Evolution of double parton distributions

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Outline

Study evolution of partonic correlations

- Introduction
- Evolution of
  - transverse ($\mathbf{y}$) dependence
  - polarization
- Conclusions

DPS cross section

- The famous pocket formula: $\sigma_{\mathrm{DPS}} \approx \frac{1}{C} \frac{\sigma_1 \sigma_2}{\sigma_{\mathrm{eff}}}$

- DPS cross section formula:
  (assuming factorization of hard scatters)

$$\frac{d\sigma}{dx_1\, dx_2\, dx_3\, dx_4} = \frac{1}{C} \sum_{p_1, p_2, p_3, p_4} \hat{\sigma}_{p_1 p_3}(x_1, x_3)\, \hat{\sigma}_{p_2 p_4}(x_2, x_4)$$
$$\times \int d^2\boldsymbol{y}\, F_{p_1 p_2}(x_1, x_2, \boldsymbol{y})\, F_{p_3 p_4}(x_3, x_4, \boldsymbol{y})$$
$$+ \ \{\text{color, flavour and fermion number interference terms}\}$$

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if? why? when?

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Suppressed at large Q

M. Mekhfi, X. Artru, 1985; A.V. Manohar, W.J. Waalewijn, 2012

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  (assuming factorization of hard scatters)

Depend on x_1, x_2, $\boldsymbol{y}$, spin, flavor and scale Q.

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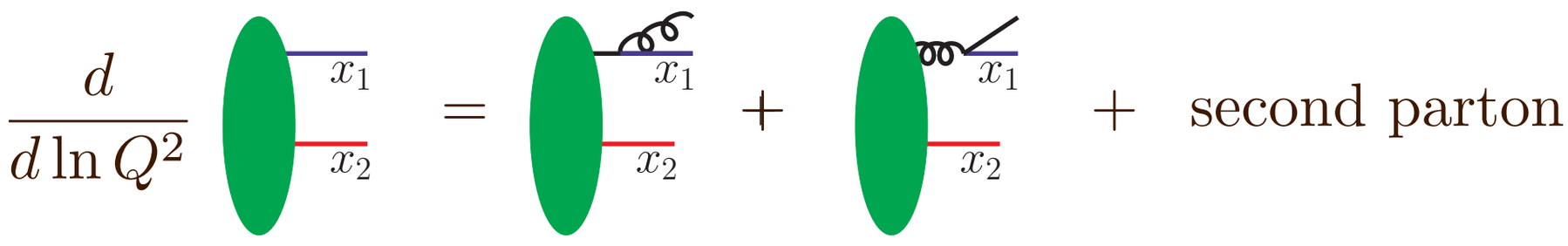
Evolution of DPDs

- Double DGLAP evolution equation - no single-feed term
- For two unpolarized quarks:

$$\frac{df_{qq}(x_1, x_2, y; Q)}{d\ln Q^2} = \frac{\alpha_s(Q)}{2\pi}\Big[P_{qq} \otimes_1 f_{qq} + P_{qg} \otimes_1 f_{gq} + P_{qq} \otimes_2 f_{qq} + P_{qg} \otimes_2 f_{qg}\Big]\,,$$



- Convolution with Altarelli-Parisi splitting kernels

$$P_{ab}(\,.\,) \otimes_1 f_{bc}(\,.\,, x_2, y; Q) = \int_{x_1}^{1-x_2} \frac{dz}{z} P_{ab}\left(\frac{x_1}{z}\right) f_{bc}(z, x_2, y; Q)\,,$$

- Analogously for polarized partons
- Separate branchings - expect evolution to wash out correlations

Evolution code

- Build on evolution code for unpolarized DPDs by J.R. Gaunt and W.J. Stirling

  J. R. Gaunt, W. J. Stirling, 2010

  - "single-feed"-term removed
  - included polarized splitting kernels and modified to handle polarized distributions (negative and zero crossings)
- Solves evolution equations stepwise in $\ln Q^2$ in VFNS, directly in x space by fourth order Runge-Kutta
- Grid in x_1, x_2 and $\ln Q^2$

Transverse dependence of DPDs

- Examine the effect evolution has on $\boldsymbol{y}$ dependence
  - Only consider unpolarized DPDs
  - Need initial condition (DPDs at initial scale
- Start from a Gaussian ansatz including interplay between longitudinal and transverse variables
- Motivated by decomposition of DPDs into GPDs

$$F_{ab}(x_1, x_2, \boldsymbol{y}) = \int d^2\boldsymbol{b} f_a(x_1, \boldsymbol{b}) f_b(x_2, \boldsymbol{b} + \boldsymbol{y})$$

$$f_a(x, \boldsymbol{b}) = f_a(x) \frac{1}{4\pi h_a(x)} \exp\left[-\frac{b^2}{4h_a(x)}\right] \qquad h_a(x) = \alpha'_a \ln\frac{1}{x} + B_a$$

- $h_a(x)$ connected to measurements of exclusive t slopes

$$f_a(x, \boldsymbol{r}) = f_a(x) \exp\left[-h_a(x)\boldsymbol{r}^2\right] \qquad \boldsymbol{r}^2 = -t$$

- Gives ansatz for transverse dependence of the DPDs at initial scale:

$$F_{ab}(x_1, x_2, \boldsymbol{y}) = f_a(x_1)\, f_b(x_2) \times \frac{1}{4\pi h_{ab}(x_1, x_2)} \exp\left[-\frac{\boldsymbol{y}^2}{4 h_{ab}(x_1, x_2)}\right]$$

$$h_{ab}(x_1, x_2) = \alpha'_a \ln\frac{1}{x_1} + \alpha'_b \ln\frac{1}{x_2} + B_a + B_b$$

- Parameters from GPD fits ($q^{\pm} = q \pm \bar{q}$)

$$\alpha'_{q^-} = 0.9\,\mathrm{GeV}^{-2}\,, \qquad B_{q^-} = 0.59\,\mathrm{GeV}^{-2}\,,$$
$$\alpha'_{q^+} = 0.164\,\mathrm{GeV}^{-2}\,, \qquad B_{q^+} = 2.4\,\mathrm{GeV}^{-2}\,,$$
$$\alpha'_{g} = 0.164\,\mathrm{GeV}^{-2}\,, \qquad B_{g} = 1.2\,\mathrm{GeV}^{-2}$$

M. Diehl, W. Kugler, 2008

M. Diehl, T. Feldmann, R. Jakob, P. Kroll, 2005

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A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, 2009

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strength of correlation

$$h_{ab}(x_1, x_2) = \alpha'_a \ln\frac{1}{x_1} + \alpha'_b \ln\frac{1}{x_2} + B_a + B_b$$

