1-2 Mass Degeneration in the Leptonic Sector

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· <u>Introduction</u>

On Mar. in 2012, non-zero θ_{13} was discovered @DAYA-BAY. Phys. Rev. Lett. **108** (2012)

Parameter	Best fit	2 σ	3 σ	tri-bi maximal
$\sin\theta_{12}^2$	0.320	0.29-0.35	0.27-0.37	1/3
$\sin\theta_{23}^2$	0.613	0.38-0.66	0.36-0.68	1/2
$\sin\theta_{13}^2$	0.0246	0.019-0.030	0.017-0.033	0
$\Delta m_{21}^2 (10^{-5}\mathrm{eV}^2)$	7.62	7.27-8.01	7.12-8.20	*
$\Delta m_{32}^2 (10^{-5}\mathrm{eV}^2)$	2.55	2.38-2.68	2.31-2.74	*

D. V. Forero et al. Phys. Rev. D **86** (2012)

θ_{13} and θ_{23} do not match the values predicted by tri-bi maximal anymore.
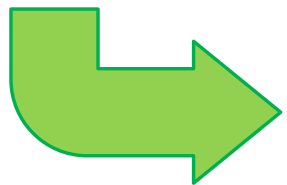
We need to reconsider about the origin of flavour structure!

Motivation

We would like to understand the origin of flavour structure
by introducing some symmetries,
BUT…

Even if such symmetries were ensured at very high energy,
it should be broken or hidden at low energy.

Furthermore, we don't know how to break these symmetries.

Spontaneously? or Explicitly?   Slightly? or Grossly?

What is the guidepost?

·<u>Motivation</u>

One of the possible guide for searching such broken symmetries

$\Longrightarrow$ 't Hooft's criterion (1980)

If a new symmetry appears in the model when arbitrary parameters set to zero, such parameters are naturally small.

For example ; Lepton number symmetry

In the limit of massless neutrinos in the effective theory, the lepton number symmetry appears.

$\dfrac{Y^2 v^2}{\Lambda_\nu} \bar{\nu}^C \nu$: this effective term violates lepton number 2 units.

- # <u>Motivation</u>

Candidates of small parameters in lepton sector

1. $\theta_{13} \ll \theta_{12}, \theta_{23}$
2. $|\theta_{23} - 45°| \ll \theta_{23}$ $\Bigg\}$ μ-τ symmetry

T. Fukuyama and H. Nishiura (1997)
G. Altarelli and F. Feruglio (1998) etc..

3. $\Delta m_{12}^2 = (m_2^\nu)^2 - (m_1^\nu)^2 \ll \Delta m_{23}^2 = |(m_3^\nu)^2 - (m_2^\nu)^2|$

4. $m_1^{u,d,\ell}, \ m_2^{u,d,\ell} \ll m_3^{u,d,\ell}$

This work

5. $\theta_{ij}^{\text{quark}} \ll \theta_{ij}^{\text{lepton}}$

What happens when we focus on these small parameters?

Motivation of Orthogonal Symmetries

Here, we take solar neutrino mass squared difference to be zero ; $\Delta m_{12}^2 = 0$.

In this limit, the $O(2)$ symmetry appears in the neutrino sector :

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} m_D & & \\ & m_D & \\ & & m_3 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} m_D & & \\ & m_D & \\ & & m_3 \end{pmatrix},$$

In other words, the 1st and 2nd generation of neutrinos
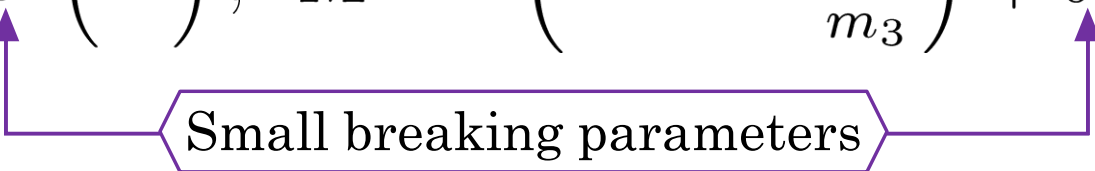are embedded into a doublet representation of $O(2)$, e.g., $\mathbf{2}_n$.

This type of mass spectra cannot apply to charged fermion sector,
because they are strongly hierarchical between the 3rd generation and others.

