Glauber gluons in the Drell-Yan Process

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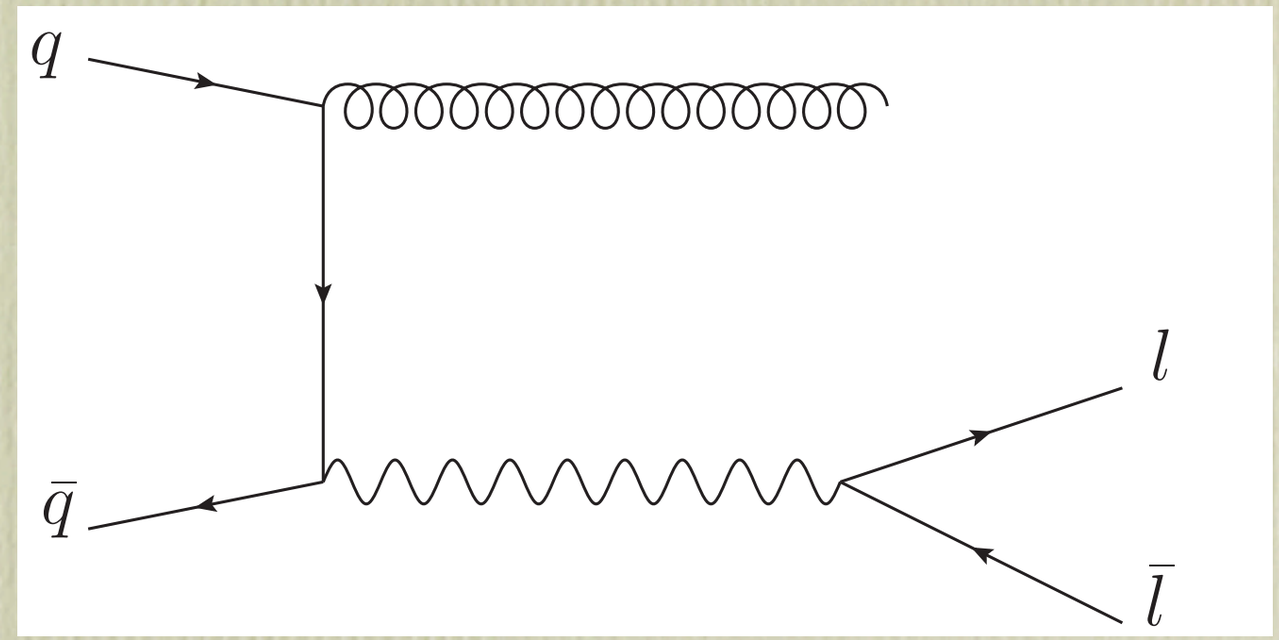
Outlines

- Introduction
- Lam-Tung relation
- Boer-Mulders function and the QCD vacuum
- Glauber gluon effect
- πp v.s. pp v.s. $p\bar{p}$

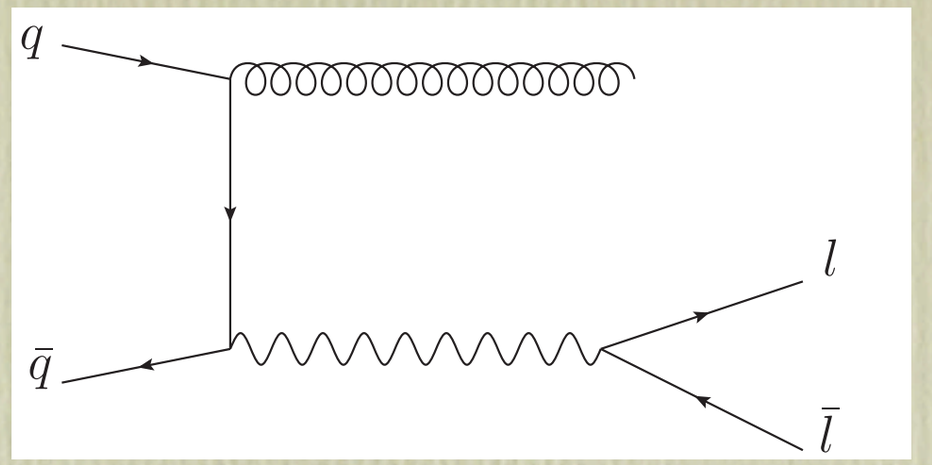
Introduction

- The Lam-Tung relation for the Drell-Yan process was derived kinematically over 20 years ago.
- Concerning the angular distribution among quarks and leptons.
- Confirmed in the Proton-Proton case
- Not for the Pion-Proton case.

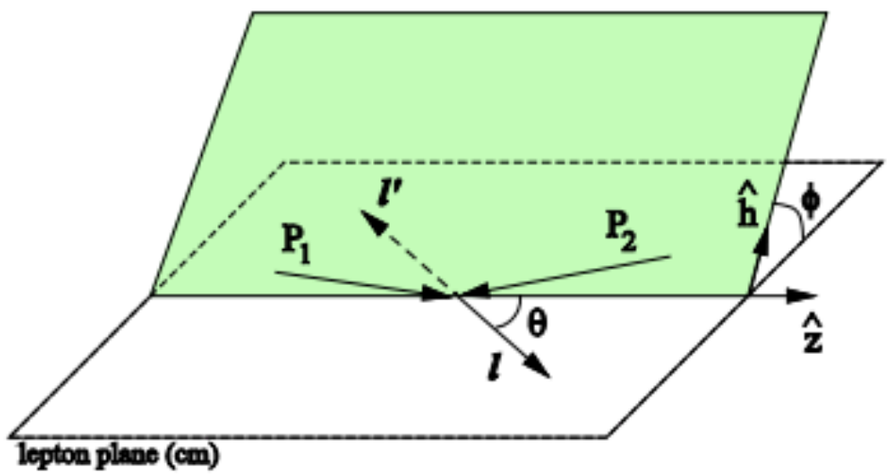
Lam-Tung relation



- Drell-Yan Process, with some real emission of gluons
- The lepton pair carries transverse momentum
- The angle between the quarks and the leptons



Drell-Yan decay angular distributions



Θ and Φ are the decay polar and azimuthal angles of the μ^+ in the dilepton rest-frame

Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

λ can differ from 1, but should satisfy $1-\lambda=2\nu$ (Lam-Tung)

Decay angular distributions in pion-induced Drell-Yan

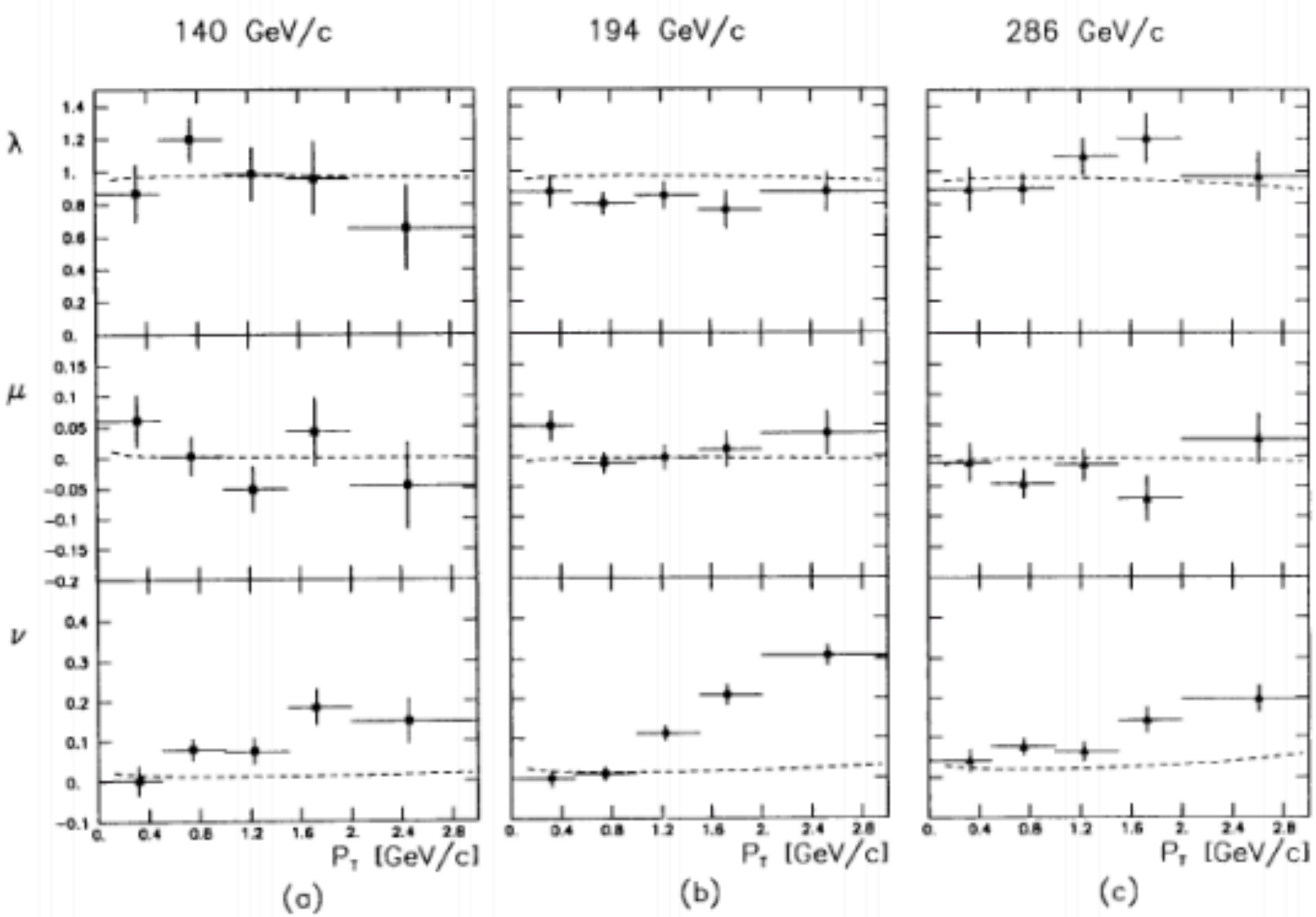


Fig. 3a–c. Parameters λ, μ, and ν as a function of P_T in the CS frame. **a** 140 GeV/c; **b** 194 GeV/c; **c** 286 GeV/c. The error bars correspond to the statistical uncertainties only. The horizontal bars give the size of each interval. The dashed curves are the predictions of perturbative QCD [3]

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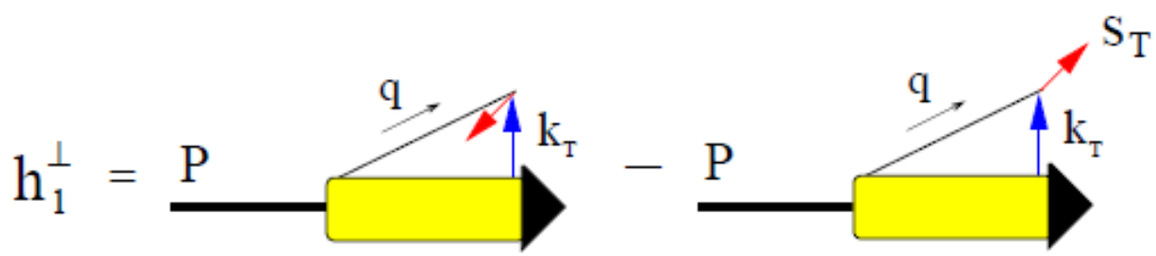
37 (1988) 545

Dashed curves are from pQCD calculations

$\nu \neq 0$ and ν increases with p_T

Boer-Mulders function

Boer (PRD 60, 014012 (1999)): Hadronic Effect

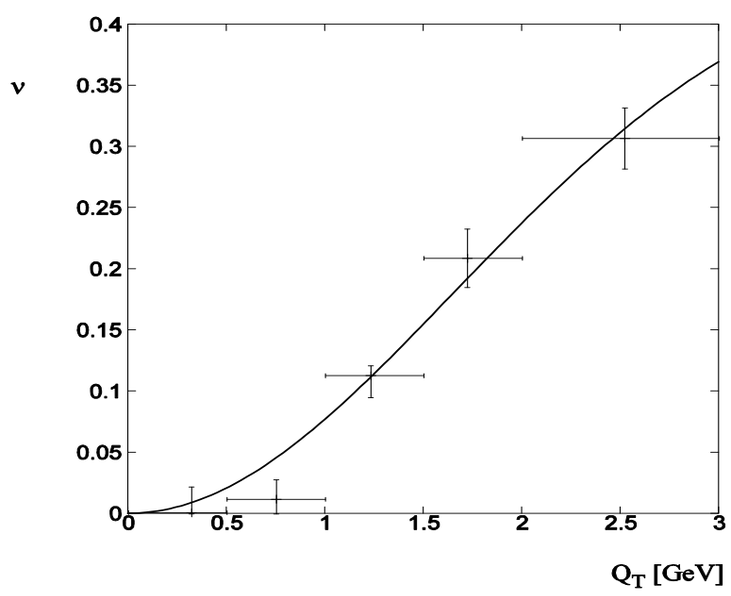


$$\propto \frac{k_T}{M}$$

hadron mass

- $h_1^\perp$ represents a correlation between quark's k_T and transverse spin in an unpolarized hadron

- $h_1^\perp$ can lead to an azimuthal dependence with $\frac{\nu}{2} \propto h_1^\perp(N)\bar{h}_1^\perp(\pi)$



$$h_1^\perp(x, k_T^2) = C_H \frac{\alpha_T}{\pi} \frac{M_C M_H}{k_T^2 + M_C^2} e^{-\alpha_T k_T^2} f_1(x),$$

