

Non-Abelian discrete flavor symmetry from magnetized extra dimensions

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collaborated with
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and Keigo Sumita

INTRODUCTION

- origin of phenomena :
Lorentz inv, gauge syms, supersymmetry, ...etc

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flavor ?

	Observed
(m_u, m_c, m_t)	$(2.3 \times 10^{-3}, 1.28, 1.74 \times 10^2)$
(m_d, m_s, m_b)	$(4.8 \times 10^{-3}, 0.95 \times 10^{-1}, 4.18)$
(m_e, m_μ, m_τ)	$(5.11 \times 10^{-4}, 1.06 \times 10^{-1}, 1.78)$
$ V_{\text{CKM}} $	$\begin{pmatrix} 0.97 & 0.23 & 0.0035 \\ 0.23 & 0.97 & 0.041 \\ 0.0087 & 0.040 & 1.0 \end{pmatrix}$

- quark, charged-lepton :

- mass hierarchies
- small mixing

- neutrino :

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$ V_{\text{MNS}} $	$\begin{pmatrix} 0.82 & 0.55 & 0.16 \\ 0.51 & 0.58 & 0.64 \\ 0.26 & 0.61 & 0.75 \end{pmatrix}$

(Particle Data Group, 2012)

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flavor symmetry as the origin of flavor structures

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- magnetic fluxes →

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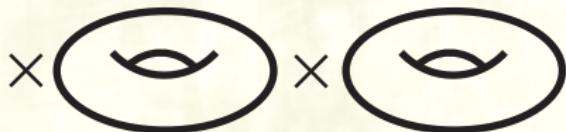
→ **flavor landscape**

(Abe, Kobayashi, Ohki, Sumita, YT, 2013)

flavor symmetry from IIB string theory ?

SPACETIME GEOMETRY

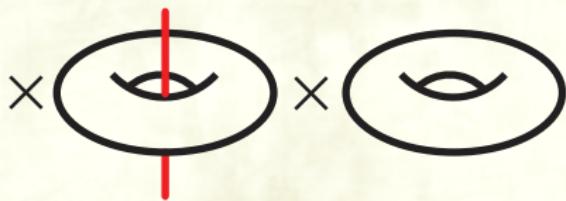
total = **4D**
spacetime = **spacetime**



no flux

SPACETIME GEOMETRY

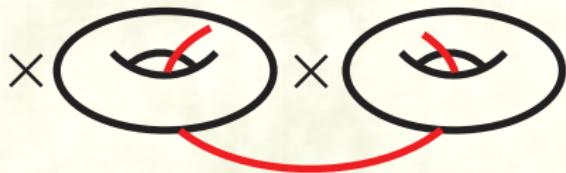
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factorizable flux

SPACETIME GEOMETRY

total = **4D**
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non-factorizable flux

Case 1

factorizable flux

MODEL WITH FACTORIZABLE FLUX

$$\begin{array}{ccc} \textbf{6D} & = & \textbf{4D} \\ \text{spacetime} & & \text{spacetime} \end{array} \times \text{ } \begin{array}{c} \text{ } \\ \text{ } \end{array}$$


- zero-mode :

$$\phi(x, z) = \sum_j \phi^j(x) \otimes \Theta^j(z)$$

$$\Theta^j(z) \equiv \mathcal{N} \cdot e^{i\pi Mz \operatorname{Im} z / \operatorname{Im} \tau} \cdot \vartheta \underbrace{\begin{bmatrix} \frac{I}{M} \\ 0 \end{bmatrix}}_{\text{Jacobi } \vartheta\text{-function}} (Mz, M\tau)$$

$$\# \text{ of } j = M \quad (\# \text{ of flux})$$

three fluxes \Rightarrow three generations

NON-ABELIAN DISCRETE FLAVOR SYMMETRY FROM FACTORIZABLE FLUX

- **Yukawa matrix:**

$$y_{ijk} \propto \sum_{m \in \mathbb{Z}_{M_3}} \delta_{k,i+j+M_3 m} \vartheta \begin{bmatrix} \frac{M_2 i - M_1 j + M_1 M_2 m}{M_1 M_2 M_3} \\ 0 \end{bmatrix} (0, \tau |M_1 M_2 M_3|)$$

