Factorization of double Drell-Yan at low transverse momentum

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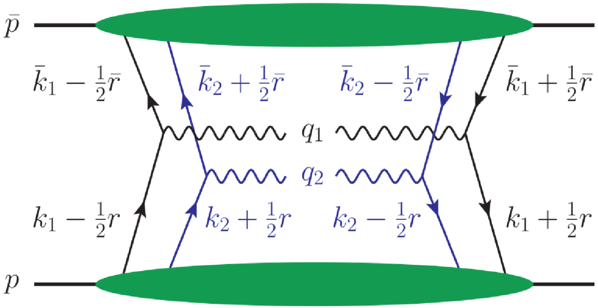
Outline

- Introduction
- Leading order analysis of double Drell-Yan
- Going beyond leading order
- Test of factorization to order α_S
- Conclusion

Introduction

consider double Drell-Yan (dDY) production of two lepton pairs with measured transverse momenta



Introduction

Why study double Drell-Yan?

- cross section of dDY estimated to be small compared to other possible processes[1]
- but: theoretically cleanest process
- no final state interactions
- under full control in the single scattering case
- problems with TMD factorization in processes with hadrons in the final state already in the single hard scattering case[2]

Double Drell-Yan serves as a good testing ground for the theory of multiparton interactions.

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[1] C.H. Kom, A. Kulesza and W.J. Stirling, "Prospects for observation of double parton scattering with four–muon final states at LHCb" [arXiv:1109.0309]

[2] Ted C. Rogers, Piet J. Mulders, "No Generalized TMD-Factorization in the Hadro-Production of High Transverse Momentum Hadrons" Phys. Rev. D **81**: 094006 (2010), [arXiv:1001.2977 [hep-ph]]

Introduction

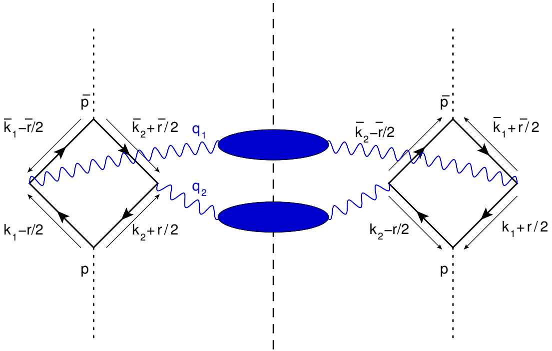
- establishing (TMD) factorization is of great importance for the predicitive power of pQCD
- several building blocks for the formulation of factorization are alreay available[3]
- factorization could be broken

⇒ do not make the approximation $d\sigma = \frac{d\sigma_1 \, d\sigma_2}{\sigma_{\text{eff}}}$
⇒ keep cross section differential in transverse momenta
⇒ study contributions to dDY with a maximum amount of correlation

[3] M. Diehl, D. Ostermeier and A. Schäfer, "Elements of a theory for multiparton interactions in QCD", [arXiv:1111.0910];
A.V. Manohar, W.J. Waalewijn, "A QCD Analysis of Double Parton Scattering: Spin and Color Correlations, Interference Effects and Evolution", [arXiv:1202.3794]

Introduction

Which kinds of diagrams yield a maximum amount of correlation between the hard scatters?



Idea: Study diagrams in which partons originate from a splitting

Introduction

There are problems with these kinds of diagrams (c.f. talk of M.Diehl)
For Dirac quarks, there are two leading regions contributing to the box diagrams:

- high k_T region: associated with one-loop correction to single parton scattering
- small k_T region: associated with double parton scattering

So far, there is no formalism to seperate these two regions, which is crucial to avoid double counting between SPS and DPS

consider a model with scalar quarks originating from a scalar particle via a pointlike coupling, scalar "photons" and SU(3) color gluons[4]

$\Rightarrow$ box diagram is dominated by low k_T region

[4] J.C. Collins, D.E. Soper, G. Sterman, "Factorization for one-loop corrections in the Drell-Yan process", Nucl. Phys. B 223 (1983) 381-421