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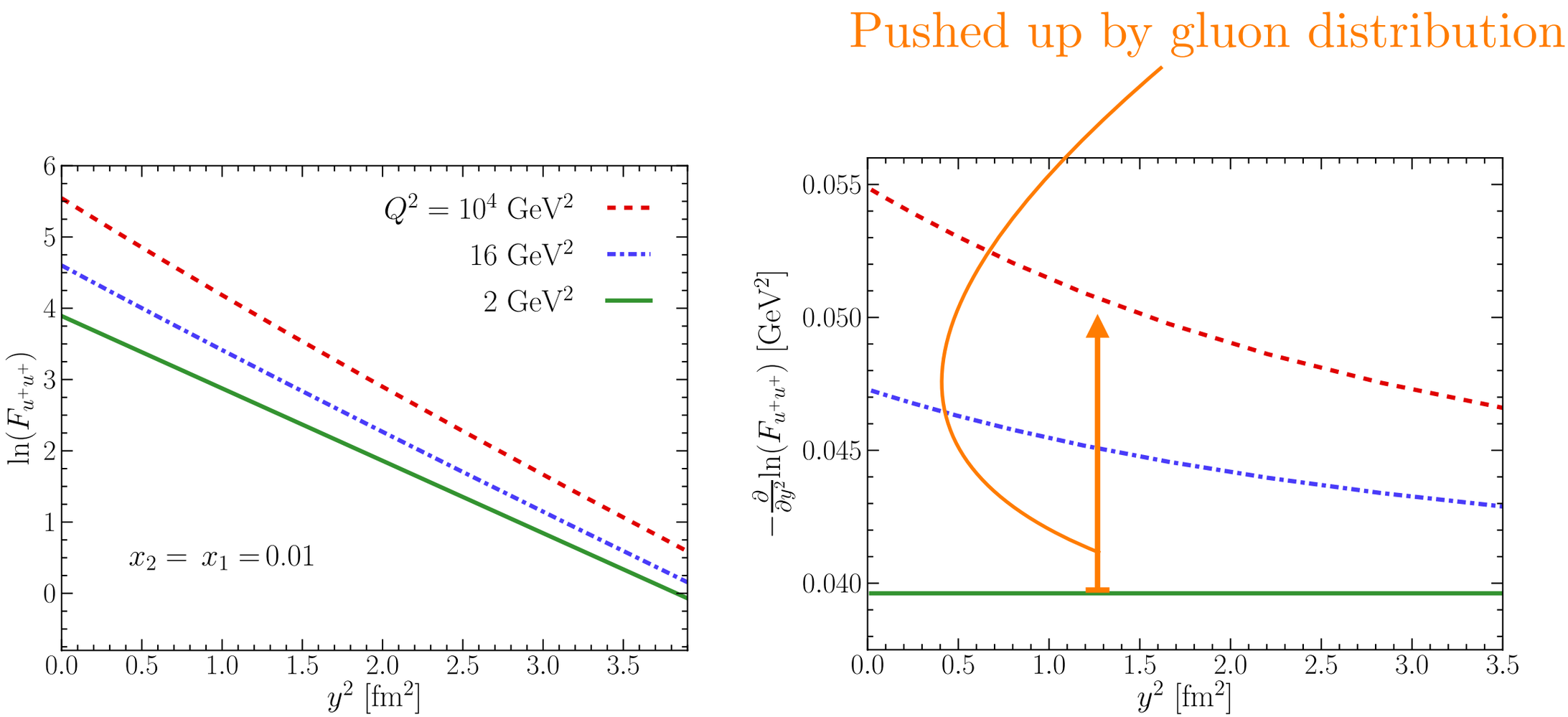
- Evolution of u^+u^+ distribution

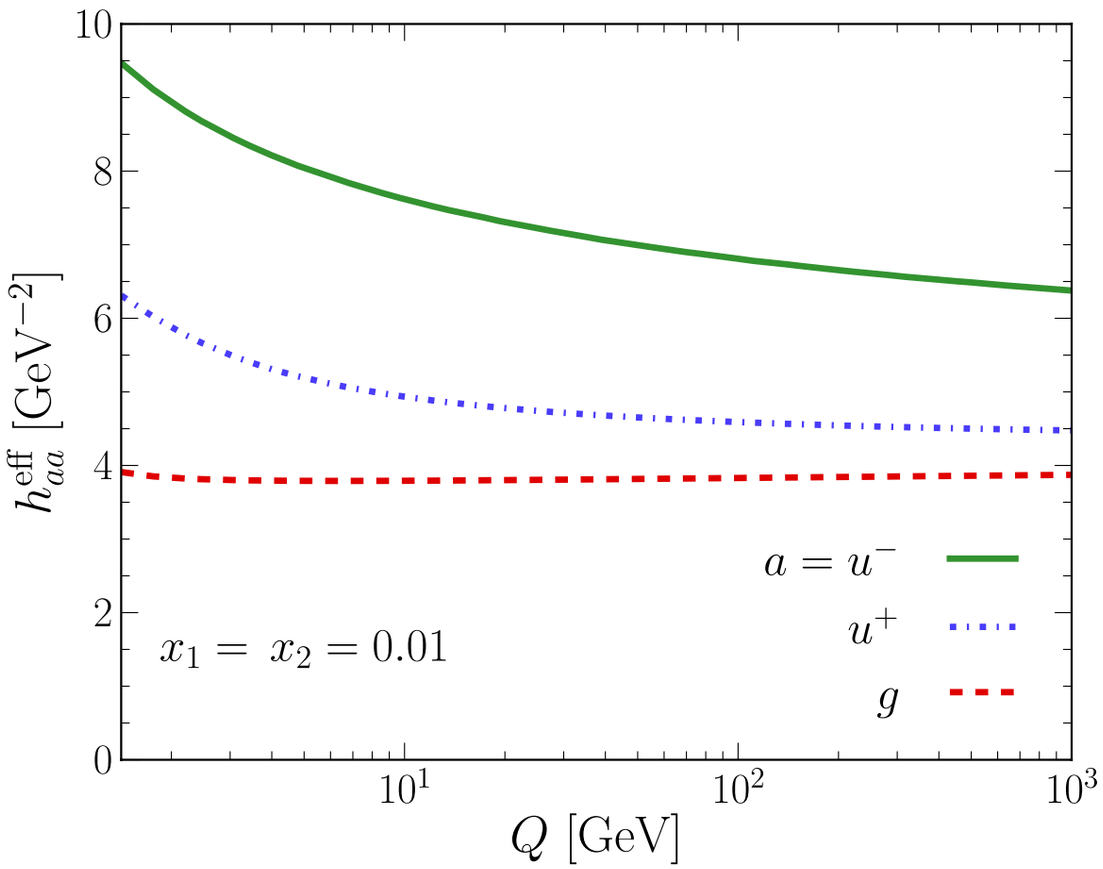


- Distributions stays approximately Gaussian up to high scale
- True also for u^- and gluon

