Jet Quenching in a Wilson Lines Formalism

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Fundamental .be Interactions

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Outline

- Transverse momentum broadening in terms of the light-like Wilson lines
- Geometrical structure of the “skewed” correlator of two space-like separated (almost) oppositely directed Wilson lines.
- Analytical continuation from the Euclidean formulation to the Minkowski light-cone geometry (leading order of the perturbative expansion)
- UV, rapidity and IR singularities, applications to the lattice simulations

Transverse Momentum Broadening

Jet quenching: observed in the heavy ion collisions at RHIC and LHC

Phenomenology: high-$p_\perp$ hadrons in the final state instead of a jet in the vacuum

Characterized by the jet quenching parameter

$$\hat{q} \equiv L^{-1} \langle k_\perp^2 \rangle = L^{-1} \int d^2 k_\perp k_\perp^2 \ P(\mathbf{k}_\perp)$$

$\rightarrow P(\mathbf{k}_\perp)$ measures probability of a fast parton to acquire a transverse momentum travelling through the medium for a distance L

Scale Hierarchy in Soft-Collinear Effective Theory

$$\lambda = Q_T/Q \ll 1$$

$$p_T - \text{broadening} \sim \lambda Q = Q_T$$

$$\text{virtuality} \sim L^{-2} \sim (\lambda Q)^2 = Q_T^2$$

$$Q_T L \approx 1$$

$$Q \gg Q_T \gg gQ_T \gg g^2 Q_T \quad \to \quad [gQ_T, Q_T] = \text{region of interest}$$

$\to$ Q_T mimics the temperature of a non-thermal medium

Jet Quenching Parameter in Terms of the Wilson Lines

$$P(\mathbf{k}_\perp; n^-) = \int d^2 z_\perp \ \mathrm{e}^{ik_\perp \cdot z_\perp} \ \tilde{P}(z_\perp)$$

$$= \int d^2 z_\perp \ \mathrm{e}^{ik_\perp \cdot z_\perp} \ \langle \star | \frac{1}{N_c} \mathrm{Tr} \ \{ \mathcal{W}^\dagger_{n^-}[0, z_\perp] \mathcal{W}_{n^-}[0, \mathbf{0}_\perp] \} | \star \rangle$$

@ [Zakharov (1996, 1997); Baier et al. (1997, 2000, 2003); Liu, Rajagopal, Wiedemann, D'Eramo (2006, 2007, 2011); Garcia-Echevarria, Idilbi, Scimemi (2011), Benzke et al. (2013); Vairo (2012)]

Correlation function of two light-like Wilson lines

$$\mathcal{W}_{n^-}[y^+, y_\perp] = \mathcal{P} \ \exp \left[ig \int_{-L/2}^{L/2} dy^- \mathcal{A}^+(y^+, y^-, y_\perp) \right]$$

What are the Wilson Lines/Loops?
Gauge-Invariant Hadronic Correlators

$$\Phi(k) = \mathrm{F.T.}\ \langle h|\ \bar{\Psi}(z)\ \mathcal{W}_\Gamma[z,0]\ \Psi(0)\ |h\rangle$$

Gauge invariance is guaranteed by the **Wilson line**, or the **gauge link**, or the **eikonal line**

$$\mathcal{W}_\Gamma = \mathcal{P}\ \exp\left[-ig\int_\Gamma d\zeta^\mu \mathcal{A}_\mu(\zeta)\right]$$

- Gauge invariance
- Path dependence and universality
- Singularities and renormalization
- Evolution and factorization

Classification of Singularities: (partially) Light-Like Wilson Lines

Let Γ be (partially) light-like
$\rightarrow$ extra singularities compared to the off-light-like case!

- Ultraviolet poles $\sim \frac{1}{\varepsilon_{\rm uv}}$
- Overlapping divergences: contain the UV and rapidity poles simultaneously $\sim \frac{1}{\varepsilon_{\rm uv}} \ \ln(\text{rapidity})$
- Pure rapidity divergences: $\sim \ln^{1,2}(\text{rapidity})$:

Most known example: Transverse-Momentum Dependent parton densities (issue of the complete evolution, generalised renormalization, etc.)

@ [ICh, Stefanis (2008, 2009, 2010); Collins (2003, 2008, 2011, 2012 etc.); Chiu, Jain, Neill, Rothstein (2011, 2012)]

@ [talks by Diehl, Ostemeier]

Mathematical issues: Generalized Loop Space formalism for the light-like Wilson lines)

@ [ICh, Mertens, Van der Vaken (2012, 2013), Mertens, Taels (2013)]

Skewed Euclidean Correlator

Definition

$$\tilde{P}(z_\perp; v, \bar{v}) = \langle \star | \frac{1}{N_c} \mathrm{Tr} \; \{\mathcal{W}^\dagger_{\bar{v}}[z_\perp] \mathcal{W}_v[\mathbf{0}_\perp]\} | \star \rangle$$

$$v_\mu = (v^0, v^z, \mathbf{0}_\perp) = L(\cos\phi/2, \sin\phi/2, \mathbf{0}_\perp)$$

$$\bar{v}_\mu = (\bar{v}^0, \bar{v}^z, \mathbf{0}_\perp) = -L(\cos\bar{\phi}/2, \sin\bar{\phi}/2, \mathbf{0}_\perp)$$

$$v^2 = \bar{v}^2 = L^2$$

Asymmetric Euclidean Function

$$\rho(v, \bar{v}) = \frac{1}{L} \int d^2k_\perp \; k_\perp^2 \; \int d^2z_\perp \; \mathrm{e}^{ik_\perp \cdot z_\perp} \; \tilde{P}(z_\perp; v, \bar{v})$$

