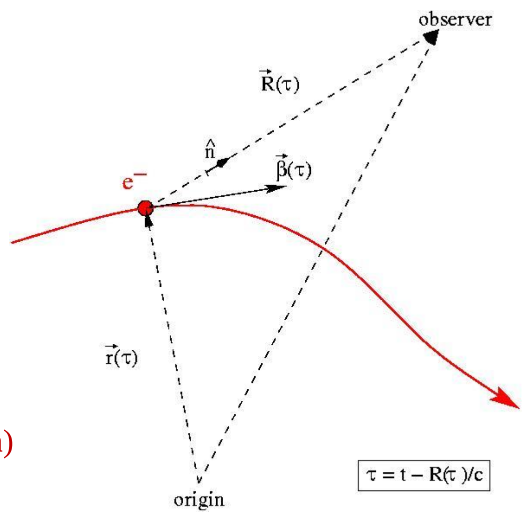


- „standard" signal theory
  - → 1-dim. signals (in time domain)
  - → system analysis with delta pulse
- imaging
  - → 2-dim. signals (in spatial domain)
  - → system analysis with point source response: PSF

Classical Synchrotron Radiation Field

- **source field:** particle field described by **Liénard-Wiechert potentials:**

$$\varphi(t)=\left(\frac{-e}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_{\tau}, \quad \vec{A}(t)=\left(\frac{-e\,\vec{\beta}}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_{\tau}$$



  - field derivation: $E(t)=-\vec{\nabla}\varphi(t)-\frac{1}{c}\dot{\vec{A}}(t), \quad \vec{H}(t)=\vec{\nabla}\times\vec{A}(t)$

$$\Rightarrow \quad \vec{E}(t)=-e\left(\cancel{\frac{\left(1-\beta^2\right)\left(\hat{n}-\vec{\beta}\right)}{R^2\left(1-\hat{n}\cdot\vec{\beta}\right)^3}}+\frac{\hat{n}\times\left[\left(\hat{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right]}{cR\left(1-\hat{n}\cdot\vec{\beta}\right)^3}\right)_{\tau}, \quad \vec{H}(t)=\hat{n}\times\vec{E}(t)$$

neglect velocity term (far field **approximation**)

  - Fourier transform:

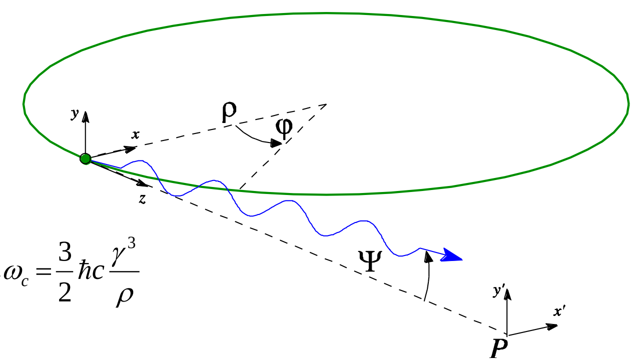
$$\vec{E}(\omega)\approx-\frac{i\omega e}{cR}\int_{-\infty}^{+\infty}\mathrm{d}\tau\left[\hat{\mathrm{n}}\times\left[\hat{\mathrm{n}}\times\vec{\beta}\right]\right]e^{i\omega(\tau+R(\tau)/c)}$$

- **special case:** charged particle moving on circular orbit

$$E_x(\omega)=E_\sigma=A_\sigma\frac{\hbar\omega}{2\hbar\omega_c}\left(1+\gamma^2\Psi^2\right)\cdot\mathrm{K}_{2/3}\left[\frac{\hbar\omega}{2\hbar\omega_c}\left(1+\gamma^2\Psi^2\right)^{3/2}\right]$$

$$E_y(\omega)=E_\pi=A_\pi\frac{\hbar\omega}{2\hbar\omega_c}\gamma\Psi\sqrt{1+\gamma^2\Psi^2}\cdot\mathrm{K}_{1/3}\left[\frac{\hbar\omega}{2\hbar\omega_c}\left(1+\gamma^2\Psi^2\right)^{3/2}\right]$$

with $\hbar\omega_c=\frac{3}{2}\hbar c\frac{\gamma^3}{\rho}$



$\Rightarrow$ analytical field description

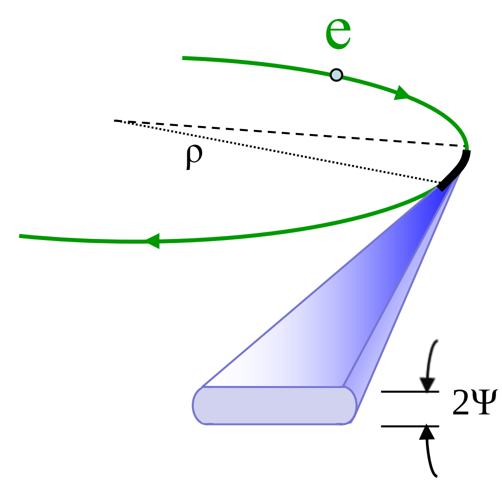
- **comments:** (i) approximative field description $\rightarrow$ far field approximation

  (ii) emission from single point on orbit $\rightarrow$ additional contributions: depth of field, orbit curvature