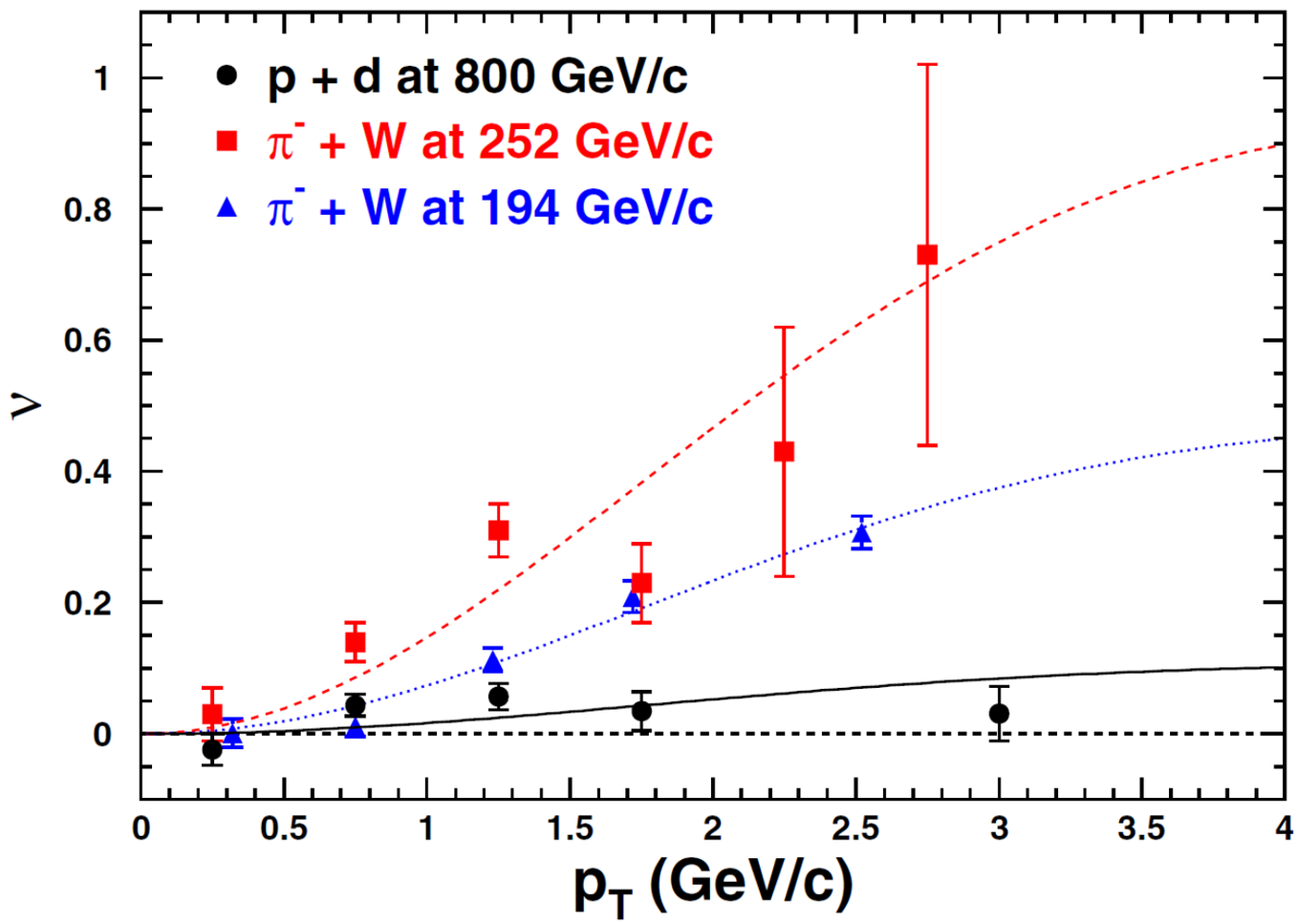
$$\nu = 16\kappa_1 \frac{p_T^2 M_C^2}{(p_T^2 + 4M_C^2)^2}, \qquad \kappa_1 = C_{H_1} C_{H_2}/2$$

$\kappa = \frac{\nu}{2} \to 0$ for large $|k_T|$

Consistency of factorization in term of TMDs

31

E866 (PRL 99 (2007) 082301): Azimuthal cos2Φ Distribution of DY events in pd



$$h_1^{\perp}(x, k_T^2) = C_H \frac{\alpha_T}{\pi} \frac{M_C M_H}{k_T^2 + M_C^2} e^{-\alpha_T k_T^2} f_1(x$$

$$\nu = 16\kappa_1 \frac{p_T^2 M_C^2}{(p_T^2 + 4M_C^2)^2},$$

$$\kappa_1 = C_{H_1} C_{H_2}/2$$

$\nu(\pi^- W \rightarrow \mu^+\mu^- X) \sim$ [valence $h_1^{\perp}(\pi)$] * [valence $h_1^{\perp}(p)$]

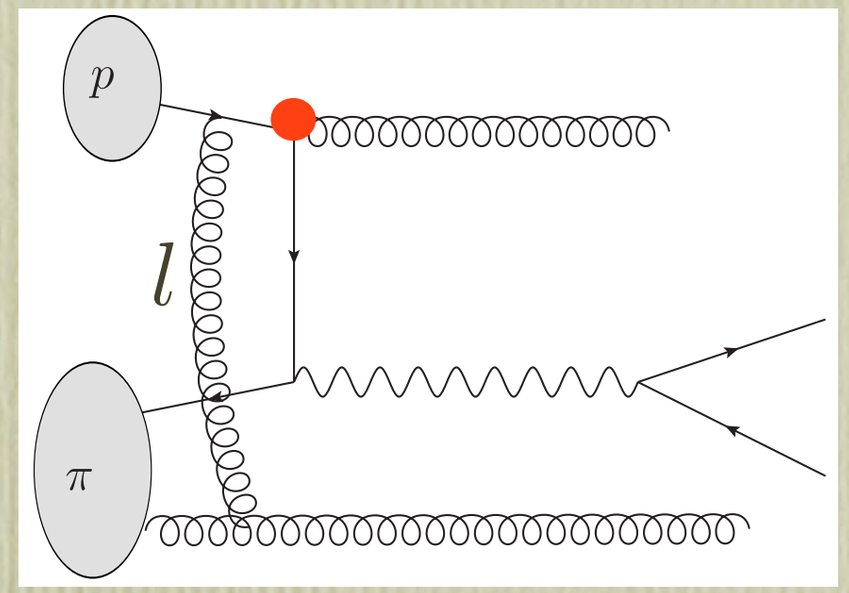
$\nu(pd \rightarrow \mu^+\mu^- X) \sim$ [valence $h_1^{\perp}(p)$] * [sea $h_1^{\perp}(p)$]

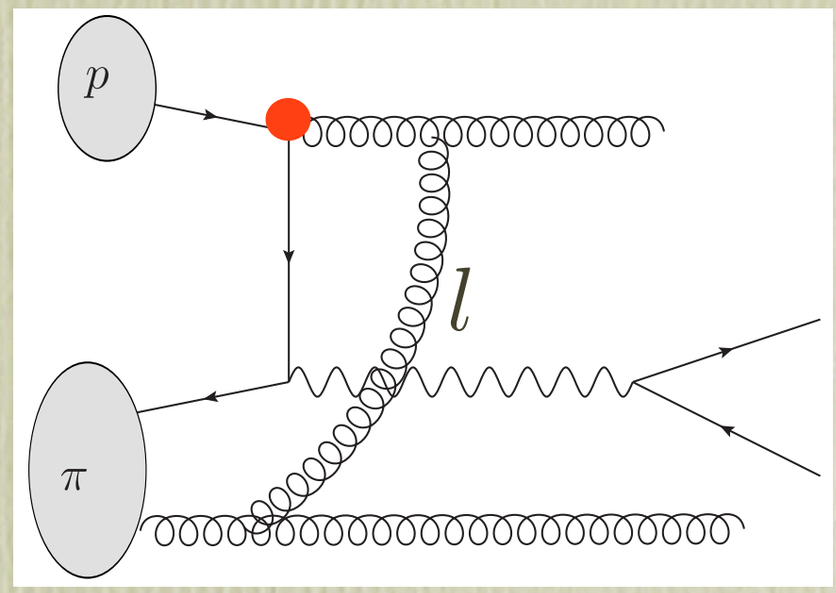
Sea-quark BM functions are much smaller than valence quarks

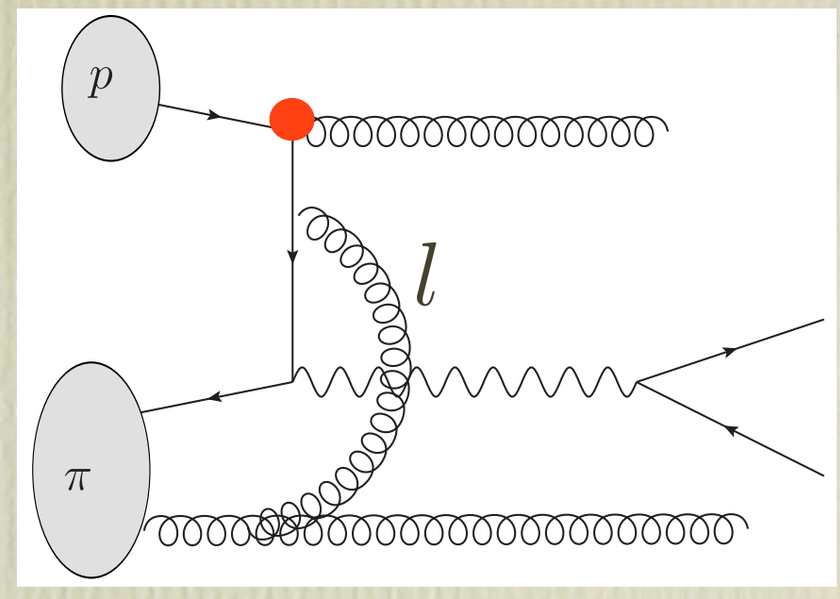
Reconciliation

- The leading Fock state of $q\bar{q}$
- The subleading Fock state of $q\bar{q}g$ forms a huge soft cloud in the pion
- How about the soft cloud?

