Statistical aspects of the Higgs rate fits : How to treat the theoretical uncertainties ?

G. Moreau
Laboratoire de Physique Théorique, Orsay, France

Based on arXiv:1303.6591 with ***A. Djouadi***
& Work in progress with ***S. Fichet***

I) Introduction

Today : The LHC has **discovered** a resonance of ~ 125.5 GeV

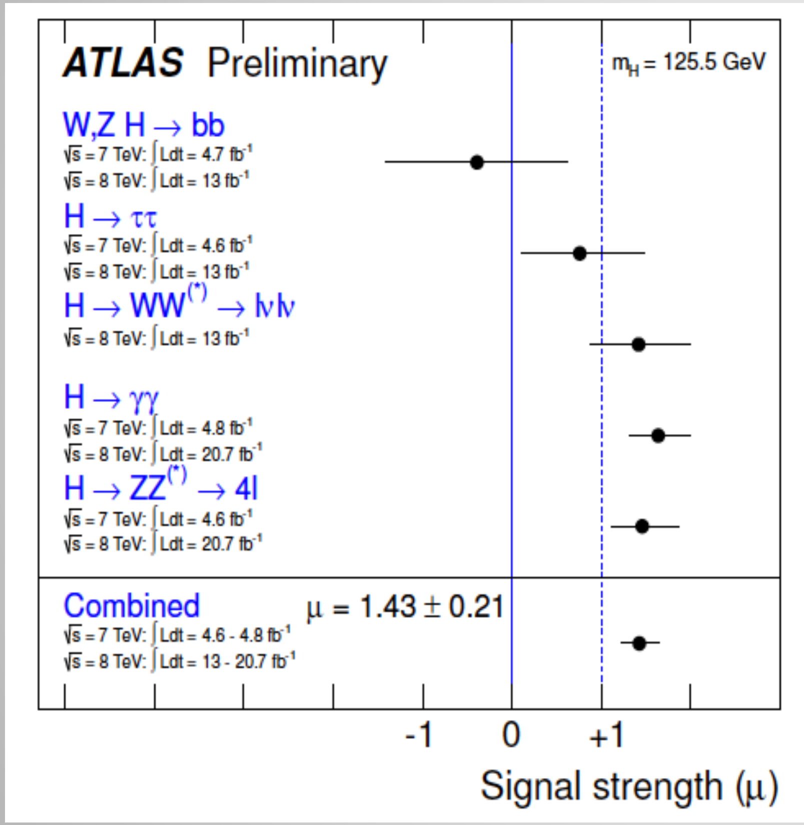
➡ *it is probably the B.E.Higgs boson => **EWSB** mechanism*

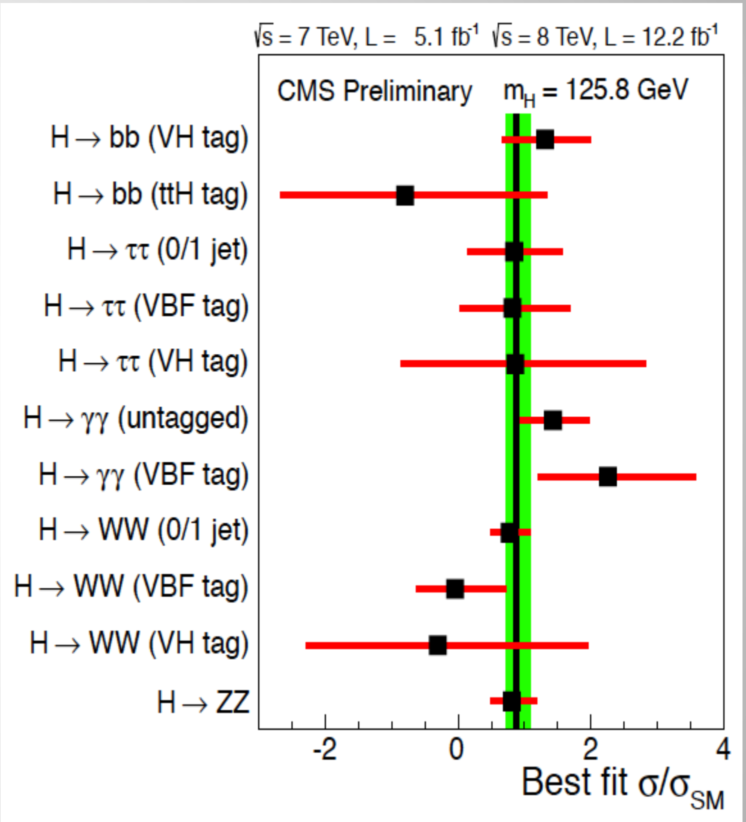
\+ Tevatron and LHC provide **60** measurements of the Higgs rates