  $\Rightarrow$ Allows us to examine the evolution of the exponent h

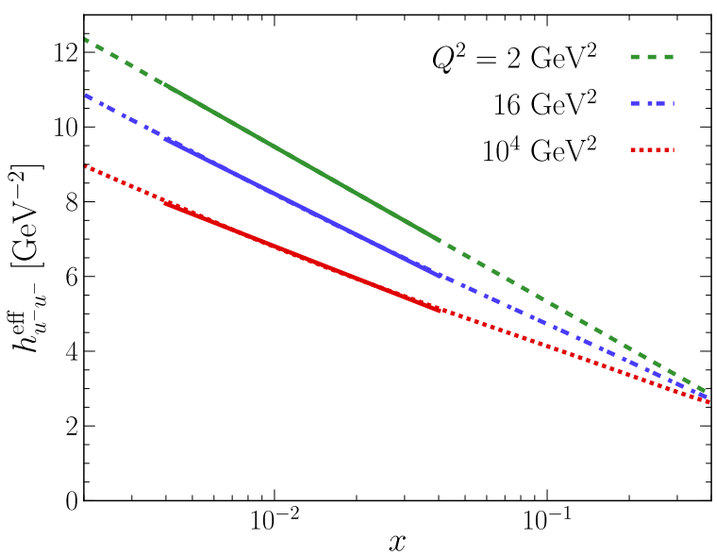
- Taking the log of the distributions gives us access to the exponential

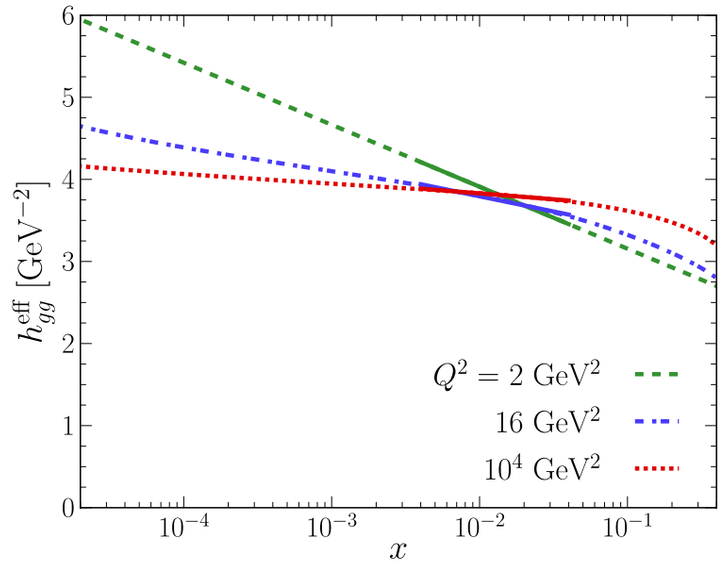
$$\frac{\ln F_{aa}(x,x,y) - \ln F_{aa}(x,x,0)}{y^2}\bigg|_{y=0.4\,\mathrm{fm}} = -\frac{1}{4h^{\mathrm{eff}}_{aa}(x,x)}$$

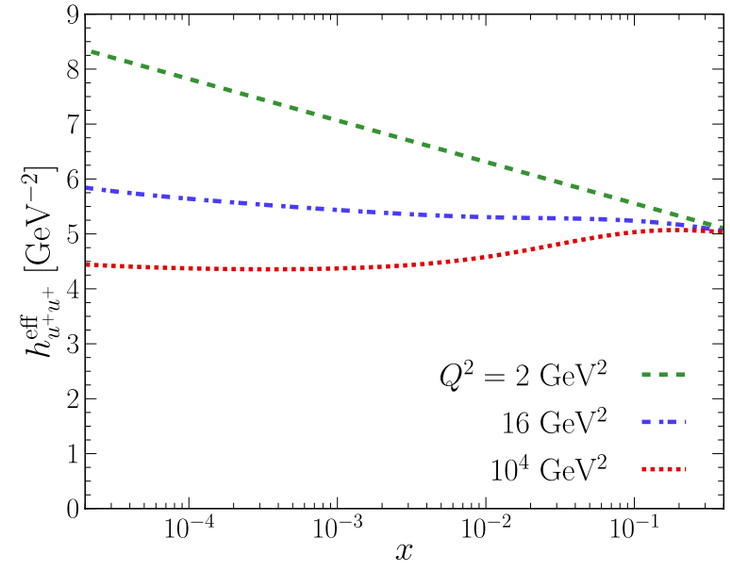


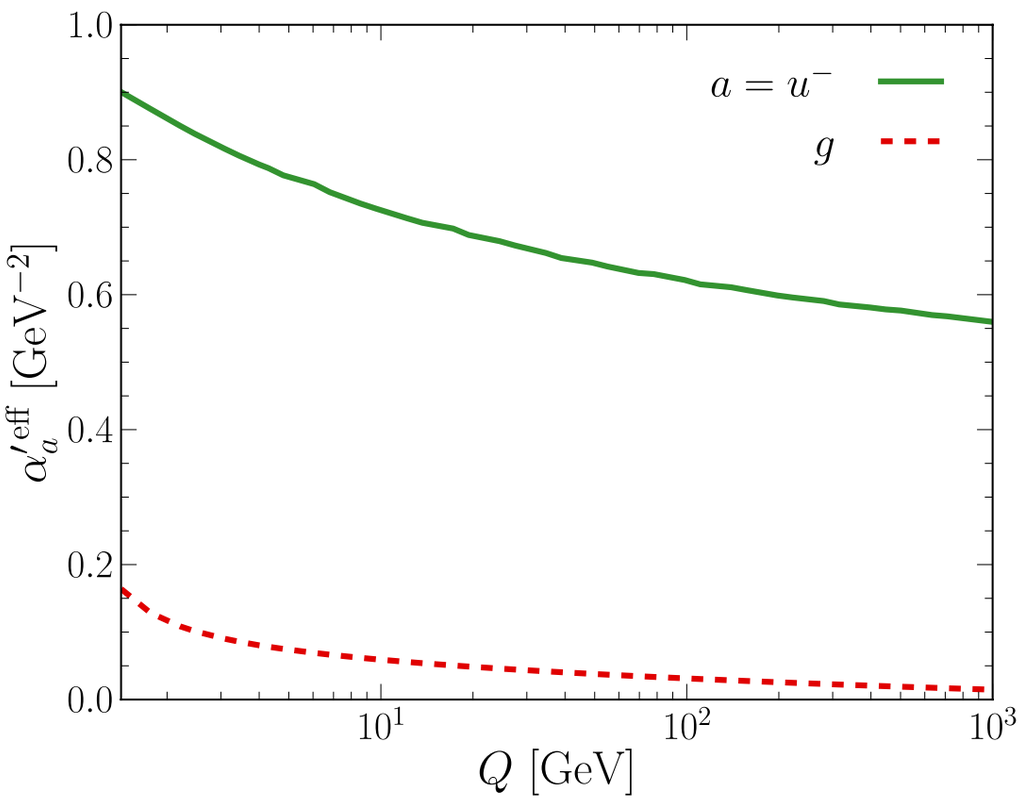
- Gluon width evolve slowly
- u^- and u^+ decrease
- Differences in transverse dependence up to large scale
  - even between u^+ and gluon

