Single Perturbative Splitting Diagrams in Double Parton Scattering.

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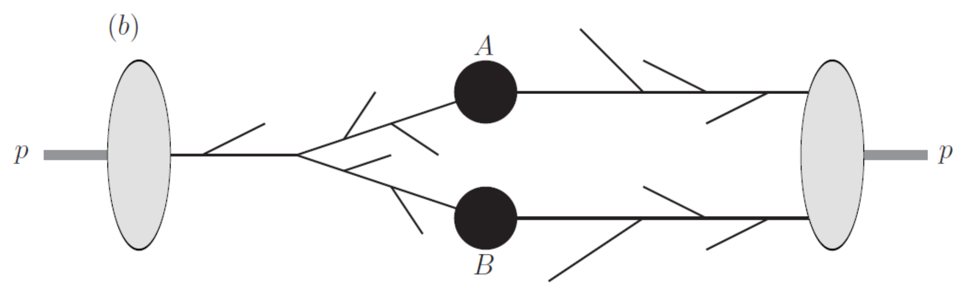
Single Perturbative Splitting Diagrams

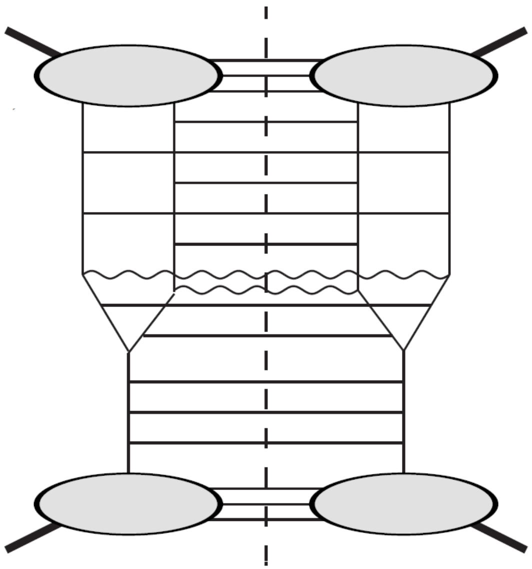
Single perturbative splitting graphs = double scattering processes in which the two partons coming from one proton have a common origin in a single **perturbative** parton:



Also known as '2v1', '3 → 4' or '2v4' graphs.

Should these graphs be included in the (LO) total cross section for double parton scattering?

Single Perturbative Splitting Diagrams



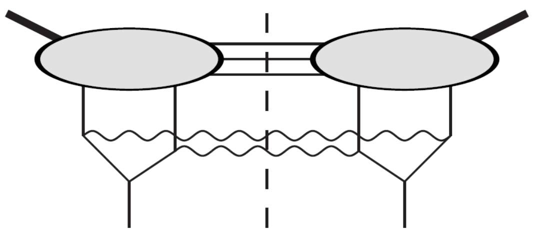
If yes, then the cross section formula corresponding to this (diagonal) 2v1 diagram with *n* QCD splitting vertices should have a piece in it proportional to:

$$\left(\frac{\Lambda^2}{Q^4}\right)\left(\alpha_S \log\left[\frac{Q^2}{\Lambda^2}\right]\right)^n$$

Power suppressed - DPS

DGLAP log for every splitting including 1 → 2 splitting

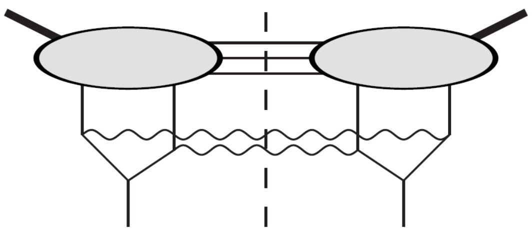
Simplest possible graph with 2v1 structure is this:



I investigated whether this graph has a piece proportional to:

$$\left(\frac{\Lambda^2}{Q^4}\right)\left(\alpha_S \log\left[\frac{Q^2}{\Lambda^2}\right]\right) \qquad (n = 1)$$

Simplest 2v1 Graph - Calculation



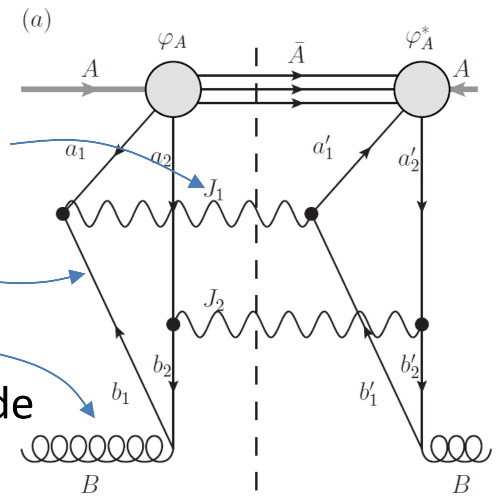
Need to use a wavefunction on the side with the two nonperturbative partons to represent the fact that the two partons are tied together in the same proton. I used formalism of Paver and Treleani (Nuovo Cim. A70 (1982) 215).