Leading order analysis

We define transverse momentum dependent double parton distributions

Definition of dTMDs

$$F(x_i, \mathbf{k}_i, \mathbf{r}) = \prod_{i=1}^{2} \int \frac{dz_i^-}{2\pi} e^{ix_i z_i^- p^+} \int \frac{d^2\mathbf{z}_i}{(2\pi)^2} e^{-i\mathbf{z}_i \cdot \mathbf{k}_i} 2p^+ \int dy^- d^2\mathbf{y}\, e^{i\mathbf{y}\cdot\mathbf{r}} \times \langle p|\mathcal{O}^*(0, z_2)\mathcal{O}(y, z_1)|p\rangle$$

with the abbreviation

$$\mathcal{O}(y, z_i) = \phi^* \left(y - \tfrac{1}{2} z_i\right) \frac{i}{2} \left(\overrightarrow{\partial} - \overleftarrow{\partial}\right)^+ \phi\left(y + \tfrac{1}{2} z_i\right) \Big|_{z_i^+ = y^+ = 0}$$

Remark: We use light-cone coordinates and display any vector as $k = (k^+, k^-, \mathbf{k})$

Leading order analysis

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Definition of dTMDs

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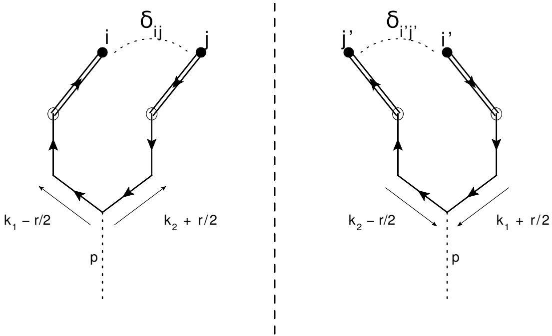
We use symmetric position and momentum variables

momentum fraction	momentum space	position space
x_1 in amplitude	$k_1 - \frac{1}{2}r$	$y + \frac{1}{2}z_1$
x_2 in amplitude	$k_2 + \frac{1}{2}r$	$+\frac{1}{2}z_2$
x_2 in conjugate amplitude	$k_2 - \frac{1}{2}r$	$-\frac{1}{2}z_2$
x_1 in conjugate amplitude	$k_1 + \frac{1}{2}r$	$y - \frac{1}{2}z_1$

Leading order analysis

Contribution of splitting to the dTMD



We take the incoming, scalar particles to be slightly off-shell with $p^2, \bar{p}^2 < 0$.
Remark: The definition of the dTMDs has to be augmented with suitably chosen Wilson lines in order to render them gauge invariant

Leading order analysis

Splitting contribution to dTMDs

$$F(x_i, \mathbf{k}_i, \mathbf{r})\Big|_{s\to q\bar{q}} = \mathcal{N}(x_1 p^+)(x_2 p^+) 2p^+ \int dk_1^- \, dk_2^- \, dr^-$$

$$\times \frac{\delta\left(p^- - (k_1^- - \frac{1}{2}r^-) - (k_2^- + \frac{1}{2}r^-)\right)\delta(p^+ - x_1 p^+ - x_2 p^+)}{[(k_1 - \frac{1}{2}r)^2 + i\varepsilon][(k_2 + \frac{1}{2}r)^2 + i\varepsilon][(k_1 + \frac{1}{2}r)^2 - i\varepsilon][(k_2 - \frac{1}{2}r)^2 - i\varepsilon]}$$

$$= \mathcal{N}\pi^2 \delta(1 - x_1 - x_2) \frac{2x_1 x_2}{[(\mathbf{k_1} - \frac{1}{2}\mathbf{r})^2 - x_1 x_2 p^2][(\mathbf{k_1} + \frac{1}{2}\mathbf{r})^2 - x_1 x_2 p^2]}$$