For now, we assume $m_D^f \to 0$ ➩ $M^f = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_3^f \end{pmatrix}$ $(f = u, d, e)$

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Motivation of Orthogonal Symmetries

To construct realistic mass matrices, we introduce small breaking terms as

$$M^f = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_3^f \end{pmatrix} + \delta^f \left(\quad \right), \quad M^\nu = \begin{pmatrix} m_D & & \\ & m_D & \\ & & m_3 \end{pmatrix} + \delta^\nu \left(\quad \right)$$

Small breaking parameters

Charged fermion sector

In this situation, both of 2-3 and 1-3 mixings depend on δ^f. Small

On the other hand, 1-2 one does not strongly depend on δ^f. Need not so small

$\Rightarrow$ Compatible with mixing pattern of CKM matrix

Neutrino sector

Because neutrino mass spectrum might be milder than charged fermion sector, the leading term is close to the identity matrix. Weakly depend on δ^ν
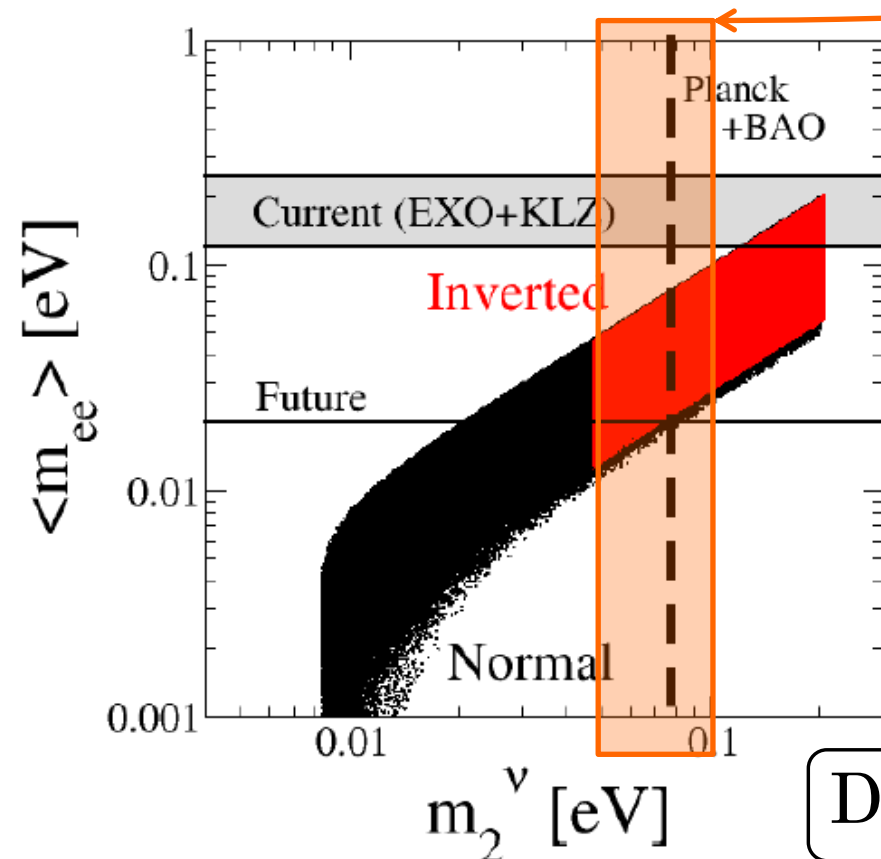
$\Rightarrow$ Large mixing in the PMNS matrix

Degeneracy of the 1st and 2nd generation seems to a good starting point.

· <u>Motivation of Degenerated Masses</u>

Neutrinoless double beta decay

If the neutrino masses are degenerated, e.g. $0.05\text{eV} < m_2^\nu < 0.1\text{eV}$, the effective mass, $\langle m_{ee} \rangle$, can be estimated as

In both case, the effective mass is finite

$0\nu\beta\beta$ decay can be observed in near future?

OR

Such degenerated region will be excluded?

Actually, such region is being excluded by Planck!

Degenerated region is very attractive!

· <u>Model</u>

Charge assignments (The simplest extension of the MSM)

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

where $I = 1\,, 2$ and $i = 1\,, 2\,, 3$ denote the indices of generations.

- L and ℓ represent the left- and right-handed leptons, respectively.

- H means SM Higgs field.

- S is SM gauge singlet scalar field.

- The VEVs of singlet scalar fields can be considered into two patterns.