Resolution: 2nd Level

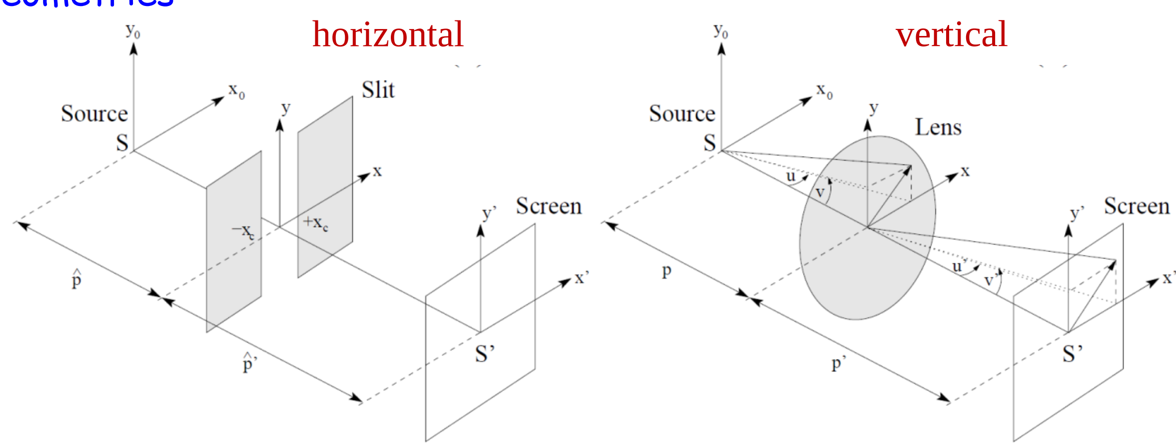
- *imaging based on analytical field description*

A. Hofmann and F. Méot, Nucl. Instr. Meth. **203** (1982) 483



  - horizontal direction
    - → radiation fan: constant intensity distribution
    - → aperture limitation: extraction mirror
    - ➡ Fraunhofer diffraction pattern: plane wave diffraction at slit
  - vertical direction
    - → radiation field: analytical expressions for E_σ, E_π
    - → action of lens: produces magnified image of source distribution
    - ➡ Fraunhofer diffraction pattern: Fourier transform of SR source field

