Outline of the third lecture

- Present essential characteristics of hard scattering observables for LHC

- Present results for hard scattering processes which are measured and calculated (at fixed order) with high accuracy
  - Z-boson production
  - Higgs production
  - Di-Photon production

- Study: perturbative convergence and phenomenology for these hadronic processes
Expectations at LHC

- Large production rates for processes with
  - Jets
  - Top-quark pairs
  - Vector bosons
- Allow precise determinations for
  - coupling constants
  - parton distributions
- Require theoretical description for hard scattering cross sections to be at least as precise as the experimental measurements
Kinematics at LHC

- Hard scattering cross section

\[ d\sigma_{P_1 P_2 \rightarrow X(s)} = \sum_{i,j} \int \, dx_1 \, dx_2 \, f_{q_i}/P_1(x_1) \, f_{q_j}/P_2(x_2) \hat{\sigma}_{ij} \rightarrow X(\hat{s}) \]

with parton-parton centre-of-mass energy \( \hat{s} = x_1 \, x_2 \, s \)

- Presence of longitudinal momentum fractions of partons: \( x_1, x_2 \)
  - Parton-parton centre-of-mass system boosted along beam axis with respect to proton-proton centre-of-mass system

- Describe final state kinematics in terms of variables that are boost-invariant (or transform trivially)
  - e.g.: transverse momenta, polar angles, rapidities
Kinematics at LHC

- **Four-momentum of a massless particle**
  \[ p^\mu = (E, p_x, p_y, p_z) = p_T (\cosh \eta, \sin \phi, \cos \phi, \sinh \eta) \]

- **Transverse momentum:** \( p_T = \sqrt{p_x^2 + p_y^2} \)

- **Pseudo-Rapidity:**
  \[ \eta = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) = -\ln(\tan(\Theta_{cm}/2)) \]

- **Polar angle:** \( \phi = \arctan \left( \frac{p_x}{p_y} \right) \)

- **Four-momentum of a massive particle**
  \[ p^\mu = (E, p_x, p_y, p_z) = (m_T \cosh y, p_T \sin \phi, p_T \cos \phi, m_T \sinh y) \]

- **Transverse mass** \( m_T = \sqrt{m^2 + p_T^2} \) and rapidity \( y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) \neq \eta \)
Kinematics at LHC

- Transverse momentum and polar angle: boost-invariant
- Rapidity transformation from parton-parton \((ab)\) frame to proton-proton \((pp)\) frame:

\[
y_{pp} = y_{ab} + \frac{1}{2} \ln \left( \frac{x_1}{x_2} \right)
\]

- Rapidity differences are boost-invariant
- Highest parton-parton centre-of-mass energy obtained for production at central rapidity \((x_1 = x_2)\)
Kinematics at LHC

- LHC probes new range in \((x, Q^2)\)
  - E.g. Z-boson production at central rapidity \((y=0)\):
    - Tevatron \((2\text{TeV})\) \(x_1=x_2\approx 0.05\)
    - LHC \((14\text{TeV})\) \(x_1=x_2\approx 0.0065\)
  - Different combinations of parton distributions dominate
    - Larger \(x\): valence quarks
    - Smaller \(x\): gluons and sea quarks
  - Phenomenology of the same process can be substantially different for LHC and Tevatron
Parton distributions at LHC

- Parton distributions determined from experimental data
  - Evolution determined by perturbative QCD (AP eqs.)
  - Input distributions are non-perturbative objects
  - Usually from global fit to multiple observables (HERA, fixed target, Tevatron, LHC)
  - Various groups (MRST, CT10, NNPDF, JR, ABM, HERAPDF, …) provide parametrizations of the distributions and errors on them
- Challenge at LHC: Probe new territory for PDF extraction
Jet observables at LHC