- Differences at the initial scale to a large extent remain after evolution up to larger scales







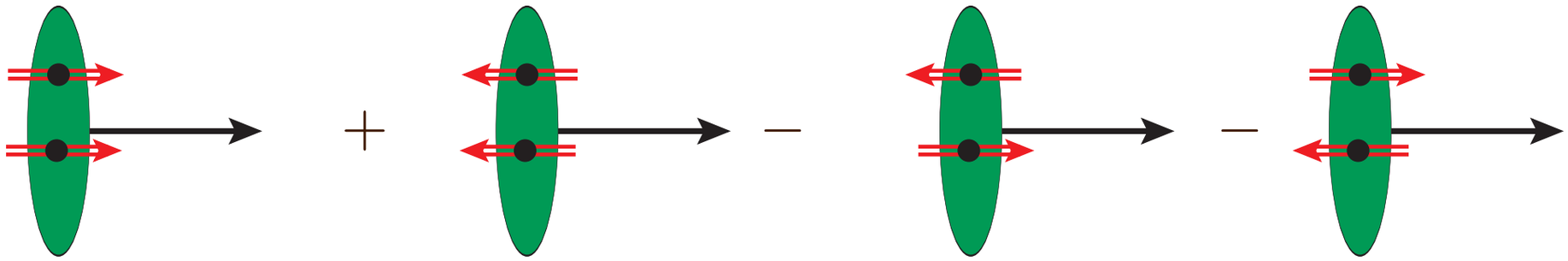


- x dependence of h
- u^- and g approx. linear in lnx (away from the large x region)
- Fit α'_a describing correlation between x_1, x_2 and $\boldsymbol{y}$
- Slow decrease, tend to 0 for gluon
- u^+ slope highly dependent on x region

- Correlations remain up to large scales

Polarized DPDs

- Example: DPD for two longitudinally polarized quarks

$f_{\Delta q \Delta q} \sim$ 

- Polarized distributions shown to be large in quark models

H.-M. Chang, A.V. Manohar, W.J. Waalewijn, 2012

- Polarization changes angular distribution of DPS cross sections

A.V. Manohar, W.J. Waalewijn, 2012; TK, M. Diehl, 2012

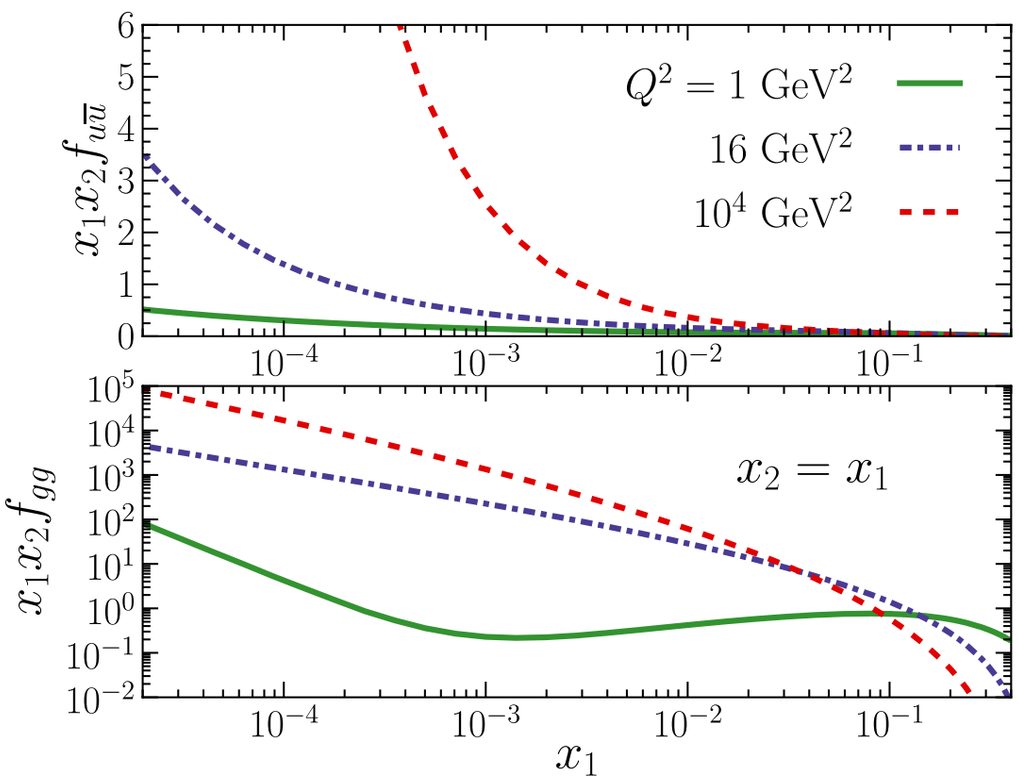
- Quarks: unpolarized q, longitudinally polarized Δq and transversely polarized δq
- Gluons: unpolarized g, longitudinally polarized Δg and linearly polarized δg

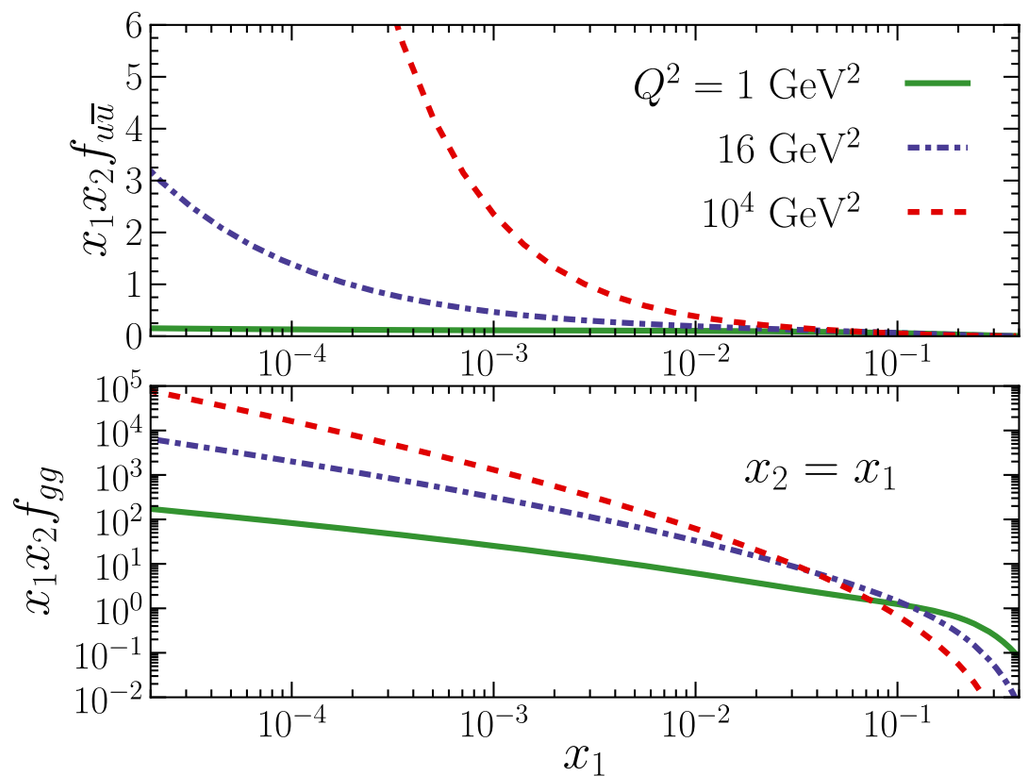
- Need an ansatz for the initial DPDs in order to study the effect of evolution on the polarization