Process explicitly considered:

Off shell photons ($Q^2 > 0$) produced

Quarks in loop

Initiated by a gluon on one side



Simplest 2v1 Graph - Calculation

$$\frac{1}{Q^2} \quad \frac{1}{Q^2}$$

Result:

$$\sigma_{1v2}(s) = \sum_{s_i s_i' t_i t_i'} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}^{s_1,t_1;s_1',t_1';\mu_1}_{\bar{q}q\to\gamma*}(\hat{s} = x_1 y_1 s)\hat{\sigma}^{s_2,t_2;s_2',t_2';\mu_2}_{q\bar{q}\to\gamma*}(\hat{s} = x_2 y_2 s)$$

$$\times \quad \Gamma_A^{s_1 s_2, s_1' s_2'}(x_1, x_2; \boldsymbol{b} = \boldsymbol{0}) \quad \underbrace{\left[\frac{\alpha_s}{2\pi} P^{\lambda\to t_2 t_1, t_2' t_1'}_{g\to q\bar{q}}(y_2)\,\delta(1 - y_1 - y_2) \int_{\Lambda^2}^{Q^2} \frac{d\boldsymbol{J_1}^2}{\boldsymbol{J_1}^2}\right]}_{1 \,\longrightarrow\, 2 \text{ splitting function}}$$

$$\propto \Lambda^2$$

2pGPD of nonperturbatively generated parton pair evaluated at **b** = **0**

Gives required logarithm

Suggests 2v1 graphs should be included in DPS cross section!

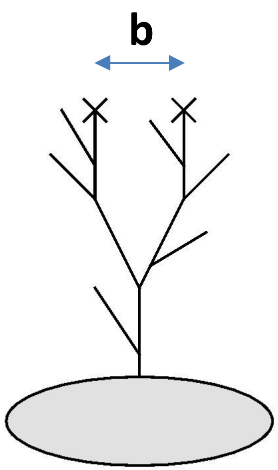
Total contribution from diagonal 2v1 graphs

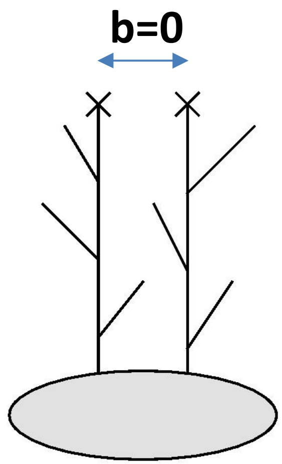
Assuming that only diagonal 2v1 diagrams contribute to the DPS cross section at leading logarithmic order, then summing up the contributions from all of the 2v1 diagrams yields the following:

$$\sigma_{(A,B)}^{D,1v2}(s) = 2 \times \frac{m}{2} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{ik\to A}(\hat{s} = x_1 y_1 s) \hat{\sigma}_{jl\to B}(\hat{s} = x_2 y_2 s)$$
$$\times \underbrace{\breve{D}_p^{ij}(x_1, x_2; Q^2)} \underbrace{\Gamma_{p,indep}^{kl}(y_1, y_2, \boldsymbol{b} = \boldsymbol{0}; Q^2)}$$

'sPDF feed part of dPDF'

$$\int d^2\mathbf{b}$$





'Independent branching' 2pGPD

Comments on the formula for σ_{1v2}

The critical requirement for the validity of the derivation on the previous page is that parton pairs connected only via nonperturbative interactions should have an ***r*** distribution that is cut off at values of order Λ_{QCD} (or a ***b*** distribution that is smooth on scales of size << proton radius). That is, the ***r*** profile of $\Gamma^{kl}_{p,indep}(y_1, y_2, \boldsymbol{\Delta}; Q^2)$ should have a width of order Λ_{QCD} .

The results of the previous slide are potentially misleading, in that they appear to indicate that 2v1 contribution to DPS probes independent branching 2pGPDs at zero parton separation. In fact, the results correspond to a broad logarithmic integral over values of $\boldsymbol{b}^2$ that are << (proton radius)^2 but >> $1/Q^2$.