= new precious source of indirect information on BSM physics

➡ *nature of the EWSB : within the **SM** or a **BSM** context !?*

The latest experimental results…





The first test to do is to fit,

$$\mu_{XX}\big|_{\rm exp} = \frac{\mathrm{N}^{\rm evts.}(pp \to H \to XX)}{\sum_i \epsilon_i^X \sigma_i(H)\ \mathrm{BR}(H \to XX)|_{\rm SM} \times \mathcal{L}}$$

with,

$$\mu_{XX}\big|_{\rm th} = \frac{\sum_i \epsilon_i^X \sigma_i(H)\ \mathrm{BR}(H \to XX)}{\sum_i \epsilon_i^X \sigma_i(H)\ \mathrm{BR}(H \to XX)|_{\rm SM}}$$

c_V

$$\begin{aligned}\mathcal{L}_h = &\ c_W\ g_{HWW}\ H\ W_\mu^+ W^{-\mu} + c_Z\ g_{HZZ}\ H\ Z_\mu^0 Z^{0\mu} \\ &- c_t\, y_t\, H\bar{t}_L t_R - c_c\, y_c\, H\bar{c}_L c_R - c_b\, y_b\, H\bar{b}_L b_R - c_\tau\, y_\tau\, H\bar{\tau}_L \tau_R + \mathrm{h.c.}\end{aligned}$$

c_f

All c=1 in the SM.

II) Why is the THEORETICAL *error crucial in fits ?*

* *The SM is in good agreement with the present data on Higgs signal strengths,* ***but it's not the end of the story…***

These tests of the SM will become more and more powerful as the EXPERIMENTAL *errors on the Higgs rates* (μ_{exp}) *will decrease.*

Then the THEORETICAL *errors on the Higgs rates can become dominant.*

* *The QCD uncertainty (P.D.F.,* α_s^2*, scale dependence) on the inclusive Higgs production cross section reaches* **~ 10%** [LHCHWG]

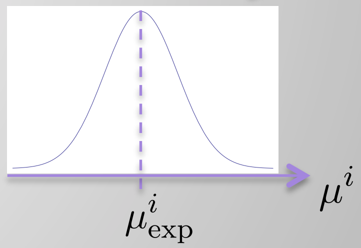
The typical deviations expected in SUSY or composite Higgs models are about **~ 1 - 20%** [R.S.Gupta, H.Rzehak, J.D.Wells, 2006]