$$f_{p_1 p_2}(x_1, x_2, \boldsymbol{y}; Q) = \tilde{f}_{p_1 p_2}(x_1, x_2; Q)\, G(\boldsymbol{y})\,,$$

Not interested in normalization and set $G(\boldsymbol{y}) = 1$

- For unpolarized DPDs $\tilde{f}_{ab}(x_1, x_2; Q_0) = f_a(x_1; Q)\, f_b(x_2; Q_0)\,,$





MSTW2008lo GJR08lo

A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, 2009; M. Glück, P. Jimenez-Delgado, E. Reya, 2007

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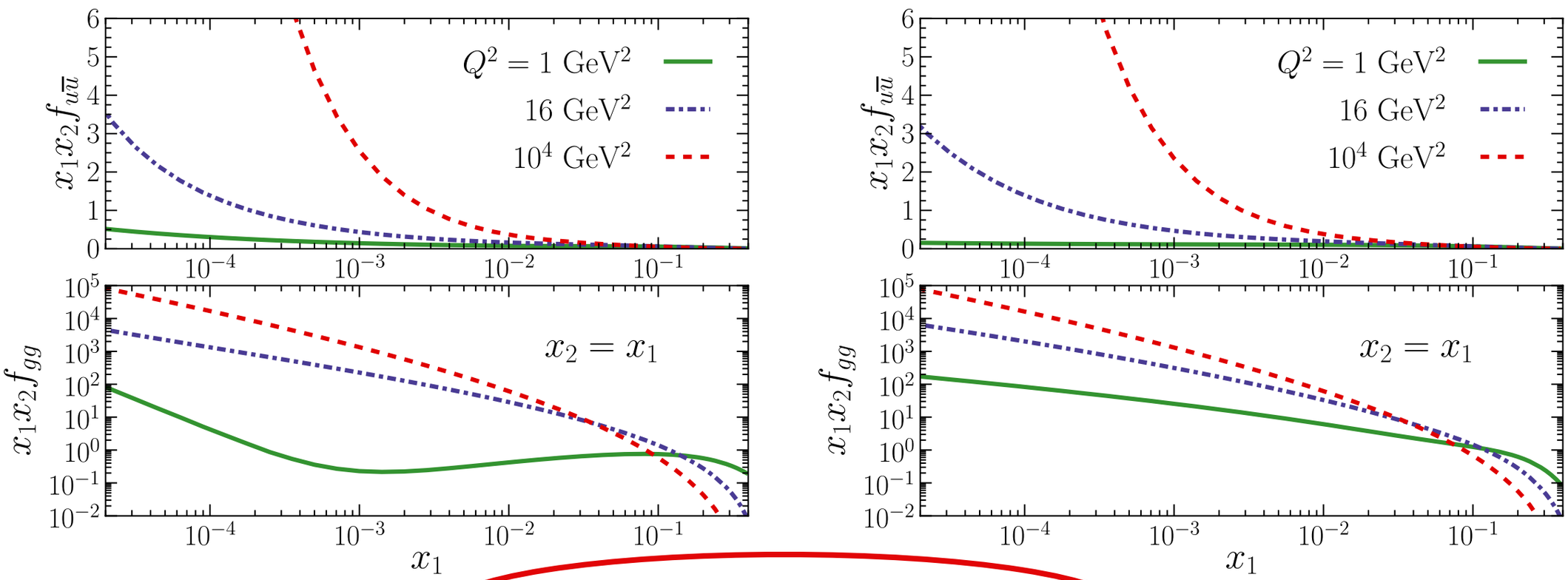

Large difference for gluon

MSTW2008lo GJR08lo

A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, 2009; M. Glück, P. Jimenez-Delgado, E. Reya, 2007

Max and *splitting* scenario

- Polarized DPDs more complicated - two different scenarios
- *Max scenario* - polarized DPDs as large as possibly allowed
  Use positivity bounds to constrain polarized DPDs

  M. Diehl, TK, 2012

  - Polarized DPDs equal to unpolarized at starting scale
- *Splitting scenario* - more dynamical, attempt to use the little information we have
  *A*t small $\boldsymbol{y}$, DPDs can be calculated in terms of PDFs

$$f_{q\bar{q}}(x_1, x_2, y) = \frac{\alpha_s}{2\pi^2} \frac{1}{y^2} \frac{f_g(x_1 + x_2)}{x_1 + x_2} T_{g \to q\bar{q}}\left(\frac{x_1}{x_1 + x_2}\right)$$

M. Diehl, D. Ostermeier, A. Schäfer, 2011

  - Analogously for other partons and polarizations
  - Expressions break down for large $\boldsymbol{y}$
  - Use ratios of splitting (*T*) factors to relate polarized DPDs to

Max and *splitting* scenario

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- *Max scenario* - polarized DPDs as large as possibly allowed
  Use positivity bounds to constrain polarized DPDs