Gluons Emitted from the Soft cloud

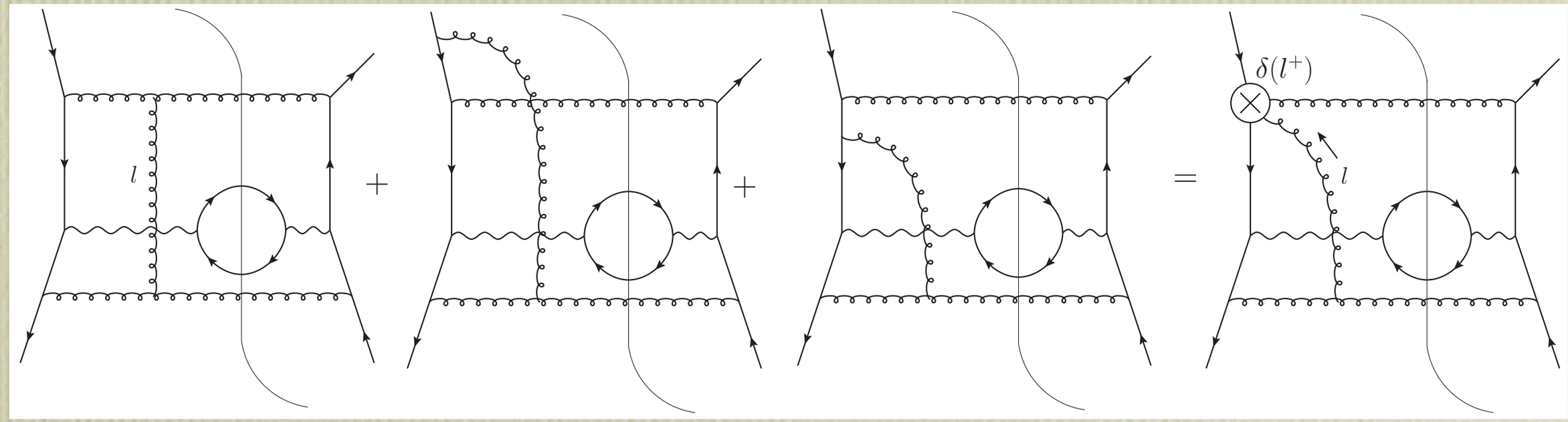




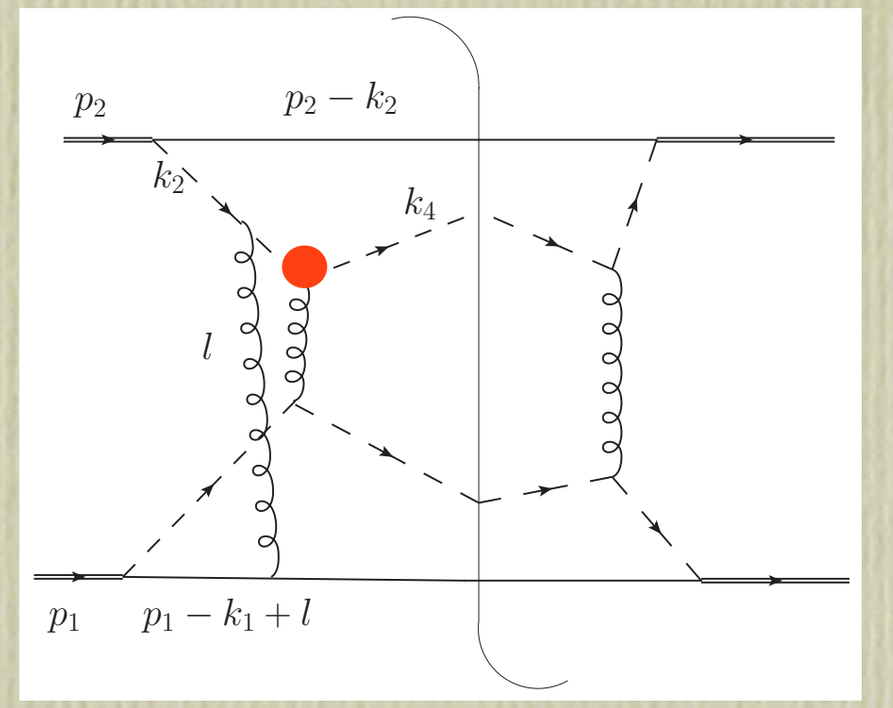


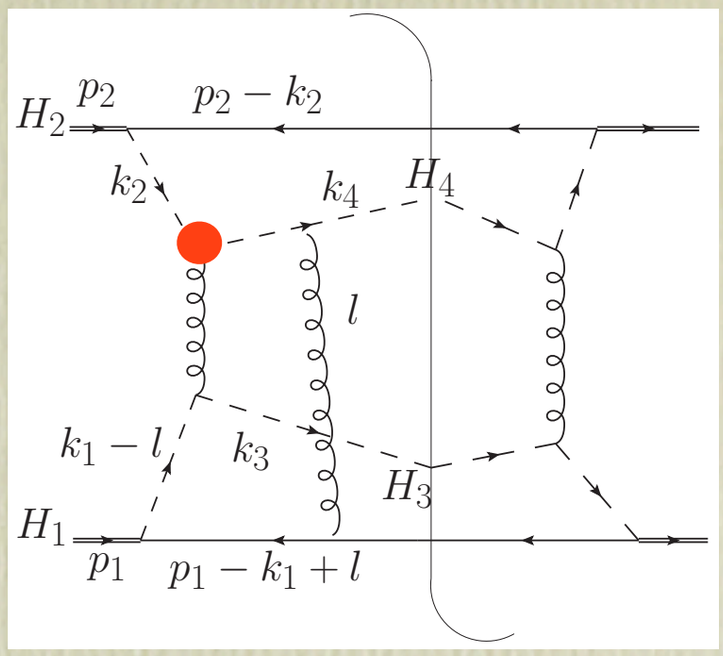
These 3 diagrams contribute a delta function $\delta(l^+)$

Gluons Emitted from the Soft cloud



Eikonalization





Assume k1 is in the plus direction and k2 is in the minus direction

$$\frac{2k_2}{(k_2+l)^2+i\epsilon} = \frac{2k_2}{k_2^2+2k_2\cdot l+l^2+i\epsilon} \approx \frac{2k_2^-}{2k_2^- l^+ + i\epsilon}$$

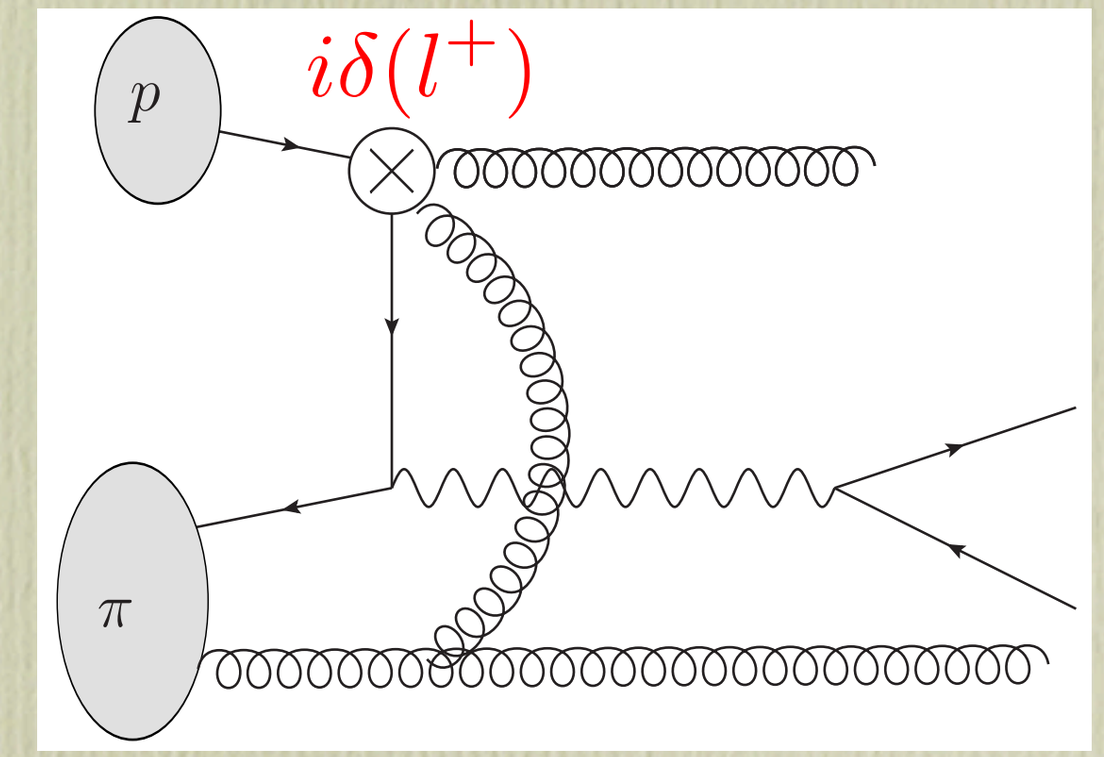
$$\approx \frac{1}{l^+ + i\epsilon} = PV\frac{1}{l^+} - i\pi\delta\left(l^+\right)$$

$$\frac{2k_4}{(k_4-l)^2+i\epsilon} \approx \frac{1}{-l^+ - i\epsilon} = -PV\frac{1}{l^+} - i\pi\delta\left(l^+\right)$$

Glauber region

- The delta function leads to the Glauber gluon, i.e., additional infrared divergence.
- Glauber region: $l^+ = 0$
- Glauber gluon is virtual
- Ordinary soft gluon can be real or virtual

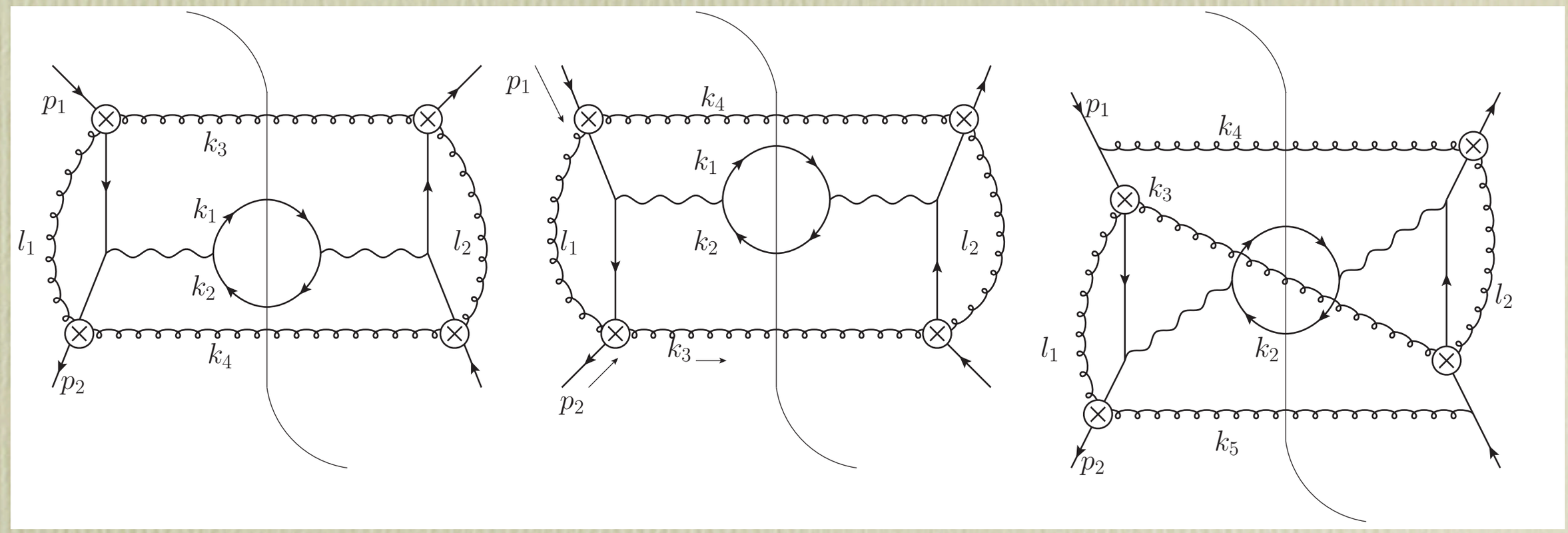
Glauber gluons



This Glauber gluon gives a factor $\cos S$ $\quad S \sim \int \frac{1}{{l_T}^2} d{l_T}^2$

if we consider all orders of emitting gluons of pion

Glauber phase



$$e^{iS} \qquad e^{-iS} \qquad 1$$

Angular coefficients after assignments of e^{iS}

$$\hat{\sigma}_0 = \left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)\left(1+\frac{1}{2}s_1^2\right)+(c_e-1)$$
$$\times\left\{2\left[\frac{E_1E_2}{k^2}c_1^2+\frac{k}{E_1}+\frac{k}{E_2}-\frac{1}{2}\left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)-2\right]\right.$$
$$\left.-\left[\frac{E_1E_2}{k^2}c_1^2-\frac{1}{2}\left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)\right]s_1^2\right\}, \qquad (3)$$
$$\hat{\sigma}_1 = \left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)\left(c_1^2-\frac{1}{2}s_1^2\right)+(c_e-1)$$
$$\times\left\{\left(\frac{E_1}{E_2}+\frac{E_2}{E_1}-2\right)c_1^2\right.$$
$$\left.+\left[\frac{E_1E_2}{k^2}c_1^2-\frac{1}{2}\left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)\right]s_1^2\right\}$$
$$\hat{\sigma}_2 = \left(\frac{E_1}{E_2}-\frac{E_2}{E_1}\right)c_1s_1+(c_e-1)$$
$$\times\left(\frac{E_2-E_1}{k}+\frac{E_1}{E_2}-\frac{E_2}{E_1}\right)c_1s_1$$
$$\hat{\sigma}_3 = \left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)s_1^2-(c_e-1)$$
$$\times 2\left[\frac{E_1E_2}{k^2}c_1^2-\frac{1}{2}\left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)\right]s_1^2,$$