III) The present TREATMENT of th. error in fits

In most experimental and theoretical papers on Higgs rate fits…

TH uncertainty added in quadrature with EXP one in the *Likelihood* as,

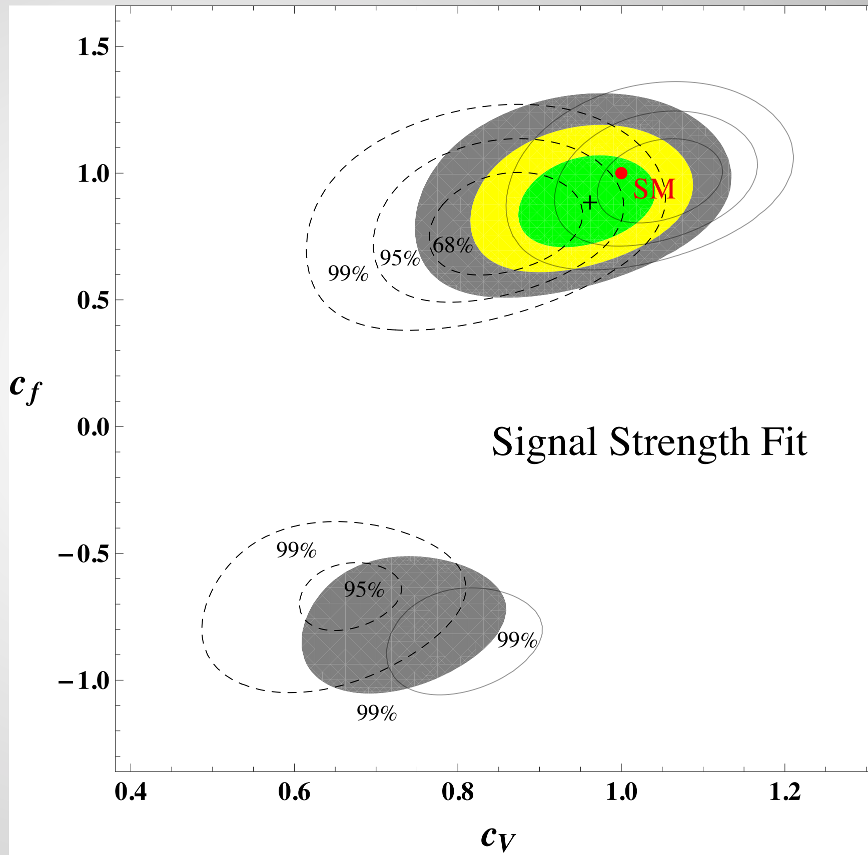
$$\mathcal{L} \;=\; pdf\left(\; \mu^i_{\rm exp}\;,\; \sqrt{(\Delta^i_{\rm exp})^2 + (\Delta^i_{\rm th})^2}\;\middle|\; \mu^i(c_f, c_V)\;\right) \;=\; \Pi_i \; pdf_i$$



➡ this reflects a decorrelation : correct

➡ it means that the TH ***pdf*** is the same as the EXP one :
a FLAT ***probability distribution*** can be better motivated by QCD

Best-fit regions :



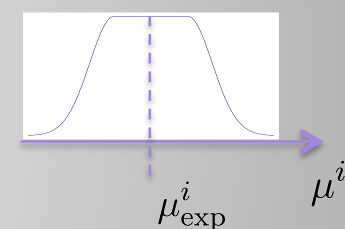
IV) MARGINALISATION of the th. uncertainty

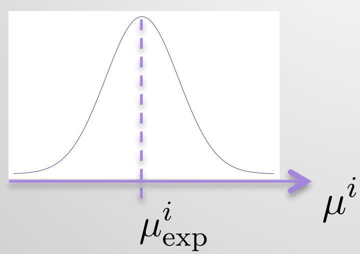
Use the rigorous Bayesian approach…

δ is a nuisance parameter :

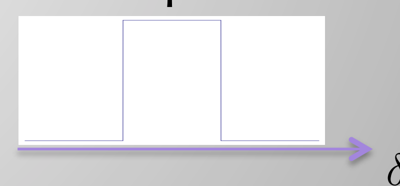
$$\mu^i_{\rm exp}(\,1 + \delta \times \Delta^i_{\rm th}\,)\ \text{with}\ \delta\ \in\ [-1, 1]$$

$$\mathcal{L}\ =\ \int d\delta\ \ pdf\left(\ \mu^i_{\rm exp}\ ,\ \Delta^i_{\rm exp}\ \middle|\ \mu^i(c_f, c_V)\ ,\ \delta\ \right)\ \times\ \pi(\delta)\qquad \approx$$