Comments on the formula for σ$_{1v2}$

Geometrical prefactor for 2v1 graphs is different to that from 2v2 or zero perturbative splitting graphs. If one assumes:

$$\Gamma^{ij}_{p,indep}(x_1, x_2, \boldsymbol{b}; Q^2) = \tilde{D}^{ij}(x_1, x_2; Q^2) F(\boldsymbol{b})$$

i.e. x – **b** factorisation for Γ_{indep}

Then:

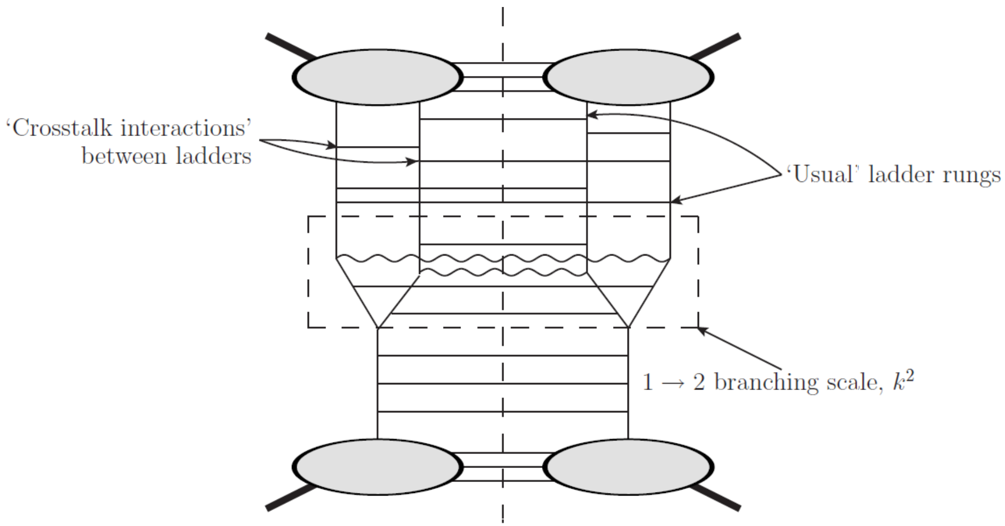
$$\frac{1}{\sigma_{eff,2v2}} \equiv \int d^2\boldsymbol{b}[F(\boldsymbol{b})]^2$$

$$\frac{1}{\sigma_{eff,1v2}} \equiv \quad F(\boldsymbol{b} = \boldsymbol{0})$$

Naive Gaussian for F(**b**) ➔ factor of two enhancement for 2v1, but dependence on precise form of F(**b**) is small – e.g. top hat, projection of hard sphere, double Gaussian give similar enhancement factors.

BDFS, Eur.Phys.J. C72 (2012) 1963
JG, JHEP 1301 (2013) 042

Crosstalk in 2v1 graphs



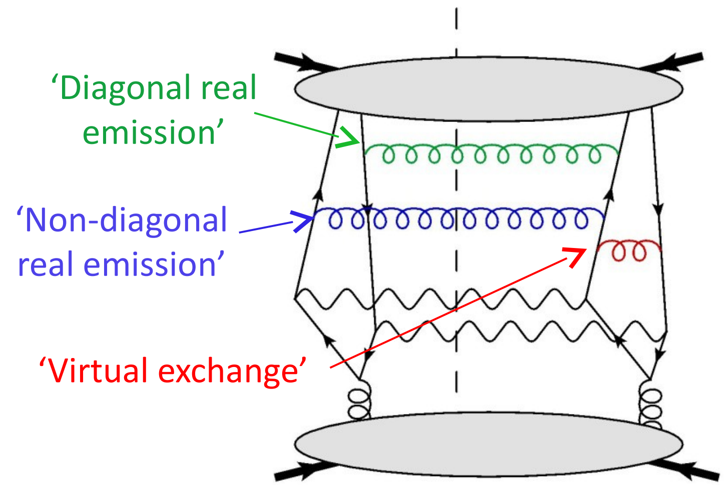
The contribution from diagonal 2v1 graphs to the DPS cross section has previously been written down in:

Ryskin and Snigirev, Phys.Rev. D83 (2011) 114047)
Blok et al., Eur. Phys. J. C72 (2012) 1963 (eq 13b)

We have discovered that there is an additional contribution to the LO 2v1 DPS cross section, associated with non-diagonal interactions ('crosstalk') on the side with two NP ladders.

Contributes at LO provided that the crosstalk on the 'two NP ladder' side occurs at lower scales than the 1→2 branching on the other side.