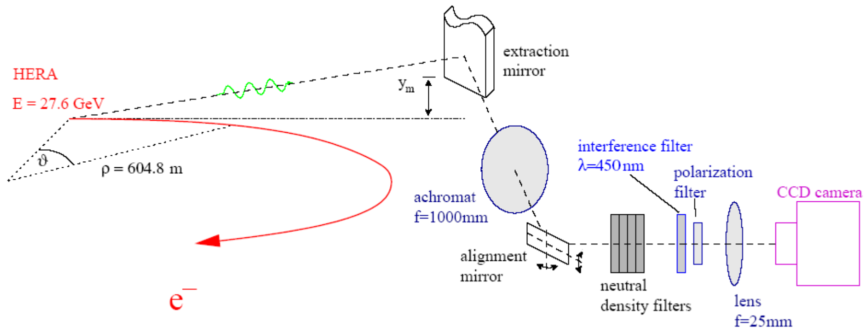
- *simplified geometries*



Diffraction Broadening

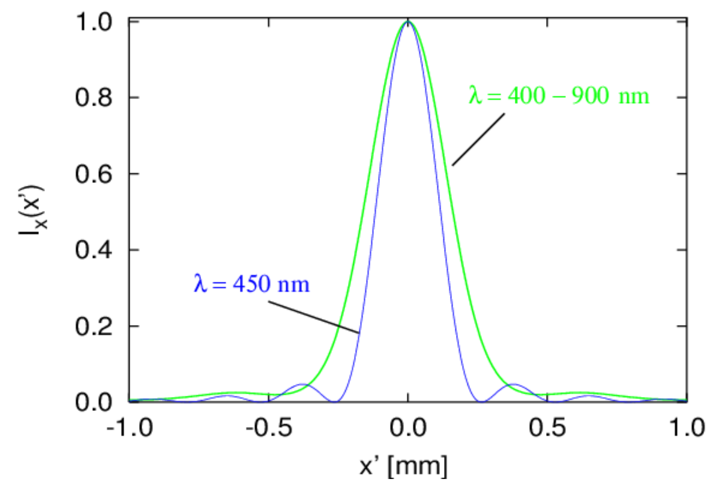
- *example:* HERAe @ DESY

G. Kube et al., Proc. BIW2004, AIP Conf. Proc. **732** (2004) 350

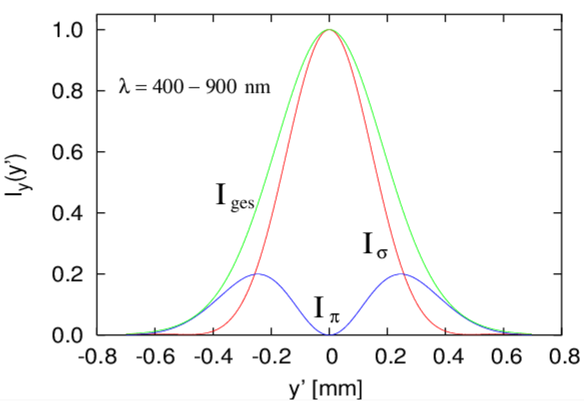


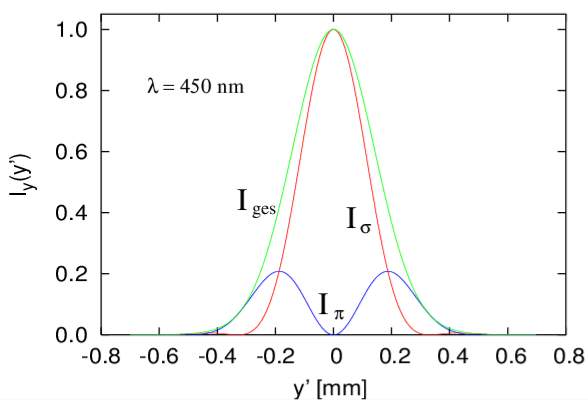
- E = 27.6 GeV
- I_{max} = 50 mA
- ρ = 604.8 m
- a = 6485.5 mm
- b = 1182.3 mm
- f = 1000 mm / 25 mm

- horizontal PSF



- vertical PSF



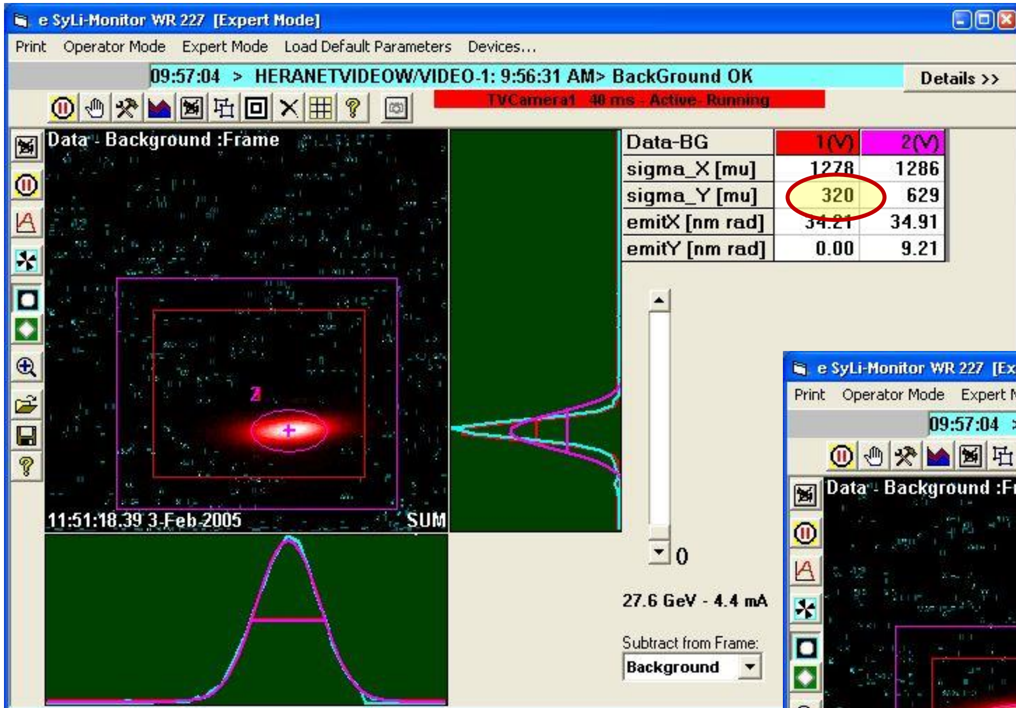


➡ good resolution:

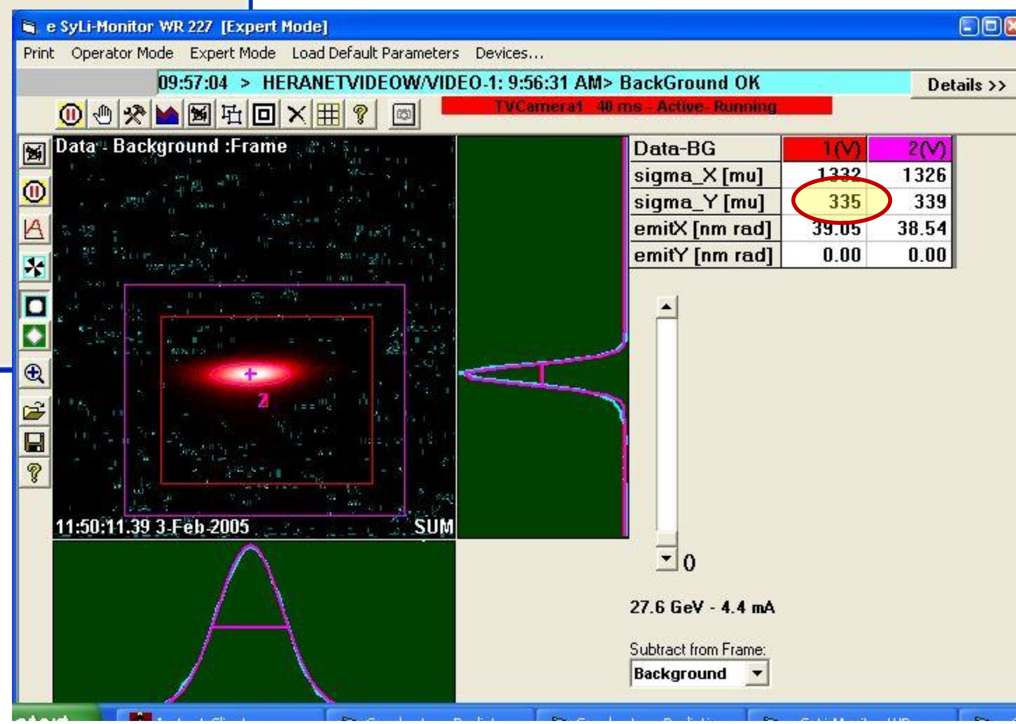
- monochromatic measurement
  → λ small
- polarization selection
  → σ-polarization