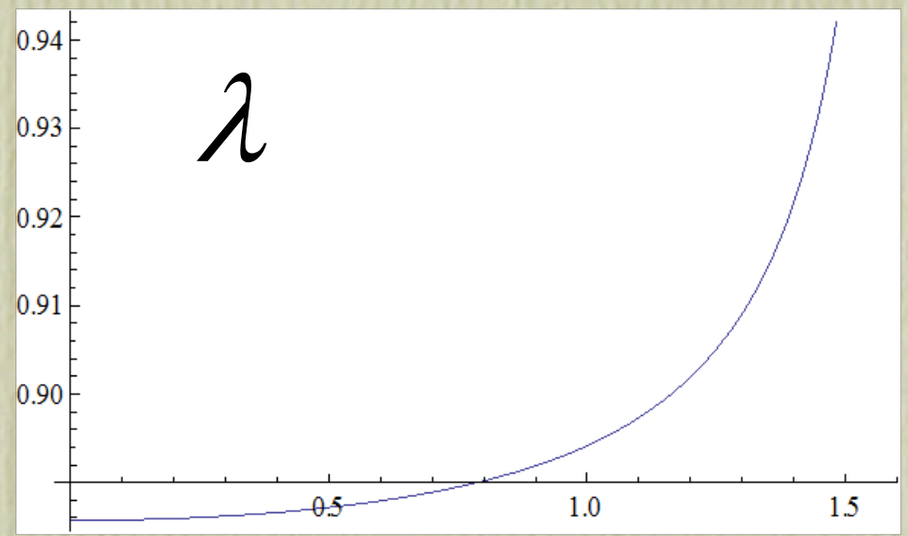
$$c_e = \cos S$$

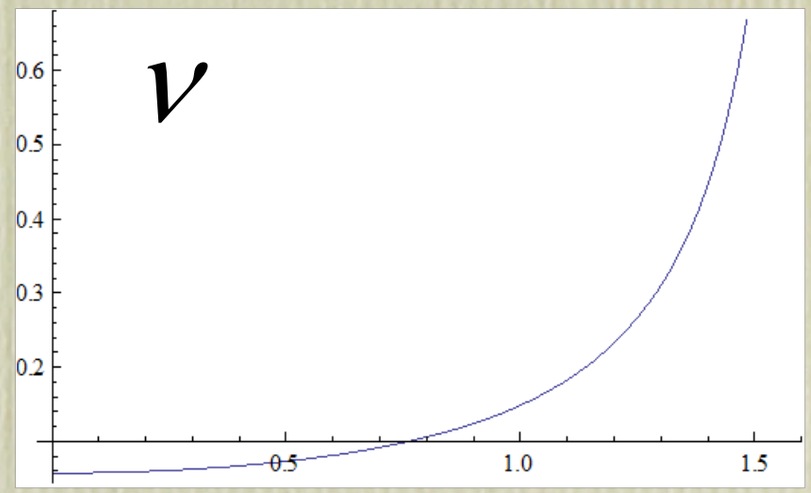
E1, E2:hadron energy
k: lepton energy

all are in CS frame

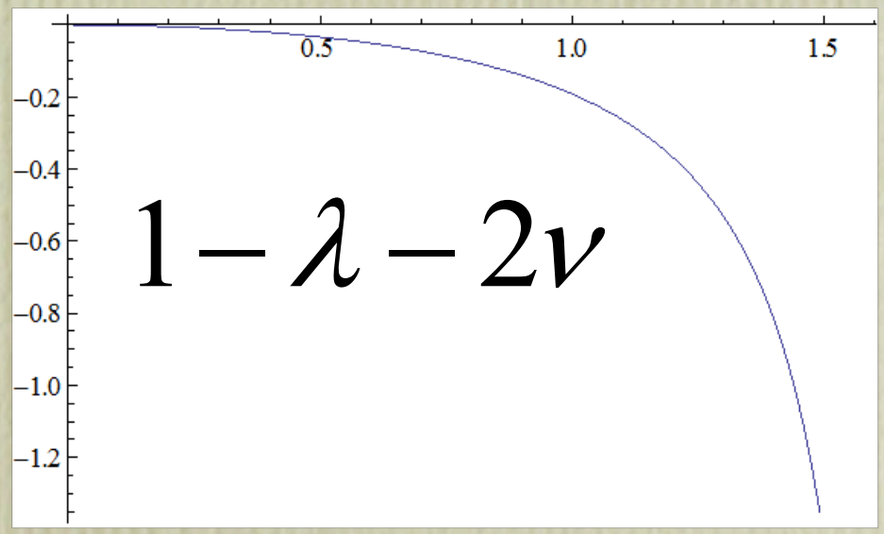
$$\lambda \sim \hat{\sigma}_1/\hat{\sigma}_0 \quad \mu \sim \hat{\sigma}_2/\hat{\sigma}_0 \quad \nu \sim \hat{\sigma}_3/\hat{\sigma}_0$$

Numerical Results





S_e



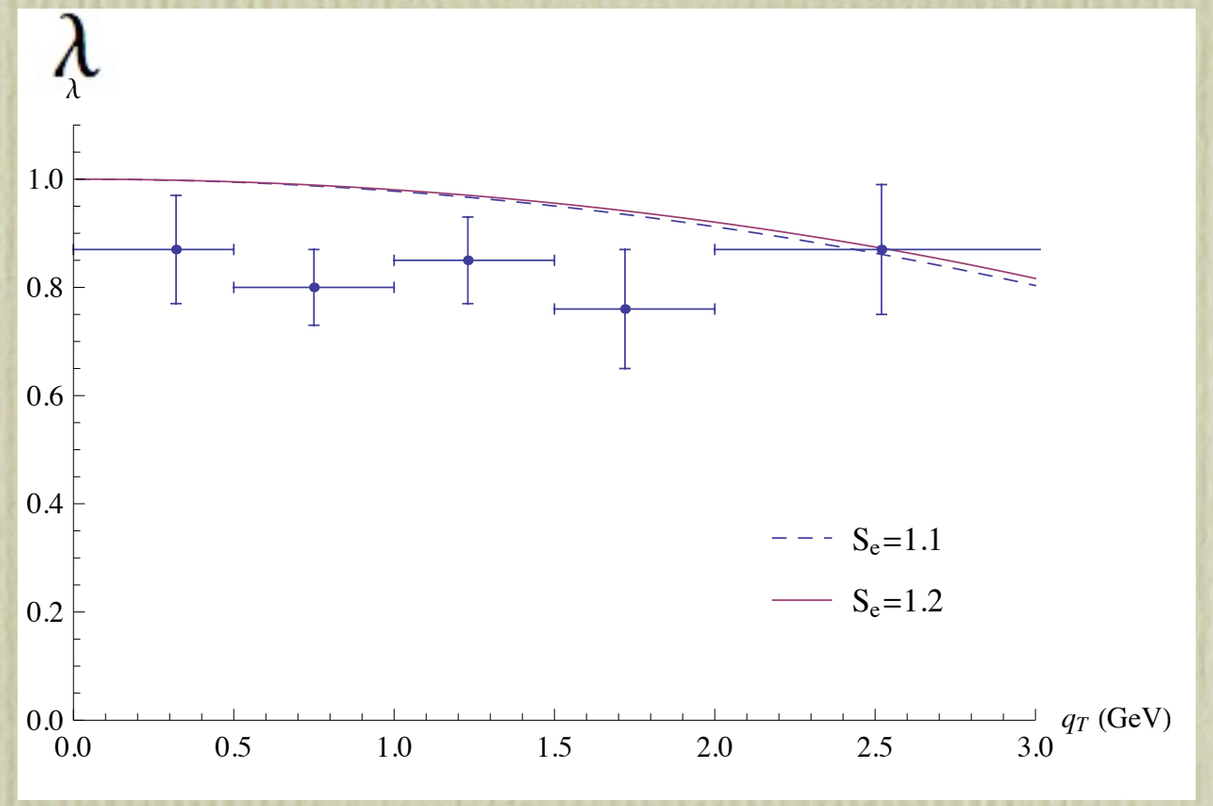
$$\sqrt{S} = 194\ GeV$$

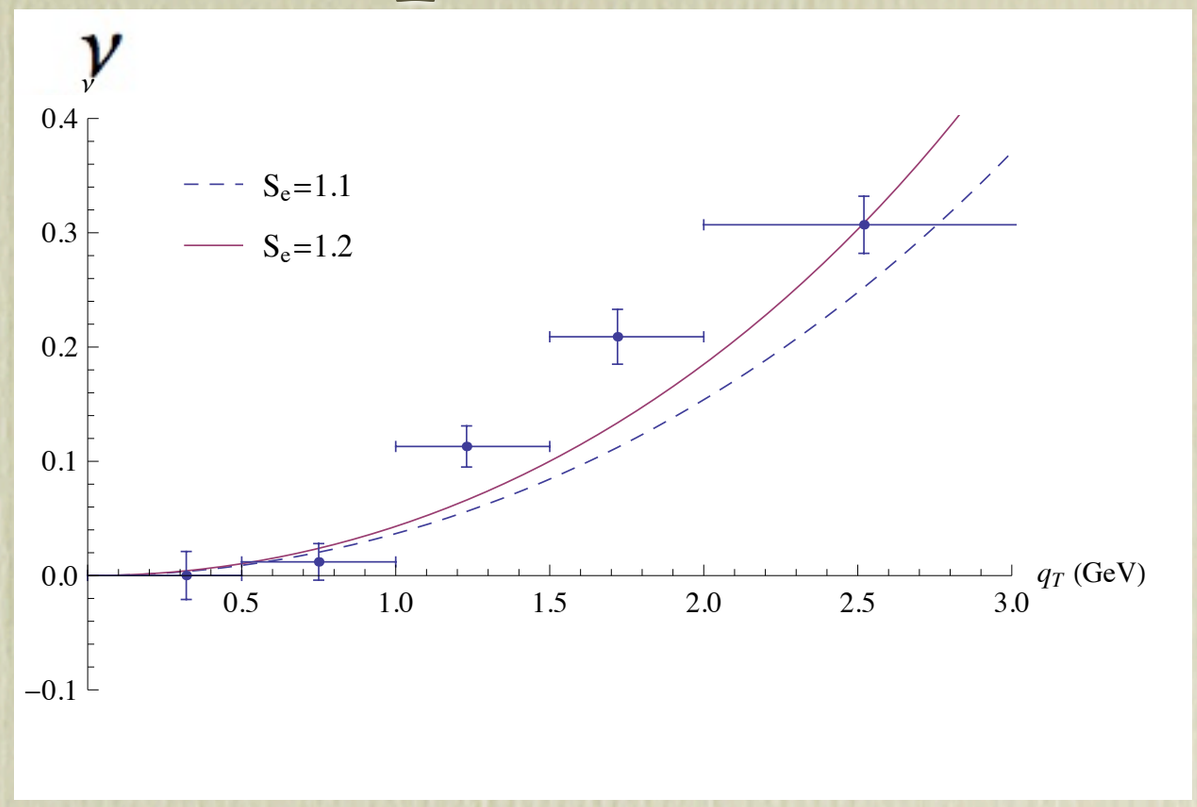
$$q_T = 2\ GeV, \quad Q = 8\ GeV$$

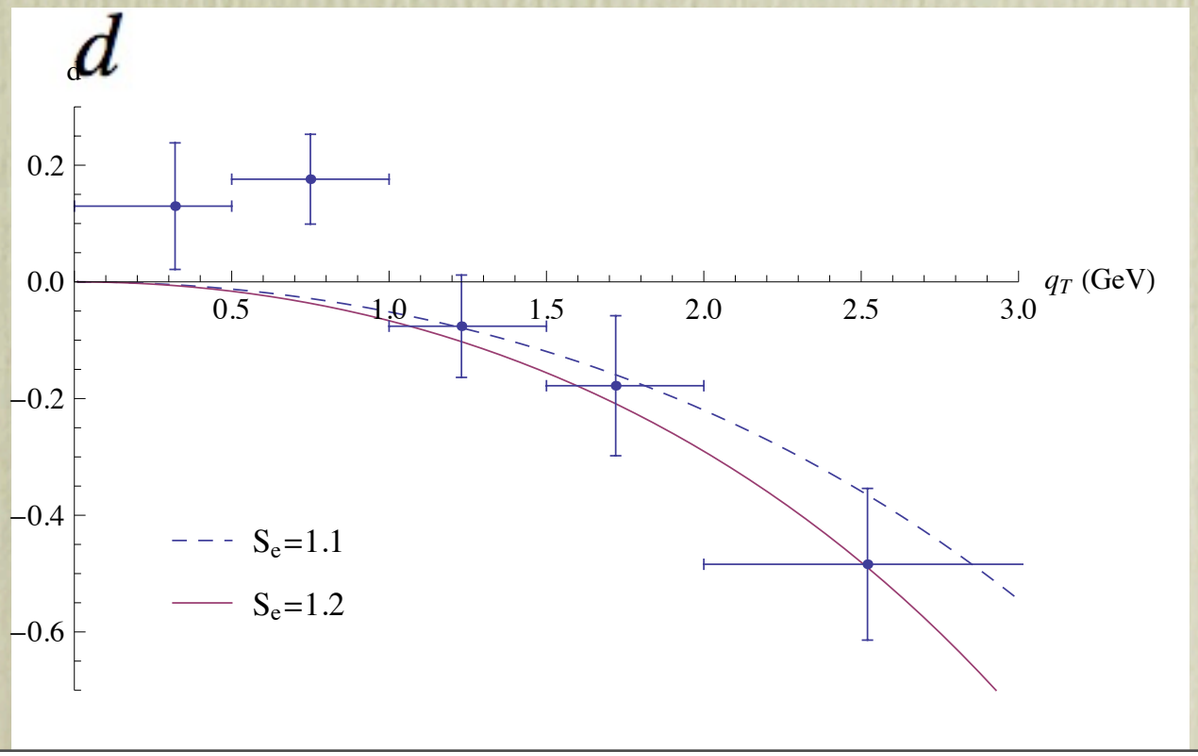
$$Se = 1.15, \quad \lambda \sim 0.9, \quad \nu \sim 0.2$$