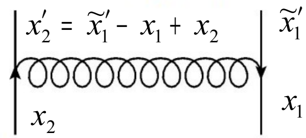
I demonstrated this by studying the simplest type of 2v1 graph containing a crosstalk interaction. Two types of crosstalk – I chose to look in detail at a diagram containing a non-diagonal real emission.

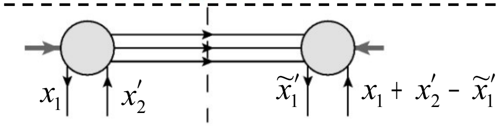


Crosstalk 2v1 graph – calculation

$$\frac{1}{Q^2}$$

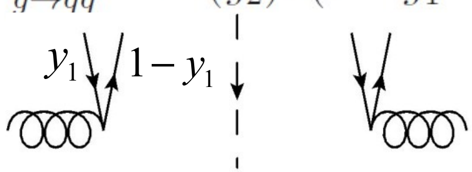
$$\sigma_{XT}(s) = \sum_{s_i\tilde{s}_i t_i \tilde{t}_i \tilde{s}'_1 s'_2} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}^{s_1,t_1;\tilde{s}_1,\tilde{t}_1;\mu_1}_{\bar{q}q\to\gamma*}(\hat{s}=x_1y_1s)\,\hat{\sigma}^{s_2,t_2;\tilde{s}_2,\tilde{t}_2;\mu_2}_{q\bar{q}\to\gamma*}(\hat{s}=x_2y_2s)$$



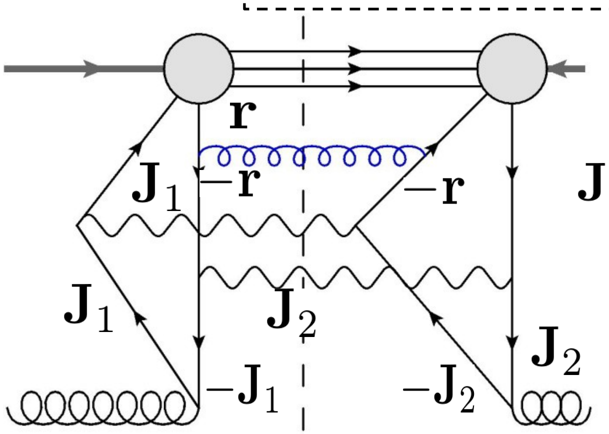


$$\left[\frac{\alpha_s}{2\pi}\int_{x_1}^{1-x_2} d\tilde{x}'_1 V^{\tilde{s}'_1 s'_2 \to \tilde{s}_1 s_2;\mu_3}_{I,q\to q}(x_1,\tilde{x}'_1,x'_2)\,\Gamma^{s_1 s'_2,\tilde{s}'_1\tilde{s}_2}_{p;q\bar{q}}(x_1,x'_2,\tilde{x}'_1)\right] \propto \Lambda^2$$

$$\left[\frac{\alpha_s}{2\pi}P^{\lambda\to t_2t_1,\tilde{t}_2\tilde{t}_1}_{g\to q\bar{q}}(y_2)\,\delta(1-y_1-y_2)\right]\int d\boldsymbol{J_1}^2 d\boldsymbol{r}^2 \frac{2\boldsymbol{\epsilon}_\lambda\cdot\boldsymbol{J}_1\boldsymbol{\epsilon}^*_\lambda\cdot(\boldsymbol{J}_1+\boldsymbol{r})}{\boldsymbol{r}^2\boldsymbol{J}_1^2(\boldsymbol{J}_1+\boldsymbol{r})^2}$$



$$\boldsymbol{r}^2 \ll \boldsymbol{J}_1^2, \boldsymbol{J}_2^2$$



$$\mathbf{J}_2 = -\mathbf{J}_1 - \mathbf{r}$$

$$\int_{\Lambda^2}^{Q^2}\frac{d\boldsymbol{J_1}^2}{\boldsymbol{J_1}^2}\int_{\Lambda^2}^{\boldsymbol{J_1}^2}\frac{d\boldsymbol{r}^2}{\boldsymbol{r}^2} = \boxed{\log^2\left(\frac{Q^2}{\Lambda^2}\right)}$$

Scale of crosstalk interaction (**r**^{2}) must be smaller than that of 1→2 splitting (**J**$_1^{\,2}$) for leading log contribution.