- **Jet production**
  - Low multiplicities: large cross section, precision QCD study
  - High multiplicities: potential signature of BSM physics
- **Use boost-invariant jet definition**
  \[
y_{ij} = \min(k_{T,i}^2, k_{t,j}^2) \frac{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}{R^2}
\]
  - \(p=1\): \(k_T\), \(p=0\): Cambridge/Aachen, \(p=-1\): anti-\(k_T\)
- **Single jet inclusive cross section**
  - Double differential: \((p_T, y)\)
  - Measured to per cent accuracy up to transverse momenta \(p_T \approx 2\) TeV

![Jet cross section graph](image-url)
Theoretical uncertainties

- Remember cross section formula for a hard scattering process

\[
d\sigma^{h_1 h_2 \rightarrow c d} = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{a,b} f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) d\hat{\sigma}^{ab \rightarrow cd}(Q^2, \mu_F^2)
\]

- Sources of uncertainty
  - Parton distributions \( f_{a/h}(x, \mu_F) \): determined from data, inherent errors
  - Partonic cross section \( d\sigma^{ab \rightarrow cd} \): expanded in perturbation theory to finite order, uncertainty from missing higher orders

- Quantify theoretical uncertainty through scale variations
  - Renormalization scale dependence: \( \alpha_s(\mu_R) \)
  - Factorization scale dependence: \( f_{a/h}(x, \mu_F), \sigma^{ab \rightarrow cd}(x, \mu_F) \)

- Expect: Scale dependence decreases including more and more higher order terms
Fixed order predictions

- LO predictions usually give a qualitative understanding of the behaviour of observables but are often not enough to describe the data accurately.
- NLO needed to
  - reduce scale uncertainty of LO theory prediction
  - Have a reliable estimation of normalization and shape as it accounts for effects of extra radiation
- Example: $Z+j$ at Tevatron
  - NLO error: $\sim 15\%$
  - substantial NLO effect
  - correction not constant
NLO calculations

- Require two principal ingredients (here: $pp \rightarrow 2j$)
  - one-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all $2 \rightarrow 2$ processes
      - known for many $2 \rightarrow 3$ processes
      - current frontier $2 \rightarrow 7$ (major challenge)
  - tree-level matrix elements
    - implicit poles from soft/collinear emission

- Infrared poles cancel in the sum (KLN, factorization theorems)

- Subtraction methods used to extract infrared poles and combine contributions to evaluate NLO observables are well-established

- Several program packages for NLO:
  MCFM, MC@NLO, POWHEG, NLOJET++ …
NNLO predictions

- **NNLO corrections needed:**
  - For processes measured to few per cent accuracy
    - jet production
    - vector boson (+jet) production
    - top quark pair production
  - For processes with potentially large perturbative corrections as new channels or new phase space regions open up
    - Higgs or vector boson production

- **Expectations for NNLO predictions:**
  - Per-cent level accuracy (as required for a meaningful interpretation of collider data and extraction of parameters)
  - First reliable estimation of theoretical uncertainties
NNLO calculations

- **Require three principal ingredients (here: \( pp \rightarrow 2j \))**
  - two-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all massless \( 2 \rightarrow 2 \) processes
  - one-loop matrix elements
    - explicit infrared poles from loop integral
    - and implicit poles from single real emission
      - usually known from NLO calculations
  - tree-level matrix elements
    - implicit poles from double real emission
      - known from LO calculations

- **Infrared poles cancel in the sum:**
- **Challenge:** combine contributions into parton-level generator
  - Need methods at NNLO to extract implicit infrared poles
Z-boson production at LHC
Z boson production at LHC

- Experimentally: Z bosons events (decaying into leptons) observed at large rate at LHC
  - At design luminosity ($10^{34}$ cm$^{-2}$s$^{-1}$): 100 Z bosons per second
  - Leptons yield clean final state signature
  - Measured with high accuracy (per cent level and below)

- Theoretically well understood
  - Perturbative corrections up to NNLO as fully differential event generator
  - Resummation of large logarithmic corrections
  - Process evaluated with per cent level accuracy

- Precision physics
  - Electroweak masses and couplings
  - Parton distributions
Z-boson production at LHC