Property of our Model

- Real scalar case

$$\frac{1}{v}M^\ell = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^0 & y_2^0 & y_3^0 \end{pmatrix} + \frac{1}{\Lambda_F^2} \begin{pmatrix} y_1\delta_1 & y_2\delta_1 & y_3\delta_1 \\ y_1\delta_2 & y_2\delta_2 & y_3\delta_2 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\frac{\Lambda_\nu}{v^2}M^\nu = \begin{pmatrix} f_\nu & 0 & 0 \\ 0 & f_\nu & 0 \\ 0 & 0 & f'_\nu \end{pmatrix} + \frac{1}{\Lambda_F^2} \begin{pmatrix} 0 & 0 & g_\nu\delta_1 \\ 0 & 0 & g_\nu\delta_2 \\ g_\nu\delta_1 & g_\nu\delta_2 & 0 \end{pmatrix} + \frac{1}{\Lambda_F^4} \begin{pmatrix} h_\nu(\delta_1^2 - \delta_2^2) & h_\nu 2\delta_1\delta_2 & 0 \\ h_\nu 2\delta_1\delta_2 & -h_\nu(\delta_1^2 - \delta_2^2) & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

where δ_i are functions of the VEVs of singlet scalar.

·Properties of our Model

·In the basis of charged lepton mass matrix is diagonalized,

$$\frac{1}{v^2}(V^\ell)^\dagger M^\ell (M^\ell)^\dagger V^\ell \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{YY^0 - \Gamma^0_{}\,s^2}{Y^0}\frac{\Lambda^4_F}{} & 0 \\ 0 & 0 & Y^0 \end{pmatrix},$$

Electron is massless

$$\frac{\Lambda_\nu}{v^2}(V^\ell)^T M^\nu V^\ell = \begin{pmatrix} \bar{M}^\nu_{11} & 0 & 0 \\ 0 & \bar{M}^\nu_{22} & \bar{M}^\nu_{23} \\ 0 & M^\nu_{32} & M^\nu_{33} \end{pmatrix}.$$

$\theta^{\mathrm{PMNS}}_{12}$ and $\theta^{\mathrm{PMNS}}_{13}$ are zero

These features are by remaining Z_2 symmetry after breaking D_N symmetry.

Smallness of electron mass and $\theta^{\mathrm{PMNS}}_{13}$ is related with this Z_2 symmetry.

·Property of our Model

- Complex scalar case

The remained Z_2 symmetry is violated by complex VEVs of S_I.

Electron mass is induced as

$$m_e^2 \simeq (\delta\phi)^2 \sum_i |Y_i'|^2 \frac{\delta_2^2}{\Lambda_F^4} v^2 \,,$$

while 12 and 13 elements of the neutrino mass matrix are given as

$$\frac{\Lambda_\nu}{v^2} \bar{M}_{12}^\nu \simeq i\delta\phi \frac{\delta_2 \delta_s}{\Lambda_F^4} \tilde{H}_\nu' + \mathcal{O}\left(\frac{(\delta\phi)^2}{\Lambda_F^4}\right) \,,$$

$$\frac{\Lambda_\nu}{v^2} \bar{M}_{13}^\nu \simeq i\delta\phi \frac{\delta_2}{\Lambda_F^2} \tilde{G}_\nu' + \mathcal{O}\left(\frac{(\delta\phi)^2}{\Lambda_F^2}\right) \,,$$

where $\tilde{H}_\nu'$ and $\tilde{G}_\nu'$ are functions of couplings and phases.

$\cdot$ <u>Conclusions</u>

- $\cdot$ We discuss a model in which 1st and 2nd generations are degenerated motivated by $\Delta m_{12}^2 = 0$.

  - $\cdot$ In this model, mixing patterns of quarks and leptons are realized naturally due to dependence on small breaking parameter δ.

- $\cdot$ To illustrate such a degenerate model, we consider a D_N flavor symmetric model which is a discrete subgroup of $O(2)$.

  - $\cdot$ We introduce one new D_N doublet and SM gauge singlet scalar, S_I.

- $\cdot$ Due to residual Z_2 symmetry, electron mass and $\theta_{13}^{\mathrm{PMNS}}$ tend to be small.

- $\cdot$ Our model has a lot of parameters therefore, we cannot predict anything. However, we need not to assume hierarchy among the coupling constants in order to reconstruct experimental data.

Thank you for your attention!

Back up

· $\underline{\mathrm{D_N\ flavour\ symmetry}}$

$\mathrm{D_N}$ flavour symmetry is a dihedral group and discrete subgroup of $O(2)$.

This symmetry has two singlet and some doublet irreducible representations.