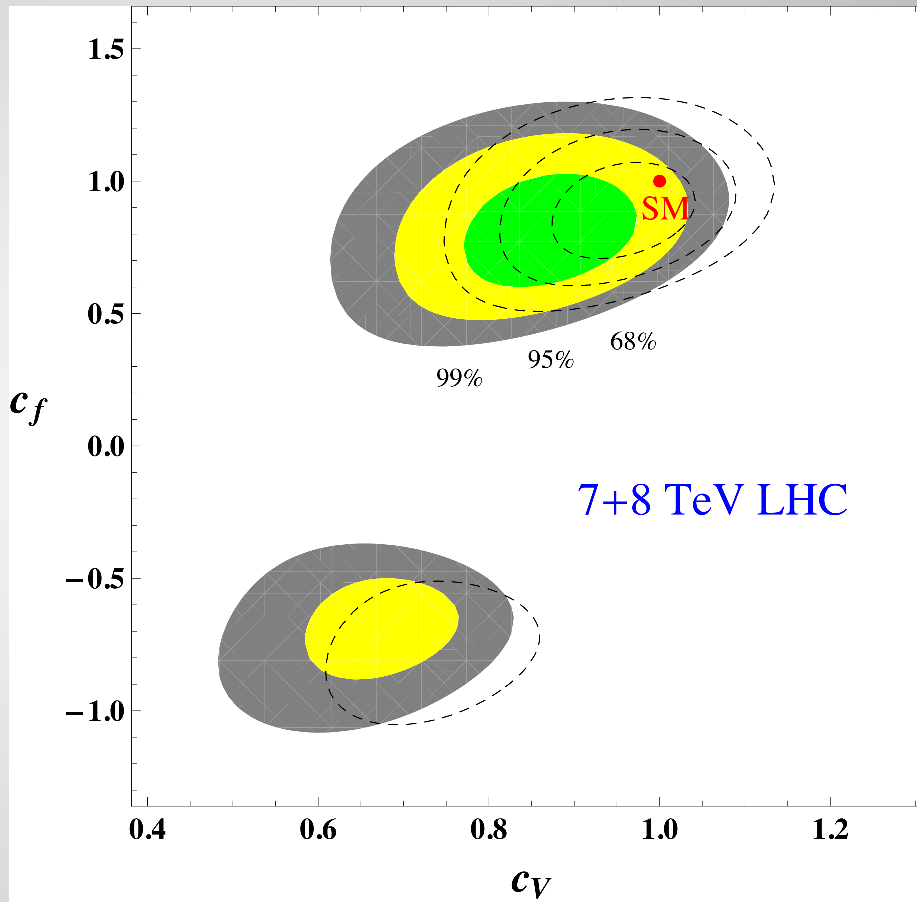
FLAT prior



*(**first** assuming TH errors purely correlated via a unique δ , for simplicity)*

Bayesian regions :

(integration over c's)



The « corresponding » Frequentist approach = Profile Likelihood :

still FLAT distribution

$$\mathcal{L}(\delta) \;=\; pdf\left(\mu^i_{\rm exp} \;,\; \Delta^i_{\rm exp} \;\middle|\; \mu^i(c_f, c_V) \;,\; \delta \right) \;\times\; \pi(\delta)$$

$$\tilde{\chi}^2 \;=\; -2\ {\rm Log}[\mathcal{L}(\delta)] \;=\; \underbrace{\sum_i \frac{[\ \mu^i_{\rm exp}(1+\delta\ \Delta^i_{\rm th}) - \mu^i(c_f, c_V)\ ]^2}{(\ \Delta^i_{\rm exp}\ )^2}}_{\chi^2} - 2\ {\rm Log}[\pi(\delta)]$$