  M. Diehl, TK, 2012

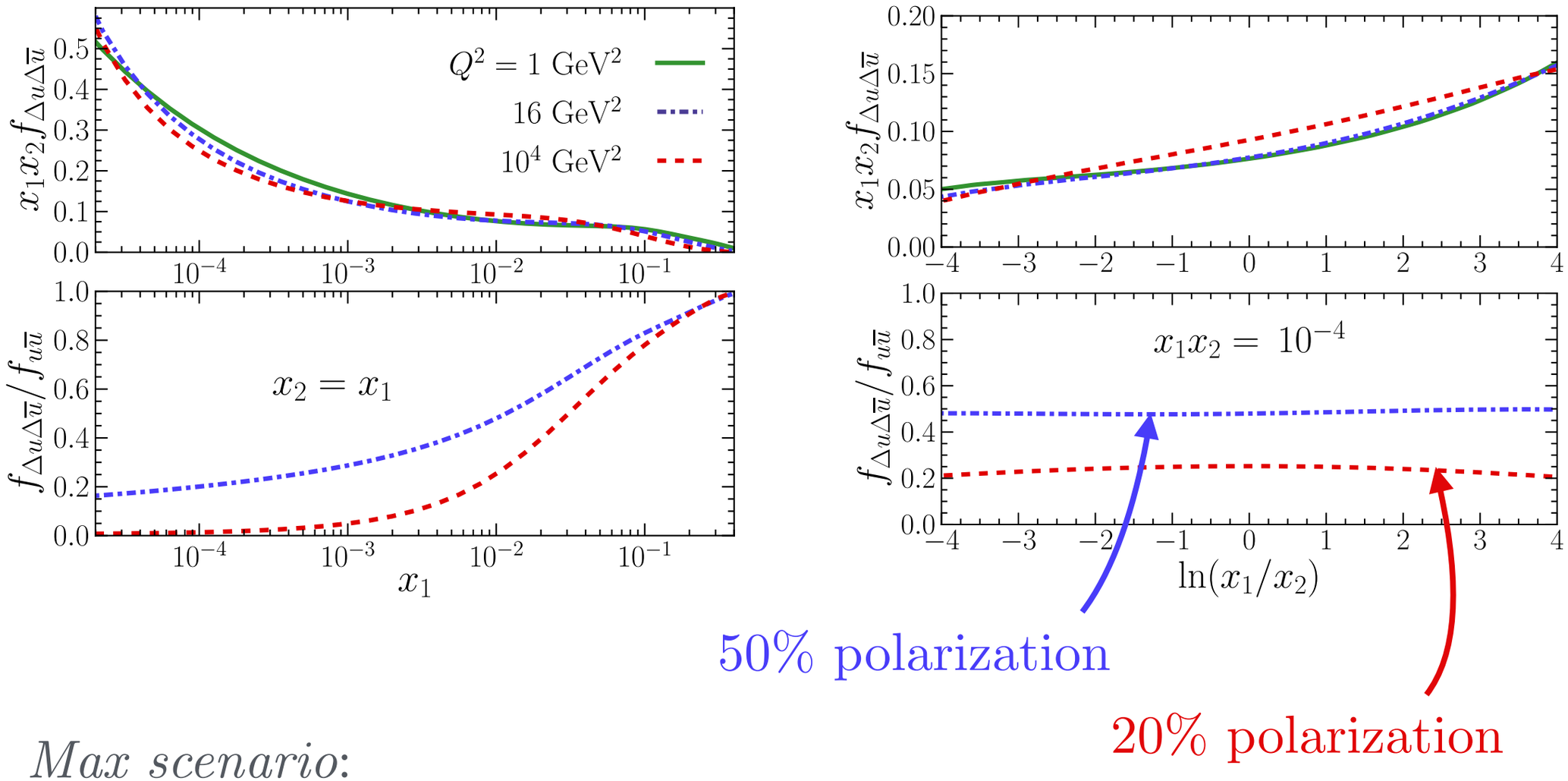
  - Polarized DPDs equal to unpolarized at starting scale
- *Splitting scenario* - more conservative, attempt to use the little information we have

$$f_{\Delta q \Delta \bar{q}} = -f_{q\bar{q}}\,, \qquad f_{\delta q \delta \bar{q}} = -\frac{2z\bar{z}}{z^2+\bar{z}^2}\, f_{q\bar{q}}\,,$$

$$f_{\Delta g \Delta g} = \frac{z\bar{z}(2-z\bar{z})}{z^2+\bar{z}^2+z^2\bar{z}^2}\, f_{gg}\,, \qquad f_{\delta g \delta g} = \frac{2z^2\bar{z}^2}{z^2+\bar{z}^2+z^2\bar{z}^2}\, f_{gg}\,,$$

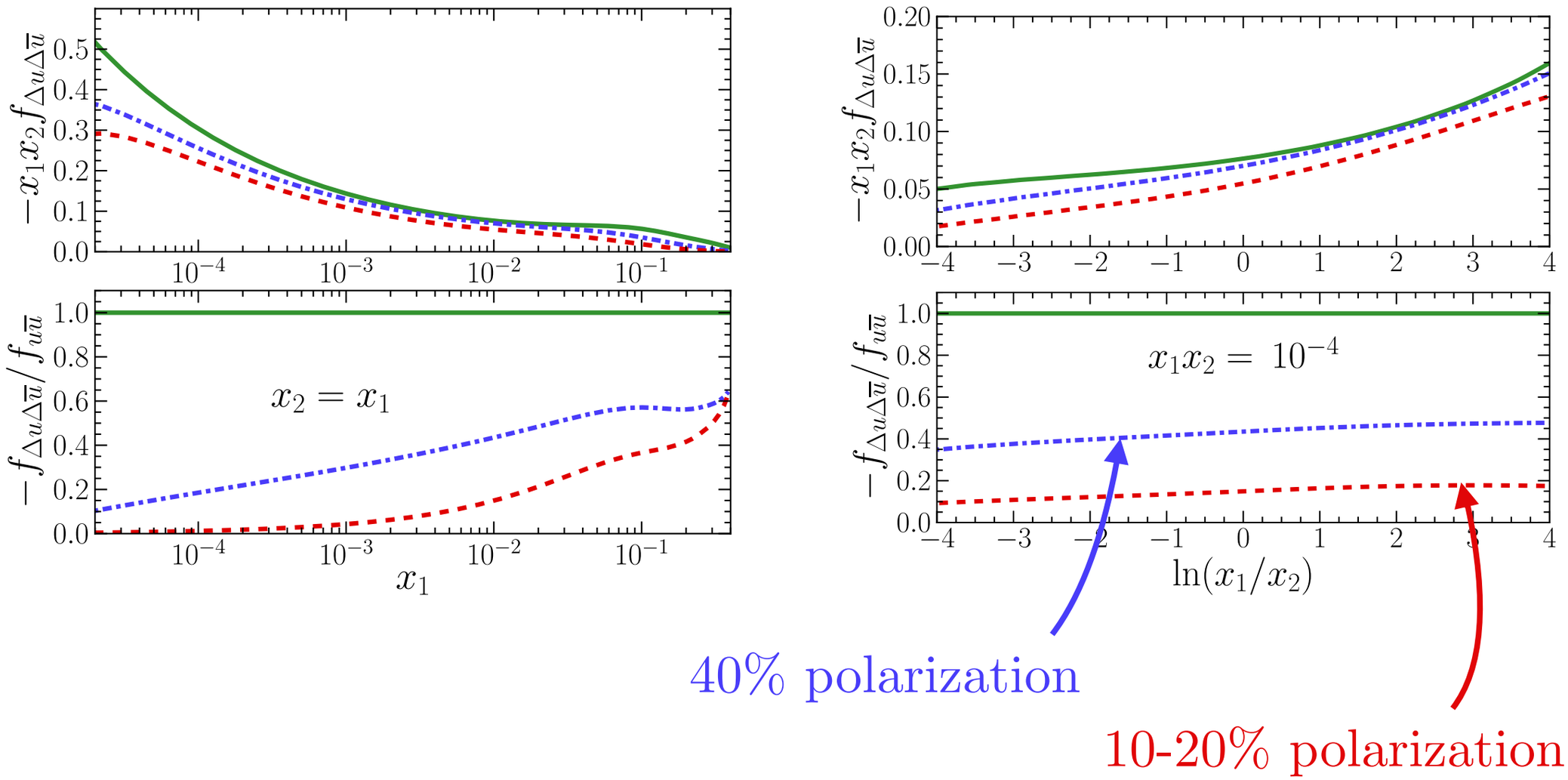
$$f_{\Delta q \Delta g} = \frac{1-z^2}{1+z^2}\, f_{qg} \qquad z = \frac{x_1}{x_1+x_2}, \quad \bar{z} = 1-z$$