$$\bar{F}(\bar{x}_i, \bar{\mathbf{k}}_i, \bar{\mathbf{r}})\Big|_{s\to \bar{q}q} =$$

$$= \bar{\mathcal{N}}\pi^2 \delta(1 - \bar{x}_1 - \bar{x}_2) \frac{2\bar{x}_1 \bar{x}_2}{[(\bar{\mathbf{k}}_\mathbf{1} - \frac{1}{2}\bar{\mathbf{r}})^2 - \bar{x}_1 \bar{x}_2 \bar{p}^2][(\bar{\mathbf{k}}_\mathbf{1} + \frac{1}{2}\bar{\mathbf{r}})^2 - \bar{x}_1 \bar{x}_2 \bar{p}^2]}$$

Leading order analysis

Plug this into the factorization formula

Factorization formula for double parton scattering

$$\frac{d\sigma}{\prod_{i=1}^{2} dx_i d\bar{x} d^2\mathbf{q}_i} = \frac{1}{C}\left[\prod_{i=1}^{2}\hat{\sigma}_i(x_i\bar{x}_i s)\right]\left[\prod_{i=1}^{2}\int d^2\mathbf{k}_i d^2\bar{\mathbf{k}}_i \delta(\mathbf{q}_i - \mathbf{k}_i - \bar{\mathbf{k}}_i)\right]$$
$$\times \int d^2\mathbf{r}\; F(x_i, \mathbf{k}_i, \mathbf{r})\bar{F}(\bar{x}_i, \bar{\mathbf{k}}_i, -\mathbf{r})$$

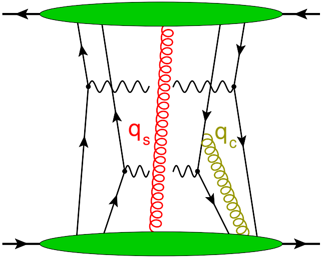
- $\hat{\sigma}_i$: partonic cross section of hard scattering i
- F: dTMD for right-moving scalar
- $\bar{F}$: dTMD for left-moving scalar

A comparison with the direct calculation of the box diagrams shows, that this reproduces the dDY cross section

Going beyond leading order

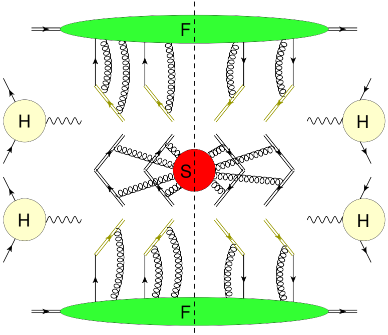
What happens if we allow for additional gluon exchange?
Arguments from single Drell-Yan case apply to large extent. Leading contributions come from

- collinear gluons: gluon momentum collinear to proton momentum
- soft gluons: all momentum components small



Going beyond leading order

- collinear gluons can be absorbed into Wilson lines in the definition of dTMDs
- soft gluons decouple from the hard and collinear part and their effect can be described by a vacuum expectation value of Wilson lines, a so-called 'soft factor'



Going beyond leading order

- collinear gluons can be absorbed into Wilson lines in the definition of TMDs
- soft gluons decouple from the hard and collinear part and their effect can be described by a vacuum expectation value of Wilson lines, a so-called 'soft factor'

This result is obtained assuming that the Glauber region (all components of gluon momentum small, but $\ell^2 \gg \ell^+ \ell^-$) does not contribute.
This is, however, not obvious and we check this by comparing the $\mathcal{O}(\alpha_S)$ result of the direct calculation of the cross section and its factorized form.