- · Multiplication rules (same as $O(2)$)

$$\underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{\mathbf{2}_n} \otimes \underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}}_{\mathbf{2}_n} = \underbrace{(x_1 y_1 + x_2 y_2)}_{\mathbf{1}} + \underbrace{(x_1 y_2 - x_2 y_1)}_{\mathbf{1'}} + \underbrace{\begin{pmatrix} x_1 y_1 - x_2 y_2 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}}_{\mathbf{2}_{2n}}$$

$$\underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{\mathbf{2}_n} \otimes \underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}}_{\mathbf{2}_m} = \underbrace{\begin{pmatrix} x_1 y_1 + x_2 y_2 \\ x_1 y_2 - x_2 y_1 \end{pmatrix}}_{\mathbf{2}_{m-n}} + \underbrace{\begin{pmatrix} x_1 y_1 - x_2 y_2 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}}_{\mathbf{2}_{n+m}}$$

These extra charges are a little bit important for the model.

· <u>Model</u>

Lagrangian invariant under D_N (up-to NNLO)

$$\mathcal{L}_f = y_i^0 \, \overline{L}_3 H \ell_i + \frac{y_i}{\Lambda_F^2} \overline{L}_I H \ell_i (S^2)_I + \frac{y_i'}{\Lambda_F^2} \overline{L}_I H \ell_i (S^{*2})_I + \frac{y_i''}{\Lambda_F^2} \overline{L}_I H \ell_i (|S|^2)_I$$

$$+ \frac{f_\nu}{\Lambda_\nu} L_I L_I H H + \frac{f_\nu'}{\Lambda_\nu} L_3 L_3 H H$$

Froggatt-Nielsen like structure due to D_N

$$+ \frac{g_\nu}{\Lambda_\nu \Lambda_F^2} L_3 L_I H H (S^2)_I + \frac{g_\nu'}{\Lambda_\nu \Lambda_F^2} L_3 L_I H H (S^{*2})_I + \frac{g_\nu''}{\Lambda_\nu \Lambda_F^2} L_3 L_I H H (|S|^2)_I$$

$$+ \frac{h_\nu}{\Lambda_\nu \Lambda_F^4} (L_J L_K)_I H H (S^4)_I + \frac{h_\nu'}{\Lambda_\nu \Lambda_F^4} (L_J L_K)_I H H (S^{*4})_I + \frac{h_\nu''}{\Lambda_\nu \Lambda_F^4} (L_J L_K)_I H H (|S|^4)_I$$

$$+ \frac{h_\nu'''}{\Lambda_\nu \Lambda_F^4} (L_J L_K)_I H H (S^2 |S|^2)_I + \frac{h_\nu''''}{\Lambda_\nu \Lambda_F^4} (L_J L_K)_I H H (S^{*2} |S|^2)_I,$$

Dimension five Weinberg operator with an energy scale Λ_ν.

A typical energy scale of D_N symmetry

After scalar fields have vacuum expectation values,

mass matrices are given as follows :

$$
\frac{1}{v}M^\ell = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^0 & y_2^0 & y_3^0 \end{pmatrix} + \frac{1}{\Lambda_F^2}\left[\begin{pmatrix} y_1\delta_1 & y_2\delta_1 & y_3\delta_1 \\ y_1\delta_2 & y_2\delta_2 & y_3\delta_2 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} y_1'\delta_1^* & y_2'\delta_1^* & y_3'\delta_1^* \\ y_1'\delta_2^* & y_2'\delta_2^* & y_3'\delta_2^* \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} y_1''\delta_a & y_2''\delta_a & y_3''\delta_a \\ y_1''\delta_b & y_2''\delta_b & y_3''\delta_b \\ 0 & 0 & 0 \end{pmatrix} \right],
$$

$$
\frac{\Lambda_\nu}{v^2}\mathcal{M}_\nu = \begin{pmatrix} f_\nu & 0 & 0 \\ 0 & f_\nu & 0 \\ 0 & 0 & f_\nu' \end{pmatrix} + \frac{1}{\Lambda_F^2}\begin{pmatrix} 0 & 0 & g_\nu\delta_1 + g_\nu'\delta_1^* + g_\nu''\delta_a \\ 0 & 0 & g_\nu\delta_2 + g_\nu'\delta_2^* + g_\nu''\delta_b \\ g_\nu\delta_1 + g_\nu'\delta_1^* + g_\nu''\delta_a & g_\nu\delta_2 + g_\nu'\delta_2^* + g_\nu''\delta_b & 0 \end{pmatrix}
$$

