

Minimal Fine Tuning in Supersymmetric Higgs Inflation

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Introduction

- Recent experimental information such as the Planck satellite results provides cosmological parameters with increasing accuracy.
- This enables us to investigate numerical aspects of concrete particle physics model of inflation.
- Among various models of inflation, we here adopt a supersymmetric Higgs inflation model to investigate its quantitative aspects.

D-flat direction

$$V_{\text{Higgs}} = \mu^2(|H_u^0|^2 + |H_u^+|^2) + \mu^2(|H_d^0|^2 + |H_d^-|^2) \quad \text{F term}$$

$$+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2$$

$$+ \frac{1}{2}g^2|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2$$

D term

$$+ m_{H_u}^2(|H_u^0|^2 + |H_u^+|^2) + m_{H_d}^2(|H_d^0|^2 + |H_d^-|^2)$$

$$+ [b(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.}]$$

soft term

$$\text{D-flat direction : } H_u = \frac{1}{\sqrt{2}}(\Phi, 0)^T, \quad H_d = \frac{1}{\sqrt{2}}(0, \Phi)^T$$

D-flat direction

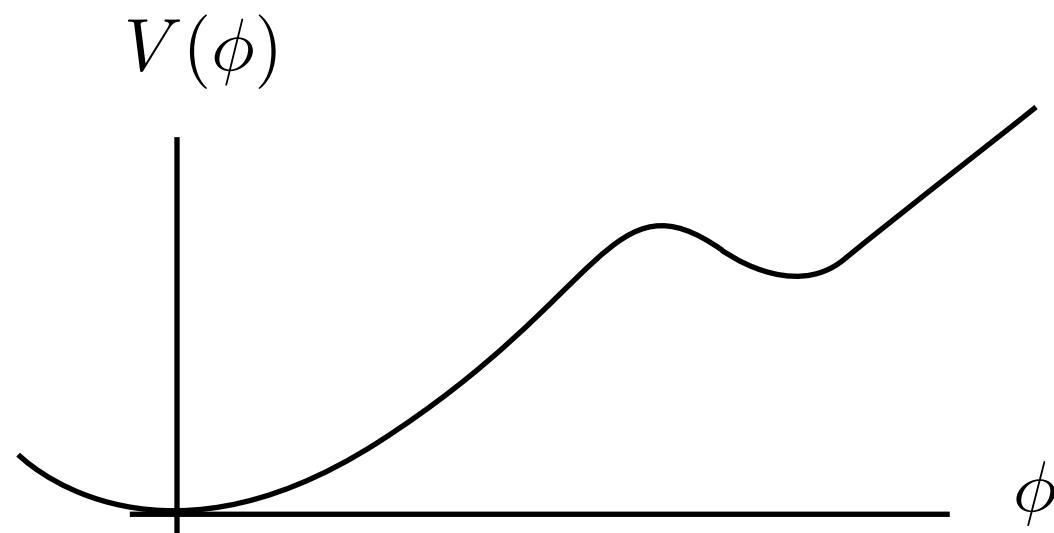
$$V_{\text{Higgs}} = \mu^2(|H_u^0|^2 + |H_u^+|^2) + \mu^2(|H_d^0|^2 + |H_d^-|^2) \quad \text{F term}$$

$$0 = \left\{ \begin{array}{l} +\frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \\ +\frac{1}{2}g^2|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 \\ +m_{H_u}^2(|H_u^0|^2 + |H_u^+|^2) + m_{H_d}^2(|H_d^0|^2 + |H_d^-|^2) \\ +[b(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.}] \end{array} \right. \begin{array}{l} \text{D term} \\ \text{soft term} \end{array}$$

$$\text{D-flat direction : } H_u = \frac{1}{\sqrt{2}}(\Phi, 0)^T, \quad H_d = \frac{1}{\sqrt{2}}(0, \Phi)^T$$

MSSM Higgs Inflation

- Configuration of the D-flat direction in Higgs potential is determined by soft terms(quadratic) and higher dim. op.
- If soft terms and higher dim. terms are cancel successfully, there exist very flat region in the potential and inflation occurs.
→fine tunings of parameters are needed.