Going beyond leading order

effect of collinear gluons

replace the quark and antiquark fields in the dTMDs in the following fashion

Replacement rule for quark and antiquark fields

$$\phi_j(\xi) \to \phi_j(\xi; y_v) = W_{jj'}(\xi; v)\phi_{j'}(\xi)$$

$$W_{jj'}(\xi; v) = \text{Pexp}\left[ig \int_0^\infty d\lambda \; vA^a(\xi - \lambda v)(t^a)_{jj'} \right]$$

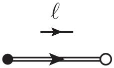
$$\phi^*_{j'}(\xi) \to \phi^*_{j'}(\xi; y_v) = \phi^*_j(\xi)\, W^\dagger_{jj'}(\xi; v)$$

$$W^\dagger_{jj'}(\xi; v) = \text{Pexp}\left[-ig \int_0^\infty d\lambda \; vA^a(\xi - \lambda v)(t^a)_{jj'} \right]$$

The definition of the dTMDs are now gauge invariant and the dTMDs depend on the rapidity $y_v = \frac{1}{2}\log\left(\left|\frac{v^+}{v^-}\right|\right)$ of the Wilson lines

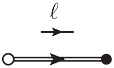
Going beyond leading order

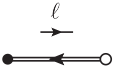
Feynman rules for Wilson lines



$$\frac{i}{\ell v + i\epsilon}$$

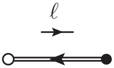
$$\frac{-i}{\ell v - i\epsilon}$$

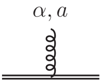




$$\frac{-i}{\ell v + i\epsilon}$$

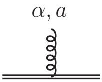
$$\frac{i}{\ell v - i\epsilon}$$





$$-igt^a v^\alpha$$

$$igt^a v^\alpha$$



Going beyond leading order

Additional problems occur:

1. The collinear and soft gluon regions overlap, therefore we have a double counting of the soft region.

2. The most intuitive choice of the Wilson lines (to have a direction along the light-cone) leads to rapidity divergences of the type

$$\int d^{2-2\varepsilon}\vec{\ell} \int_0^1 d\alpha \, \frac{1}{\alpha} \, \frac{(1-\alpha)}{[\vec{\ell}^2 - \alpha(1-\alpha)k^2 - i\epsilon]}$$

Going beyond leading order

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This cannot be cured by finite quark masses, a gluon mass or dimensional regularization!

Going beyond leading order

We follow the method advocated by Collins[5] to solve these problems:
Subtract a certain combination of soft factors from the dTMDs given the following conditions

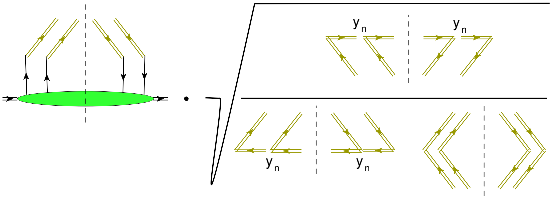
- rapidity divergences cancel
- use only one non-lightlike direction
- the dTMDs depend on only one additional parameter
- dependence on this parameter is governed by the modified Collins-Soper evolution equation
- the definition of the dTMDs is boost invariant
- no explicit soft factor in the factorization formula

[5] J.C. Collins, "The Foundations of Perturbative QCD"

Going beyond leading order

The subtracted dTMDs are, somewhat symbolically, defined as

$$F^{sub}(x, z_i, y; y_n) = F(x, z_i, y; -\infty)\sqrt{\frac{S(\mathbf{z}_i, \mathbf{y}; y_n, +\infty)}{S(\mathbf{z}_i, \mathbf{y}; -\infty, y_n)S(\mathbf{z}_i, \mathbf{y}; -\infty, +\infty)}}$$



rapidities y_n of Wilson lines: $y_n = \frac{1}{2}\log\left(\left|\frac{v^+}{v^-}\right|\right)$, all spacelike or lightlike as a limit from spacelike region

Going beyond leading order

The subtracted dTMDs are, somewhat symbolically, defined as

$$F^{sub}(x, z_i, y; y_n) = F(x, z_i, y; -\infty)\sqrt{\frac{S(\mathbf{z}_i, \mathbf{y}; y_n, +\infty)}{S(\mathbf{z}_i, \mathbf{y}; -\infty, y_n)S(\mathbf{z}_i, \mathbf{y}; -\infty, +\infty)}}$$

Remark:

- in general dTMDs have a non-trivial color structure
- choose color-singlet and -octet as a basis
- F^{sub} and F are vectors $\begin{pmatrix} {}^{1}F \\ {}^{8}F \end{pmatrix}$
- soft factors are matrices $\begin{pmatrix} {}^{11}S & {}^{18}S \\ {}^{81}S & {}^{88}S \end{pmatrix}$ leading to mixing between color-singlet and -octet

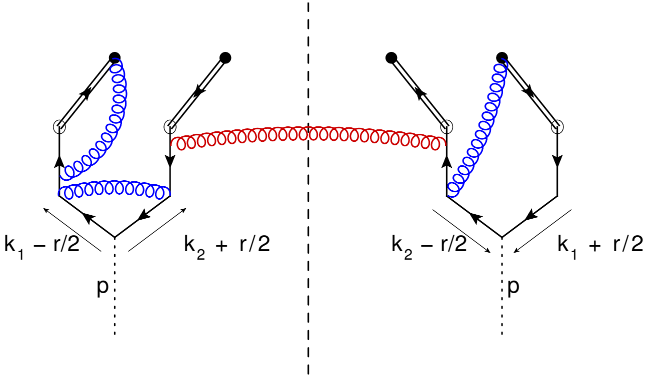
Test of factorization to order α_S

With these buidling blocks at hand, we can make a check of the following factorization formula to order α_S

Factorization formula

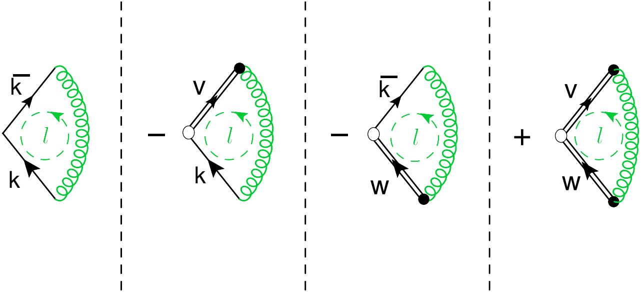
$$\frac{d\sigma}{\prod_{i=1}^{2} dx_i d\bar{x}_i d^2\mathbf{q}_i} = \frac{1}{C}\left(\prod_{i=1}^{2}\hat{\sigma}_i(x_i\bar{x}_i s)\right)\left(\prod_{i=1}^{2}\int d^2\mathbf{k}_i d^2\bar{\mathbf{k}}_i \delta^{(2)}(\mathbf{q}_i - \mathbf{k}_i - \bar{\mathbf{k}}_i)\right)$$
$$\times \int d^2\mathbf{r}\,\bar{F}^{sub}(\bar{x}_i, \bar{\mathbf{k}}_i, -\mathbf{r})\,F^{sub}(x_i, \mathbf{k}_i, \mathbf{r})$$

Test of factorization to order α_S



We did a calculation of all virtual diagrams...

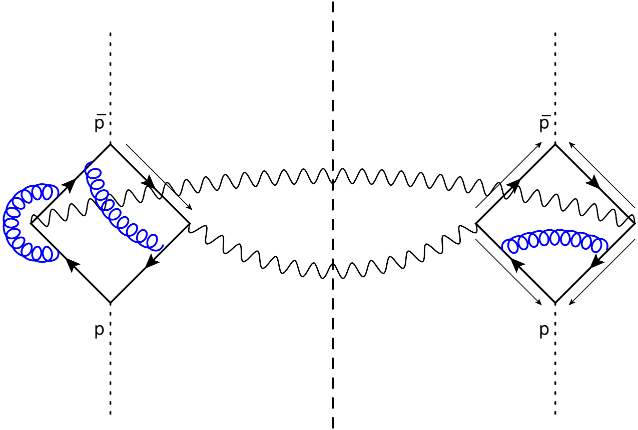
Test of factorization to order α_S



...and the hard parts...