$$
+ \frac{1}{\Lambda_F^4}\begin{pmatrix} h_\nu\epsilon_1 + h_\nu'\epsilon_1^* + h_\nu''\epsilon_a + h_\nu'''\epsilon_x + h_\nu''''\epsilon_x^* & h_\nu\epsilon_2 + h_\nu'\epsilon_2^* + h_\nu''\epsilon_b + h_\nu'''\epsilon_y + h_\nu''''\epsilon_y^* & 0 \\ h_\nu\epsilon_2 + h_\nu'\epsilon_2^* + h_\nu''\epsilon_b + h_\nu'''\epsilon_y + h_\nu''''\epsilon_y^* & -(h_\nu\epsilon_1 + h_\nu'\epsilon_1^* + h_\nu''\epsilon_a + h_\nu'''\epsilon_x + h_\nu''''\epsilon_x^*) & 0 \\ 0 & 0 & 0 \end{pmatrix},
$$

where, δ s and ϵ s are some functions of s_i and ϕ_i .($\langle S_I \rangle = (s_1 e^{i\phi_1} \quad s_2 e^{i\phi_2})^T$.)

There are so many parameters = No prediction.

But we note that there is no strong hierarchy among each couplings.

Feature of mass matrices

The mass matrix for charged lepton sector

$$\frac{1}{v}M^\ell = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^0 & y_2^0 & y_3^0 \end{pmatrix} \text{ rank=1} \quad \Longrightarrow \quad m_\tau$$

$$+ \frac{1}{\Lambda_F^2} \left[\begin{pmatrix} y_1\delta_1 & y_2\delta_1 & y_3\delta_1 \\ y_1\delta_2 & y_2\delta_2 & y_3\delta_2 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} y_1'\delta_1^* & y_2'\delta_1^* & y_3'\delta_1^* \\ y_1'\delta_2^* & y_2'\delta_2^* & y_3'\delta_2^* \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} y_1''\delta_a & y_2''\delta_a & y_3''\delta_a \\ y_1''\delta_b & y_2''\delta_b & y_3''\delta_b \\ 0 & 0 & 0 \end{pmatrix} \right],$$

If VEV of S_I is real, the second term is also rank=1 matrix, therefore we can not obtain the electron mass.

Here, we suppose complex VEVs for the scalar field, the second term is rank=2 matrix.

• Feature of mass matrices

This model has a relationship between electron mass and $\theta_{13}^{\mathrm{PMNS}}$.

When $\phi_2 = \phi_1 + \delta\phi \ (\delta\phi \ll 1)$,
the second term of charged lepton mass matrix can be divided into

$$\left(\begin{bmatrix} y_1 e^{2i\phi_1} + y_1' e^{-2i\phi_1} + y_1'' \end{bmatrix} (s_1^2 - s_2^2) \quad \cdots \quad \cdots \\ \begin{bmatrix} y_1 e^{2i\phi_1} + y_1' e^{-2i\phi_1} + y_1'' \end{bmatrix} 2s_1 s_2 \quad \cdots \quad \cdots \\ 0 \qquad\qquad \cdots \quad \cdots \end{bmatrix} \right) + i\delta\phi \left(\begin{bmatrix} -\begin{bmatrix} y_1 e^{2i\phi_1} - y_1' e^{-2i\phi_1} \end{bmatrix} 2s_2^2 \quad \cdots \quad \cdots \\ \begin{bmatrix} y_1 e^{2i\phi_1} - y_1' e^{-2i\phi_1} \end{bmatrix} 2s_1 s_2 \quad \cdots \quad \cdots \\ 0 \qquad\qquad \cdots \quad \cdots \end{bmatrix} \right) ,$$

Each term is rank = 1 matrix.

Correspond to muon mass   Correspond to electron mass

Small electron mass is induced by spontaneous CP violation, that is, non-vanishing $\delta\phi$.

· <u>Parameter sets (Normal hierarchy case)</u>

$$\frac{1}{v}M^\ell = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^0 & y_2^0 & y_3^0 \end{pmatrix} + \frac{1}{\Lambda_F^2}\left[\begin{pmatrix} y_1\delta_1 & y_2\delta_1 & y_3\delta_1 \\ y_1\delta_2 & y_2\delta_2 & y_3\delta_2 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} y_1'\delta_1^* & y_2'\delta_1^* & y_3'\delta_1^* \\ y_1'\delta_2^* & y_2'\delta_2^* & y_3'\delta_2^* \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} y_1''\delta_a & y_2''\delta_a & y_3''\delta_a \\ y_1''\delta_b & y_2''\delta_b & y_3''\delta_b \\ 0 & 0 & 0 \end{pmatrix}\right],$$