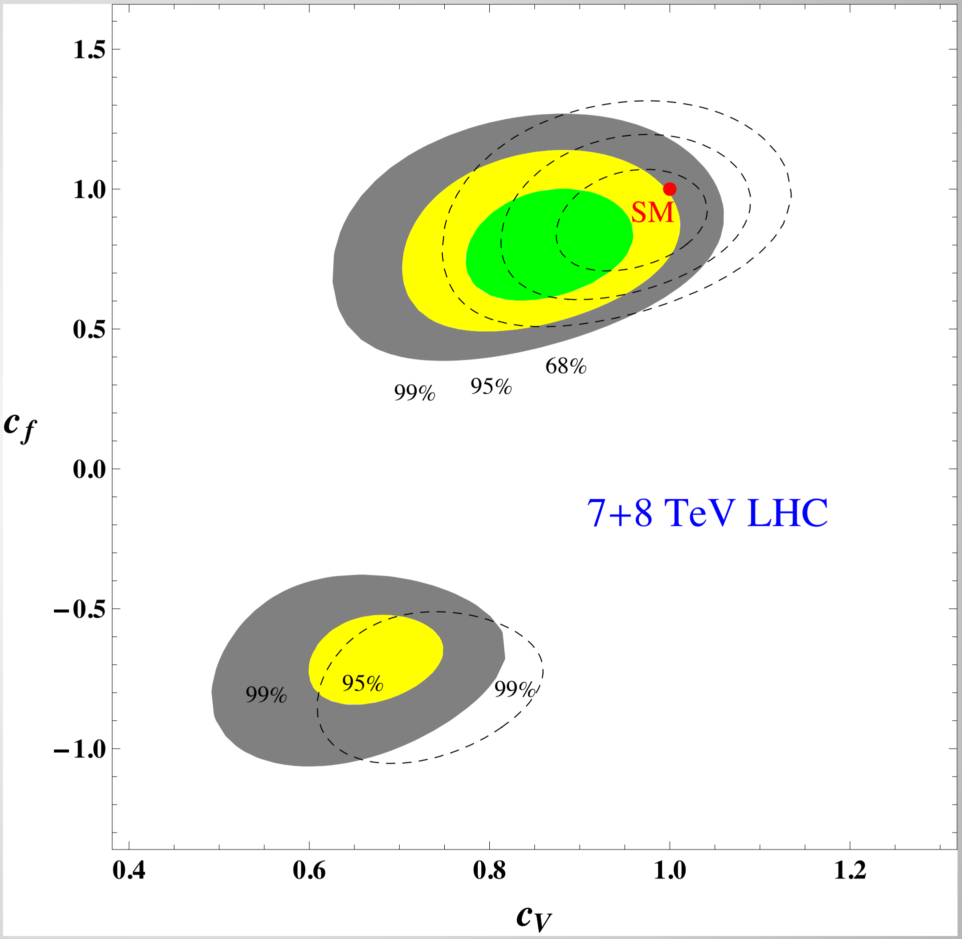
$$\Delta\tilde{\chi}^2 = \tilde{\chi}^2 - \tilde{\chi}^2_{\rm min}(\delta, c_f, c_V|_{\rm opt})$$

➡ « Preferred error » : $\delta|_{\rm opt} = -1$

i.e. Higgs fit used to determine the QCD uncertainty !

Frequentist regions :

(w.r.t. best-fit point)

