

Neutrino mass texture with neutrino mass ratio and Cabibbo angle

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based on:

Y. S., R. Takahashi (Hokkaido U.),
M. Tanimoto (Niigata U.), PTEP **2013** 6, 063B02.



1. Introduction

Experiments indicate large $\theta_{13}!!$

- Experimental result by Daya Bay @Neutrino 2012

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010 \text{ (stat)} \pm 0.005 \text{ (syst).}$$

- Consistent with RENO, Double Chooz, and T2K experiments.
- Global fit of the neutrino oscillation:

D. V. Forero, M. Tortola and J. W. F. Valle, arXiv:1205.4018 [hep-ph].

parameter	best fit	2σ	3σ
$\sin^2 \theta_{12}$	0.320	0.29-0.35	0.27-0.37
$\sin^2 \theta_{23}$	0.613 (0.427) 0.600	0.38-0.66 0.39-0.65	0.36-0.68 0.37-0.67
$\sin^2 \theta_{13}$	0.0246 0.0250	0.019-0.030 0.020-0.030	0.017-0.033
Δm_{sol}^2 [10^{-5}eV^2]	7.62	7.27-8.01	7.12-8.20
$ \Delta m_{\text{atm}}^2 $ [10^{-3}eV^2]	2.55 2.43	2.38-2.68 2.29-2.58	2.31-2.74 2.21-2.64

- $\sin \theta_{13}$ is nearly Cabibbo angle:

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- The relation between masses and flavor mixing angles:

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- Neutrino masses and flavor mixing angles are related each other!!

$$\sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}} = 0.173 \simeq \frac{\lambda}{\sqrt{2}} \quad \Rightarrow \quad \sin \theta_{13}.$$

- Before reactor experiments were reported θ_{13} ($|U_{e3}| \equiv \sin \theta_{13}$), the tri-bimaximal mixing (TBM) $V_{\text{tri-bi}}$ was good scheme.

$$U_{\text{PMNS}} = V_{\text{tri-bi}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$|U_{e2}| = \frac{1}{\sqrt{3}}, \quad |U_{e3}| = 0, \quad |U_{\mu 3}| = \frac{1}{\sqrt{2}}.$$

- The left-handed Majorana neutrino mass matrix:

$$M_\nu^{\text{TBM}} = \frac{m_1 + m_3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2 - m_1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1 - m_3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

- TBM is realized by non-Abelian discrete group.
H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. S., and M. Tanimoto,
Prog. Theor. Phys. Suppl. **183** (2010) 1; Lect. Notes Phys. **858** (2012) 1.

2. Large θ_{13} and neutrino mass matrix

We can discuss three cases as 1-2, 1-3, 2-3 mixing deviation from tri-bimaximal one.

W. Rodejohann and H. Zhang, Phys. Rev. D **86** (2012) 093008.

A. Damanik, arXiv:1206.0987 [hep-ph].

- Case I: 1-2 mixing deviation from tri-bimaximal one.
In this case $|U_{e3}| = 0$, then it is **unfavored**.
- Case II: 1-3 mixing deviation from tri-bimaximal one.
In this case $\sin^2 \theta_{12} > \frac{1}{3}$ which is unfavored.
- Case III: 2-3 mixing deviation from tri-bimaximal one.

$$U_{\text{PMNS}} = V_{\text{tri-bi}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{pmatrix},$$

$$|U_{e2}| = \left| \frac{\cos \phi}{\sqrt{3}} \right|, \quad |U_{e3}| = \left| \frac{\sin \phi}{\sqrt{3}} \right|, \quad |U_{\mu 3}| = \left| \frac{\cos \phi}{\sqrt{2}} + \frac{\sin \phi}{\sqrt{3}} \right|.$$

In this case, $\sin^2 \theta_{12} < \frac{1}{3}$.

- We consider the framework of Split Seesaw:

A. Kusenko, F. Takahashi and T. T. Yanagida, Phys. Lett. B **693** (2010) 144.

$$M_{R1} \sim \mathcal{O}(\text{keV}) \ll M_{R2}, \quad M_{R3} \sim \mathcal{O}(10^{12} \text{ GeV}),$$

$$Y_{1i}^D \ll Y_{2i}^D, \quad Y_{3i}^D.$$

- M_{R1} is the sterile neutrino: Dark matter candidate.
- Realized in 5D theory compactified on S^1/Z_2 .
- We can separate the neutrino mass matrix:

$$M_R^{3 \times 3} = \begin{pmatrix} M_{R1}^{1 \times 1} & 0 \\ 0 & M_R^{2 \times 2} \end{pmatrix}, \quad M_D = (Y_{3 \times 1}^D \quad Y_{3 \times 2}^D) v.$$

- By using seesaw mechanism:

$$M_\nu = Y_{3 \times 2}^D (M_R^{2 \times 2})^{-1} (Y^D)_{2 \times 3}^T v^2 + \sum_i Y_{1i}^D M_{R1}^{-1} (Y^D)_{i1}^T v^2.$$



Flavor mixing



No effect on flavor mixing

- We can consider "Minimal Texture".