- **selection rule**
- **the character** in ϑ -function

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→ three Z_3 generators $X = Z, Z', C$ s.t. $X \Theta^j :$

$$\omega = e^{2\pi i / 3}$$

$$Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad Z' = \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

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Δ(27) symmetry in any 3-point coupling

Case 2

factorizable and non-factorizable flux

MODEL WITH NON-FACTORIZABLE FLUX

$$\begin{array}{ccc} \textbf{8D} & = & \textbf{4D} \\ \textbf{spacetime} & & \textbf{spacetime} \end{array} \times \begin{array}{c} \text{two circles} \\ \text{with flux} \end{array}$$

- zero-mode :

$$\phi(x, \vec{z}) = \sum_{\vec{j}} \phi^{\vec{j}}(x) \otimes \Theta^{\vec{j}}(\vec{z})$$

$$\Theta^{\vec{j}}(\vec{z}, \Omega) = \mathcal{N} \cdot e^{i\pi(\mathbb{N} \cdot \vec{z})(\text{Im } \Omega)^{-1} \text{Im } \vec{z}} \cdot \underbrace{\vartheta \begin{bmatrix} \vec{j} \\ 0 \end{bmatrix} (\mathbb{N} \cdot \vec{z}, \mathbb{N} \cdot \Omega)}_{\text{Riemann } \vartheta\text{-function}}$$

$$\# \text{ of } \vec{j} = \det \mathbb{N} \quad (\text{flux matrix})$$

$\det \mathbb{N} = 3 \Rightarrow$ three generations

GENERATION TYPES (THREE GENS ON T^4)

type 1 :

$$\vec{j} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/3 \\ 0 \end{pmatrix}, \begin{pmatrix} 2/3 \\ 0 \end{pmatrix}$$

type 2 :

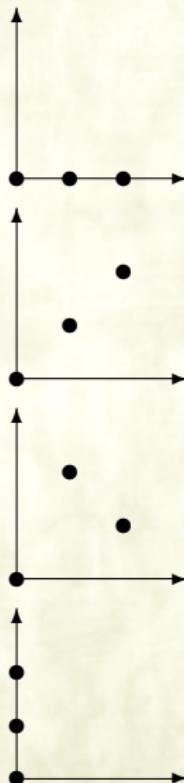
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type 4 :

$$\vec{j} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1/3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2/3 \end{pmatrix}$$



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- Yukawa matrix :

$$Y_{\vec{i}\vec{j}\vec{k}} \propto \sum_{\vec{m}} \delta_{\vec{k}, \mathbb{N}_H^{-1}(\mathbb{N}_L \vec{i} + \mathbb{N}_R \vec{j} + \mathbb{N}_L \vec{m})} \\ \times \int d^2y \left[e^{-\pi \vec{y} (\tilde{\Omega}_L + \mathbb{N}_R \tilde{\Omega}_R + \mathbb{N}_H \Omega) \cdot \vec{y}} \cdot \vartheta \begin{bmatrix} \vec{K} \\ 0 \end{bmatrix} (i \vec{Y} | i \vec{Q}) \right]$$

- selection rule
- the character in \vec{K}

$$(\vec{i} - \vec{j} + \vec{m}) \frac{\mathbb{N}_L (\mathbb{N}_L + \mathbb{N}_R)^{-1} \mathbb{N}_R}{\det \mathbb{N}_L \det \mathbb{N}_R}$$

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$\Delta(27)$ symmetry only if

- same generation type
- $3n$ pairs Higgs model

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Thank you !