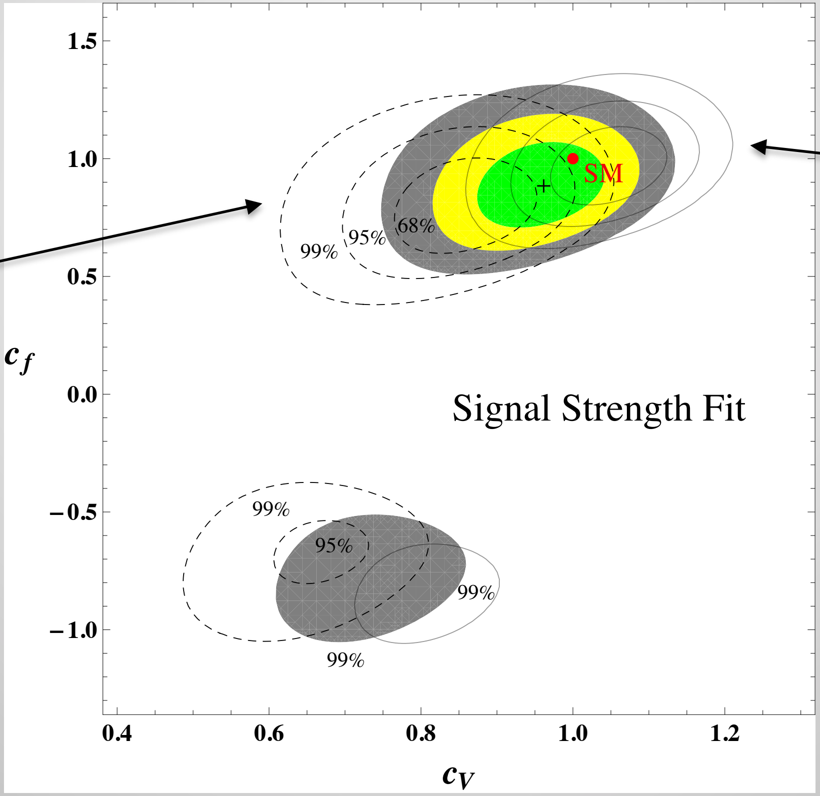


V) The th. uncertainty as a BIAS

Consider the two extreme values of δ to illustrate the QCD error effect on fits :

(no preferred δ value from the Higgs fit)

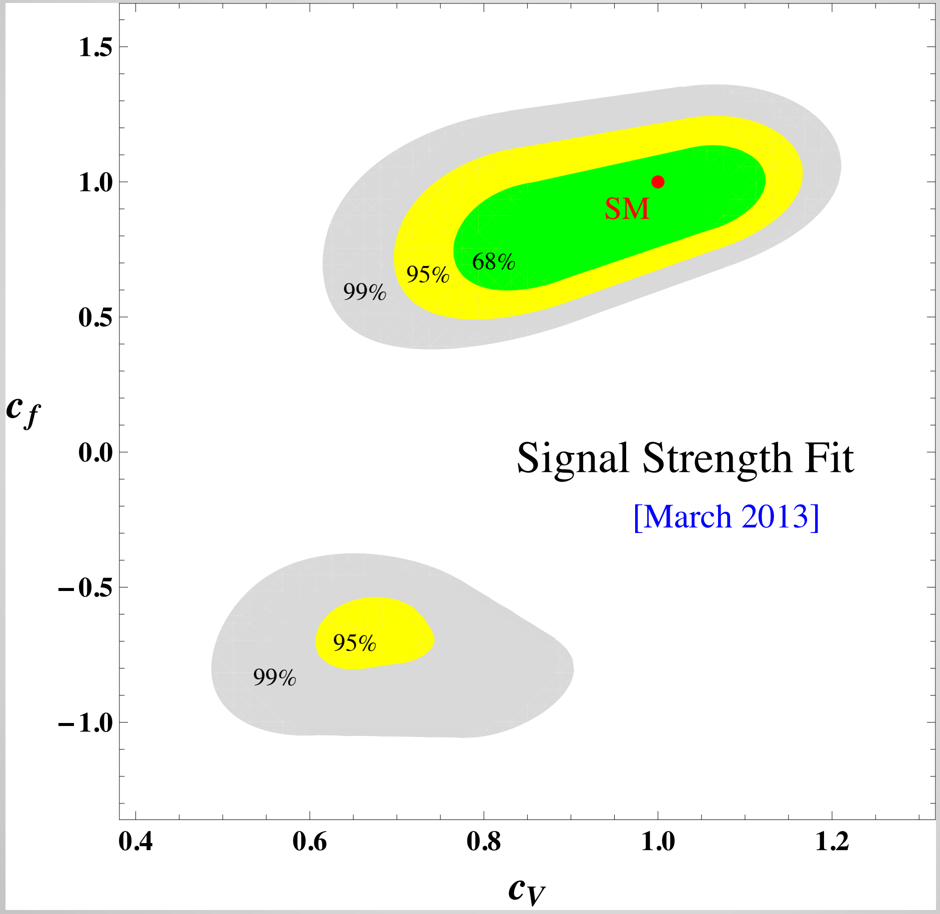
$\delta = -1$



$\delta = +1$

(Frequentist)