Longitudinal quark polarization



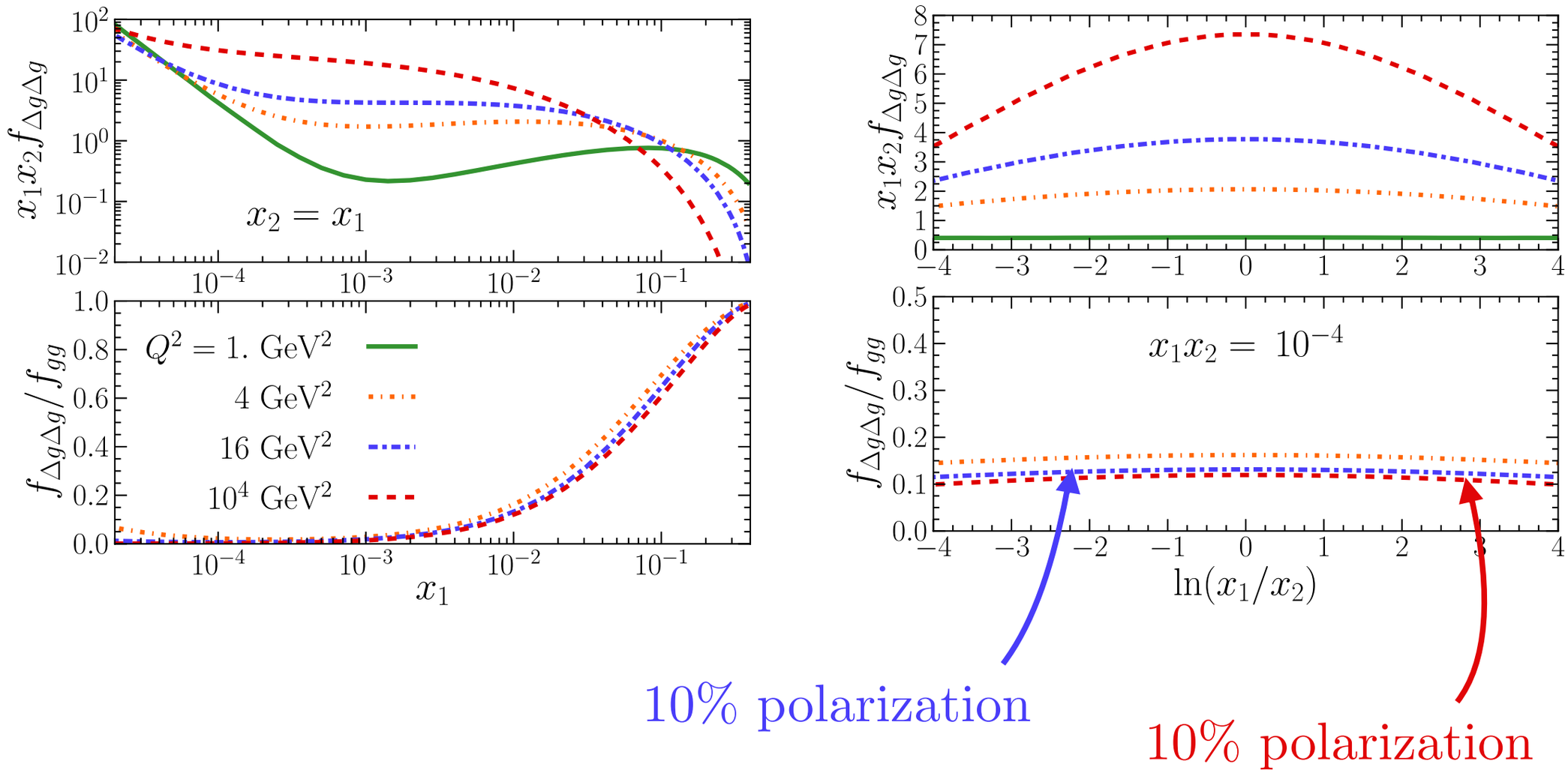
- *Max scenario*:
  - Large longitudinal polarization up to high scales in wide range of x_i
  - Degree of polarization flat in rapidity - generic feature in *max scenario*

Longitudinal quark polarization



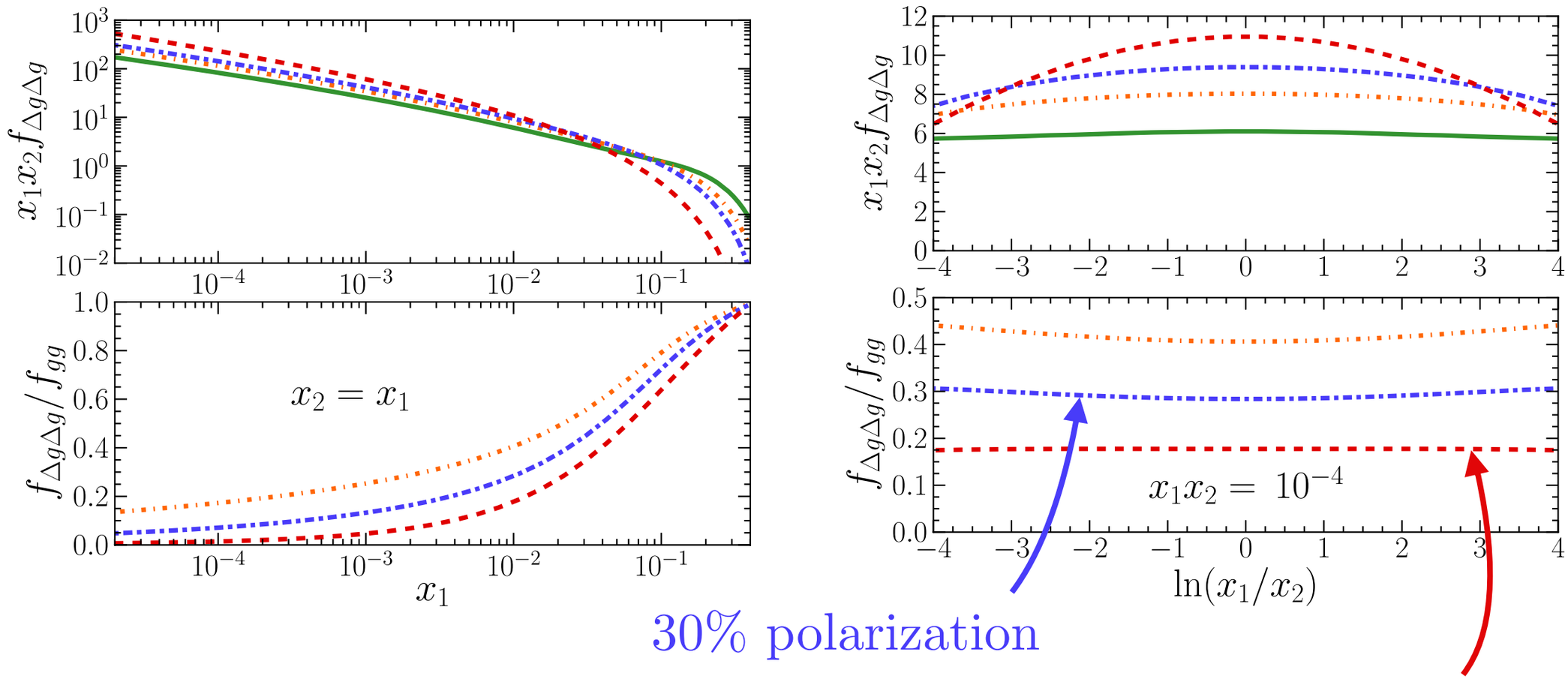
- *Splitting scenario*:
  - Similar to *max scenario* - sizeable polarization and flat rapidity
  - Somewhat smaller polarization, in particular at large x_i