$$y_3^0 = 1.0,\ y_1^0 = y_2^0 = 1.2,\ y_1 = -y_2 = y_3 = y_1' = y_2' = -y_3' = 0.8,$$
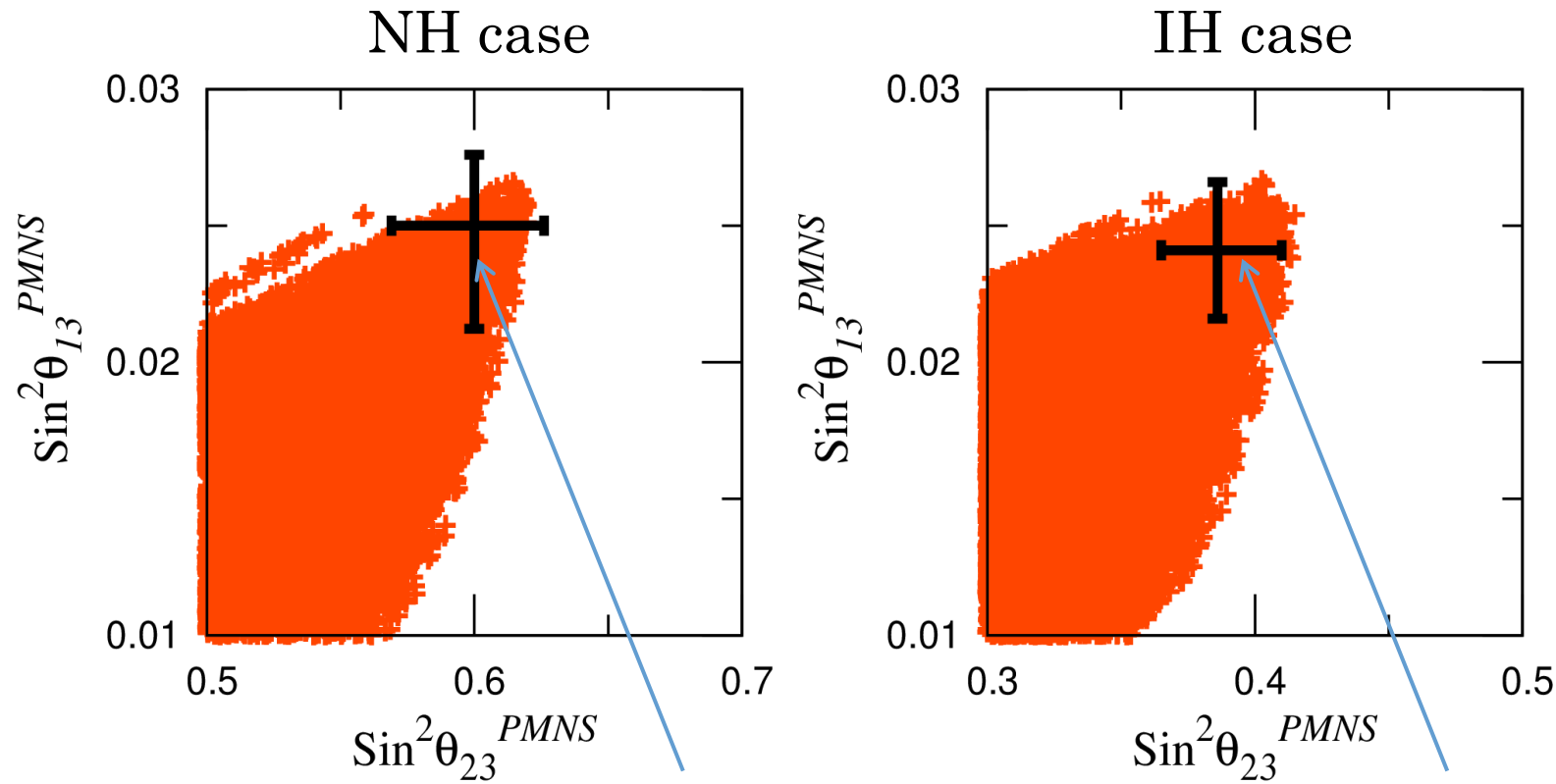
$$y_1'' = -y_2'' = -y_3'' = 0.8 \sim 1.3,$$

$$\frac{\Lambda_\nu}{v^2}\mathcal{M}_\nu = \begin{pmatrix} f_\nu & 0 & 0 \\ 0 & f_\nu & 0 \\ 0 & 0 & f_\nu' \end{pmatrix} + \frac{1}{\Lambda_F^2}\begin{pmatrix} 0 & 0 & g_\nu\delta_1 + g_\nu'\delta_1^* + g_\nu''\delta_a \\ 0 & 0 & g_\nu\delta_2 + g_\nu'\delta_2^* + g_\nu''\delta_b \\ g_\nu\delta_1 + g_\nu'\delta_1^* + g_\nu''\delta_a & g_\nu\delta_2 + g_\nu'\delta_2^* + g_\nu''\delta_b & 0 \end{pmatrix}$$