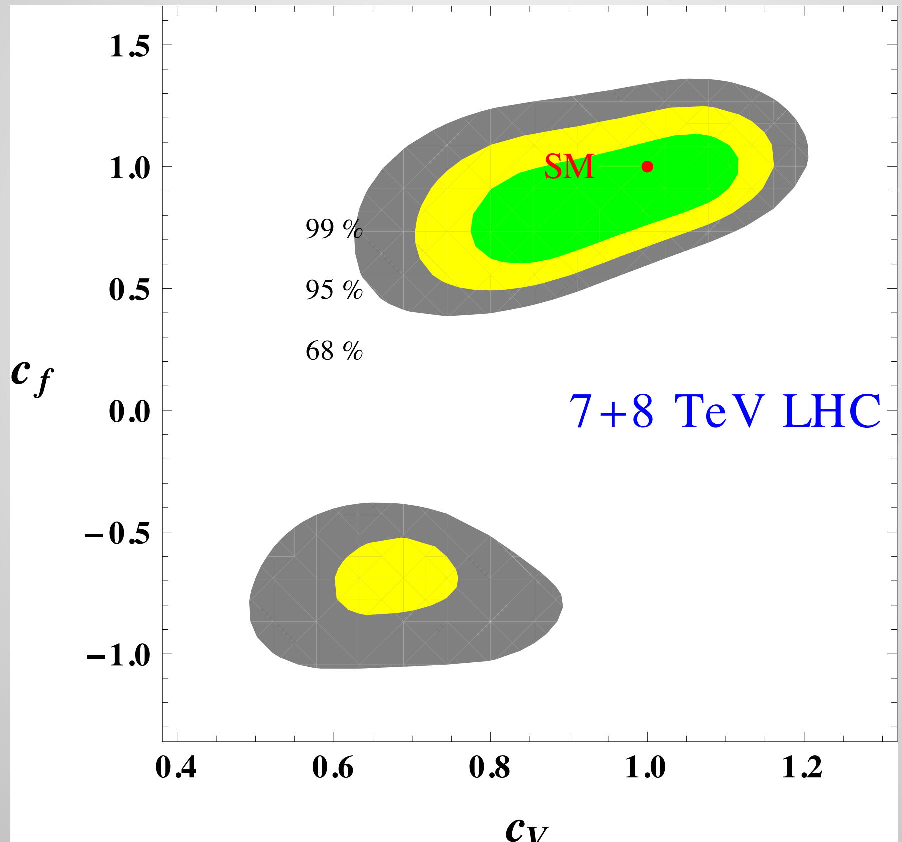
Varying δ continuously …



(Frequentist)

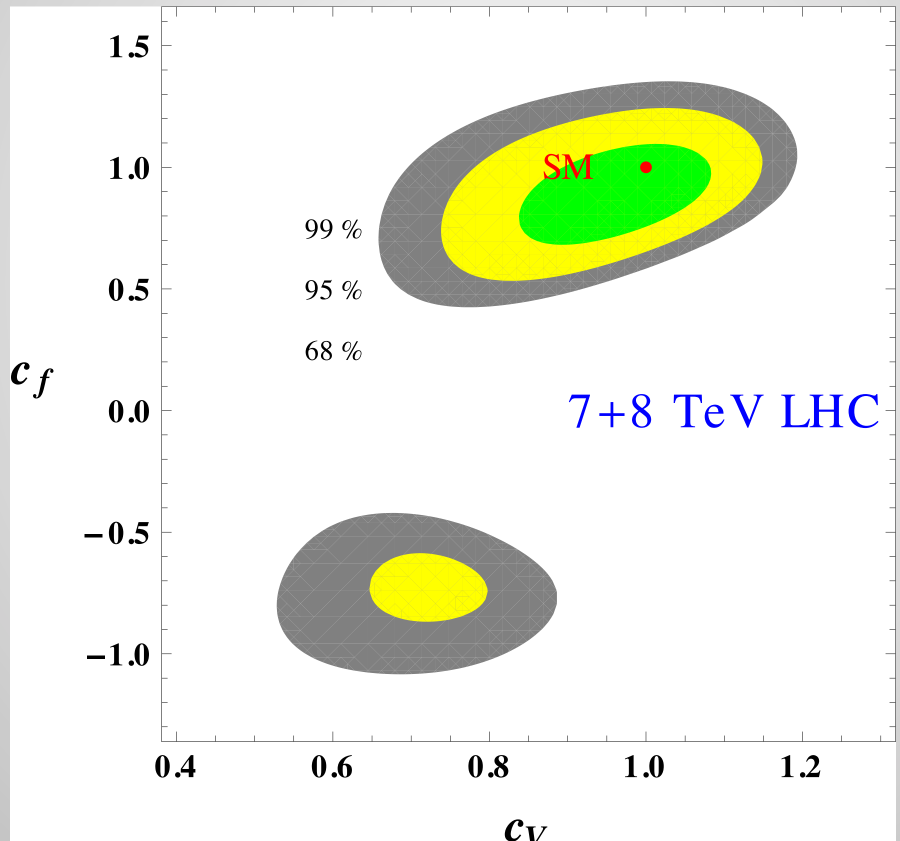
Obtaining this envelop directly through :