- Only one partonic contribution at LO: $q\bar{q} \rightarrow Z$

$$\sigma_{pp\rightarrow Z}^{\text{LO}} = \sum_i \int dx_1 dx_2 f_{q_i}(x_1, \mu_F^2) f_{\bar{q}_i}(x_2, \mu_F^2) \hat{\sigma}_{0,q_i\bar{q}_i\rightarrow Z}(x_1p_1, x_2p_2)$$

- Including NLO and NNLO corrections

$$\sigma_{pp\rightarrow Z+X}^{\text{NNLO}} = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \left[ \hat{\sigma}_{0,i,j\rightarrow Z}(x_1, x_2) + \alpha_s(\mu_R)\hat{\sigma}_{1,i,j\rightarrow Z+X}(x_1, x_2, \mu_F) \right.$$

$$+ \left. \alpha_s^2(\mu_R)\hat{\sigma}_{2,i,j\rightarrow Z+X}(x_1, x_2, \mu_F, \mu_R) \right].$$

- Sum over the flavours $i,j$ of the initial partons
- Include gluon-induced processes

- Inclusive cross section: $Z+X$ production
  - Any number of jets is allowed with the $Z$ boson
Z-boson production at LHC

- **New channels open up at NLO and NNLO**
  - $qg$-induced processes at NLO
  
  - $gg$-induced processes at NNLO

Predictions known up to NNLO in pert. QCD!
Using FEWZ and MSTW2008, at 7 TeV:
Z-boson production at LHC

- Scale dependence
  - $\mu_F = \mu_R$ varied between $[M_Z/2; 2M_Z]$
  - Scale variation is reduced at each order
    - LO: 30%, NLO 6%, NNLO <1%
    - But: LO uncertainty band underestimates higher orders

- Origin of large NLO corrections
  - New partonic channel $qg \rightarrow Zq$
  - Large gluon luminosity leads to NLO corrections of 15-30% (depending on rapidity)

- Reliable estimate of theoretical uncertainty only at NNLO
Higgs Boson production at the LHC
Higgs boson production at LHC

- **Standard model Higgs particle**
  - scalar boson, $m_H = 125$ GeV
  - couples directly to massive particles, proportional to mass
  - couples indirectly to gluons and photons via loops
  - Dominant production mechanism: gluon fusion $gg \rightarrow H$

![Diagram of Higgs production mechanisms](image-url)

**Figure 10.41:** Higgs production cross section as a function of Higgs mass.

**Figure 10.42:** Higgs branching ratios (a) and total width (b).
Higgs boson production at LHC

- Higgs discovery (2012) in multiple decay channels
  - $\text{BR}(\gamma \gamma) \approx 10^{-3}$
  - $\text{BR}(WW^*) \approx 0.1$
  - $\text{BR}(ZZ^*) \approx 0.02$
  - $\text{BR}(\tau\tau) \approx 0.08$
- Dominant channel: not yet observed due to large background
  - $\text{BR}(bb) \approx 0.8$
Higgs boson production at leading order

- For gluon fusion process: \( gg \rightarrow H \)
  - LO cross section: already 1 loop!
  - 2\( \rightarrow 1 \) process: very simple kinematics

\[
PS \sim \delta(\hat{s} - M_H^2) \quad \hat{s} = x_1 x_2 s_{pp}
\]

- Parton level cross section

\[
\sigma^{LO}_{gg \rightarrow H}(z) = \frac{G_F \alpha_s^2}{288 \pi \sqrt{2}} \left| \frac{3}{4} \sum_Q \mathcal{F}_{1/2}(\tau_Q) \right|^2 \delta(1 - z)
\]

- Quark loop factor \( \mathcal{F}(\tau) \): smooth and finite function

\[
\tau_Q = \frac{M_H^2}{4 m_Q^2} \quad z = \frac{m_H^2}{\hat{s}}
\]
Higgs production at NLO