- The right-handed Majorana and Dirac neutrino mass matrices:

$$M_R^{2 \times 2} = \begin{pmatrix} M_{R2} & 0 \\ 0 & M_{R3} \end{pmatrix}, \quad M_D = Y_{3 \times 2}^D v = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} v.$$

- The condition of 2-3 mixing deviation (normal hierarchy):

$$2a - b - c = 0, \quad \text{and} \quad 2d - e - f = 0.$$

- In this condition, left-handed Majorana neutrino mass matrix:

$$M_\nu = V_{\text{TBM}}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4} \left(\frac{(b+c)^2}{M_{R2}} + \frac{(e+f)^2}{M_{R3}} \right) & \frac{\sqrt{\frac{3}{2}}((b-c)(b+c)M_{R3} + (e-f)(e+f)M_{R2})}{2M_{R2}M_{R3}} \\ 0 & \frac{\sqrt{\frac{3}{2}}((b-c)(b+c)M_{R3} + (e-f)(e+f)M_{R2})}{2M_{R2}M_{R3}} & \frac{(b-c)^2M_{R3} + (e-f)^2M_{R2}}{2M_{R2}M_{R3}} \end{pmatrix} v^2 V_{\text{TBM}}.$$

- In this case, Majorana mass can rescale, then the right-handed Majorana and Dirac neutrino mass matrices:

$$M_R^{2 \times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} v.$$

One zero texture in normal hierarchy

- $b + c = 0$: $M_R^{2 \times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & \frac{e+f}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix} v.$

- The left-handed Majorana neutrino mass matrix:

$$M_\nu = M_D (M_R^{2 \times 2})^{-1} M_D^T = \begin{pmatrix} \frac{1}{4}(e+f)^2 & \frac{1}{2}e(e+f) & \frac{1}{2}(e+f)f \\ \frac{1}{2}e(e+f) & \frac{1}{2} + e^2 & -\frac{1}{2} + ef \\ \frac{1}{2}(e+f)f & -\frac{1}{2} + ef & \frac{1}{2} + f^2 \end{pmatrix} \frac{v^2}{M_R}.$$

- Rotating tri-bimaximal mixing matrix V_{TBM} :

$$M_\nu = V_{\text{TBM}}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}(e+f)^2 & \frac{1}{2}\sqrt{\frac{3}{2}}(e-f)(e+f) \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}}(e-f)(e+f) & 1 + \frac{1}{2}(e-f)^2 \end{pmatrix} \frac{v^2}{M_R} V_{\text{TBM}},$$

$$V_{\text{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

- The neutrino mass eigenvalues: $\frac{m_2}{m_3} \simeq \frac{3}{4}(e + f)^2 \equiv r$.
- The lepton mixing:

$$U_{\text{PMNS}} = V_{\text{TBM}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{pmatrix}, \quad \tan 2\phi \simeq \sqrt{\frac{3}{2}}(e-f)(e+f) \equiv \sqrt{6}\lambda.$$

- The relevant mixing matrix elements:

$$|U_{e2}| = \left| \frac{\cos \phi}{\sqrt{3}} \right| \simeq \sqrt{\frac{1}{3} - \frac{\lambda^2}{2}}, \quad |U_{e3}| = \left| \frac{\sin \phi}{\sqrt{3}} \right| \simeq \frac{\lambda}{\sqrt{2}},$$

$$|U_{\mu 3}| = \left| \frac{\cos \phi}{\sqrt{2}} + \frac{\sin \phi}{\sqrt{3}} \right| \simeq \sqrt{\frac{1}{2} - \frac{3\lambda^2}{4}} + \frac{\lambda}{\sqrt{2}}.$$

- Reparametrization including phases:

$$e = \frac{2re^{2i\alpha} - 3\lambda e^{i\delta}}{2\sqrt{3}re^{2i\alpha}}, \quad f = \frac{2re^{2i\alpha} + 3\lambda e^{i\delta}}{2\sqrt{3}re^{2i\alpha}}.$$