Skewed Euclidean Correlator

Leading Non-Trivial Order

$$\tilde{P}^{(1)}(z_\perp; v, \bar{v}) = -(ig)^2 \ (v_\mu \bar{v}_\nu) \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\sigma'$$

$$\cdot \mathrm{Tr} \frac{1}{N_c} \ \langle \star | \ \mathcal{P}[\mathcal{A}_\mu(v\sigma + z_\perp)\mathcal{A}_\nu(\bar{v}\sigma')] | \star \rangle$$

Two-gluon non-thermal correlator: simplest Ansatz

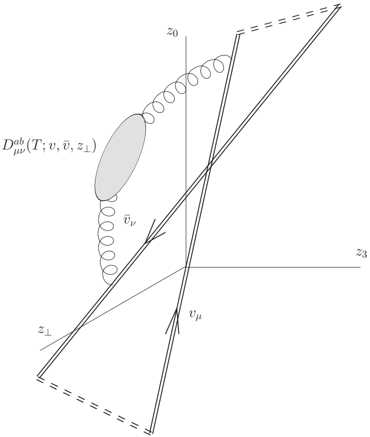
$$\langle \star | \ [A^a_\mu(v\sigma + z_\perp)A^b_\nu(\bar{v}\sigma')] | \star \rangle = \delta^{ab} \ D_{\mu\nu}(v\sigma - \bar{v}\sigma' + z_\perp) \ ,$$

$$D_{\mu\nu}(z) = g_{\mu\nu}\partial^2 D_1(z^2) - \partial_\mu \partial_\nu D_2(z^2)$$

$$= g_{\mu\nu}(2\omega\partial_u + 4z^2\partial_u^2)D_1(z^2) - (2g_{\mu\nu}\partial_u + 4z_\mu z_\nu \partial_u^2)D_2(z^2)$$

Skewed Euclidean Correlator

Leading Non-Trivial Order



Skewed Euclidean Correlator

Result in Terms of the Generic Two-Point Function

$$P^{(1)}(z_\perp; v, \bar{v}) = g^2 C_{\rm F} 2\pi \frac{\cos\Delta\phi}{|\sin\Delta\phi|} \left[(\omega - 2) D_1(z_\perp^2; \Lambda) + 2 z_\perp^2 D_1'(z_\perp^2; \Lambda)\right]$$

Covariant Angular Factor

$$K(v, \bar{v}) \equiv \frac{\cos\Delta\phi}{|\sin\Delta\phi|} = \frac{(v \cdot \bar{v})}{\sqrt{v^2 \bar{v}^2 - (v \cdot \bar{v})^2}}$$

Ill-defined in the light-cone limit in the Minkowski!
However: skewed layout

$$v \to v_{\rm LC} = \frac{L}{2}\left({\rm e}^{\psi_1/2}, -{\rm e}^{\psi_1/2}, \mathbf{0}_\perp\right) \ , \ \bar{v} = -L\left(\cosh\frac{\psi_2}{2}, -\sinh\frac{\psi_2}{2}, \mathbf{0}_\perp\right)$$

$$K(v_{LC}, \bar{v}) = \frac{v_{\rm LC} \cdot \bar{v}}{\sqrt{v_{\rm LC}^2 \bar{v}^2 - (v_{\rm LC} \cdot \bar{v})^2}} = i$$

Skewed Euclidean Correlator

Gaussian Ansatz

$$D_1(z^2) = \mathrm{e}^{-\kappa(g^2, Q_T) z_\perp^2}$$

Infrared cutoff = Debye mass

$$\kappa(g^2, Q_T) = m_E^2 \sim g^2 Q_T^2$$

Scale dependence

$$\rho_{\mathrm{NP}}^{(1)}(Q_T; v, \bar{v}) = g^2 Q_T \ C_{\mathrm{F}} \ 32\pi m_E^2 \frac{\cos\Delta\phi}{|\sin\Delta\phi|} \sim g_E^2 m_E^2$$

$$g_E^2 \sim g^2 Q_T \ , \ m_E \sim g Q_T$$

Comparison

$$\hat{q}(k_\perp^*; g_E, m_E) = \frac{g_E^2 m_E^2 C_{\mathrm{F}}}{2\pi} \ln \frac{|k_\perp^*|}{m_E}$$

@ [Caron-Huot (2009), Laine (2012)]

Conclusions and Outlook

- Presence of low energy scales $Q_T, gQ_T, g^2 Q_T$ is essential for the gauge-invariant formulation of the jet quenching process in terms of the light-like Wilson lines
- Analytical continuation from the skewed Euclidean geometry to the light-cone Minkowskian layout is justified
- Leading-order result for a generic two-gluon correlation function at zero temperature is obtained

$$\langle\star|\ [A^a_\mu(z)A^b_\nu(z')]|\star\rangle$$

- Further program:
  - Nonperturbative contributions (models of the QCD vacuum)
  - Thermal gluon correlators (the Ansatz is more complicated)
  - Next-to-leading calculations (rapidity divergences arise)
- Technical details

@ [ICh, Lauwers, Taels: arXiv:1307.5518]