$$1-\lambda-2\nu \sim -0.3$$

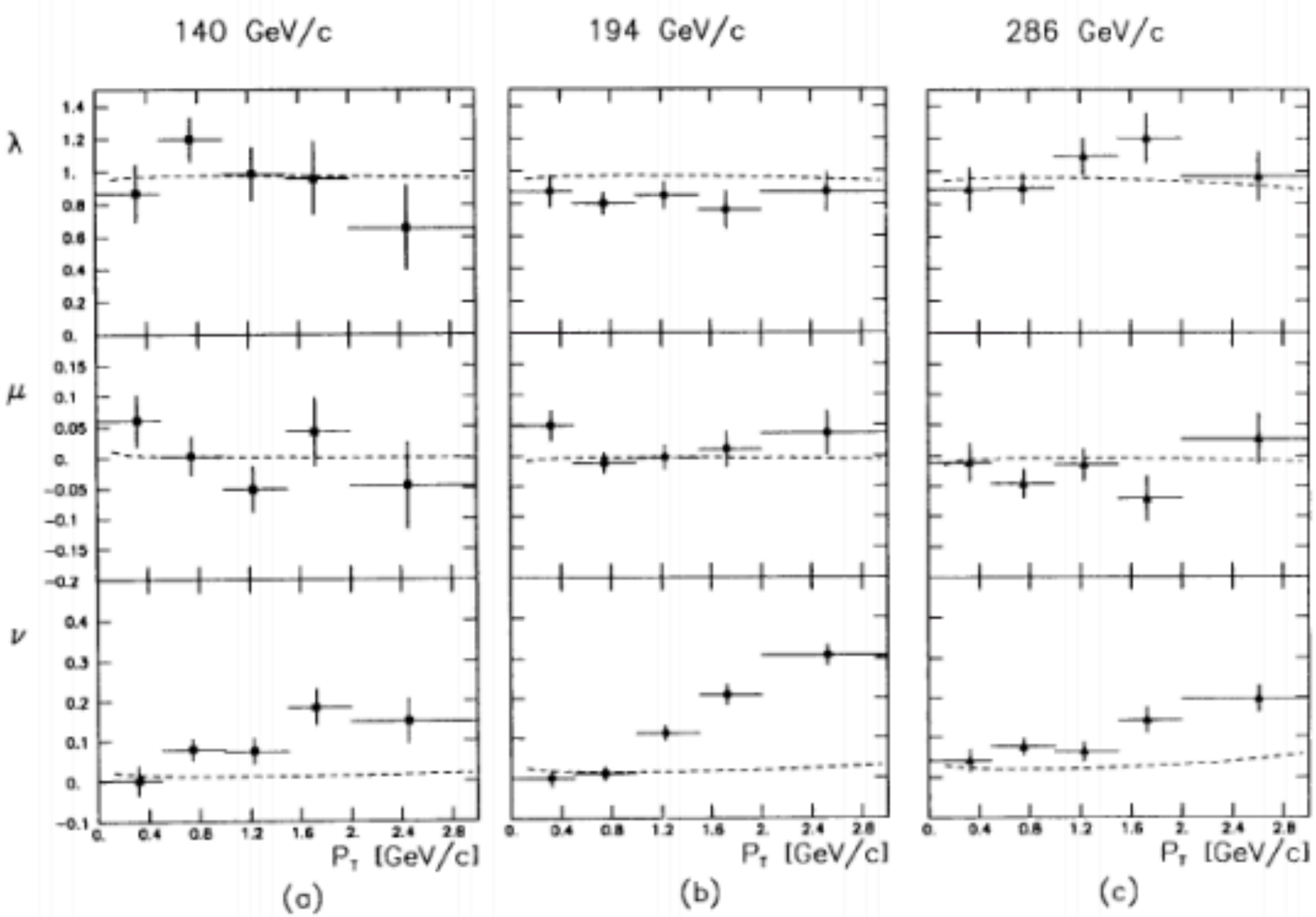
coefficients vs q_T







Decay angular distributions in pion-induced Drell-Yan



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Dashed curves are from pQCD calculations

$\nu \neq 0$ and ν increases with p_T

Speculation

- Lam-Tung relation holds at $p_T = 0$, in which the configuration contains only two partons (no real gluon emission)
- Lam-Tung relation breaks at $p_T > 0$, in which the configuration contains at least 3 partons
- Violation of L-T relation caused by Glauber gluon?
- Glauber effect important only in pion due to the relatively large soft cloud?

How to verify?

- Using $p\bar{p}$, since the $\bar{q}$ is a valence one.
- In Boer's prediction (VV>VS), LT should be broken.
- We can use this exp. to discriminate the two proposals (BM effect or Soft cloud nature).

Conclusions

- Glauber gluons appear in k_T factorization for complicated QCD processes.
- The Glauber gluons in the pion may break the Lam-Tung relation, which was still not solved at present.
- Proposed the exp. of $p\bar{p}$ collision to JPARC, to discriminate the different resolutions.
- Try to use the same method to study h to bbar jets

Thank you for holding till the last second.
Enjoy the Banquet!

Lightcone Coordinate

$$k^+ = \frac{1}{\sqrt{2}}\left(k^0 + k^3\right)$$

$$k^- = \frac{1}{\sqrt{2}}\left(k^0 - k^3\right)$$

$$\mathbf{k_T} = \left(k^1, k^2\right)$$

$$k^2 = 2k^+k^- - \left|\mathbf{k_T}\right|^2$$

$$k\cdot l = k^+l^- + k^-l^+ - \mathbf{k_T}\cdot\mathbf{l_T}$$

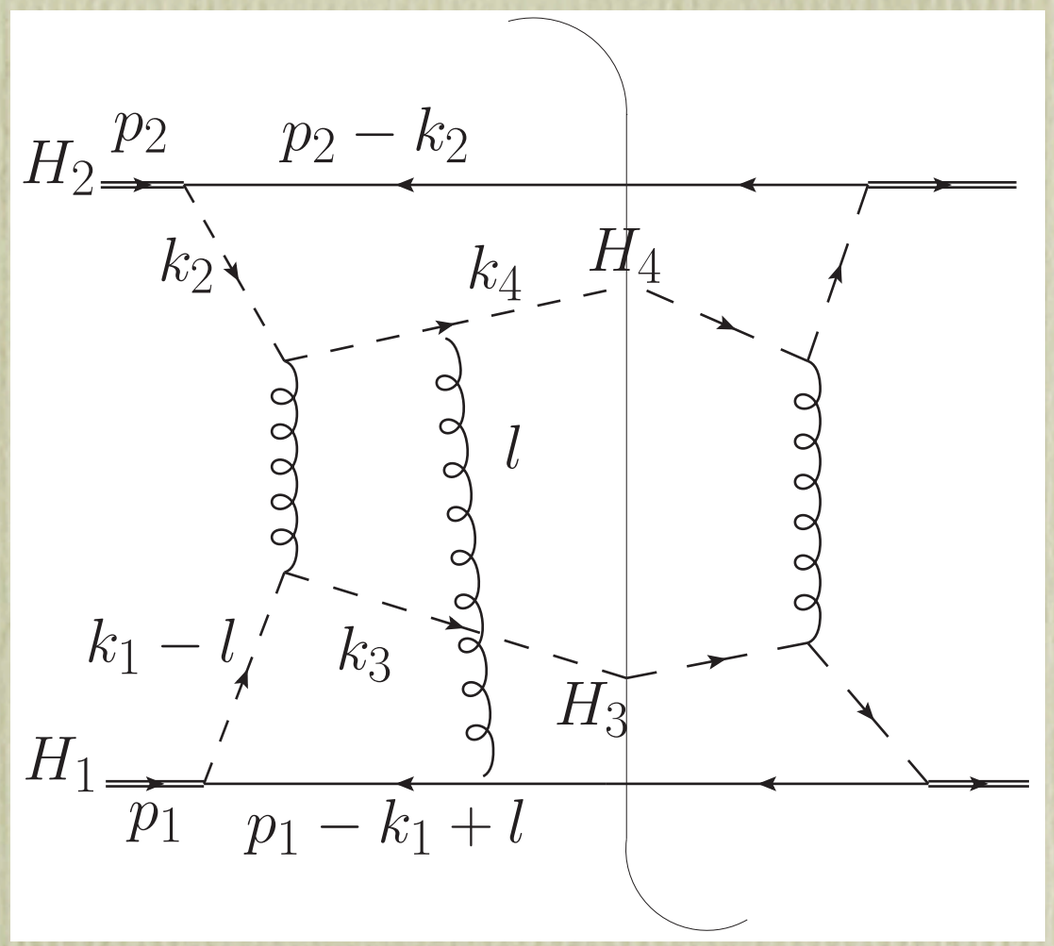
Glauber Divergence

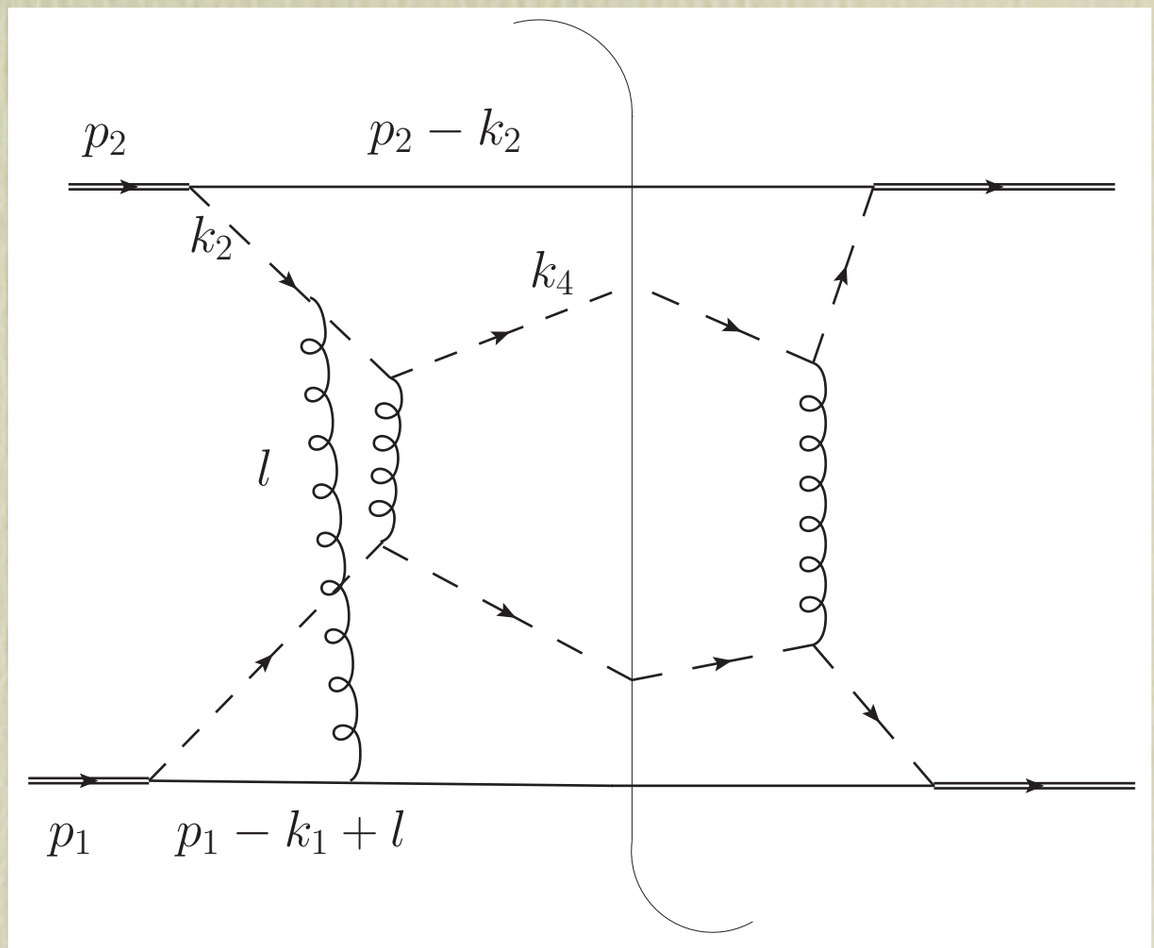
$$H_1(p_1) + H_2(p_2) \to H_3(p_3) + H_4(p_4) + X$$

$$E_3 E_4 \frac{d\sigma}{d^3\mathbf{p}_3 d^3\mathbf{p}_4} = \sum \int d\sigma_{i+j\to k+l} \; f_{i/1} f_{j/2} d_{3/k} d_{4/l}$$

- The problem is ...
  - Soft gluons from parton 1
  - Coupling to parton 2
  - Factorization breakdown?

$H_1 + H_2 \to H_3 + H_4 + X$



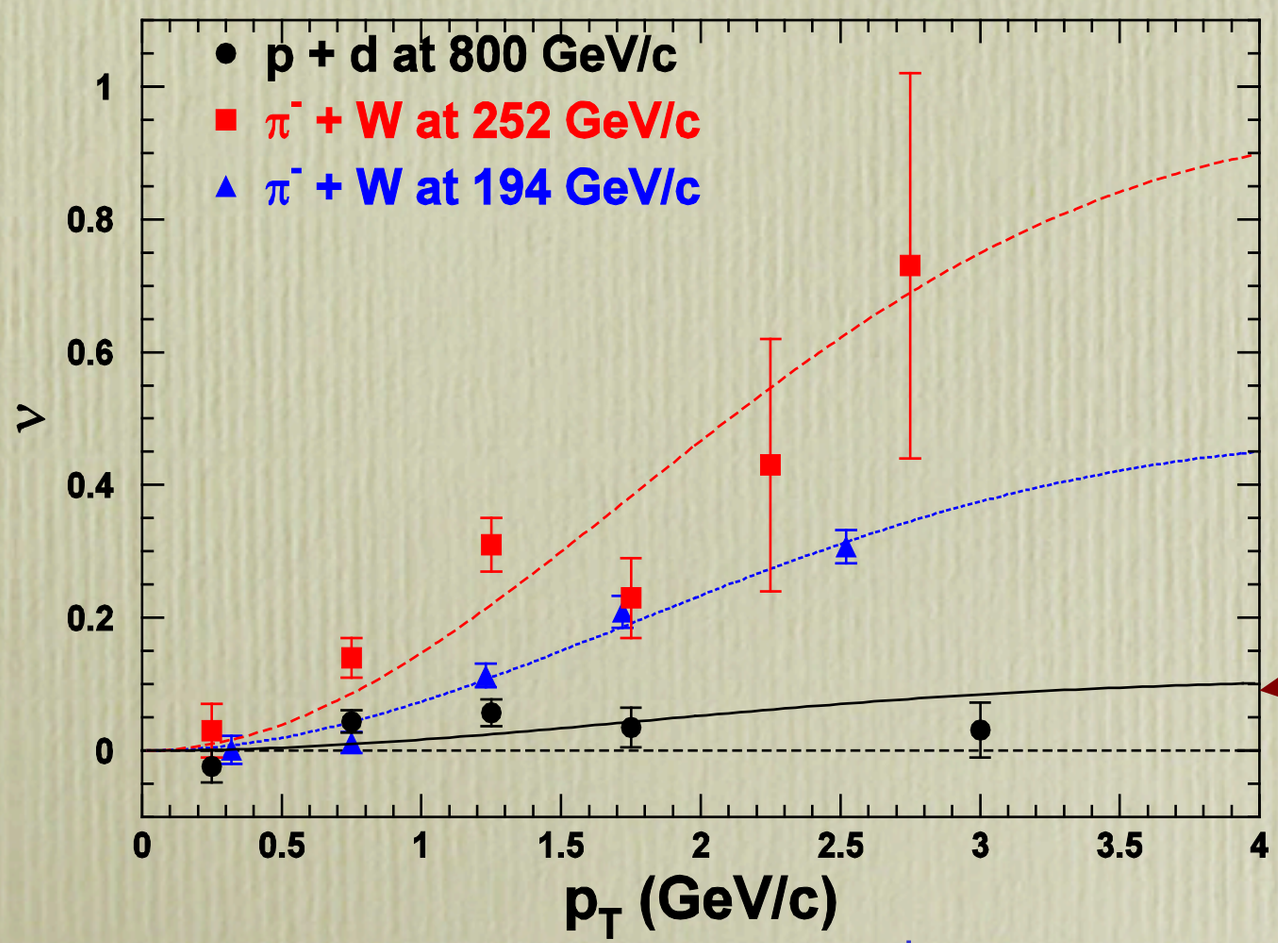


$$|\mathcal{M}|^2 = |\mathcal{M}^{(0)} + \mathcal{M}^{(1)} + \cdots|^2.$$
$$= |\mathcal{M}^{(0)}|^2 + 2\mathrm{Re}[\mathcal{M}^{*(0)}\mathcal{M}^{(1)}] + \cdots$$

Proton and Pion

- Proton - proton => OK

E866 Collab., Lingyan Zhu et al.,
PRL 99 (2007) 082301; PRL 102 (2009) 182001



Small ν is observed for p+d and p+p D-Y

Pion

- Massless behavior as a Goldstone boson in the exact chiral symmetry $m_{u,d} = 0$
- Effective potential between the 2 quarks is linear for large distances. => Massive
- Pion is unique
- Resolve strange behavior for Pion-Proton Process?