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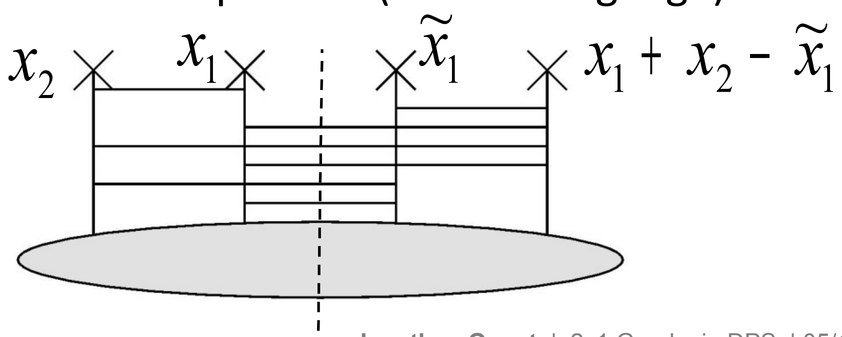
2v1 cross section including crosstalk

Summing up the leading logarithmic contributions from all 2v1 graphs, including those with crosstalk on the side with two 'nonperturbatively generated' partons:

$$\sigma^{D,2v1}_{(A,B)}(s) = 2 \times \frac{m}{2} \sum_{l i_i j_i i'_i j'_i} \int_{\Lambda^2}^{Q^2} dk^2 \frac{\alpha_s(k^2)}{2\pi k^2} \int dx_1 dx_2 dy_1 dy_2 \frac{dx'_1}{x'_1}\frac{dx'_2}{x'_2}\frac{dy'_1}{y'_1}\frac{dy'_2}{y'_2}$$
$$\times \hat{\sigma}_{i_1 j_1 \to A}(\hat{s} = x_1 y_1 s)\hat{\sigma}_{i_2 j_2 \to B}(\hat{s} = x_2 y_2 s)$$
$$\times \frac{D^l_h(y'_1 + y'_2, k^2)}{y'_1 + y'_2} P_{l \to j'_1 j'_2}\left(\frac{y'_1}{y'_1 + y'_2}\right) D^{j_1}_{j'_1}\left(\frac{y_1}{y'_1}; k^2, Q^2\right) D^{j_2}_{j'_2}\left(\frac{y_2}{y'_2}; k^2, Q^2\right)$$
$$\times D^{i_1}_{i'_1}\left(\frac{x_1}{x'_1}; k^2, Q^2\right) D^{i_2}_{i'_2}\left(\frac{x_2}{x'_2}; k^2, Q^2\right) \Gamma^{i'_1 i'_2}_h(x'_1, x'_2; x'_1, k^2)$$

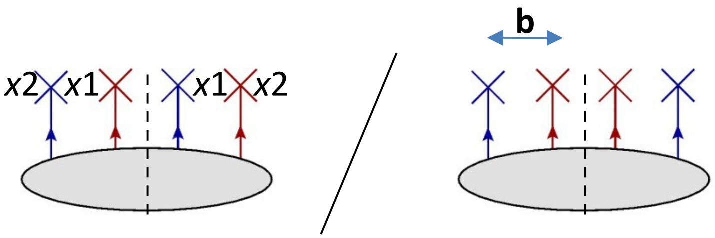
$\Gamma^{i'_1 i'_2}_h(x_1, x_2; \tilde{x}_1, \mu^2)$ is a four-parton 'twist 4' matrix element whose evolution involves all possible exchanges between these partons (in an axial gauge).



Colour in the evolution of $\Gamma(x_1,x_2;\tilde{x}_1)$

Recall that for the 2pGPD with finite **b**, every distribution which does not have the partons with the same lightcone mtm fractions paired up into colour singlets is Sudakov suppressed:

M. Mekhfi and X. Artru, Phys.Rev. D37 (1988) 2618–2622
Diehl, Ostermeier and Schafer (JHEP 1203 (2012) 089)
A. V. Manohar and W. J. Waalewijn, Phys.Rev. D85 (2012) 114009



$$\sim \exp\left(\frac{\alpha_s}{2\pi}\left(C_R^I - C_V^I\right)\ln^2\left(\mathbf{b}^2Q^2\right)\right)$$

$\left(C_R^I - C_V^I\right) = -\frac{1}{2}C_A$ for the two quark case considered here.

In axial gauge: arises from incomplete cancellation of soft region between real emission diagrams and virtual self-energy corrections in colour interference distributions.