$$+ \frac{1}{\Lambda_F^4}\begin{pmatrix} h_\nu\epsilon_1 + h_\nu'\epsilon_1^* + h_\nu''\epsilon_a + h_\nu'''\epsilon_x + h_\nu''''\epsilon_x^* & h_\nu\epsilon_2 + h_\nu'\epsilon_2^* + h_\nu''\epsilon_b + h_\nu'''\epsilon_y + h_\nu''''\epsilon_y^* & 0 \\ h_\nu\epsilon_2 + h_\nu'\epsilon_2^* + h_\nu''\epsilon_b + h_\nu'''\epsilon_y + h_\nu''''\epsilon_y^* & -(h_\nu\epsilon_1 + h_\nu'\epsilon_1^* + h_\nu''\epsilon_a + h_\nu'''\epsilon_x + h_\nu''''\epsilon_x^*) & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$f_\nu' = 1.0,\ f_\nu = 0.90 \sim 0.95,\ g_\nu = g_\nu'' = 0.9,\ g_\nu' = 0.8 \sim 1.3,$$

$$h_\nu = h_\nu' = h_\nu'' = -h_\nu''' = -h_\nu'''' = 0.8 \sim 1.3,$$

$$\frac{s_{1,2}}{\Lambda_F} = 0.15 \sim 0.30,\ \phi_{1,2} = 0 \sim 2\pi,$$

Numerical Analysis (θ_{13}-θ_{23} plane)

NH case

IH case

Fogli *et al.* 1205.5254

Forero *et al.* PRD **86**

Constraints

- m_e/m_μ, m_μ/m_τ and θ_{12} in 1σ range
- $\Sigma m_i < 0.6\text{eV}$

Nearly maximal θ_{23} and relatively large θ_{13} are successfully obtained.

• Possibility of Spontaneous CP Violation

We assume that the singlet scalar fields were completely decoupled from the theory at high energy.

➡ We investigate only the potential of the singlet scalar

Scalar potential

$$V_S = \alpha_S(s_1^2 + s_2^2) + \alpha_S'(s_1^2 \cos 2\phi_1 + s_2^2 \cos 2\phi_2) + \beta_S^a(s_1^2 + s_2^2)^2$$

$$- 4\beta_S^b s_1^2 s_2^2 \sin^2(\phi_1 - \phi_2) + \beta_S^c\{(s_1^2 - s_2^2)^2 + 4s_1^2 s_2^2 \cos^2(\phi_1 - \phi_2)\}$$

$$+ \beta_S'\left[s_1^4 \cos 4\phi_1 + s_2^4 \cos 4\phi_2 + 2s_1^2 s_2^2 \cos[2(\phi_1 + \phi_2)]\right]$$

$$+ \gamma_S(s_1^2 + s_2^2)(s_1^2 \cos 2\phi_1 + s_2^2 \cos 2\phi_2).$$

· <u>Possibility of Spontaneous CP Violation</u>

The minimization conditions respect with the phases are given as

$$\frac{\partial V_S}{\partial \phi_1} = -2s_1^2 \left[\left\{ \alpha_S' + \gamma_S \left(s_1^2 + s_2^2 \right) \right\} \sin 2\phi_1 + 2(\beta_S^b + \beta_S^c)s_2^2 \sin \left[2 \left(\phi_1 - \phi_2 \right) \right] \right.$$

$$\left. + 2\beta_S' \left(s_1^2 \sin 4\phi_1 + s_2^2 \sin \left[2 \left(\phi_1 + \phi_2 \right) \right] \right) \right] = 0 \,,$$

$$\frac{\partial V_S}{\partial \phi_2} = -2s_2^2 \left[\left\{ \alpha_S' + \gamma_S \left(s_1^2 + s_2^2 \right) \right\} \sin 2\phi_2 - 2(\beta_S^b + \beta_S^c)s_1^2 \sin \left[2 \left(\phi_1 - \phi_2 \right) \right] \right.$$

$$\left. + 2\beta_S' \left(s_2^2 \sin 4\phi_2 + s_1^2 \sin \left[2 \left(\phi_1 + \phi_2 \right) \right] \right) \right] = 0 \,.$$