$$\Delta\bar{\chi}^2(c_f, c_V) = \mathrm{Min}|_\delta \left\{ \chi^2(c_f, c_V, \delta) - \chi^2_{\mathrm{min}}(c_f, c_V|_{\mathrm{opt}}, \delta) \right\}$$



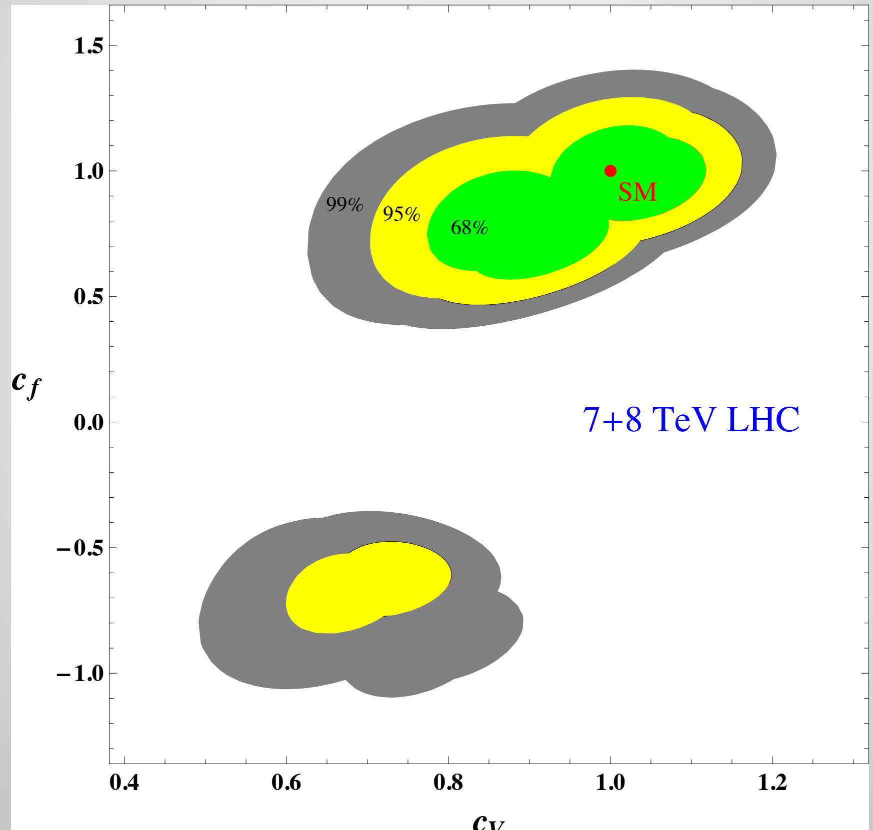
(Frequentist)

The « corresponding » Bayesian approach :



Beyond the 100% correlated case …

Dominant QCD error is from PDF+α_s^2 => $\delta_{ggF,tth} \neq \delta_{VBF,Vh}$



(Frequentist)

Conclusion

The **determination of the Higgs couplings** - *through the signal strength fit* - **depends** significantly on the **statistical** treatment adopted for the **theoretical uncertainties** *(QCD errors on σ_h's & B_h's)*.