Polarization Dependency

- screen shot: HERAe @ DESY



σ polarization



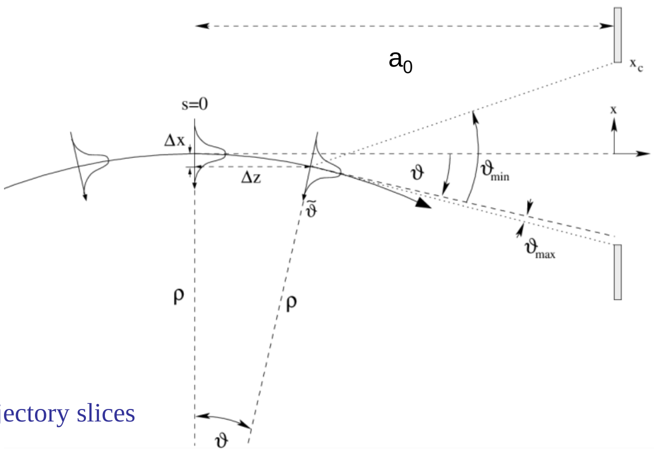
π polarization

Depth of Field and Curvature

- intensity summation along particle trajectory
  - emission angles (ϑ, Ψ) normal distributed in source plane
  - beam sizes (σ_x , σ_y) normal distributed in source plane
  - trajectory element: $ds = \rho d\vartheta$

A. Andersson and J. Tagger, Nucl. Instr. Meth. **A364** (1995) 4

G. Kube et al., Proc. BIW2004, AIP Conf. Proc. **732** (2004) 350



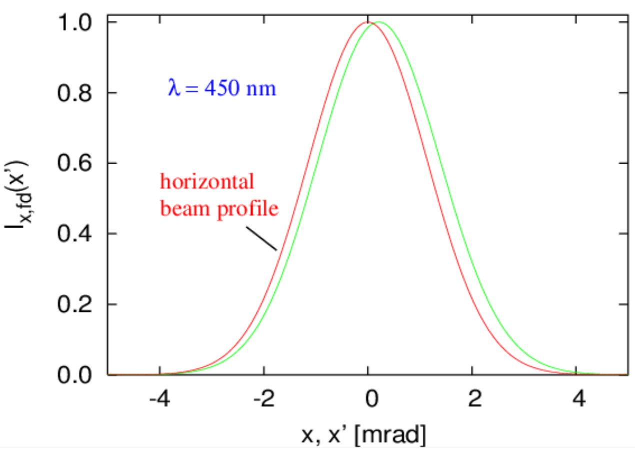
$$I_{y,dof}(y') = I_0 \int_{-\vartheta_{\lim}}^{+\vartheta_{\lim}} d\vartheta \frac{a_0 \rho\, h(\vartheta)}{b_0 \sqrt{2\pi} \sqrt{\sigma_y^2 + (\rho\vartheta\sigma_\Psi)^2}} \cdot \exp\left(- \frac{(a_0 y')^2}{2b_0^2 \left\{\sigma_y^2 + (\rho\vartheta\sigma_\Psi)^2\right\}} \right)$$

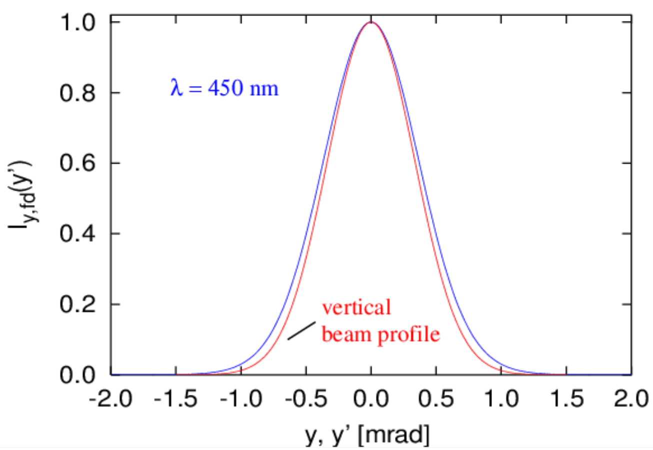
$$I_{x,dof}(x') = I_0 \int_{-\vartheta_{\lim}}^{+\vartheta_{\lim}} d\vartheta \frac{a_0 \rho\, h(\vartheta)}{b_0 \sqrt{2\pi} \sqrt{\sigma_x^2 + (\rho\vartheta\sigma_\vartheta)^2}} \cdot \exp\left(- \frac{a_0^2 (x' - \rho\vartheta^2)^2}{2b_0^2 \left\{\sigma_x^2 + (\rho\vartheta\sigma_\vartheta)^2\right\}} \right)$$