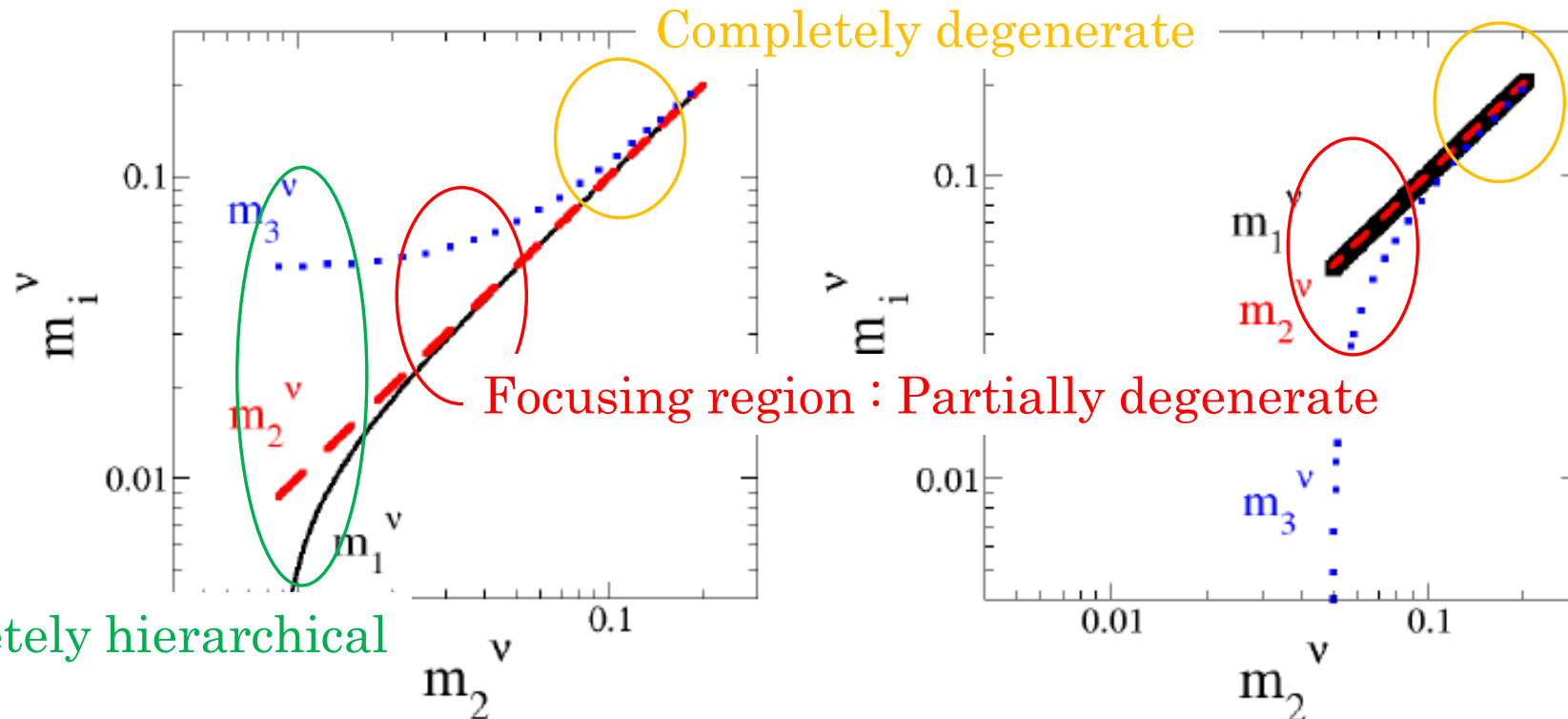For simplicity, we set $\alpha_S' = \beta_S' = \gamma_S = 0$.
Then we get,

$$2(\beta_S^b + \beta_S^c)s_2^2 \sin[2(\phi_1 - \phi_2)] = 0, \quad 2(\beta_S^b + \beta_S^c)s_1^2 \sin[2(\phi_1 - \phi_2)] = 0.$$

These condition suppose $\phi_1 = \phi_2$.

· <u>Hierarchy among neutrino masses</u>

Normal Hierarchy

Inverted Hierarchy

Completely degenerate

Focusing region : Partially degenerate

Completely hierarchical

Fixed: $\Delta m_{12}^2 = 7.54 \times 10^{-5}\ eV^2$ $\Delta m_{23}^2 = 2.43(2) \times 10^{-3}\ eV^2$

[Fogli, *etal*, PRD86(2012)]

T. Araki @26th neutrino conference