- Effective Lagrangian approach \((m_{\text{top}} \rightarrow \infty)\) limit
  - Top-quark loop reduces to a point-like interaction
  - Coupling to gluons described by an effective Lagrangian:
    \[
    \mathcal{L} = \frac{1}{4u} C_1 Z_1 G^a_{\mu \nu} G^{a\mu\nu} H,
    \]
  - gluon field strength \(G^a_{\mu \nu}\)
  - Higgs field \(H\)
  - Wilson coefficient \(C_1\), renormalization \(Z_1\), both have series in \(\alpha_s\)
  - Higgs field vacuum expectation value \(v \approx 246 \text{ GeV}\)
  - Feynman rules for the \(ggH, gggH, ggggH\) vertices can be deduced from this effective Lagrangian
Effective Lagrangian approach

- Factorises QCD effects (dynamics of gluons) from heavy particle effects
  - heavy quark loop described with Wilson coefficient $C_1$
- Simplifies calculation of QCD corrections considerably
  - Reduces the number of loops by one at each order
  - Turns a two-scale problem $(m_t, m_H)$ into a one-scale problem
- LO cross section: only proportional to $ggH$ vertex

\[
\hat{\sigma}_0(z) = \sigma_0 \delta(1 - z) = \frac{\pi}{576 \nu^2} \left( \frac{\alpha_s}{\pi} \right)^2 \delta(1 - z)
\]
Higgs production at NLO

- **Virtual corrections to** $gg \rightarrow H$
  - Matrix-element $2 \text{Re}(M_0^* M_1)$
  - After UV renormalisation of coupling $\alpha_s$ and Wilson coefficient $C_1$

$$\hat{\sigma}_V = \sigma_0^{(d)} \left( \frac{\mu^2}{m_H^2} \right)^\epsilon \frac{\alpha_s}{\pi} \left\{ \left[ C_A \left( -\frac{1}{\epsilon^2} \right) + \text{Finite} \right] - \frac{1}{\epsilon} \left( C_A \frac{11}{6} - n_f \frac{1}{3} \right) + \mathcal{O}(\epsilon) \right\} \delta(1-z)$$

- Pole terms: $1/\epsilon^2$ (soft) and $1/\epsilon$ (collinear, $\propto \beta_0$)
- Cancel partly with real contributions ($1/\epsilon^2$, $1/\epsilon$) and partly with mass factorization of incoming gluon distribution involving

$$P_{gg}(z) = 2C_A \left( \frac{z}{(1-z)_+} + \frac{1-z}{z} \right) + \left( \frac{11}{6} C_A - \frac{2}{3} T_F n_f \right) \delta(1-z)$$
Higgs production at NLO

- **Real corrections** $gg \rightarrow Hg$
  - Matrix element singular if the outgoing gluon is either soft or collinear to one of the incoming gluons
  - Parametrise final state phase space to map these singularities in variables $(z, \lambda)$ (Soft limit $z \rightarrow 1$, collinear limits $\lambda \rightarrow (0,1)$)
    \[ p_g = \frac{\hat{s}(1-z)}{2} \left(1, 2\sqrt{\lambda(1-\lambda)}, 0, 1 - 2\lambda\right) \]
  - yields phase space regulator $(1 - z)^{1-2\epsilon} \int_0^1 d\lambda [\lambda(1-\lambda)]^{-\epsilon}$
  - Extraction of poles: expansion in distributions (like in DIS)
    \[ \hat{\sigma}_R = \sigma_0^{(d)} \left(\frac{\mu^2}{m_H^2}\right) \frac{\alpha_s}{\pi} C_A \left\{ \left(1 + \frac{1}{\epsilon^2}\right) - \frac{2}{\epsilon} \left(\frac{z}{1-z} + \frac{1-z}{z}\right) + \text{Finite} + O(\epsilon) \right\} \]
  - Real + virtual corrections: universal left-over $1/\epsilon \, P_{gg}(z)$: mass factorisation of initial-state collinear gluon singularity used
Higgs production at NNLO

- State-of-the-art theory for Higgs production: NNLO
  (FeHIP: C. Anastasiou, K. Melnikov, F. Petriello; HNNLO: S. Catani, M. Grazzini)

- Bin integrated Higgs boson rapidity distributions
  - with and without a jet veto

- Hard radiation at higher orders suppressed with a jet veto
- large NLO corrections, perturbative convergence observed at NNLO
- scale variation at LO underestimates missing higher orders
Total di-photon production cross section normalized to branching fraction