Higgs Potential

Superpotential : $\mathcal{W} = \mu \mathcal{H}_u \cdot \mathcal{H}_d + \frac{\lambda}{2} \frac{(\mathcal{H}_u \cdot \mathcal{H}_d)^2}{M_{\text{pl}}}$

Flat direction : $H_u = \frac{1}{\sqrt{2}}(\phi e^{i\theta}, 0)^T, \quad H_d = \frac{1}{\sqrt{2}}(0, \phi e^{i\theta})^T$

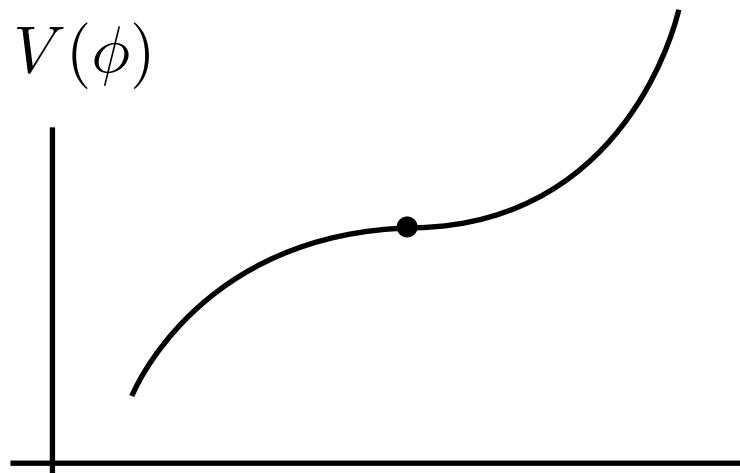
Higgs potential : $V(\phi, \theta) = \frac{1}{2}m^2(\theta)\phi^2 - 2\lambda\mu\cos(2\theta)\phi^4 + 2\lambda^2\phi^6$
 $m^2(\theta) = \frac{1}{2}(2\mu^2 + m_{H_u}^2 + m_{H_d}^2 - 2b\cos 2\theta)$

Along the angular direction, potential is minimized at $\theta = 0$

Higgs potential : $V(\phi) = \frac{1}{2}m_0^2\phi^2 - \frac{\lambda\mu}{4M_{\text{pl}}}\phi^4 + \frac{\lambda^2}{32M_{\text{pl}}^2}\phi^6$

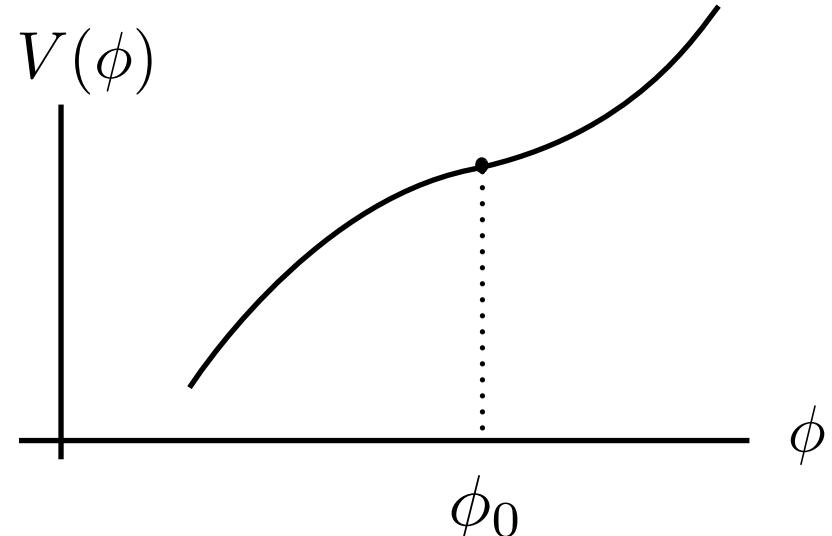
$$m_0^2 = \frac{1}{2}(2\mu^2 + m_{H_u}^2 + m_{H_d}^2 - 2b) \sim m_{\text{soft}}^2$$

Higgs Potential



$$V(\phi) = \frac{1}{2}m_0^2\phi^2 - \frac{\lambda\mu}{4M_{\text{pl}}}\phi^4 + \frac{\lambda^2}{32M_{\text{pl}}^2}\phi^6$$

A saddle point exist when the parameter relation $3m_0^2 = 4\mu^2$ holds.



We define α as

$$3m_0^2 = 4\mu^2(1 + 8\alpha^2)$$

If $\alpha^2 \ll 1$, slow-roll parameters for inflation are very small near the inflection point ϕ_0 .

MSSM Higgs Inflation

With the parameters of $X \equiv \alpha(\frac{M_P}{m_0})$ and λ

A. Chatterjee and A. Mazumdar, (2011)

$$\text{Power Spectrum : } \sqrt{A_s} \simeq \frac{1}{72\sqrt{6}\pi} \frac{1}{X^2\lambda} \sin^2(6\sqrt{6}\mathcal{N}X\lambda)$$

$$\text{Spectral index : } n_s \simeq 1 - 24\sqrt{6}X\lambda \cot(6\sqrt{6}\mathcal{N}X\lambda)$$

To realize $\sqrt{A_s}_{\text{exp}} \simeq 4.69 \times 10^{-5}$

we need.. $X \lesssim 10^4$ $\lambda \sim 10^{-7}$

Allowed value of X has an upper bound !