A. V. Manohar and W. J. Waalewijn, Phys.Rev. D85 (2012) 114009

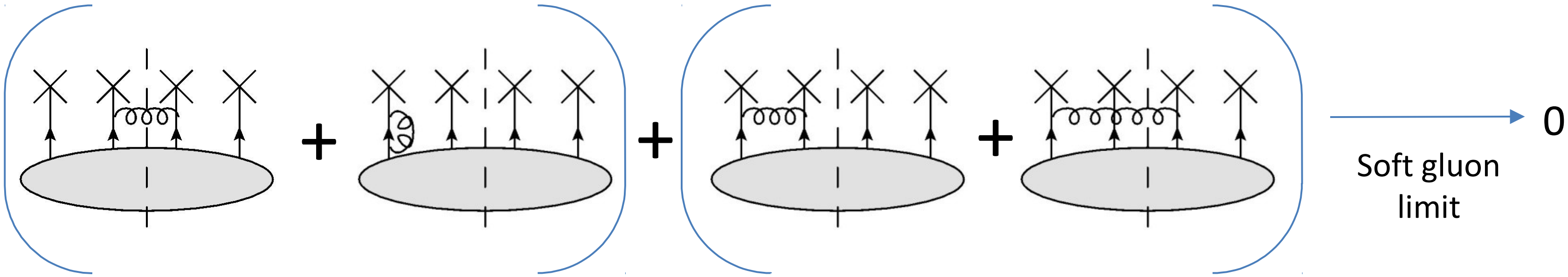
Physically: occurs because these distributions involve a movement of colour by the large transverse distance **b** in the hadron.

M. Mekhfi and X. Artru, Phys.Rev. D37 (1988) 2618–2622

Colour in the evolution of $\Gamma(x_1,x_2;\tilde{x}_1)$

There is no such Sudakov suppression of colour interference distributions for the twist-four distribution Γ. The extra 'crosstalk' diagrams that are allowed in the evolution of this distribution provide extra soft-gluon divergences that cancel those from 'diagonal' real emission and virtual self energy corrections.

Schematically:



Physically: All four partons in this distribution are close together in transverse space (much closer together than $1/\mu$ in transverse space). Soft longwave gluons can only resolve total colour of all four partons, which is constrained to be 0 since the proton is colourless).

Levin, Ryskin, Shuvaev, Nucl.Phys. B387 (1992) 589–616
Gribov, Dokshitzer, Troyan, Khoze Sov. Phys. JETP 88 (1988) 1303

Colour in crosstalk and small x

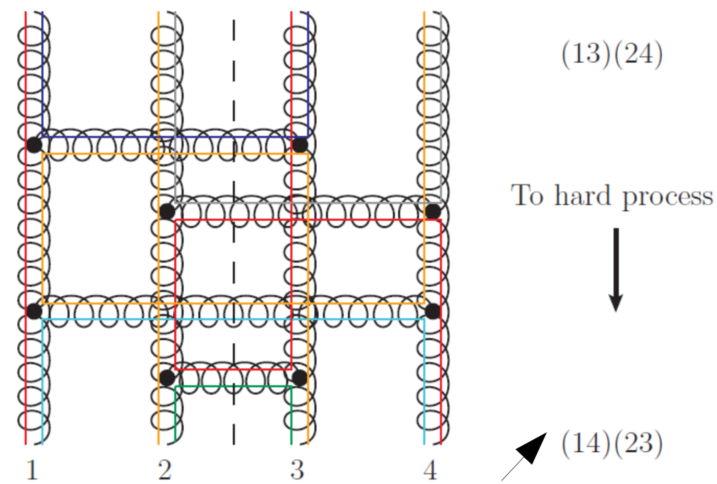
Consider small *x* (perhaps most relevant region for DPS processes at the LHC). Gluons dominant → we'll consider only these partons.

At low x, anomalous dimensions prefer colour configurations in which there are two colour singlet pairs of partons in the twist-4 distribution.

Levin, Ryskin, Shuvaev, Nucl.Phys. B387 (1992) 589–616
Diehl, Ostermeier and Schafer (JHEP 1203 (2012) 089)

At the scale of the 1→2 splitting in the 2v1 diagram, k^2, the nonperturbatively generated partons with identical *x* fractions should be in a colour singlet state to avoid Sudakov suppression in subsequent evolution.

Using two off-diagonal + two diagonal real emissions, can alter the way in which the legs are grouped into colour singlets at scales < k^2 - 'colour recombination'.