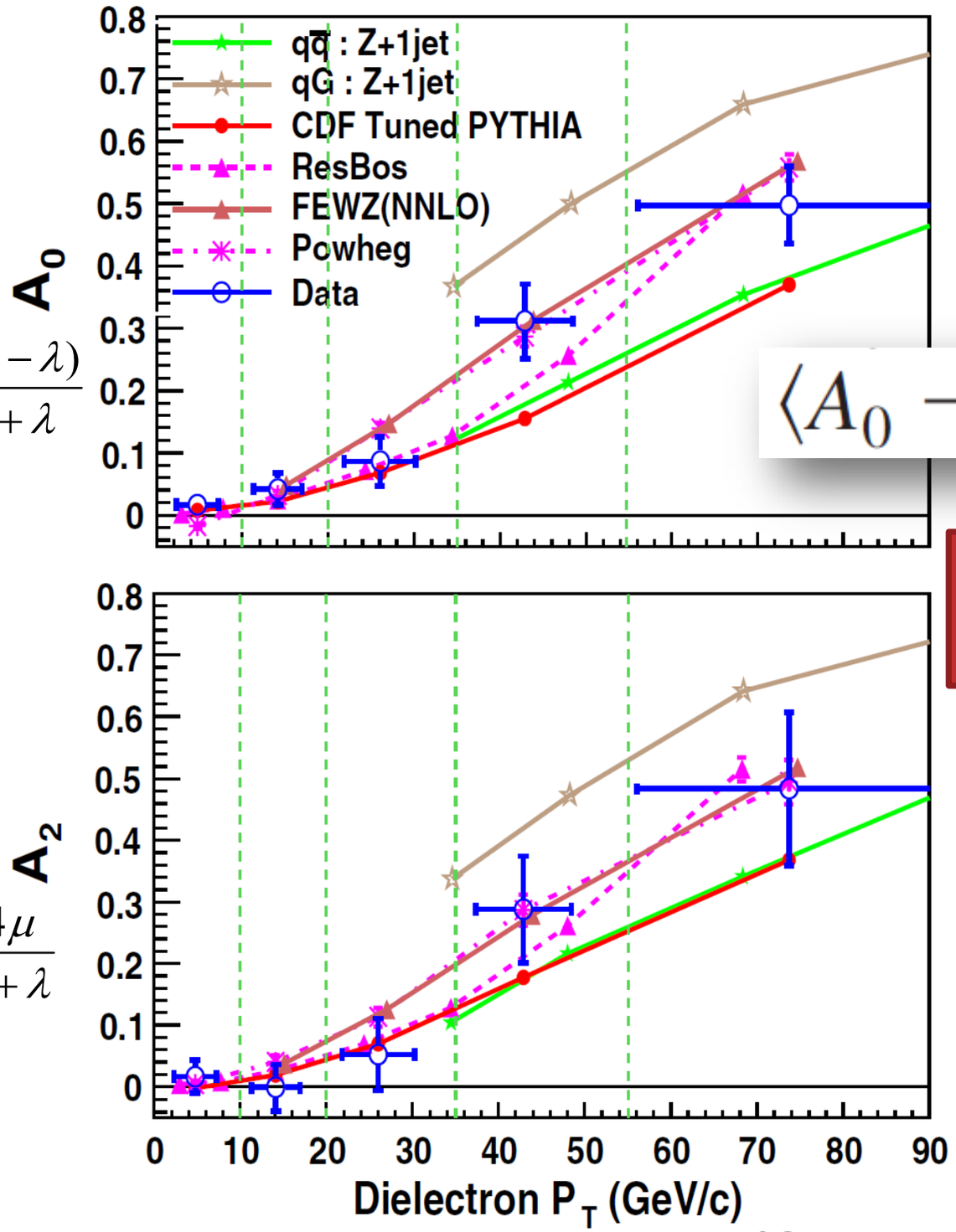
Glauber gluon?

- Glauber gluon breaks the universality of PDF of hadron 1, and kT factorization
- Glauber gluon exists if three or more hadrons are involved
- Glauber gluon does not exist if only one or two hadrons are involved. No delta function

CDF (PRL 106, 241801 (2011)): Angular Distribution of DY events at Z pole in p-pbar

$$A_0 = \frac{2(1-\lambda)}{3+\lambda}$$

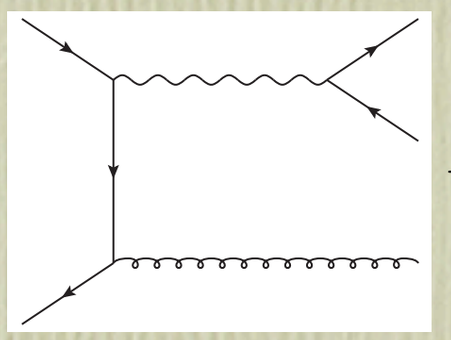
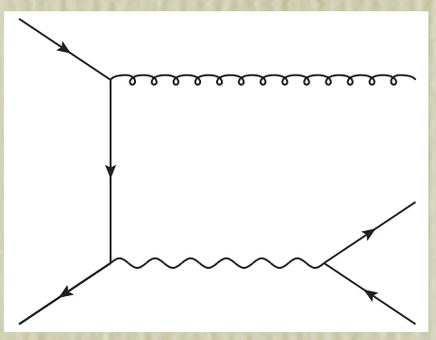
$$A_2 = \frac{4\mu}{3+\lambda}$$

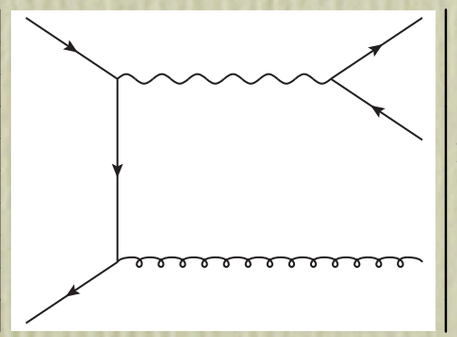
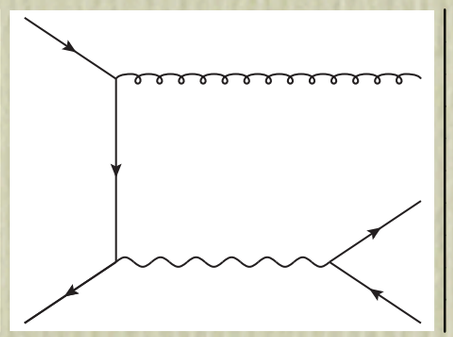
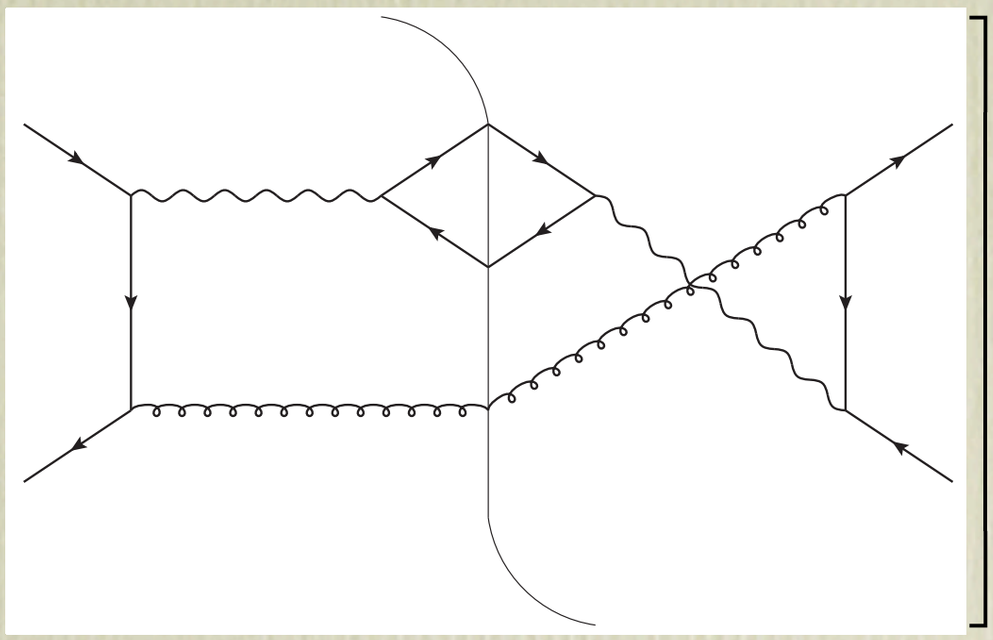


$$\langle A_0 - A_2 \rangle = 0.02 \pm 0.02$$

A good agreement with the Lam-Tung relation A_0-A_2=0

Phase between diagrams

$\Big|$  $+ \quad e^{i\alpha}$  $\Big|^2$

$= \Big|$  $\Big|^2 + \Big|$  $\Big|^2 + 2\,\mathrm{Re}\Big[e^{-i\alpha}$  $\Big]$

Reconciliation

- The leading Fock state of $q\bar{q}$ is tight in the pion to reduce the mass
- The subleading Fock state of $q\bar{q}g$ forms a huge soft cloud in the pion to fit its role of Nambu-Goldstone boson

Inserting the phase alpha

$$\text{const} \quad \left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)\left(1+\frac{1}{2}s_1^2\right)+(1-\cos\alpha)\left[2\left(-\frac{E_1}{E_2}-\frac{E_2}{E_1}+\frac{k}{E_1}+\frac{k}{E_2}+\frac{E_1}{k}+\frac{E_2}{k}-3\right)-s_1^2\frac{E_1+E_2-k}{k}\right]$$

$$\lambda \quad \cos^2\theta\left\{\left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)\left(c_1^2-\frac{1}{2}s_1^2\right)+(1-\cos\alpha)\left(-2c_1^2+\frac{E_1+E_2-k}{k}s_1^2\right)\right\}$$

$$\mu \quad \sin 2\theta\left\{\left(\frac{E_1}{E_2}-\frac{E_2}{E_1}\right)c_1s_1+(1-\cos\alpha)\frac{E_2-E_1}{k}c_1s_1\right\}$$