- with cuts on the two photons, to improve signal/background ratio
- Observe: perturbative corrections to di-photon signal follow same pattern as the inclusive Higgs production cross section
  - NLO/LO large, NNLO/NLO moderate
  - scale dependence: stabilized at NNLO
  - NNLO corrections smallest for $\mu = M_H/2$

Results for $pp \to H \to \gamma\gamma$

- $\gamma\gamma$ + X
- $\sqrt{s} = 14$ TeV
- $m_H = 120$ GeV
- MRST2001 pdfs
- $p_t > 40$ GeV
- $p_t > 25$ GeV
- $R = 0.4$, $Rt < 15$ GeV
- $|\eta| < 2.5$
Results for $pp \rightarrow H \rightarrow \gamma\gamma$

- Di-photon transverse momentum ($p_T^{\gamma\gamma}$) distribution
  - consider average $p_T$ of the two photons
  - Observe: large perturbative corrections close to the leading order kinematical bound: $p_T^{\gamma} < m_H/2$

- General feature of fixed order predictions: not accurate at phase space edges
  - At low $p_T^{\gamma\gamma}$: resummation is required
  - Background has completely different kinematical features
    - can use transverse momentum distribution as discriminator
Higgs+jet production at the LHC

- Essential to establish the properties of the newly discovered Higgs boson

- Experiments select events according to number of jets
  - Different backgrounds for different jet multiplicities
  - $H+0\text{jet}$ and inclusive $H$ production known at NNLO
    (C. Anastasiou, K. Melnikov, F. Petriello; S. Catani, M. Grazini)
  - $H+1\text{jet}$ and $H+2\text{jet}$ known at NLO
  - $H+0\text{jet}$ and $H+1\text{jet}$ samples of comparable sizes

- NNLO for $H+1\text{jet}$ needed
  - gluons-only total cross section just completed recently
    (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
  - Full calculation and differential distributions in progress
Di-Photon production at the LHC
Di-Photon production at the LHC

- Di-photon production: irreducible background for $H \rightarrow \gamma\gamma$
  - at present determined from sideband data fits
- Discrepancy between NLO theory and data in some distributions
  - Require precise theoretical predictions (NNLO)

![Graph showing di-photon differential cross section](image)

- $pp \rightarrow H \rightarrow \gamma\gamma$
- $pp \rightarrow \gamma\gamma$

**Figure 7:** (Left) Diphoton differential cross section as a function of the azimuthal angle between the two photons, $D_{jj}^{gg}$, from data (points) and from theory (solid line) for the photon pseudorapidity range $|\eta| < 2.5$. (Right) The difference between the measured and theoretically predicted diphoton cross sections, divided by the theory prediction, as a function of $D_{jj}^{gg}$. In both plots, the inner and outer error bars on each point show the statistical and total experimental uncertainties. The 4% uncertainty on the integrated luminosity is not included in the error bars. The dotted line and shaded region represent the systematic uncertainties on the theoretical prediction from the theoretical scales and the PDFs, respectively.

**Figure 8:** (Left) Diphoton differential cross section as a function of the azimuthal angle between the two photons, $D_{jj}^{gg}$, from data (points) and from theory (solid line) for the photon pseudorapidity range $|\eta| < 1.44$. (Right) The difference between the measured and theoretically predicted diphoton cross sections, divided by the theory prediction, as a function of $D_{jj}^{gg}$. In both plots, the inner and outer error bars on each point show the statistical and total experimental uncertainties. The 4% uncertainty on the integrated luminosity is not included in the error bars. The dotted line and shaded region represent the systematic uncertainties on the theoretical prediction from the theoretical scales and the PDFs, respectively.
Photon isolation

- Photons need to be isolated from hadrons in events
  - Suppress secondary photons from hadron decays
  - Complete isolation not infrared safe, nor exp. well-defined