→ each slice shifted by $\Delta x = \rho\vartheta^2$ against position at s=0 (curvature of trajectory)

→ weight function h(ϑ): accounts for amount of light extracted by mirror from different trajectory slices

- example: HERAe @ DESY





Synchrotron Radiation Field

- **second representation:** starting point again **Liénard-Wiechert potentials**

O.Chubar and P.Elleaume,
Proc. EPAC96, Stockholm (1996) 1177

$$\varphi(t)=\left(\frac{-e}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_\tau , \quad \vec{A}(t)=\left(\frac{-e\,\vec{\beta}}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_\tau$$

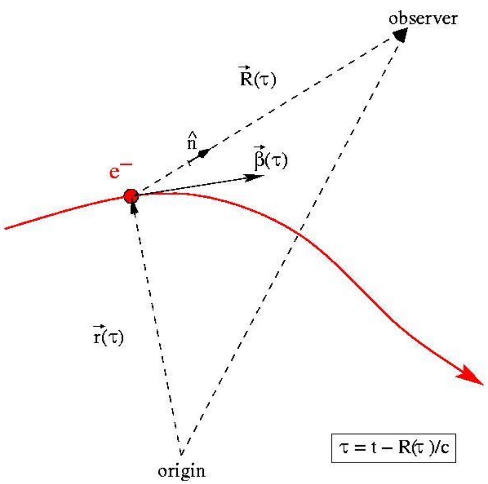
  - Fourier transform of potentials:

$$\varphi(\omega)=-e\int_{-\infty}^{+\infty}\mathrm{d}\tau\,\frac{1}{R(\tau)}\,e^{i\omega(\tau+R(\tau)/c)} , \quad \vec{A}(\omega)=-e\int_{-\infty}^{+\infty}\mathrm{d}\tau\,\frac{\vec{\beta}(\tau)}{R(\tau)}\,e^{i\omega(\tau+R(\tau)/c)}$$

  - field derivation:

$$\boxed{\vec{E}(\omega)=-\frac{i\omega e}{c}\int_{-\infty}^{+\infty}\mathrm{d}\tau\left[\frac{\left(\vec{\beta}-\hat{n}\right)}{R(\tau)}-\frac{ic}{\omega}\,\frac{\hat{n}}{R^2(\tau)}\right]\mathrm{e}^{i\omega(\tau+R(\tau)/c)}}$$

with $\tau=\int_0^z\frac{\mathrm{d}z}{c\beta_z(z)}=\frac{1}{c}\int_0^z\mathrm{d}z\left[1+\frac{1+(\gamma\beta_x)^2+(\gamma\beta_y)^2}{2\gamma^2}\right]$



➡ knowledge of arbitrary particle orbit: $\vec{E}(\omega)$ determined

➡ arbitrary magnetic field configuration: determines orbit and $\vec{E}(\omega)$

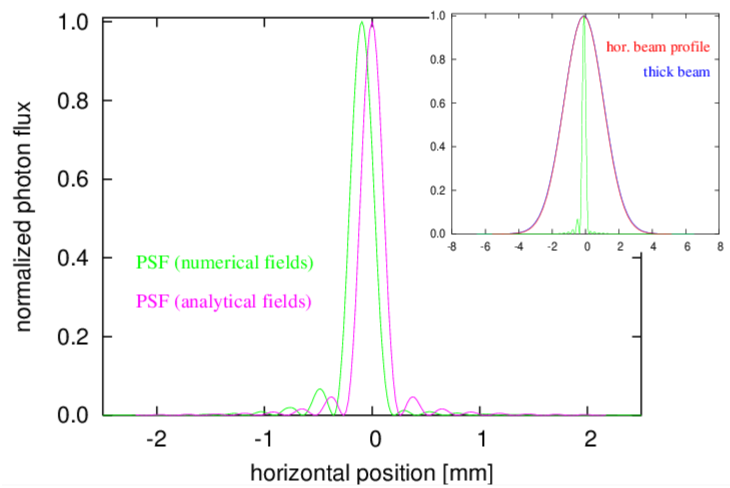
- **comments:**
  (i) exact field description → numerical near field calculation
  (ii) includes depth of field & curvature → no additional contributions, only field propagation
  (iii) free codes available → easy field calculation, even field propagation!