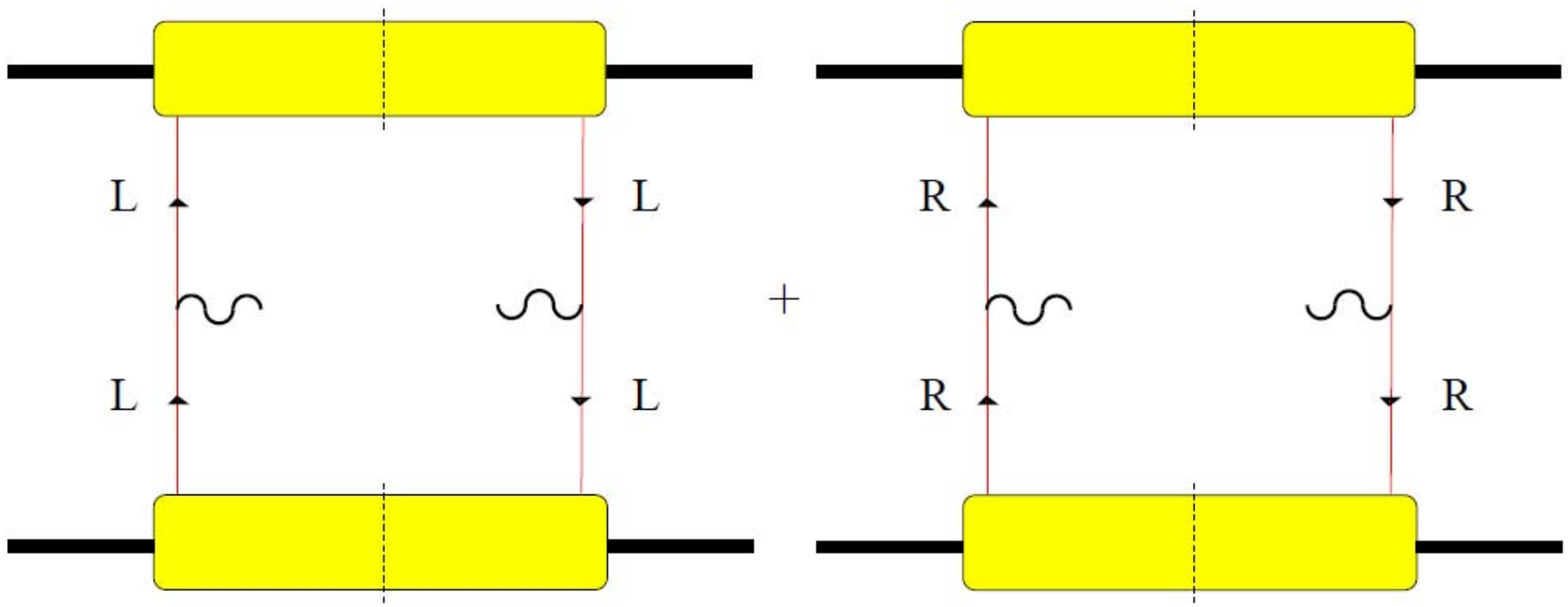
$$\nu \quad \frac{1}{2}\sin^2\theta\cos 2\phi\left\{\left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)s_1^2+(1-\cos\alpha)(-2)\frac{E_1+E_2-k}{k}s_1^2\right\}$$

$$1-\lambda-2\nu \;=\; \frac{(1-\cos\alpha)\left[1-\frac{E_1E_2}{2k^2}s_1^2+\frac{E_1E_2}{k^2}\sin^2(2\theta_1)\right]}{\left(\frac{E_1}{E_2}+\frac{E_2}{E_1}\right)\left(1+\frac{1}{2}s_1^2\right)+(1-\cos\alpha)\left(\frac{E_1E_2}{2k^2}s_1^2-1\right)\cos(2\theta_1)}$$

$$\sin\theta_1 \sim \frac{p_T}{Q}$$

Boer

$$\rho^{(q,\bar{q})} = \frac{1}{4}\{\mathbf{1} \otimes \mathbf{1}\}$$

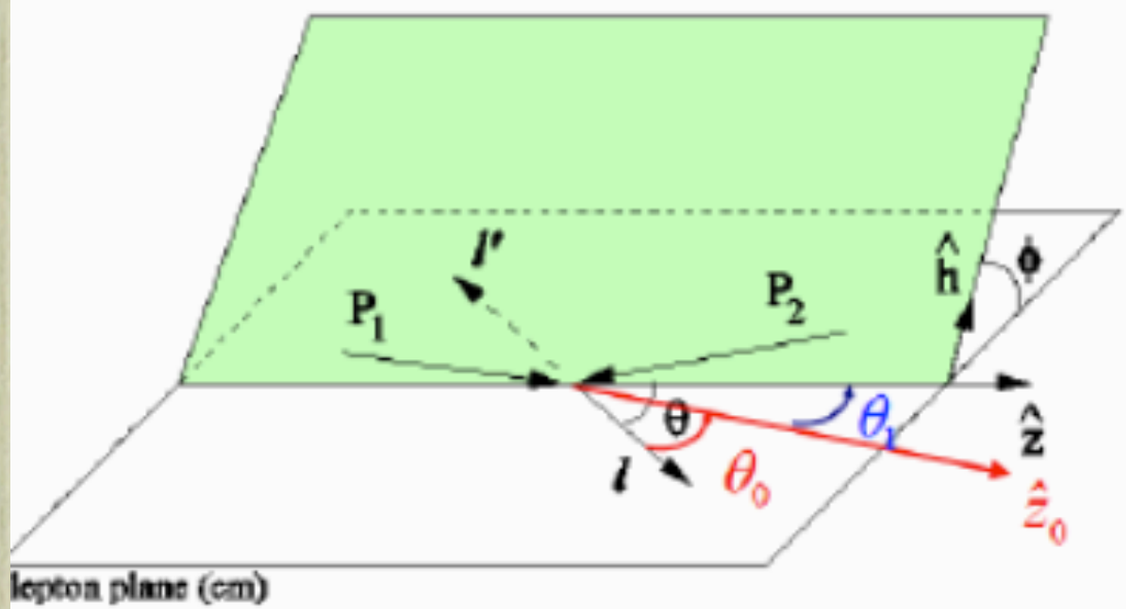


transversely polarized photon, structure W_T only

Miniworkshop on Dihadron Fragmentation Functions (DiFF), Pavia, Sept 7, 2011

A simple geometric derivation of the generalized Lam-Tung relation (a la Oleg Teryaev)

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$



In the γ^* rest frame:

$\hat{z}$ signifies the Collins-Soper frame

$\hat{z}_0$ is along the collinear $q-\bar{q}$ axis

Leptons are emitted with uniform azimuthal distribution, and with θ_0 dependence:

$d\sigma \sim 1+\lambda_0\cos^2\theta_0$

($\lambda_0 = 1$ for spin-1/2 quark;

$\lambda_0 = -1$ for spin-0 quark)

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos\phi$$

$$\begin{aligned} d\sigma &\sim 1+\lambda_0(\cos\theta\cos\theta_1+\sin\theta\sin\theta_1\cos\phi)^2 \\ &= [1+(\lambda_0/2)\sin^2\theta_1]+\cos^2\theta[\lambda_0\cos^2\theta_1-(\lambda_0/2)\sin^2\theta_1] \\ &+\sin 2\theta\cos\phi[(\lambda_0/2)\sin 2\theta_1]+\sin^2\theta\cos 2\phi[(\lambda_0/2)\sin^2\theta_1] \end{aligned}$$

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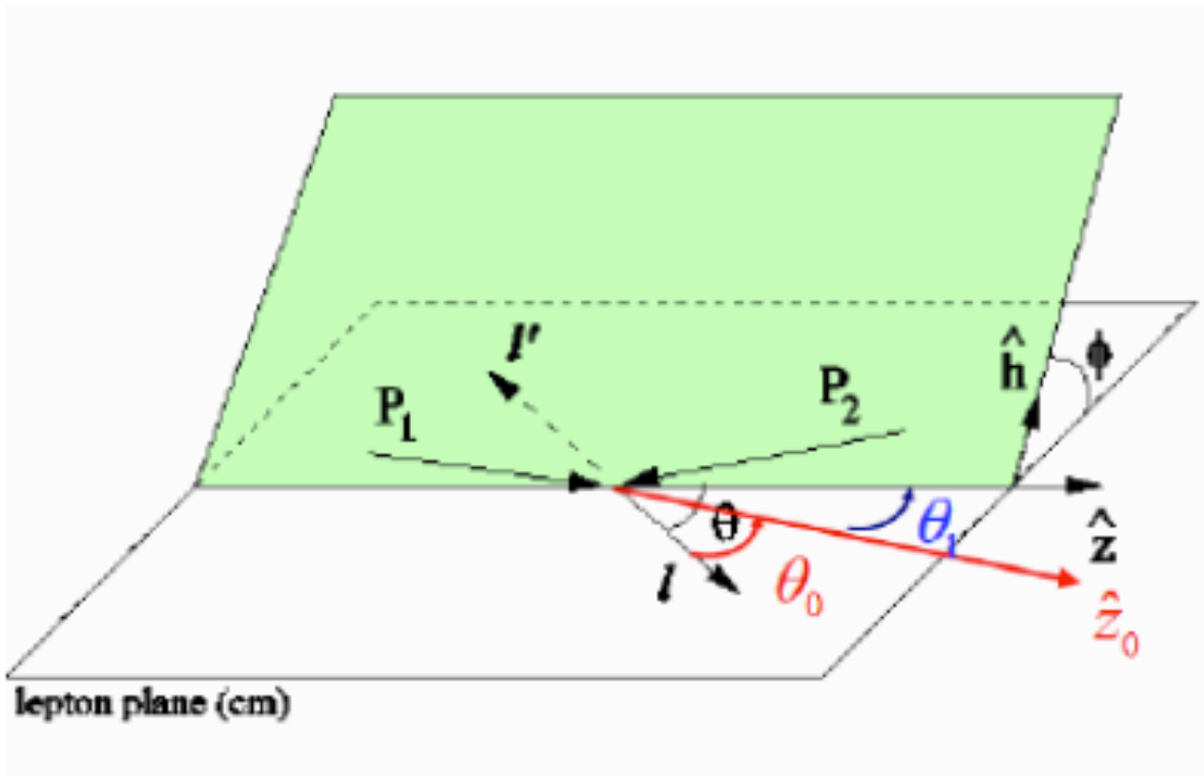


Therefore, we have

$$\lambda=\lambda_0\frac{2\text{-}3\sin^2\theta_1}{2+\lambda_0\sin^2\theta_1}$$

$$\mu=\lambda_0\frac{\sin 2\theta_1}{2+\lambda_0\sin^2\theta_1}$$

$$\nu=\lambda_0\frac{2\sin^2\theta_1}{2+\lambda_0\sin^2\theta_1}$$

and

$$\lambda_0=\frac{\lambda+\frac{3}{2}\nu}{1-\frac{1}{2}\nu}$$ (Generalized Lam-Tung relation)

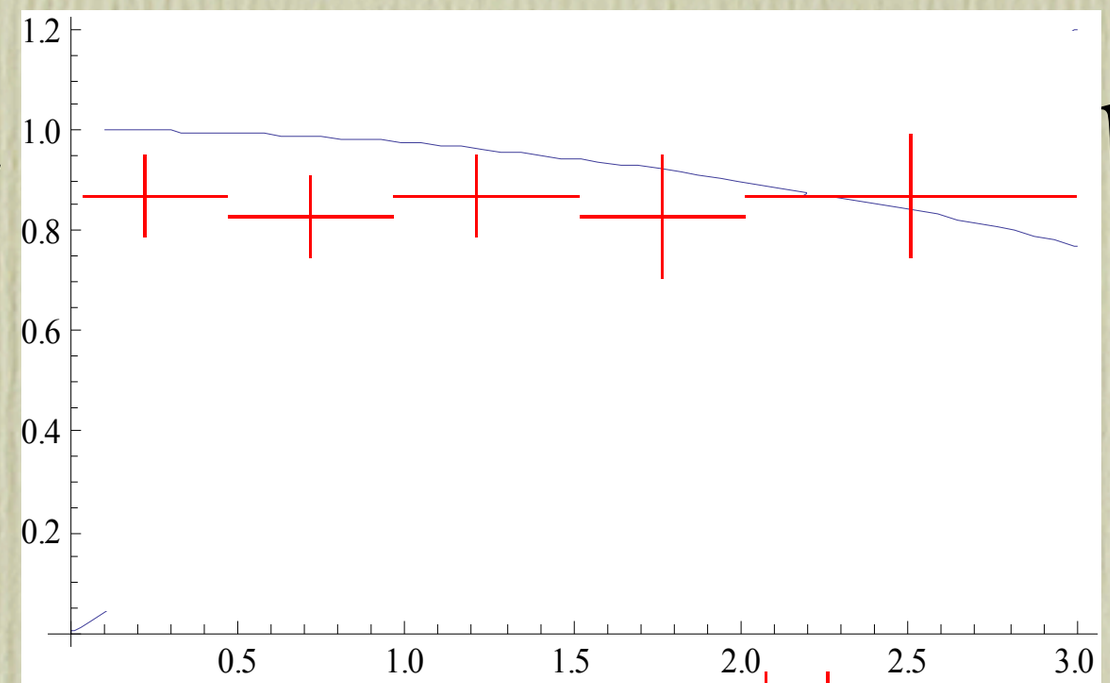
If $\lambda_0=1$, we have $2\nu=1-\lambda$ (Lam-Tung relation)

If $\lambda_0=-1$ (spin-0 quark), we have $-\nu=1+\lambda$

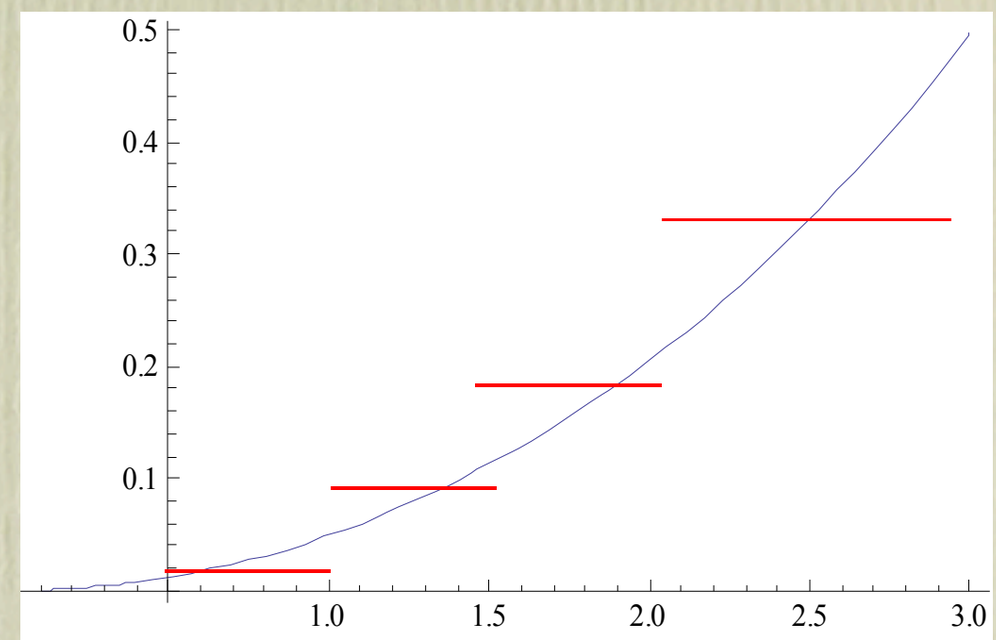
$$\begin{aligned}d\sigma &\sim 1+\lambda_0(\cos\theta\cos\theta_1+\sin\theta\sin\theta_1\cos\phi)^2\\ &=[1+(\lambda_0/2)\sin^2\theta_1]+\cos^2\theta[\lambda_0\cos^2\theta_1-(\lambda_0/2)\sin^2\theta_1]\\ &+\sin 2\theta\cos\phi[(\lambda_0/2)\sin 2\theta_1]+\sin^2\theta\cos 2\phi[(\lambda_0/2)\sin^2\theta_1]\end{aligned}$$