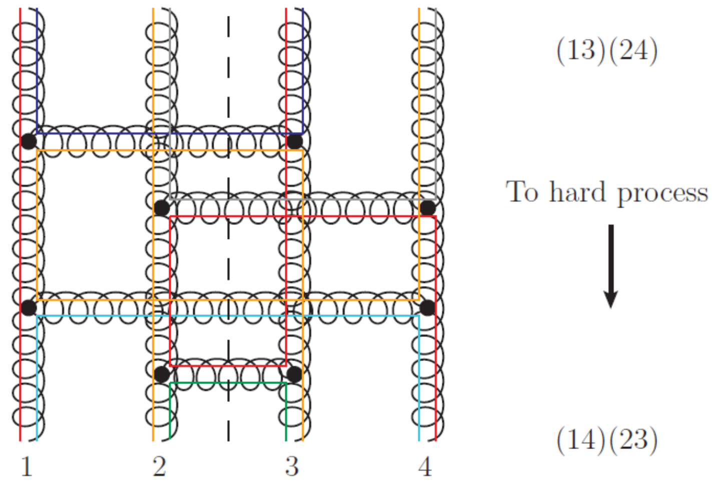


Colour singlet pairings

Colour in crosstalk and small x

The colour recombination process is non-planar – therefore colour recombination vertex is suppressed by $\boxed{1/\left(N_c^2-1\right)}$

J. Bartels, Phys.Lett. B298 (1993) 204–210
J. Bartels, Z.Phys. C60 (1993) 471–488
J. Bartels and M. Ryskin, Z.Phys. C60 (1993) 751–756
J. Bartels and M. Ryskin, arXiv:1105.1638.



Are crosstalk effects numerically small due to this colour suppression?

Colour in crosstalk and small x

Not necessarily – Bartels and Ryskin demonstrated using a DLLA calculation in the context of shadowing corrections to DIS that crosstalk effects can be significant provided they have enough 'evolution space' to occur in:

$$K_{BR}(Y,\xi) = 1 + 2\sqrt{\pi\delta}\left(\frac{4N_C}{\pi b} Y \ln\left(\frac{\xi - \xi_\Lambda}{-\xi_\Lambda}\right)\right)^{1/4}$$

$Y = \ln(1/x)$, $\xi = \ln(Q^2/Q_0^2)$, $\xi_\Lambda = \ln(\Lambda_{QCD}^2/Q_0^2)$

$\delta \sim 1/N_c^4$ Q_0^2 = evolution start scale

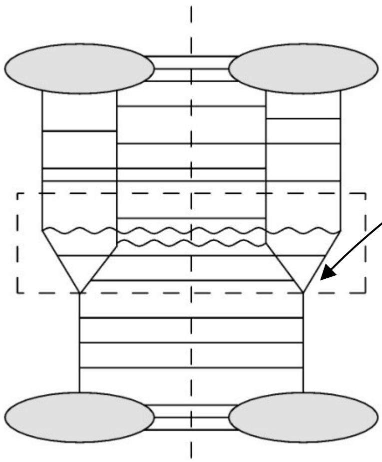
e.g. for $x = 0.001$
Q = 5 GeV
Q_0 = 1 GeV

(N_C = 3, Λ = 0.359 GeV, only gluons)