SRW: http://www.esrf.eu/Accelerators/Groups/InsertionDevices/Software/SRW (Chubar & Elleaume, ESRF)

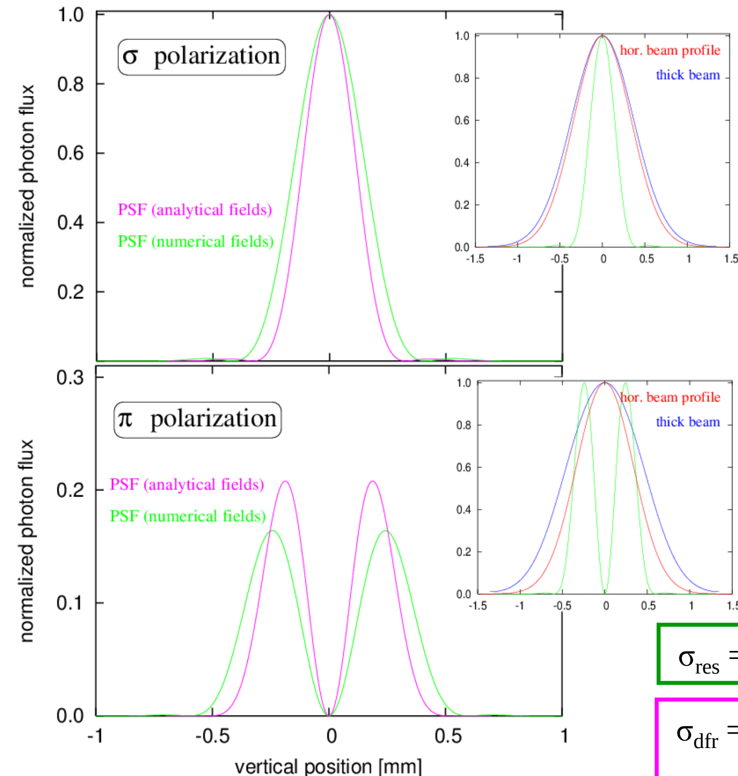
Spectra: http://radiant.harima.riken.go.jp//spectra/index.html (Tanaka & Kitamura, SPring8)

Comparison

- resolution broadening effects for the HERAe emittance monitor
  - calculation of spatial SR intensity distribution including beam emittance
  - quadratical subtraction of beam size (σ_x = 1175 µm, σ_y = 260 µm)



- near field description: σ_{res} = 203 µm
- analytical description:
  σ_{dfr} = 188 µm
  σ_{dof} = 275 µm
  → σ_{res} = 333 µm



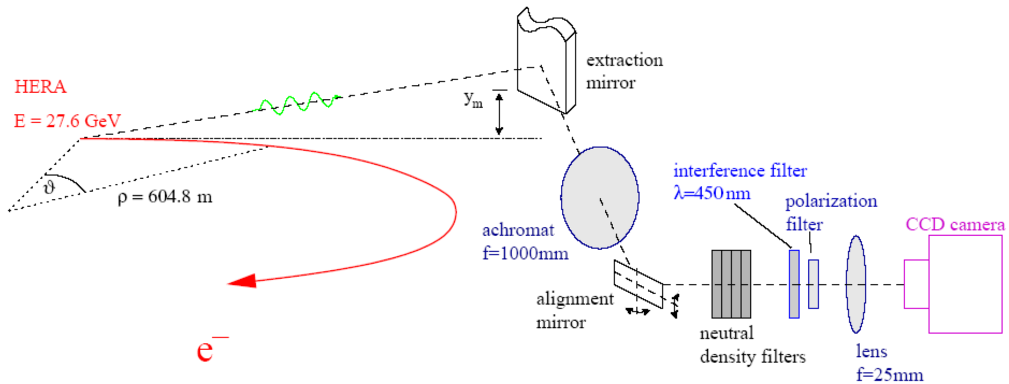
σ_{res} = 138 µm

σ_{dfr} = 106 µm
σ_{dof} = 134 µm
→ σ_{res} = 171 µm

G.Kube et al., Proc. BIW2004, AIP Conf. Proc. **732** (2004) 350 G.Kube et al., Proc. DIPAC‘05, Lyon, France (2005) 202 (σ polarization)

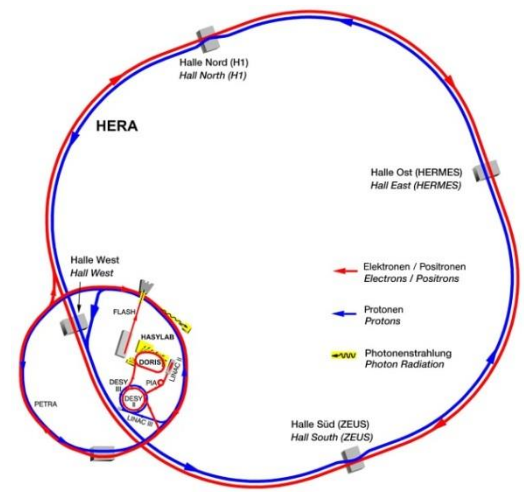
SyLi Monitor for HERAe @ DESY

- monitor setup



- HERA @ DESY

27.6 GeV $e^+(e^-)$ / 920 GeV p



- HERA e beam size: σ_{hor} = 1175 µm, σ_{vert} = 260 µm

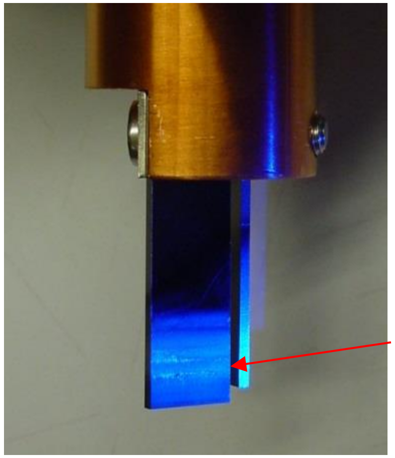
  → resolution with **optical SR** sufficient