Numerical Results

λ

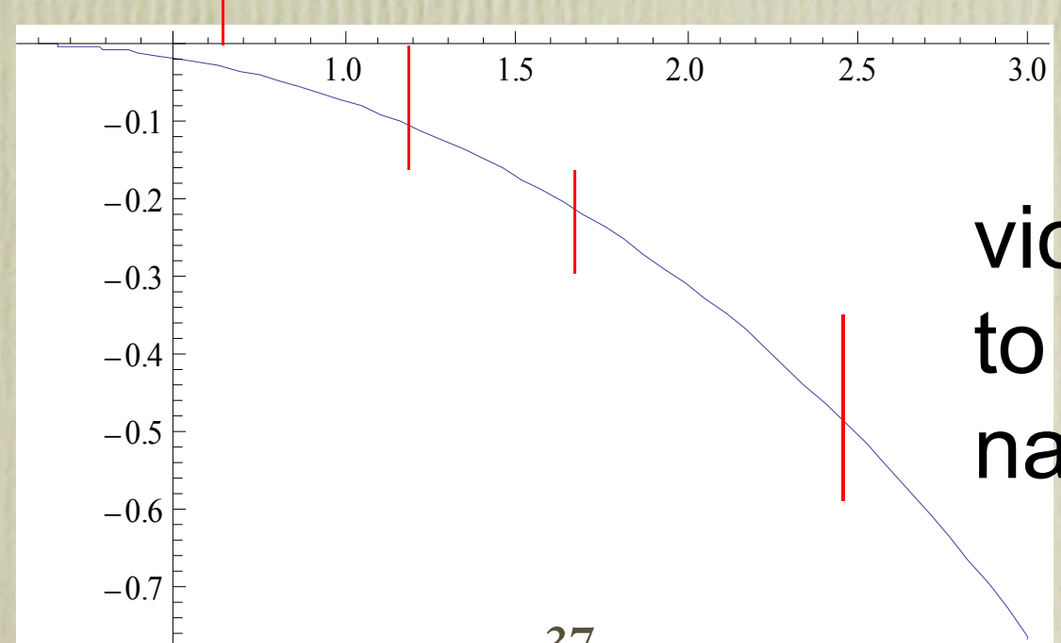


ν



q_T

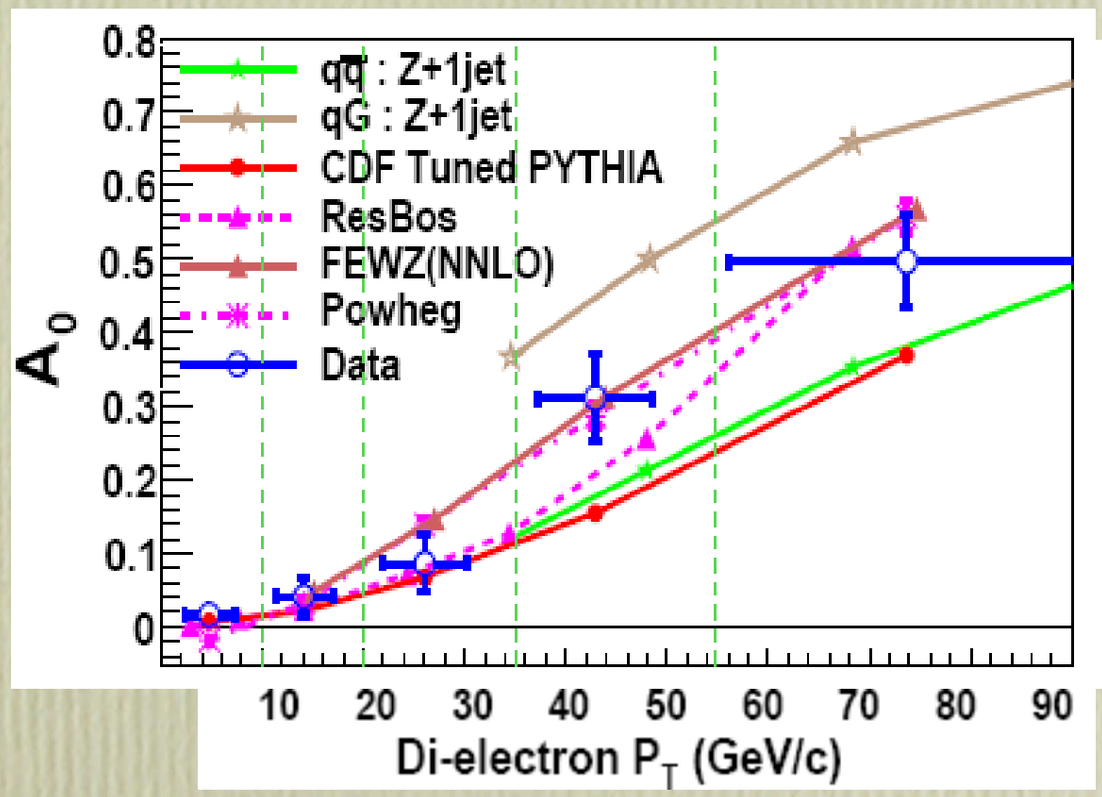
$1-\lambda-2\nu$

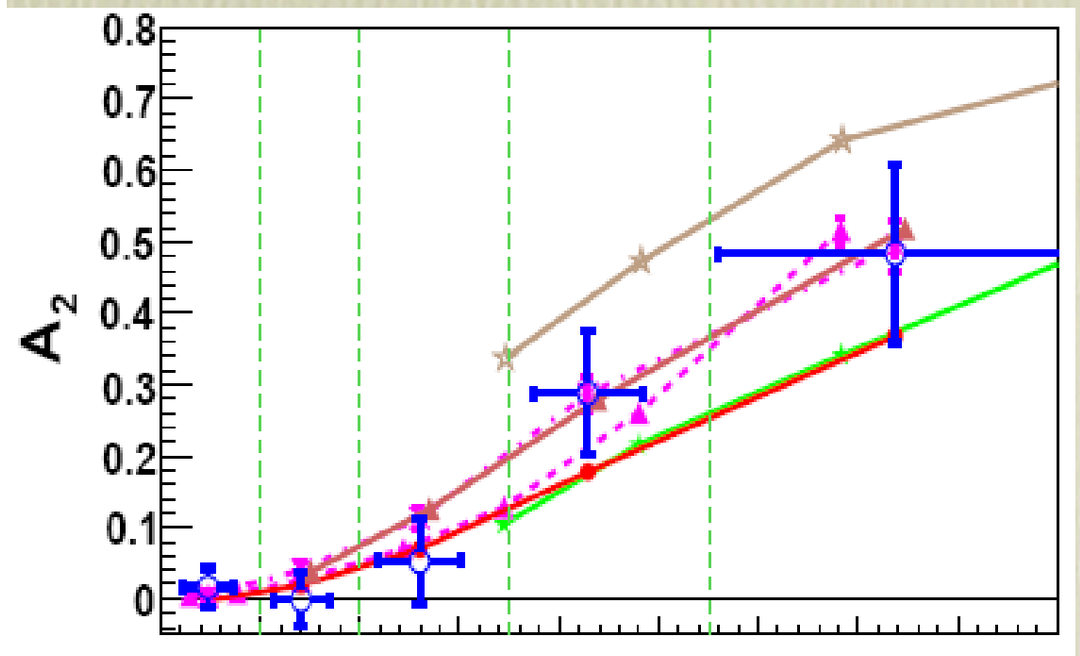


violation proportional to $\sin^2\theta_1$, vanishes naturally as q_T -> 0

CDF

A0 and A2 are functions of lambda and nu
When A0 = A2, The LT relation holds



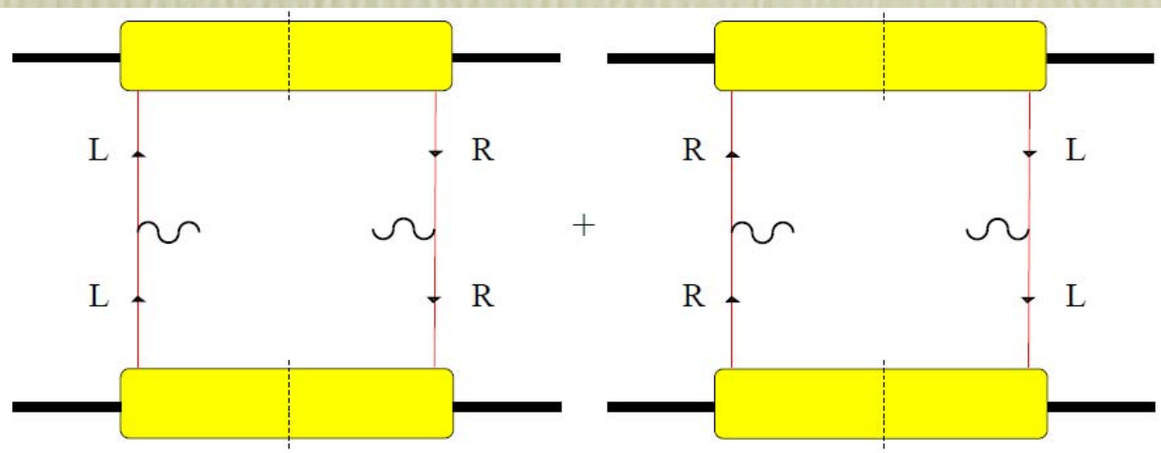


CDF, 1103.5699

may not discriminate BM or Glauber gluons due to high p_T

Angular asymmetry requires helicity flip

- Asymmetry $\cos 2\phi$ comes from an interference between +1 and -1 photon helicities

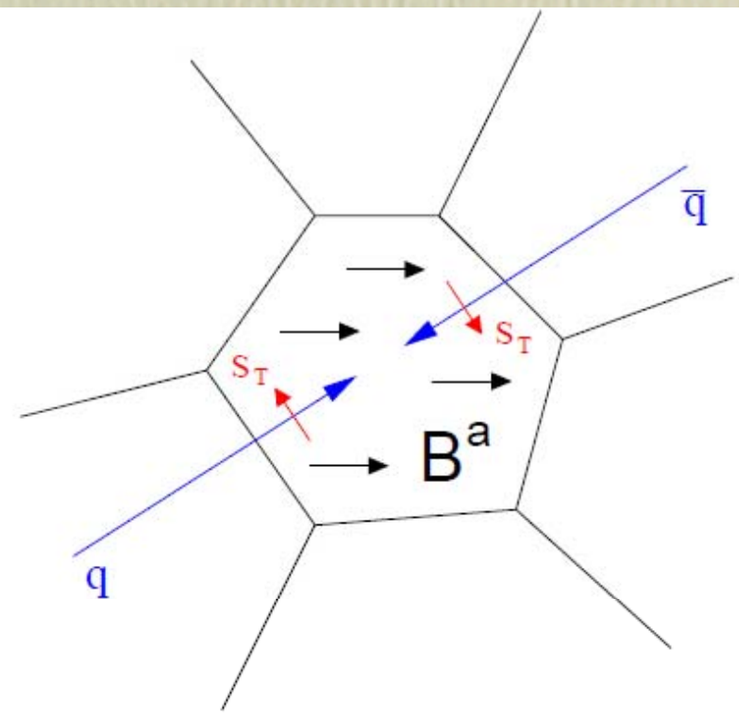


This requires transversely polarized quark-antiquark annihilation

then parton transverse d.o.f comes in to play
sin term appears, which breaks LT relation

Explanation as a QCD vaccum

The QCD vacuum can induce a spin correlation between an annihilating $q\,\bar{q}$



Chromo-magnetic Sokolov-Ternov effect: spin-flip gluon synchrotron emission leading to a correlated polarization of q and qbar.

The spin density matrix becomes:

$$\rho^{(q,\bar{q})} = \frac{1}{4}\{\mathbf{1}\otimes\mathbf{1} + F_j\,\boldsymbol{\sigma}_j\otimes\mathbf{1} + G_j\,\mathbf{1}\otimes\boldsymbol{\sigma}_j + H_{ij}\,\boldsymbol{\sigma}_i\otimes\boldsymbol{\sigma}_j\}$$

Lam-Tung relation could be violated

$$1-\lambda-2\nu=-4\kappa=-4\frac{H_{22}-H_{11}}{1+H_{33}}$$