For example..

$$X_{\max} = 2.051 \times 10^4 \quad \text{for } \mathcal{N} = 50$$

MSSM Higgs Inflation

$$V(\phi) = \frac{1}{2}m_0^2\phi^2 - \frac{\lambda\mu}{4M_{\text{pl}}}\phi^4 + \frac{\lambda^2}{32M_{\text{pl}}^2}\phi^6 \quad 3m_0^2 = 4\mu^2(1 + 8\alpha^2) \quad X \equiv \alpha\left(\frac{M_P}{m_0}\right)$$

X	$\lambda_2^{(1)}$	$n_s^{(1)}$	$\lambda_2^{(2)}$	$n_s^{(2)}$
2.0×10^4	0.675×10^{-7}	0.948	0.914×10^{-7}	0.975
1.8×10^4	0.592×10^{-7}	0.937	1.189×10^{-7}	1.000
1.6×10^4	0.555×10^{-7}	0.932	1.468×10^{-7}	1.022
1.4×10^4	0.531×10^{-7}	0.928	1.807×10^{-7}	1.044
1.2×10^4	0.515×10^{-7}	0.926	2.247×10^{-7}	1.069
1.0×10^4	0.503×10^{-7}	0.924	2.861×10^{-7}	1.099
1.0×10^3	0.481×10^{-7}	0.920	3.837×10^{-6}	1.678
1.0×10^2	0.481×10^{-7}	0.920	(4.133×10^{-5})	(3.334)
1.0×10^1	0.481×10^{-7}	0.920	(4.229×10^{-4})	(8.501)

$$X_{\max} = 2.051 \times 10^4 \longrightarrow n_s = 0.960$$

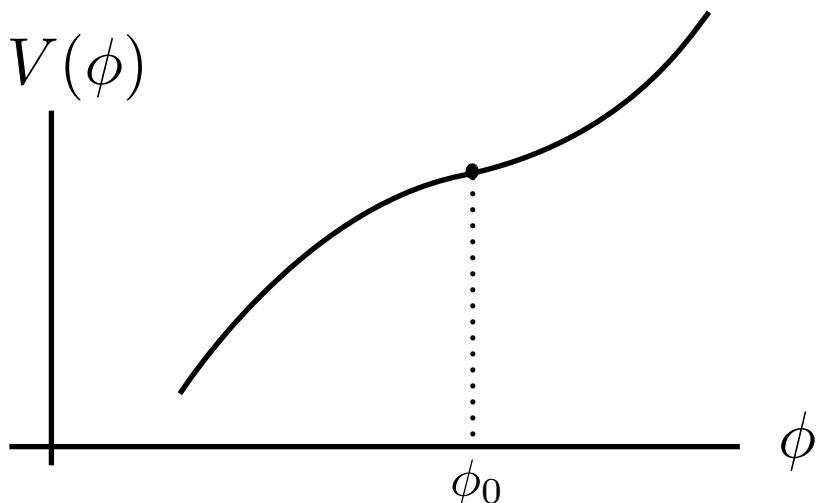
Summary

- We performed a simple case study of supersymmetric Higgs inflation model
- The observed tilt of the spectral index implies that the tuning is minimal to realize the sizable amplitude of density fluctuations.
- Once we take into account the inflationary fine tuning, small m_0/m_Z does not reduce the total degree of fine tuning including electroweak hierarchy.

$$X \equiv \alpha\left(\frac{M_P}{m_0}\right) = \alpha\left(\frac{m_Z}{m_0}\right)\left(\frac{M_P}{m_Z}\right) \sim 10^4$$

10/10

Inflection Point



$$\frac{d^2V}{d\phi^2} = 0$$

$$\phi_0 = \left(\frac{4M_P}{\sqrt{3}\lambda} m_0 \right)^{\frac{1}{2}} + \dots$$

$$V(\phi) = V(\phi_0) + \beta_1(\phi - \phi_0) + \frac{1}{3!}\beta_3(\phi - \phi_0)^3 + \dots$$

$$V(\phi_0) = \frac{1}{6}m_0^2\phi_0^2 + \dots$$

Hubble parameter :

$$\beta_1 = 8\alpha^2 m_0^2 \phi_0 + \dots$$

$$H \simeq \sqrt{\frac{V_0}{3M_P^2}} = \frac{1}{3\sqrt{2}} \frac{m_0 \phi_0}{M_P}$$

$$\beta_3 = 8 \frac{m_0^2}{\phi_0} + \dots$$

Higgs Potential

Along the angular direction, potential is minimized at $\theta = 0$

Higgs potential :

$$V(\phi) = \frac{1}{2}m_0^2\phi^2 - \frac{\lambda\mu}{4M_{\text{pl}}}\phi^4 + \frac{\lambda^2}{32M_{\text{pl}}^2}\phi^6$$

$$m_0^2 = \frac{1}{2}(2\mu^2 + m_{H_u}^2 + m_{H_d}^2 - 2b) \sim m_{\text{soft}}^2$$