Longitudinal gluon polarization



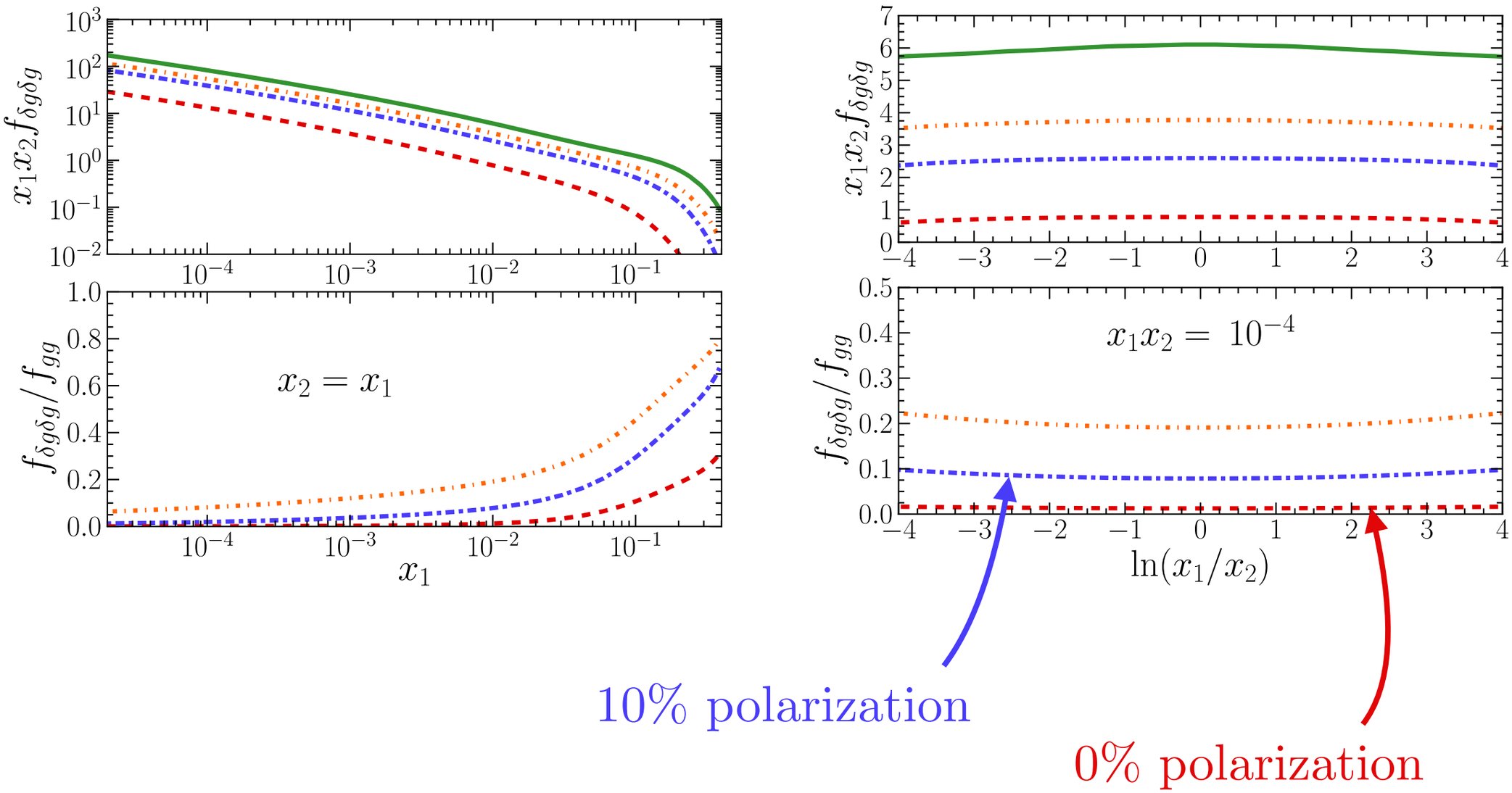
- *Max scenario* starting from MSTW distributions
  - f_{gg} rises much faster than $f_{\Delta g \Delta g}$
  - Longitudinal gluon polarization quickly washed out

Longitudinal gluon polarization



- *Max scenario* starting from GJR distributions
  - Much larger degree of polarization than with MSTW;
    - by factor 2-3 at larger scales
  - Difference due to the unpolarized gluon PDF at low scales
  - Smaller degree of polarization than for quarks and antiquarks

Linear gluon polarization



- Double linear gluon polarization rapidly suppressed by evolution
  - even in most positive scenario

Conclusions

- Transverse dependence:
  - Evolution approximately preserved Gaussian shape
  - Slow evolution of Gaussian width
  - Slow decrease in correlation between x_1, x_2 and $\boldsymbol{y}$
  - Differences between transverse dependence of DPDs preserved by evolution
- Polarization:
  - Study show when we need to worry about polarization
  - Double linear gluon polarization rapidly suppressed by evolution
  - Longitudinal gluon polarization can remain sizeable for not too small x and not too large Q - large dependence on PDF set
  - Quark/antiquark polarization can be large up to large scales