- <u>problem:</u> **heat load** on **extraction mirror** (X-ray part of SR)

  → material with low absorption coefficient (Be)

  → cooling of extraction mirror

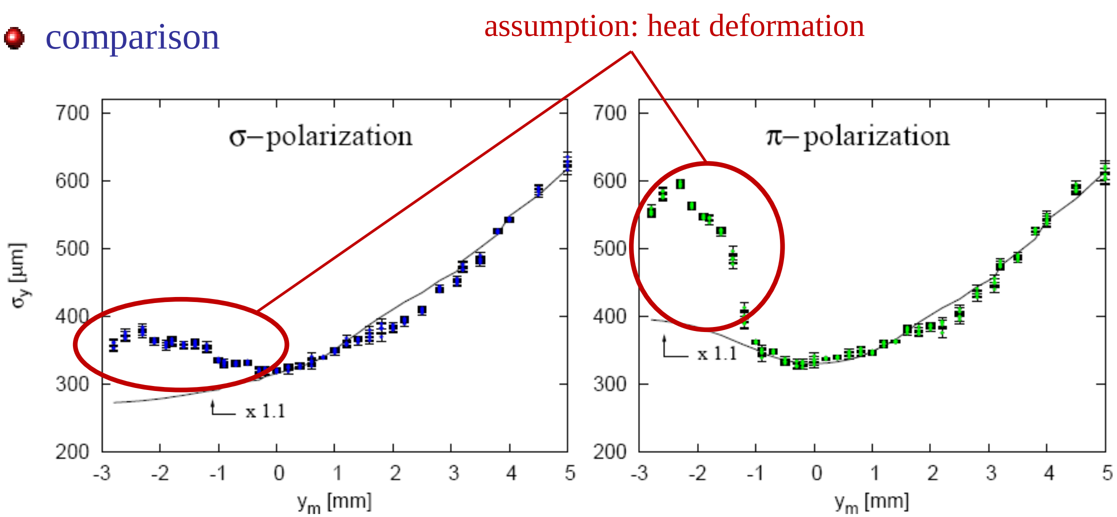
  → **not sufficient to prevent image distortion…**

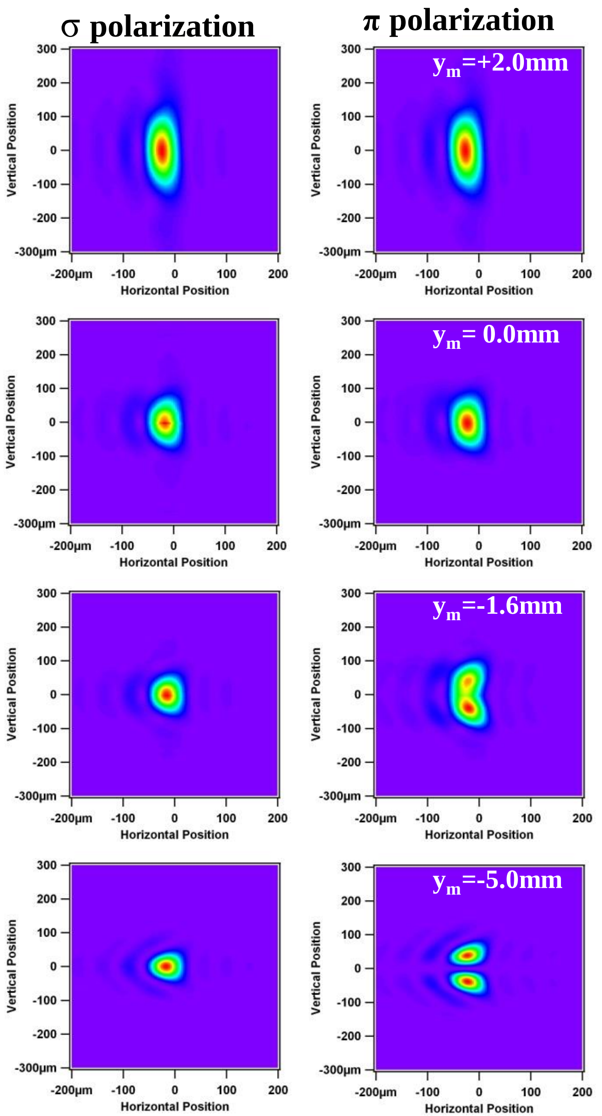


T_{max} = 1200°C

SyLi Monitor for HERAe @ DESY

- solution: **observation out of orbit plane**
  - → move mirror above orbit plane
  - → X-ray part of SR is emitted close to beam axis and will not hit the mirror surface
  - → optical SR components have larger angular distribution and are reflected at mirror surface
  - influence on monitor resolution ?
- resolution calculation (for single particle)
  - → numerical near field calculation with SRW code
- comparison