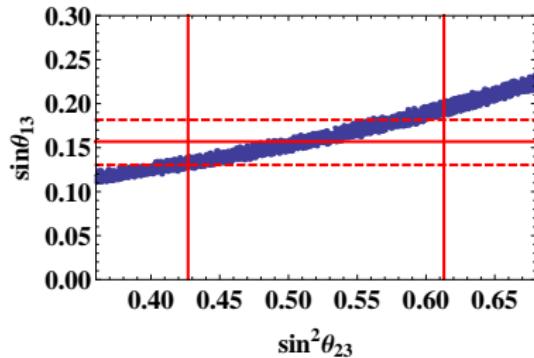
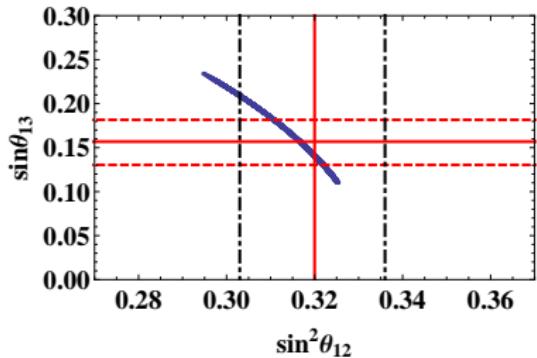
Numerical result

- $\lambda/\sqrt{2} = r, \delta = 2\alpha$:

$$e = \frac{(2 - 3\sqrt{2})\sqrt{r}e^{i\alpha}}{2\sqrt{3}}, \quad f = \frac{(2 + 3\sqrt{2})\sqrt{r}e^{i\alpha}}{2\sqrt{3}}.$$

- Neutrino Texture:

$$M_R^{2 \times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & \frac{1}{\sqrt{3}}\sqrt{r}e^{i\alpha} \\ \frac{1}{\sqrt{2}} & \frac{2-3\sqrt{2}}{2\sqrt{3}}\sqrt{r}e^{i\alpha} \\ -\frac{1}{\sqrt{2}} & \frac{2+3\sqrt{2}}{2\sqrt{3}}\sqrt{r}e^{i\alpha} \end{pmatrix} v.$$



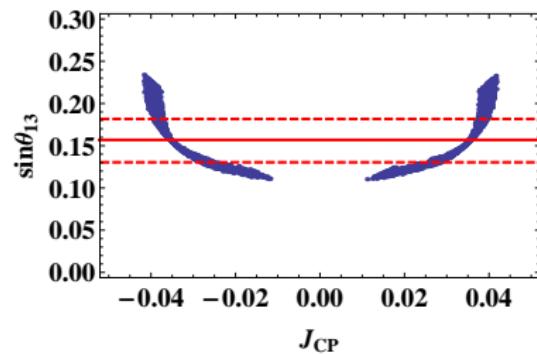
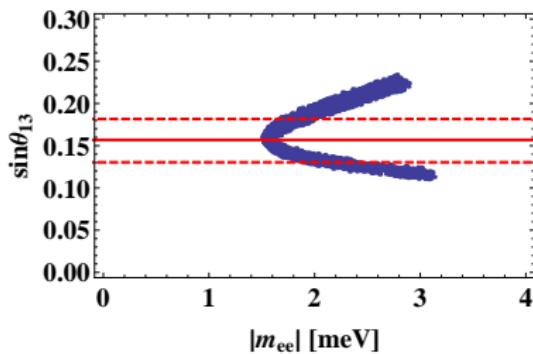
$\nu 0\beta\beta$ and CP violation

- $\nu 0\beta\beta$:

$$|m_{ee}| = \sum_i^3 |m_i U_{ei}^2|.$$

- CP violation:

$$J_{CP} = \text{Im} [U_{\mu 3} U_{\tau 3}^* U_{\mu 2}^* U_{\tau 2}] .$$



3. Summary

Conclusion

- The large θ_{13} is given impact for us.
- We propose minimal texture which makes the connection between masses and mixing angles.

$$r = \sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}} = \frac{\lambda}{\sqrt{2}} = \sin \theta_{13}.$$

- These textures are motivated for model building.

Future work

- We want to consider contribution from charged lepton sector.
- What is symmetry or mechanism introducing these texture ???

One zero texture in normal hierarchy

- The general texture:

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} v.$$

One zero texture in normal hierarchy

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$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} v.$$

- (1) $b + c = 0$:

$$M_D = \begin{pmatrix} 0 & \frac{e+f}{2} \\ b & e \\ -b & f \end{pmatrix} v \rightarrow \begin{pmatrix} 0 & \frac{e+f}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix} v.$$

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- (2) $c = 0$:

$$M_D = \begin{pmatrix} \frac{b}{2} & \frac{e+f}{2} \\ b & e \\ 0 & f \end{pmatrix} v \rightarrow \begin{pmatrix} \frac{1}{2} & \frac{e+f}{2} \\ 1 & e \\ 0 & f \end{pmatrix} v.$$

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- (3) $b = 0$:

$$M_D = \begin{pmatrix} \frac{c}{2} & \frac{e+f}{2} \\ 0 & e \\ c & f \end{pmatrix} v \rightarrow \begin{pmatrix} \frac{1}{2} & \frac{e+f}{2} \\ 0 & e \\ 1 & f \end{pmatrix} v.$$

One zero texture in normal hierarchy

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- We focus on (1) texture.