Test of factorization to order α_S

... and compared with the full $\mathcal{O}(\alpha_S)$ double Drell-Yan process:



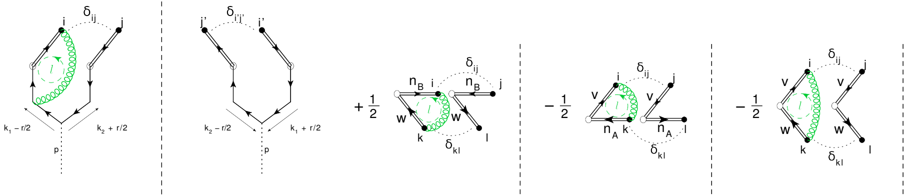
Test of factorization to order α_S

Calculation of the diagrams:

- nonzero gluon mass to regulate infrared divergences
- take incoming targets slightly off-shell $(p^2, \bar{p}^2 < 0)$
- dimensional regularisation for UV divergences
- real diagrams are zero due to color factor

Test of factorization to order α_S

Example: Wilson line "vertex" correction



$$I = \frac{4\pi\alpha_S C_F \mu^{2\varepsilon}}{(2\pi)^{4-2\varepsilon}} \int d^{4-2\varepsilon}\ell \; \frac{1}{[\ell^2 - \lambda^2 + i\epsilon][\ell^+ + i\epsilon]} \times$$
$$\times \left[\frac{2k^+ - \ell^+}{[(\ell - k)^2 + i\epsilon]} - \frac{1}{[-\ell^- + \ell^+ e^{-2y_n} + i\epsilon]} \right]$$

Soft subtraction terms combine to one effective subtraction factor

Test of factorization to order α_S

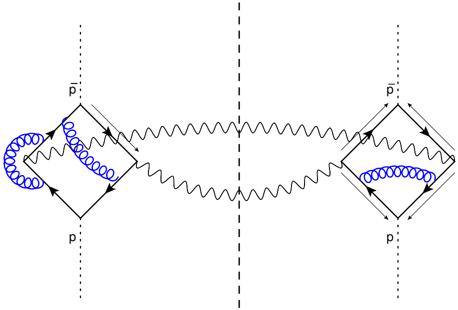
The result is

$$I = \frac{\alpha_S C_F}{4\pi} \frac{1}{4} \Bigg[\mathrm{Li}_2\left(\frac{\zeta^2}{\lambda^2}\right) - 2\mathrm{Li}_2\left(\frac{k^2}{\lambda^2}\right) - \frac{\pi^2}{3} + \log\left(\frac{\mu^2}{-k^2}\right)$$
$$+\frac{1}{\varepsilon} - \gamma_E + \log(4\pi) + i\pi + 2$$
$$+\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-2\gamma_E + \log(4\pi) + i\pi - \log\left(\frac{\zeta^2}{\mu^2}\right)\right)$$
$$+\log\left(\frac{\zeta^2}{\mu^2}\right)\left(2\gamma_E - \log(4\pi) - i\pi + \frac{1}{2}\log\left(\frac{\zeta^2}{\mu^2}\right)\right)$$
$$-\frac{\pi^2}{3} + 2\gamma_E(\gamma_E - i\pi) + \frac{1}{2}\log(4\pi)(\log(4\pi) + 2i\pi) - 2\gamma_E \log(4\pi)\Bigg]$$

where $k = k_1 - \frac{1}{2}r$ and $\zeta^2 = 2x_1^2(p^+)^2 e^{-2y_n}$

Test of factorization to order α_S

Results:



- All two- and three point contributions factorize
- Currently calculating direct four point contributions

Conclusion

- As a step towards establishing TMD factorization of double Drell-Yan, we have studied one loop corrections to box-type double Drell-Yan diagrams
- Final results are to appear soon[6]
- An extension to real QCD will have to wait untill the problems with the box diagrams are solved
- Possible extension: include additional spectators

[6] M. Diehl, D. Ostermeier and A. Schäfer, "Factorization of double Drell-Yan at low transverse momentum", in preparation