G. Kube et al., Proc. of DIPAC05, Lyon, France (2005) 202

Proton Synchrotron Radiation

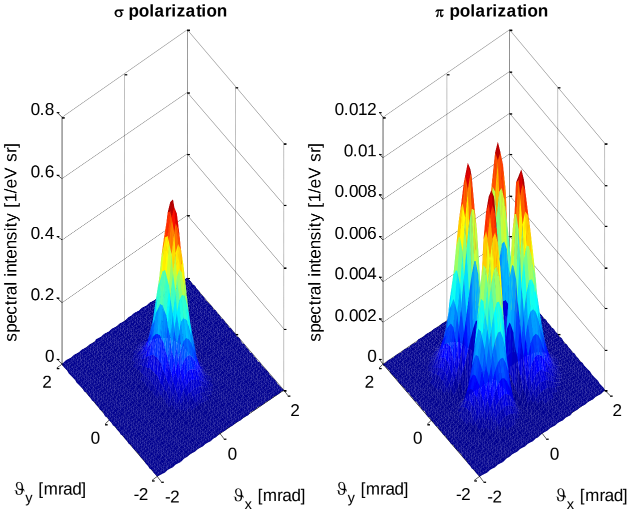
- analytical approach

R. Coisson, Phys. Rev. **A20** (1979), pp.524-528

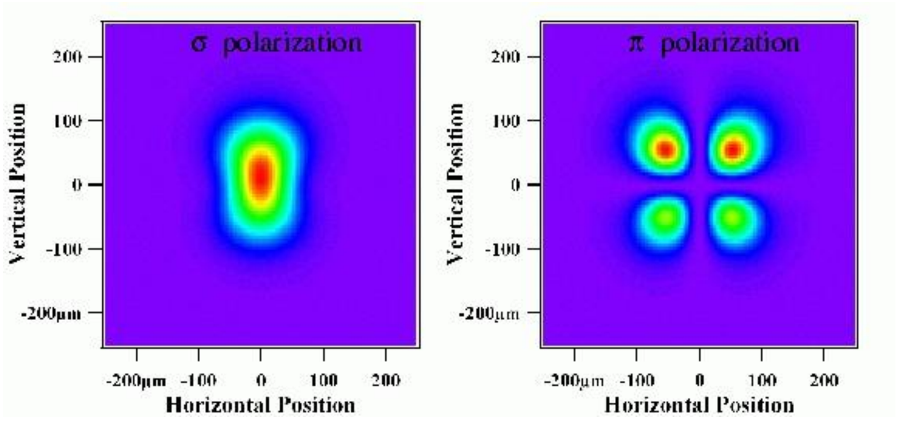
  - fringe field parametrization: $B(z) = \frac{B_0}{2}\left(1 + \mathrm{erf}\,(z/L)\right)$

$$\frac{d^2N}{d\Omega\, d(\hbar\omega)} = \frac{8\alpha}{\pi^2}\mu_N^2 \frac{\gamma^6}{(\hbar\omega)^3} B_0^2\, f^2 \exp\left[-2\left\{\frac{L(1+\vartheta^2\gamma^2)}{4\gamma^2\,\hbar c}\hbar\omega\right\}^2\right]$$

$$f^2 = f_x^2 + f_y^2 \quad \text{with} \quad f_x = \frac{\gamma^2(\vartheta_x^2+\vartheta_y^2)-1}{(1+\vartheta^2\gamma^2)^3}, \quad f_y = \frac{2\gamma^2\vartheta_x\vartheta_y}{(1+\vartheta^2\gamma^2)^3}.$$



- numerical approach
  - el.field fully determined via particle orbit
  - takes into account exact field distribution
  - calculation based on SRW code



Summary and Conclusion

- resolution in terms of geometrical optics
  - good resolution: λ small
  - balance acceptance angle ϑ: σ_{dfr} vs. σ_{dof}

J.A. Clarke, Proc. EPAC94, London (1994) 1643

R. Littauer, Proc. 2nd SLAC Summer School (1982) 902

A. Hofmann and K.W. Robinson, IEEE Trans.Nucl.Sci.**18** (1971) 937

- resolution based on analytical approach
  - good vertical resolution: σ polarization
  - calculation of broadened beam profiles in image plane
  - optimization of ϑ possible

A. Hofmann and F. Méot, Nucl. Instr. Meth.**203** (1982) 483

A. Andersson and J. Tagger, Nucl. Instr. Meth. **A364** (1995) 4

G. Kube et al., Proc. BIW2004, AIP Conf. Proc. **732** (2004) 350

A. Hofmann, CAS 1996, 2003, …

- numerical near field calculations
  - detailed resolution information
  - curvature and depth of field included

    $\rightarrow$ disturbed wave front (no more spherical)
  - possible for arbitrary magnetic configurations

O. Chubar and P. Elleaume, Proc. EPAC96, Stockholm (1996) 1177

T. Tanaka and H. Kitamura, J. Synchrotron Rad. **8** (2001) 1221

G. Kube et al., Proc. of DIPAC05, Lyon, France (2005) 202

- analytical approach including depth of field/curvature
  - based on paraxial Green's function
  - other

    $\rightarrow$ disturbed wave front characterized by phase factor

G. Geloni et al., DESY preprint 05-032

O. Chubar et al., Nucl. Instr. Meth. **A435** (1999) 495

R.A. Bosch, Nucl. Instr. Meth. **A431** (1999) 320