- Isolation criteria
  - Fixed cone isolation
    \[ \sum_{\delta < R} E_T^h < E_T^{max} \]
  - Smooth cone isolation (S.Frixione)
    \[ \sum_{\delta < R} E_T^h < E_T^{max} \left( \frac{1 - \cos(\delta)}{1 - \cos(R)} \right)^n \]
    - only soft radiation allowed close to photon
    - experimental implementation difficult (finite detector resolution)
Photon production mechanisms

- Direct process: photon produced in hard interaction
  - perturbatively calculable
  - collinear quark-photon contributions present
- Fragmentation of parton into photon:
  - described by a non-perturbative parton-to-photon fragmentation function
  - absorbs collinear singularities from direct process
  - requires non-perturbative input
- Fixed cone isolation
  - both processes contribute
  - fragmentation contributions reduced but not eliminated
- Smooth cone isolation
  - No fragmentation contribution, direct process contribute without collinear part
Di-photon production at the LHC

- **NNLO calculation:** $2\gamma_{\text{NNLO}}$
  (S. Catani, L. Cieri, D. de Florian, G. Ferrera, M. Grazzini)
- Parton-level event generator, based on $q_T$-subtraction
  - Analytic cancellation of infrared poles
  - Using a smooth isolation criterion to define photons
  - Includes all $O(\alpha_s^2)$ corrections to direct photon production $pp \to \gamma\gamma$
- First fully consistent inclusion of the gluonic box contribution

![Diagram of di-photon production]

$O(\alpha_s^2)$, gluon luminosity comparable size to $O(\alpha_s^0)$, qq luminosity

- Box also included in NLO-type codes (DIPHOX+gamma2MC, MCFM)
Di-photon production at the LHC

- Partonic contributions to $pp \rightarrow \gamma\gamma$ up to NNLO

LO: $q\bar{q} \rightarrow \gamma\gamma$

NLO: $q\bar{q} \rightarrow \gamma\gamma(\text{+}g)$
$qg \rightarrow \gamma\gamma q$

NNLO: $q\bar{q} \rightarrow \gamma\gamma(\text{+}gg)$
$qg \rightarrow \gamma\gamma q(\text{+}g)$
$gg \rightarrow \gamma\gamma$

- At NLO and NNLO: $qg$ channel dominant
Di-photon production at NNLO

- **Invariant-mass distribution with staggered photon cuts**

- **At LO:** both $\gamma$ produced at equal $p_T (40 \text{ GeV})$
  - New phase space region ($25 \text{ GeV} < p_T^{\gamma \text{ soft}} < 40 \text{ GeV}$) opens at NLO

- **NNLO corrections large for low $M_{\gamma\gamma}$ region**
  - Main contribution from $qg$ channel (dominant channel at NLO)
  - Box contribution of similar size than LO

- **At LO:** both $\gamma$ produced at equal $p_T (40 \text{ GeV})$
  - New phase space region ($25 \text{ GeV} < p_T^{\gamma \text{ soft}} < 40 \text{ GeV}$) opens at NLO

- **NNLO corrections large for low $M_{\gamma\gamma}$ region**
  - Main contribution from $qg$ channel (dominant channel at NLO)
  - Box contribution of similar size than LO

$$
p_T^{\gamma \text{ hard}} \geq 40 \text{ GeV}
$$

$$
p_T^{\gamma \text{ soft}} \geq 25 \text{ GeV}
$$

$$
|\eta^{\gamma}| \leq 2.5
$$

$$
20 \text{ GeV} \leq M_{\gamma\gamma} \leq 250 \text{ GeV}
$$

**Table 1:**

Cross sections for $pp \rightarrow \gamma\gamma + X$ at the LHC ($\sqrt{s} = 14 \text{ TeV}$). The applied cuts are described in the text.

**Figure 1:**

Invariant mass distribution of the photon pair at the LHC ($\sqrt{s} = 14 \text{ TeV}$): LO (dots), NLO (dashes) and NNLO (solid) results. We also present the results of the box and NLO+box contributions. The inset plot shows the corresponding $K$-factors.
Inclusion of NNLO corrections resolves discrepancy between NLO-type prediction and data
End of Lecture 3