$$\frac{\Gamma_{\text{crosstalk}}}{\Gamma_{\text{no crosstalk}}} = 1.96$$

J. Bartels and M. Ryskin, Z.Phys. C60 (1993) 751–756

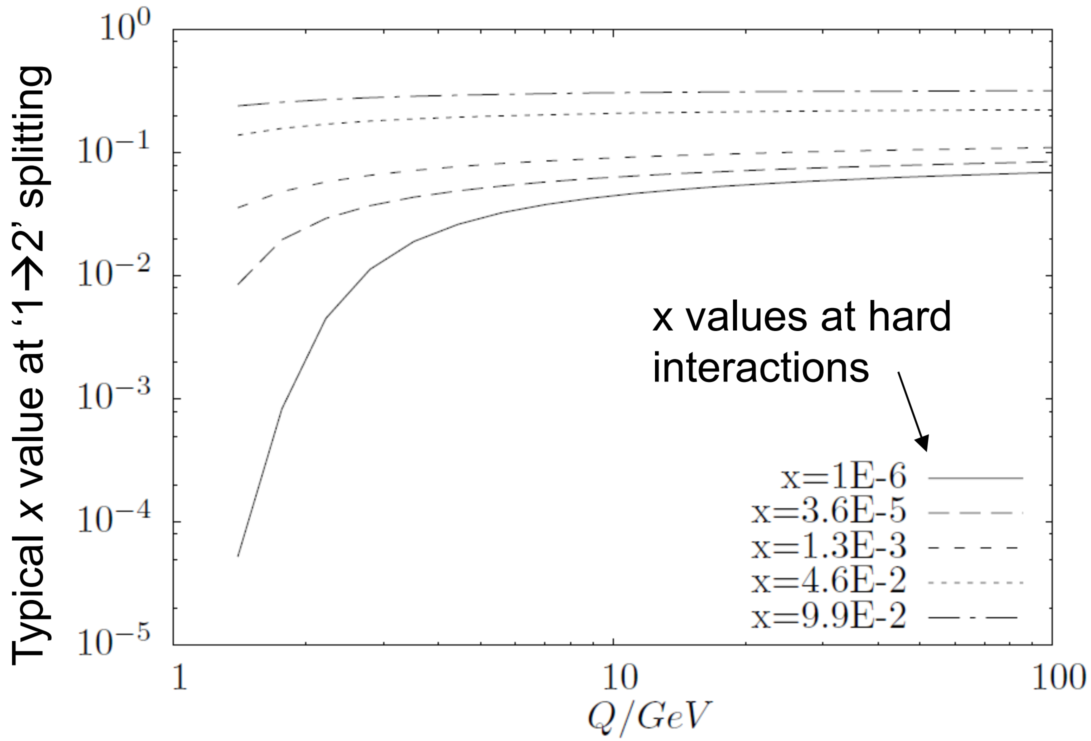
Numerical size of crosstalk effects



What sets the size of the evolution space for the crosstalk effects? A: The x and k^2 values of the 1→2 splitting. What are these for given values of x and Q^2 at the hard scale?

Numerical investigation:
(only gluons considered)

For Q > 10 GeV, typical value of x for 1→2 splitting, or typical value of x at which crosstalk interactions finish is ~10^{-2}-10^{-1}



(at hard interactions)

This is the region of x where PDFs do not change much with scale, or decrease → crosstalk interactions likely have little numerical impact due to this fact.

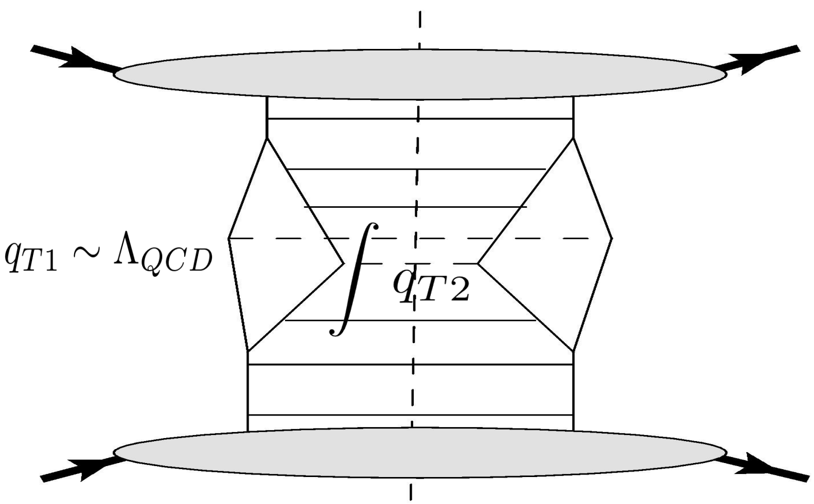
Comment: 1v1 graphs in DPS

It has been suggested in several papers that 'double perturbative splitting' or '1v1' graphs should not be included in DPS cross section and resummed, and instead taken into account in a fixed order way as higher order corrections to the SPS cross section.

Consequences:
- Operators in DPS involve both protons at once.
- No 'factorisation' in usual sense of the word.

Manohar and Waalewijn (Phys.Lett. B713 (2012) 196–201).

Blok, Dokshitzer, Frankfurt, Strikman [arXiv:1306.3763]



One can think of at least one scenario where this is not satisfactory – production of two sets of final states where we integrate over transverse momentum of one set of final state particles, and measure the other at small $q_T \sim \Lambda_{QCD}$

Then this type of 1v1 graph has a natural piece with same power as 'canonical' DPS (and also SPS) and a DGLAP log for every splitting – shouldn't we resum this in DPS?

Summary

- We have closely studied the contribution to the LO pp DPS cross section from 2v1 diagrams.
- We found:
  - That diagonal 2v1 graphs contribute to the LO DPS cross section with a different geometrical prefactor from 1v1 graphs.
  - That 2v1 graphs in which the two NP ladders exchange partons with one another contribute to the DPS cross section, provided that the 'crosstalk' takes place at lower scales than the perturbative $1 \rightarrow 2$ splitting.
- Crosstalk interactions between the two NP ladders are suppressed by colour effects. At low *x* the most likely sort of crosstalk interaction is a 'colour recombination' – this is suppressed by $1/(N_c^2 - 1)$.
- However, it is not colour effects that suppress the crosstalk contribution to the 2v1 cross section – it is a lack of 'evolution space' for the crosstalk effects to occur in.
- Should 1v1 graphs be completely removed from the DPS cross section?